A Preliminary Statistical Evaluation of GPS Static Relative Positioning

M. Filomena Teodoro and Fernando M. Gonçalves

Abstract The objective of this work is to evaluate GPS static relative positioning (Hofmann-Wellenhof et al, GNSS-Global Navigation Satellite Systems GPS, GLONASS, Galileo, and more. Springer Verlag-Wien, New York, 2008; Kaplan and Hegarty, Understanding GPS: Principles and Applications. Artech House, Norwood, 2006; Leick, GPS Satellite Surveying. John Wiley & Sons, New Jersey, 2004), regarding accuracy, as the equivalent of a Real Time Kinematic (RTK) network and to address the practicality of using either a continuously operating reference stations (CORS) or a passive control point for providing accurate positioning control. The precision of an observed 3D relative position between two global navigation satellite systems (GNSS) antennas, and how it depends on the distance between these antennas and on the duration of the observing session, was studied. We analyze the performance of the software for each of the six chosen ranges of length in each of the four scenarios created, considering different intervals of observation time. The relation between observing time and baseline length is established. In this work are applied different statistical techniques, such as data analysis and elementary/intermediate inference level techniques (Tamhane and Dunlop, Statistics and Data Analysis: From Elementary to Intermediate. Prentice Hall, New Jersey, 2000) or multivariate analysis (Turkman and Silva, Modelos Lineares Generalizados da teoria a prática. Sociedade Portuguesa de Estatística, Lisboa, 2000; Anderson, An Introduction to Multivariate Analysis. Jonh Wiley & Sons, New York, 2003).

M.F. Teodoro (✉)
CINAV, Center of Naval Research, Naval Academy, Portuguese Navy, Almada, Portugal

CEMAT, Center for Computational and Stochastic Mathematics, Instituto Superior Técnico, Lisbon University, Lisbon, Portugal
e-mail: maria.alves.teodoro@marinha.pt

F.M. Gonçalves
NGI, Nottigham Geospatial Institute, University of Nottingham, Nottingham, UK
e-mail: geofactor@hotmail.com

1 Introduction

RTK networks are common in Europe but this is not the case in emerging economies where huge construction projects are running requiring geodetic support. In such cases, the easiest way to ensure that kind of support still is the static relative positioning using a single reference station. This technique provides surveyors the ability to determine the 3D coordinates of a new point with centimeter-level accuracy relative to a control point located several hundred kilometers away, which in turn can be associated with another GNSS receiver of a CORS operated by some institution.

Today the global navigation satellite systems play a fundamental role in the way that surveyors measure positional coordinates. It is now possible to determine the 3D coordinates of a new point with centimeter-level accuracy relative to a control point located several hundred kilometers away, which in turn can be associated with another GNSS receiver of a CORS operated by some institution. Examples of such networks are the ordnance survey (OS) Network across the UK [7] or, globally, the International GNSS Service Network [4].

In this research the coordinates of the OS active stations were used as 'true' values to address the practicality of using either a CORS or a passive control point for providing accurate positioning control and, implicitly, the performance of the software used. The precision of an observed 3D relative position between two GNSS antennas, and how it depends on the distance between these antennas and on the duration of the observing session, was studied. These results were attained through using commercial software LGO to process 105 single baselines, ranging from 61 to 898 km, according to observing sessions of varying lengths. ABEP was used as a reference station, with fixed coordinates, and the values obtained for the rover stations compared with those provided by OS. Also, to address the differences between using broadcast or precise ephemerides and computing the tropospheric effects or for simply applying a tropospheric model, the data processing was repeated for all different strategies.

Generally results show, whatever the strategy followed, that the length of the baseline matters, regarding the rate of successful baselines processed for a priori given values of 1D (ellipsoidal height accuracy) and 2D (compound of longitude and latitude accuracy). While distance matters, under the conditions of this experiment, the results also indicate that the duration of the observing session does not present the same pattern for 1D and 2D. In addition to the length of the baseline and the duration of the observing session, positioning precision depends on several other factors, including the methodology and the software used for processing GPS data, in this case the LGO. Biases associated with meteorological effects (ionosphere and troposphere) also play an important role in the total error budget of positioning precision.

This work investigates the performance of commercial software LGO when baselines are processed in static mode [10]. The parameter to be tested is the time of observation needed to achieve a given accuracy (1D and 2D) for a set of baseline length ranges. Four different scenarios were created, as follow:

- Broadcast ephemerides and Hopfield model (BH);
- Broadcast ephemerides and Computing the troposphere (BC);
- Precise ephemerides and Hopfield model (PH);
- Precise ephemerides and Computing the troposphere (PC).

Summarizing, the present work is comprised of introduction and conclusion sections, a section with background information, another describing the methodology adopted and two sections containing specific tests and results.

2 GNSS Overview

In this section, we provide an introduction of GPS, the navigation system used in this research. As there are a number of relevant references available, e.g. [3, 5, 6, 10], only a very brief discussion on the basics of the system will be given, with particular emphasis on the parts which are relevant to observation modeling of systematic biases and errors affecting GPS measurements. The various types of GPS observables of interest on baseline determination in static relative positioning are also described, as are some of their possible combinations. The possible usefulness of Precise Ephemerides, in terms of the increased accuracy in long baselines, is also evaluated.

There are numerous sources of measurement errors that influence GPS performance. Both observables types, code and phase, are affected by many systematic biases and errors, different in their source and suitable method of treatment. The most important of these biases and errors are briefly reviewed here. The orbital errors and tropospheric effects will be discussed later with more detail.

The sum of all systematic biases and errors contributing to the measurement error is referred to as a range bias. In [2] Bingley argues that this bias is caused by a physical phenomenon, as is the case, for example, in ionospheric or tropospheric delays, and error is the quantity remaining after the bias has been mitigated to some extent, which is the case, for example, for errors in broadcast ephemerides. According to the same author, the systematic biases and errors affecting GPS measurements can be grouped into three main categories: satellite related, atmospheric related and station related.

3 The Data

OS active stations were used to investigate the relation between time of observation and length of the baseline. A total of 105 baselines were processed using LGO, separated into six range groups (R_i, $i = 1, \ldots, 6$,) according with their lengths in kilometers:

- $R_1 = [000 - 100] \rightarrow$(5 baselines)
- $R_2 = [100 - 200] \rightarrow$(14 baselines)
- $R_3 = [200 - 300] \rightarrow$(27 baselines)
- $R_4 = [300 - 400] \rightarrow$(29 baselines)
- $R_5 = [400 - 500] \rightarrow$(14 baselines)
- $R_6 = [500 - 900] \rightarrow$ (16 baselines)

All the stations are permanent stations of clear sky visibility and with low multipath conditions. The quality of the data is therefore expectedly high. Day 13/06/2013 of receiver independent exchange (RINEX) data of GPS week 1744 was downloaded from the data archive of the active GPS network of Ordnance Survey (OS Net) for each of the 106 stations (http://www.ordnancesurvey.co.uk/gps/os-net-rinex-data/). These RINEX data include phase measurement of the carrier waves L_1 and L_2, P_1, P_2 and C/A pseudo-range code at a 30 s interval.

For this experiment, 24 h of dual-frequency GPS carrier phase observations for each of 105 baselines formed by ABEP, chosen as reference station, and all the other active stations, designated as rover, from OS Network were used. These 105 baselines range in length from 61 *km* to 898 *Km* and correspond to all active stations considered 'healthy' on the 13th June 2013. The data for each baseline comprised the same 24 h session that was further subdivided into periods of time of 1, 2, 3, 4, 6, 8, 12 and 24 h as follow, where the two first digits represent the beginning of the observation period and the last two the end:

- 1 h periods: [0001], [0607], [1213], [1819];
- 2 h periods: [0002], [0608], [1214], [1820];
- 3 h periods: [0003], [0609], [1215], [1821];
- 4 h periods: [0004], [0408], [0812], [1216], [1620], [2024];
- 6 h periods: [0006], [0612], [1218], [1824];
- 8 h periods: [0008], [0816], [1624];
- 12 h periods: [0012], [1224];
- 24 h period: [0024].

The division of time in this way was done in order to evaluate the performance of the software for different lengths of observation time and for similar lengths but at different times of the day experiencing diverse atmospheric conditions.

A preliminary experiment shows that to obtain high accurate relative positioning 3D coordinates for long baselines in static mode with LGO at least 4 h of observation are recommended. Therefore, special focus was given to periods of this magnitude and over. These cover the whole day in non overlapping periods, whereas for the 1, 2 and 3 h intervals only representative samples were chosen.

The criteria followed to select the reference station were primarily based on location. Thus ABEP, on the west coast of England, was chosen, because of its high altitude and location, providing a well distributed range of radial vectors to all the other active stations, either in latitude and longitude. Its 3D positional coordinates were fixed to the official values adopted by OS.

In order to evaluate at what range of baseline lengths the use of precise ephemerides become worthwhile, both results using broadcast and precise ephemerides are presented as well. The corresponding SP3 files were downloaded from the data archive of IGS (http://igscb.jpl.nasa.gov/components/prods_cb.html). These include precise ephemerides at a sampling interval of 15 min and the high-rate precise satellite clocks with a sampling of 30 s.

Hence, the four different scenarios can be compared as follow: direct comparison of the results obtained using the broadcast ephemerides and the precise ephemerides (BH versus PH and BC versus PC); direct comparison of the results obtained using Hopfield model and computing the troposphere (BH versus BC and PH versus PC).

At starting points 1D, 2D and 3D accuracy criteria were established for each baseline, as only successful processed baselines are of interest for this research. The chosen values were set to 1D and 2D accuracies to be better than 3 cm and 3D better than 4.5 cm. These are realistic values, as the OS active stations have 1D accuracy of about 2 cm in magnitude and close to 1 cm in 2D. Therefore, assuming the 3 cm as 1D and 2D threshold seems to be reasonable due the fact that this tolerance allows for the 'absorption' of errors inherent to the coordinates of the stations. Despite how perfectly the baseline was calculated an error of up to 4 cm in height and 2 cm in plan could arise due to the uncertainty associated with the coordinates.

The published coordinates of each of these stations (in Cartesian format on the header of the corresponding RINEX file) are assumed as 'true' and used to compute the errors (1D, 2D and 3D) in the solutions processed by LGO.

In Fig. 1 we present the percentage of successful baselines in 1D (dashed lines) and 2D (full lines). There is a clear trend for fewer successful baselines as the length increases, regardless of the strategy adopted, either in 1D or 2D.

In a preliminary approach, it was found that the different ranges led to significantly different results. Were used parametric tests to compare proportions (t-test). With some small samples in certain ranges, were also applied some nonparametric tests that allow us to compare location measures, or a chi-square test and a Kruskal-Wallis to evaluate if the proportions of success are the same in the different ranges; a chi-square independence test were also used to evaluate the relation between the proportion of success and range. In Table 1 are described the $p - values$ obtained when it is tested the difference of the proportion of success for different ranges.

In general, different strategies conduce to similar results: almost all comparisons have the same conclusion—the proportions of success in different ranges are not equal except when the ranges are sequential of each other. Also were performed similar tests comparing different strategies considering the same range. Generally, the proportions of success for the same range, but with different strategies conducted to significant tests, meaning that there is statistical evidence of different proportions of success per different strategies for same range.
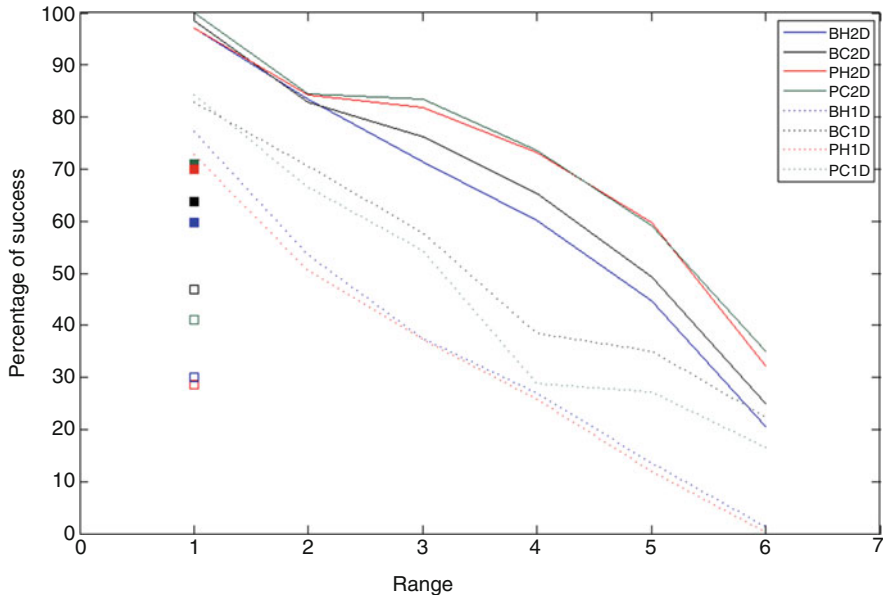
Fig. 1 Percentage of successful baselines in 1D (*dashed lines*) and 2D (*full lines*)

In Fig. 2 the yellow triangles represent successful baselines in 1D and 2D simultaneously; the red triangles represent successful baselines only in 1D considering the 24 h session. Figure 2 (left) shows that, considering the 24 h session, Hopfield is only acceptable up to a certain distance (Wales and Southern England). On the other hand, the computed option is acceptable over greater distances (Wales, Southern England, North England and Southern Scotland), as can be seen in Fig. 2 (right).

4   Conclusions

This work studied the relation for single baselines between lengths ranges an between the different ranges and the observation time required to obtain high-accurate positioning, using commercial software LGO. The analysis for different observation time is not reproduced in this paper. The results are valid for this specific software and under the conditions of the experiments. Four different strategies were established and evaluated through the processing of a total of 11,760 baselines. The data processing and testing used several options concerning the best thresholds for accuracy. The LGO results were compared with the published coordinates by Ordnance Survey and the baselines passing the accuracy criteria were isolated. Other test and techniques were developed, inquiring about the significance of the hour of the day, the amplitude of time interval of exposure, considering the four strategies.

Table 1 Tests of proportions of success difference for distinct ranges. Strategies[a] BH, BC, PH and PC

| Strategies	Ranges	R_2	R_3	R_4	R_5	R_6
BH	R_1	0.3204	0.0679	0.0204	0.0073	0.0008
	R_2		0.3271	0.0988	0.0200	0.0007
	R_3			0.4097	0.0745	0.0006
	R_4				0.2767	0.0052
	R_5					0.2140
BC	R_1	0.5652	0.2053	0.0267	0.0367	0.0065
	R_2		0.4100	0.0395	0.0530	0.0054
	R_3			0.1452	0.1585	0.0158
	R_4				0.8250	0.2491
	R_5					0.4492
PH	R_1	0.3640	0.1159	0.0357	0.0122	0.0017
	R_2		0.4222	0.1208	0.0229	0.0008
	R_3			0.3533	0.0538	0.0003
	R_4				0.2541	0.0033
	R_5					0.1900
PC	R_1	0.4018	0.1237	0.0048	0.0116	0.0018
	R_2		0.4451	0.0168	0.0320	0.0034
	R_3			0.0502	0.0844	0.0071
	R_4				0.9174	0.3316
	R_5					0.4812

[a]Four strategies considering 1D
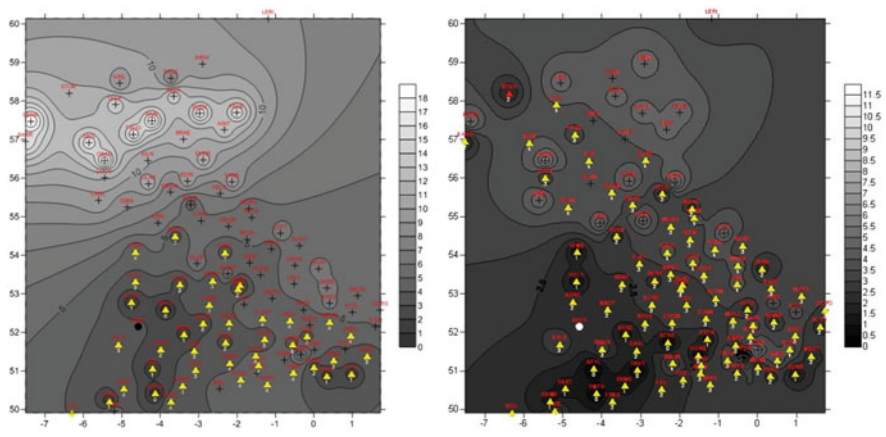
Fig. 2 Successful baselines simultaneously in 1D and 2D (*yellow*) or just in 1D (*red*). Strategy: BH (*left*), BC (*right*)

MANOVA was applied with several factors such as range, strategies, amplitude of interval time exposure [8]. Also was developed a GLM model [1, 9]. Such statistical approach details will be found in a future extended version of this manuscript.

References

1. Anderson, T.W.: An Introduction to Multivariate Analysis. Jonh Wiley & Sons, New York (2003)
2. Bingley, R.M.: GNSS Principles and Observables: Systematic Biases and Errors. Short Course, University of Nottingham, Nottingham Geospatial Institute, Nottingham (2013)
3. Hofmann-Wellenhof, B., Lichtenegger, H., Wasle, H.: GNSS-Global Navigation Satellite Systems GPS, GLONASS, Galileo, and more. Springer Verlag-Wien, New York (2008)
4. IGS: International GNSS Service. http://igscb.jpl.nasa.gov/ (2016). Cited 19/08/2016
5. Kaplan, E.D., Hegarty, C.J.: Understanding GPS: Principles and Applications. Artech House, Norwood (2006)
6. Leick, A.: GPS Satellite Surveying. John Wiley & Sons, New Jersey (2004)
7. OS Net Business and Government: Ordnance Survey. http://www.ordnancesurvey.co.uk/oswebsite/products/os-net/index.html (2016). Cited 19/08/2016
8. Tamhane, A.C., Dunlop, D.D.: Statistics and Data Analysis: From Elementary to Intermediate. Prentice Hall, New Jersey (2000)
9. Turkman, M.A., Silva, G.: Modelos Lineares Generalizados da teoria a prática. Sociedade Portuguesa de Estatística, Lisboa (2000)
10. Wells, D.E., et al.: Guide to GPS Positioning. Canadian GPS Associates, Fredericton. http://plan.geomatics.ucalgary.ca/papers/guide_to_gps_positioning_book.pdf (1986). Cited 1/10/2016