Chapter 39
Scaffolding Complex Modelling Processes: An In-Depth Study

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Abstract The support of students during their work on complex modelling problems is an ambitious process, especially if the students work as autonomously as possible. Scaffolding as a theoretical construct to describe how teachers should act in these situations, so that students can solve the problem as independently as possible, has proven to be adequate for empirical studies. In the research project presented, the activities analysed were those of future teachers working as tutors supporting students working on complex problems over 3 days. The tutors were educated beforehand in pre-service teacher seminars and had learned special scaffolding measure activities for this small group work. Based on an analysis of videotaped modelling processes, examples of successful and unsuccessful teacher activities are analysed. Finally, examples of appropriate strategic scaffolding measures are presented.

Keywords Complex modelling problems • Modelling days • Scaffolding • Scaffolding measure • Teacher intervention • Minimal support • Adaptivity

39.1 Introduction

Already for decades, the competency to solve real-world problems with mathematics is emphasised as one of the core competencies in mathematics education in many curricula around the world, and various approaches for the implementation of mathematical modelling in schools are proposed. Although there exists consensus on the relevance of mathematical modelling in schools, modelling examples still do not play a high role in everyday teaching in many parts of the world, amongst other reasons, due to the fact that teaching and learning processes become more difficult and less predictable and the design of the learning environment is more ambitious

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(for an overview, see Kaiser 2016). Especially, when complex, authentic modelling problems are treated, the role of the teacher becomes highly demanding. It is still an open question, how the teacher can, and will, support students within their modelling processes, especially when independent working of the students is to be fostered as called for in many curricula. As previous studies have shown, adaptive support by teachers guiding the students on their own way can seldom be identified in classrooms (Leiss 2007).

In the following, we will describe a study, in which students solved complex modelling problems within a learning environment fostering their independency. The students tackled the problem together with other students working in small groups, only tutored by (future) teachers, educated for adaptive support. As the theoretical framework for the present study, the scaffolding approach has been chosen, which seems to be especially appropriate for the kind of adaptive teacher activities necessary for independent modelling activities.

39.2 Theoretical Background of the Study

39.2.1 Scaffolding as Theoretical Basis of the Study

The concept of *scaffolding* was originally introduced by Wood et al. (1976), who described it as a form of fostering a problem-solving process of a single child by a single tutor. As the aim of this scaffolding process, they described that the child solved a problem as independently as possible and received support from an experienced person only in situations where independent work due to non-existing knowledge or skills of the child was not possible. Within a problem-solving phase, the tutor intensified or reduced her interventions, depending on the child's ability to work further on independently or not (Wood et al. 1976, p. 92).

The term scaffolding has been extended and adapted over time in a variety of ways, which is described in the extensive survey paper by Van de Pol et al. (2010) with an overview on the current discussion on scaffolding. Their model relies on three important aspects of scaffolding, namely, *contingency* (responsiveness, tailored, adjusted, differentiated or calibrated support), *fading* (gradual withdrawal of the scaffolding) and *transfer of responsibility* to the learner, while diagnostic strategies play an important role in the whole process. The development of the ability to take over responsibility only develops over a longer period of time, and according to this, *fading* should be seen as a long-term process.

39.2.2 Teacher Activities to Promote Independent Student Activities

An approach developed by Zech (1996) within the problem-solving discussion proposed a step-by-step approach to support students with minimal help at five different levels:

1. Students are motivated only in a general way.
2. Positive feedback is given based on successful intermediate results.
3. Strategic support is given which takes the form of hints that refer on how to proceed without addressing content-related issues.
4. Content-related strategic support is offered; these are interventions, which also relate to the procedure, but content-related issues are involved.
5. A content-related intervention is completely related to the content of the task and contains the core of the solution.

In this differentiation, the first two supportive measures mainly encourage the students, while the last three interventions give support related to solution methods or the content of the task. Of these three interventions, the strategic support plays a prominent role as the students are only supported to find a way to go on, but the solution itself must still be developed by the students themselves. The use of strategic support is for the intention of partly independent work by the students and is one possible approach to realise the aspects of scaffolding (fading and transfer of responsibility) and even contingency if the strategic supports rely on subtle diagnosis.

39.2.3 Mathematical Modelling in School

As already mentioned, mathematical modelling plays an important role in mathematics education all over the world. There exist various conceptions of mathematical modelling; in this study, we understand mathematical modelling as comprising the following important steps from the real-world situation into the mathematics: a real-world problem is coming up, which has to be understood and simplified. This leads to a real-world problem, which is then translated into a mathematical model. Mathematical work within the mathematical model leads to mathematical results, which are translated back into the real world. The real-world result is validated, whether the result answers the original problem adequately or not. If this is not the case, the modelling cycle has to be carried out again until a satisfactory solution is produced. This approach can be visualised with a modelling cycle (Fig. 39.1), which can serve a metacognitive means for classroom activities.

While working through the modelling cycle different sub-competencies are needed (for a detailed description, see Kaiser and Brand 2015), amongst others:

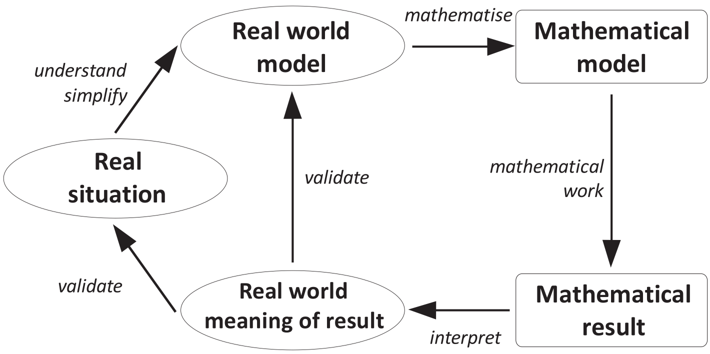

Fig. 39.1 Modelling cycle (Kaiser and Stender 2013, p. 227)

- Competence to understand a real-world situation,
- Competence to develop a real-world model,
- Competence for establishing a mathematical model out of a real model,
- Competence to solve mathematical problems within a mathematical model,
- Competence to interpret mathematical results in a real-world model or a real-world situation,
- Competence to validate the solution in the real-world model or the real situation and, if necessary, do another loop in the modelling process.

In addition, the metacognitive competence to understand your own work and to control your own work is central. For acquiring these competences, it is essential that students work independently on their own. However, when students begin modelling independently, the task to do everything on their own is too complex, so monitoring by a teacher is indispensable. The previously described approaches to scaffolding are an important concept for the implementation of this kind of support.

39.3 Research Aim, Design of the Study and Modelling Problem Used

The aim of this study is to identify empirically appropriate teacher interventions for scaffolding in situations where teachers are tutoring students who are solving complex, realistic modelling problems. As the research environment, the so-called 'modelling' days were established, in which the students work independently supported by tutors. The 'modelling days' are a learning environment where students (15 years old) work for 3 days on one single modelling problem chosen by themselves, and the work takes place in small groups of students. Two future teachers having been educated in special master seminars acted as tutors for two groups.

In the in-depth study presented here, students and teachers worked on the following problem: *Roundabout Versus Traffic Light*. At what kind of an intersection can more cars pass a crossing? This problem allows different approaches. If there are any intersections nearby the school, traffic counts at these crossings could be done, but analytical considerations can also be carried out, which was suggested in the observed learning groups. Two fundamentally different assumptions for the work on the problem can be made:

- The maximum number of cars passing the intersection or roundabout depends on how fast the vehicles drive *through* the intersection area.
- The maximum number of cars passing the intersection or roundabout depends on how fast the vehicles drive *into* the intersection area.

In a first approach, it makes sense to assume the maximal possible symmetry in the situation, that is, from all directions come the same number of cars and the drivers want to go in all directions with equal probability, with velocities and accelerations being the same for all cars. The crossing is a simple four-road intersection, where the traffic light gives way only for one direction at a time. The restrictive assumptions can be reduced during the modelling process, to obtain a more sophisticated solution.

For the case of the traffic lights using the first approach, students have to identify the possible ways through the intersection and then calculate the time a car needs to pass the crossing in three different directions that means with different radii of curvature. These calculations are mostly done with constant speed in the first approach considering acceleration in further work. Students may then calculate the average time to drive through the intersection based on three times calculated before. Using this average time and estimated times for red, yellow and green phases, the number of cars passing the intersection in a certain time is calculated and seen as the capacity of the intersection.

In the second approach focusing on the time until a car is entering the intersection, one has to calculate the starting time of a whole queue of cars, when the traffic light switches to green, which leads to the number of cars that can enter the intersection during one green phase. This calculation deals with constant and accelerated movements and the time a car has to wait until the necessary distance to the car before occurs. One also needs to take into account that the cars at the end of the line drive with constant speed according to the speed limit after a phase of acceleration. The processes in the roundabout are quite complex and can be simulated, which leads to a deeper understanding of the roundabout process (for further details, see Stender and Kaiser 2016).

39.4 Methods

In an in-depth study (Beutel and Krosanke 2012), the entire working process of one group of students was analysed and reconstructed. The reconstruction provided important knowledge about the students' solving process and about teachers' behaviour in modelling processes. The effects of interventions in the context of the complete modelling process were analysed, distinguishing short-term and long-term effects. Based on transcribed video-recordings, the material was analysed using the methodological approach of qualitative content analysis (Mayring 2015).

The segmentation of the entire solution process at various levels, which is the result of the analyses, has been visualised over time (see exemplarily in Fig. 39.2). To ensure reliability of the methodical approach two researchers coded the material separately and discussed non-conformities.

The main categories of analysis were developed inductively and deductively (these are shown in the seven rows of Fig. 39.2):

- Thematic topics related to the modelling problem (code: paraphrase of the work);
- Type of intersection the students are working on (codes: students are working on the traffic light or on the roundabout or on both comparing them);
- Subtopic (code: according to subtopic coding scheme developed inductively);
- Phases of the modelling cycle (codes: six phases of the modelling cycle);
- Timeline in minutes (Fig. 39.2 shows an example lasting 12 min);
- Working behaviour (codes: independent student activities, reluctant participation, nonworking phases);

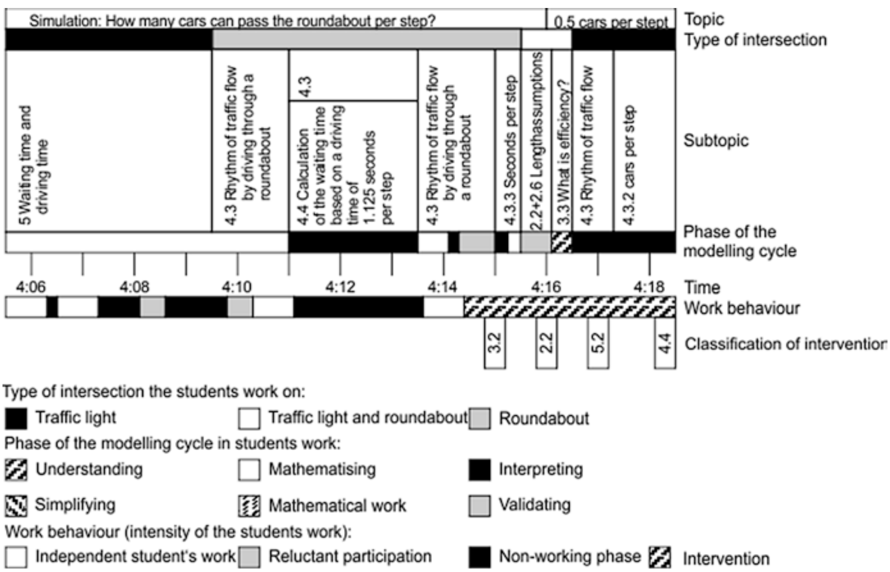

Fig. 39.2 Visualisation of the reconstructed modelling process

- Classification of the intervention (code: according to coding scheme of intervention, e.g. invasive or responsive, non-verbal intervention, motivational help, feedback help, general-strategic help, content-oriented strategic help, content help, organisational matters, discipline problems);
- Intervention as a trigger of metacognitive processes such as procedural and/or declarative (not included in the visualisation shown in Fig. 39.2).

The visualisation helps to identify specific characteristics of the solving process by considering only one level of the reconstruction at a time. On the other hand, the different levels can be considered at the same time to identify interactions between the students' modelling process and tutors' behaviour. Due to the restriction of the observation to only one small group of students with five students, the generalisability of the results is limited.

39.5 Results

In the following, we will concentrate on the interactions between students' solution processes and tutors' behaviour.

39.5.1 Effects of Tutors' Interventions on the Working Behaviour of Students

The analyses show that non-verbal interventions had no negative impact on the way students work in the group. This means that just the fact that tutors looked for the kind of work the students were doing without speaking to them, but noticed by the student, did not disturb the students' work.

Mostly success can be observed, that is, students take up their work again after a standstill; however, several special incidents happened. For example, in the middle of the second day of modelling, a long period of nonworking occurred, which arose after a special intervention. The students had completed their first modelling cycle, and the tutor wanted them to validate their solution and to start a new modelling cycle. The tutor therefore pointed to the validation phase in a modelling diagram hanging on the wall telling them that they were in that phase of the modelling process and asked them to think about the results once more and then start again. This intervention was followed by long nonworking phases of the students and a dominance of intervention phases compared to independent activities. Apparently, the students were not able to improve their model by going through the modelling cycle another time or were not motivated enough to restart the modelling process, maybe because they felt their previous work was not valued enough. This incident highlights that restarting a modelling cycle after validating the results achieved is a complex phase within modelling activities and needs sensitive teacher interventions. Furthermore, this incident shows that considering only the working behaviour directly after an intervention as an indicator of successful assistance is not differentiated enough. It must be taken into account that interventions may have as a long-term process other effects, showing the success of minimal and adaptive support over time. In the following, we report the effects of teacher interventions differentiating between short-term and long-term effects of the tutors' interventions.

39.5.2 Short-Term and Long-Term Effects of Interventions

Two interventions that took place within the first hour of the modelling process observed seem adequate for the analysis of long-term and short-term effects. The students were working on understanding the modelling problem and on making assumptions about the flow of traffic at crossroads. The analysis of the incident shows that the students were unable to assess whether they were allowed to make assumptions and, accordingly, for which variables of the specific modelling problem it was adequate to make assumptions.

Eric: Are we allowed to come up with numbers? (all students looking to the tutor)
Tutor: Yes. So what kind of numbers do you want to come up with?
Eric: Oh, well – there is an intersection and from the right the cars come from the countryside driving into the town. That are eight percent. Okay?
Tutor: If you all agreed on it this is okay in the first run.
Eric: Yes (nodding).
Tutor: You together have to decide that.
Eric: So eight percent (addressing his group) – even it's not so good for calculation?
Anna: No.

This dialogue led to various assumptions by the students. Analysis of students' activities immediately after this intervention showed the response could be assessed as successful, because the students continued to work and developed assumptions which they could not have made before. Finally, the students' assumptions led to a real-world model, which was too complex to solve, so the concentration on work decreased and the intervention was rated as unsuccessful. This last nonworking phase led to another intervention: The students presented their work, and the tutor expounded the problems of the students' assumptions, not by rating them but in a way that the students were in need of an explanation of the assumptions made. This is what we call a *strategic intervention* and in terms of scaffolding transfers responsibility of the work to the students. Within this explanation, the students realised that their assumptions had a high complexity and were thus encouraged by the tutor to simplify them further. Due to this intervention, reasonable assumptions of the general conditions at one crossroad were developed, but the students also estimated the number of cars that pass through the roundabout per minute. As this influential factor needs to be calculated and not estimated, this was not yet an adequate simplification of the real situation and led to a phase of uncertainty. The effect of this intervention can again be rated differently, namely, as success in the short term but with problematic long-term consequences. From the perspective of scaffolding, the students took over responsibility and worked independently for a while, so this strategic help was successful with respect to scaffolding.

There are also interventions which were completely ignored in the short term by the students but were taken up in the long run. For example, prepared material for the simulation of the roundabout was handed out by the tutor, but the students did not use it immediately. After a short while, the material was used independently and the results were very important for the modelling process. This intervention had hardly any short-term effects but had a long-term success.

To summarise, these results indicate that the success of teachers' interventions can be evaluated differently considering short- or long-term developments. The tutors have to consider these different aspects while intervening, which points to the difficulty of real adaptive interventions. In an attempt to develop a definition of success of interventions at different levels, metacognitive processes, which have been triggered by an intervention, have proven to be important. Effects at the declarative level (concerning learning strategies, person and task characteristics) as well as at the procedural level of metacognition can be identified. Strategic interventions are mostly the trigger of such processes, but they can also be an effect of content-related help and feedback. If feedback was identified as the trigger of metacognitive processes, feedback had been given in combination with a content-related help.

39.5.3 Improvement of the Competence 'Simplifying'

The students dealt with complex modelling problems for the first time, that is, they were modelling novices. This meant, amongst other things, that they were not used to dealing with complex modelling problems. As already mentioned, they had especially no experience with adequate simplifying. Many difficulties in understanding which assumptions and simplifications were adequate and which were not can be identified. After the interventions described in Sect. 39.5.2, 11 other interventions concerning simplifying could be identified. In more than half of these interventions, only already existing assumptions were presented, and no new ones were developed. Even in other interventions observed, mainly no simplifying activities could be identified. However, on the second day, the students gained competency in simplifying and to know which assumptions may be adequate during a modelling process. For example, the assumption of how many cars can pass the roundabout per minute, which had been used on the first day, was explicitly rejected on the second day. To conclude, low external influence – that is, only strategic help by the tutor – and a long time of working autonomously on one problem seem to be adequate to increase the competency to simplify in the group and thus the transfer of responsibility to the students was successful.

39.5.4 Focus on the Intervention: 'Present Status of Work'

The intervention *to present the status of the work and the results* is the intervention, which was identified as given the most in the present study. As in previous studies (Kaiser and Stender 2013), its high potential for students' metacognitive activities and for teachers was confirmed. One consequence of this particular kind of intervention can be the promotion of reflecting and structuring the current results and current activities. Success was identified in terms of progress in the modelling process by the following effects: the solution of a partial problem, the realisation of the importance of obtained results and thus their inclusion into the modelling process and the verbalisation of previously intuitive insights. For example, the students formulated the result of their mathematical simulation of traffic flow on a roundabout, which is essential for the solution process, within the intervention *present status of work* for the first time. In another situation, the students tried to use their previous results to solve the interim problem: 'How many cars can pass through the roundabout per minute?' After a long time of nonworking, the intervention *present status of work* helped the students to remember an important result, which had already been obtained earlier. A short time later, the students were able to solve their interim problem adequately.

39.6 Conclusion

To summarise, the study presented confirms the usefulness of special kinds of intervention as a diagnostic tool for teachers. The study confirms the high relevance of diagnosis activity in order to give adaptive assistance to the students' problems. The importance of diagnosis-based support could be identified in exemplary interventions. For example, a wrong diagnosis led to the breakdown of the work by the students, although the work may have made sense in terms of working independently. In addition, a misdiagnosis may lead to an inadequate assistance by the tutor, which might cause much confusion with negative impact on the working behaviour or may lead to boredom by students, because the tutor refers to aspects already considered sufficiently by the students. In conclusion, the intervention 'present status of work' can be considered by teachers as a powerful scaffolding measure at the beginning of every intervention in complex modelling processes, because it has the potential for a positive impact on the solution process in various respects.

References

Beutel, M., & Krosanke, N. (2012). *Rekonstruktion von Handlungsabläufen in komplexen Modellierungsprozessen – Schülerprobleme und Lehrerverhalten* (Masters thesis). University of Hamburg.

Kaiser, G. (2016). The teaching and learning of mathematical modeling. In J. Cai (Ed.), *First compendium for research in mathematics education*. Reston: National Council of Teachers of Mathematics.

Kaiser, G., & Brand, S. (2015). Modelling competencies – Past development and further perspectives. In G. A. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modelling in education research and practice: Cultural, social and cognitive influences* (pp. 129–149). Cham: Springer.

Kaiser, G., & Stender, P. (2013). Complex modelling problems in co-operative self-directed learning environments. In G. A. Stillman, G. Kaiser, W. Blum, & J. P. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (pp. 277–293). Dordrecht: Springer.

Leiss, D. (2007). Hilf mir es selbst zu tun. In *Lehrerinterventionen beim mathematischen Modellieren*. Hildesheim: Franzbecker.

Mayring, P. (2015). Qualitative content analysis: Theoretical background and procedures. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education* (pp. 365–380). Dordrecht: Springer.

Stender, P., & Kaiser, G. (2016). Fostering modeling competencies for complex situations. In C. Hirsch & A. R. McDuffie (Eds.), *Annual perspectives in mathematics education 2016: Mathematical modeling and modeling mathematics* (pp. 107–115). Reston: National Council of Teachers of Mathematics.

Van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in teacher-student interaction: A decade of research. *Educational Psychology Review, 22*(3), 271–296.

Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *The Journal of Child Psychology and Psychiatry, 17*, 89–100.

Zech, F. (1996). *Grundkurs Mathematikdidaktik: Theoretische und praktische Anleitungen für das Lehren und Lernen von Mathematik*. Weinheim: Beltz.