Chapter 34
The Dual Modelling Cycle Framework: Report on an Australian Study

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Abstract The aim of this study was to investigate how 23 students from one Year 6 class in an Australian primary school engaged with two modelling tasks using the dual modelling cycle framework. This framework is designed to assist students who do not find a solution to a modelling task by introducing a second similar yet simpler modelling task in a second cycle. Students participated in 2 × 60 min lessons over 2 days. Results indicate they benefitted from the modelling approach theorised by the Dual Modelling Cycle Framework. While students demonstrated an inability to find a solution for the first task, they were fully engaged in Task 2. They enjoyed this cognitively demanding yet stimulating approach that provided all students with opportunities to participate in an orchestrated discussion where they were able to find solutions for Task 1 and justify their findings using evidence from their concrete models.

Keywords Dual Modelling Cycle Framework (DMCF) • Oil Tank Task • Toilet Paper Tube Task • Primary school

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34.1 Introduction

The Australian Curriculum Mathematics is designed to develop capabilities necessary for all Australian school-age students to fully engage in daily life (ACARA 2015, para. 1). In order to achieve this, "The curriculum focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, logical reasoning, analytical thought and problem-solving skills" (para. 2). With this curriculum, there have been calls for teachers to change their pedagogical approach to embrace the intent of the Australian Curriculum Mathematics (Galbraith 2013). The work by Stillman and Galbraith (1998) continues to be relevant to supporting such a change as they argue for an emphasis on context to ensure sense making is promoted, and it is here that the lesson launch is important (Jackson et al. 2012). In addition, tasks need to be cognitively demanding (Lampert et al. 2013) yet suitable for the differentiated classroom (Boaler and Staples 2008). Pulling aspects of the lesson together is a skillful orchestration of the discussion (Stein et al. 2008) where students are "pressed" to make connections while justifying their perspectives with evidence. While recognising these and other previous research, Galbraith (2013) called for an emphasis on mathematical modelling as one way to create balance within conventional classroom mathematics, in an effort to support Australian teachers as they go about implementing the Australian Curriculum Mathematics.

34.2 Theoretical Framework

Mathematical modelling is widely used with realistic problem-solving contexts as a way to empower modeller independent use of mathematical knowledge in thoughtful and creative ways. This approach requires opportunities for multiple solution paths with the orchestration of discussion around the *best* solution in comparison to the conventional approach to mathematics problem-solving that looks at *the* solution. This approach has been captured by the cognitive theoretical framework developed by Blum and Leiß (2007, p. 225) where modellers move through a cycle of steps that requires them to access both the real and mathematical worlds. This single modelling cycle is sufficient if modelling is proceeding successfully. While many researchers draw on this model, research does indicate that students will move between the real and mathematical worlds while in the process of finding a solution (e.g., Stillman and Galbraith 1998; Matsuzaki 2007, 2011). When this process stalls and modellers do not know how to proceed to find a solution, one way forward is for them to be guided to a *similar yet simpler modelling task* that will aid the development of a solution for the original problem. In this chapter, we explore a theoretical extension to Blum and Leiß's (2007) model with a view to facilitating the teaching of mathematical modelling that considers a diversity of modeller abilities. Here Saeki and Matsuzaki's (2013) extended theoretical modelling framework, the Dual

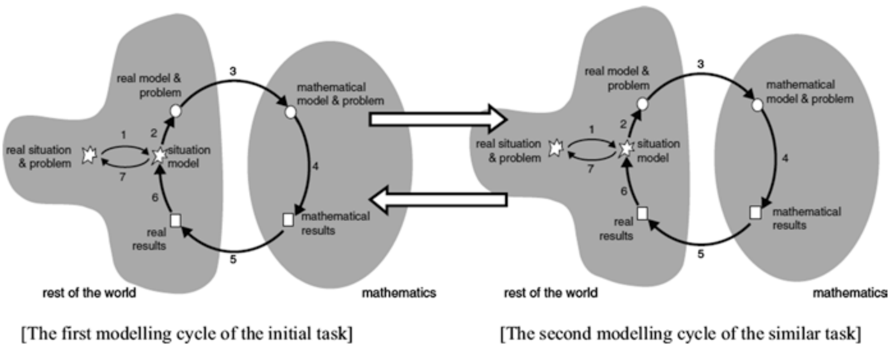


Fig. 34.1 Dual modelling cycle framework (Saeki and Matsuzaki 2013, p. 94)

Modelling Cycle Framework (DMCF) (see Fig. 34.1), is designed to cater for a diversity of learners.

The theoretical propositional basis of the DMCF is that it requires two tasks, the initial task, Task 1, which is located in the first modelling cycle, and Task 2, which is located in the second modelling cycle. When students cannot progress their solution to Task 1, they are guided by their teacher to move to cycle 2 where they are introduced to a similar, yet simpler task (Polya 1945). The selection of the second task is critical as its role is specifically designed to develop student understanding that will assist with the solution of Task 1. The intention with the DCMF is therefore that by moving from the initial modelling task, Task 1, to a similar and simpler modelling task, Task 2, they are more likely to experience success in both modelling cycles.

Research by Matsuzaki and Saeki (2013) identified that teachers play an important role in facilitating switching between cycles and tasks to ensure successful outcomes for all students. Their research implemented experimental modelling lessons with undergraduate students in Japan that led to the identification of three stages in the DMCF: (1) transition from the first modelling cycle to the second modelling cycle, (2) modelling within the second modelling cycle, and (3) transition from the second modelling cycle back to the first modelling cycle. Kawakami et al. (2012, 2015) moved the use of the DMCF from undergraduate students to Year 5, elementary school students in Japan. It was the use of the DMCF in the elementary setting that captured the interest of Australian researchers, as this framework was seen as a way to assist teachers to implement the Australian Curriculum Mathematics answering Galbraith's (2013) call for greater use of mathematical modelling and at the same time cater for a wide diversity of student ability (Lamb et al. 2014). The research questions that guide this research are: *How do students in this Australian school, who are experiencing difficulty with Task 1, respond when their teacher switches to Task 2? And, how does this influence student modelling response to Task 1?*

34.3 Research Design

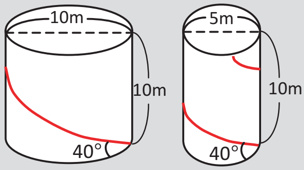
One primary school in Brisbane, Australia, participated in the DMCF project. Although this school was a sample of convenience, it is also typical of most primary schools within Brisbane with year levels from Prep to Year 6. Participants described in this paper involved 23 students (age 11 or 12). The students participated in two lessons (60 min × 2) over 2 days in the second week of their school year. All researchers named on this chapter attended the lessons. These two lessons were designed to cycle through the three stages of the DMCF identified by Matsuzaki and Saeki (2013). The tasks the students completed are outlined in Fig. 34.2, and these tasks were the same tasks that had been completed previously in the Japanese studies.

The tasks were designed to assist in developing student understanding of the geometric structure of an ordinary helix on the outside of a cylinder (see Fig. 34.2 below). The delivery of the lessons used the following structure as in the Japanese studies (Matsuzaki and Saeki 2013; Kawakami et al. 2012, 2015). Initially a picture of oil tanks was shown where the tanks had differing diameters and a spiral staircase from the ground to the top. The photograph included several fire trucks with firemen and engineers in discussion at the foot of the oil tanks. The context was presented as the firemen needing to know which spiral stair would get them to the top first, as they needed to climb to the top of one of the tanks as quickly as possible to cool them because they were in danger of exploding. It was clear to all that there were several types of oil tanks with their heights equal but their diameters different. The students were asked, "Were the lengths of the spiral stairs on these oil tanks the same or not?" It was explained that the angle of the spiral stairs around each tank climbed at 40°. Task 1 in Fig. 34.2.2 was then presented and those participating were asked to produce 2D drawings of the 3D model. Following this modelling, Task 2, the *Toilet Paper Tube Task*, was introduced. The purpose of this task was to model the oil tank, but this model permitted the toilet paper tube to be cut up along the slit to assist in identifying a second 2D model. After this task, the students were asked to again consider Task 1.

Collected data included lesson video-recordings, iPad audio-recordings of each group's discussion, each student's worksheets, lesson artefacts and field notes. Lesson artefacts included digital images of student modelling, while field notes were kept by researchers noting any critical insights or issues as they emerged throughout the lessons. These data were analysed in two ways. First the analysis looked for evidence of student independent engagement with each modelling cycle, their transition to the second modelling cycle and how the second modelling situation informed the first, and if this led to enhanced potential in mathematical proficiency. Second, the predicted models that the student would draw for Task 1 were the rectangular model and the parallelogram models, with the expectation that most will draw the rectangular model as this had been the case when Japanese students had attempted this task (see Kawakami et al. 2015). Analysis of Task 2 was expected to focus on the parallelogram model where student mathematics to explain the rela-

Oil Tank Task (TASK1)

There are several types of oil tanks. Their heights are equal but their lengths of diameters are different. Is the length of the spiral stairs on these oil tanks equal or not? As the angle of the spiral stairs climbed at 40° for each.



Toilet Paper Tube Task (TASK2)

It is impossible to open along the actual spiral stair of the oil tank. We can use a toilet paper tube as a similar shape to an oil tank as it can be opened along its slit to show the 2D shape. Consider what the shape of an opened toilet paper tube would be.

Fig. 34.2 Teaching material based on DMCF (Kawakami et al. 2015, p. 197)

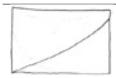
tionship between the parallelogram model and the rectangular model would be drawn out.

34.4 Results and Discussion

34.4.1 Student Experiences with the Modelling Tasks

While the 23 Year 6 students enthusiastically engaged with Task 1, only 11 students drew the anticipated model, the rectangular model. This result meant that the researchers had to modify their analysis protocol for Task 1. For the 11 rectangular models, each was drawn with a curved line to represent the stairs. See Table 34.1 for models A and B being variations in the rectangular model. Note Model A did not indicate reaching the top of the oil tank and Model B did not accurately represent the transition of the wrap around spiral stairs from front to back. The remaining 12 students reproduced the 3D model (Models C and D) suggesting that they did not know how to produce 2D drawings from the 3D models. Our initial interpretation of Models C and D was that the students had reproduced the problem. On greater reflection, this model and that of Model D, do include a 2D net of the oil tank, but also include additional features. Clear evidence of the front and back view of the stairs in Model D suggests students' earlier learning of orientation where they have been required to visualise and draw the view from the top, front, back and sides of various shapes. This finding resulted in a reclassification where Models C and D were classified as examples of an *orientation model* which we consider is in the *grey zone* incorporating some aspects of the 2D model and some aspects of an orientation model. Nonetheless, it was evident that these models were not going to assist the students to provide a solution to the problem as all students had experienced some form of difficulty with Task 1. The teacher then intentionally switched the students over to Task 2, the similar but simpler task.

Table. 34.1 Student Task 1, drawings of 3D model of the oil tanks – Models A, B, C and D

2D models		Orientation models	
Model A	Model B	Model C	Model D
			

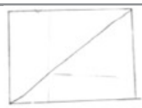
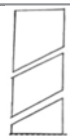
As a result of Task 1, part of the intentional switching to Task 2 was to get the students to predict, through visualisation, what a toilet paper tube would look like when cut along the slit. A toilet paper tube was selected as it is a similar shape to the oil tank, and the slit can readily represent the spiral stairs assisting visualisation. Also, it is easy to cut the toilet paper along the slit to disclose the shape.

Table 34.2 displays the variety of students' 2D models. Again, the students did not provide the models predicted by the researchers, and as a consequence, the analysis had to change again. Worthy of note is that three students drew models that were a combination of 2D and 3D models. The model displayed in Table 34.2 seems to indicate that the students have provided a side view producing a further orientation model. The 3D model is close to the parallelogram model but the student has drawn it in 3D. Both these models were analysed as being within the *grey zone* incorporating 2D and orientation features.

No students used mathematics to establish the relationship between the *Oil Tank Task* and *Toilet Paper Tube Task*. To assist in finding the relationship between these tasks the students were given two toilet paper tubes and asked to cut one toilet tube straight up from bottom to top to confirm the rectangle model of the oil tank and to cut around the slit of the second toilet paper tube to confirm the parallelogram model. These activities concluded the first lesson.

The second lesson commenced by reinforcing the rectangle and parallelogram models from the day before by using large concrete materials to model the oil tanks. One model was cut straight up from bottom to top to produce the rectangle model and the other along the slit to produce the parallelogram model. Following this activity, the students were intentionally switched back to Task 1 and asked, "Are the staircases the same length or not?" In trying to solve this problem, it was noted in the researchers' field notes that the students enthusiastically engaged in collaboratively constructing models to represent the 5 m and 10 m diameter oil tanks. When they cut these models up they were able to provide evidence they needed to convincingly argue through an orchestrated discussion, that the staircases on the tanks were the same length. Following this realisation the students were again stretched by being asked to explain the relationship between the rectangle and parallelogram models. Using the concrete models created at the beginning of the lesson, the students were able to overlay the parallelogram model of the 5 m diameter oil tank over the 10 m diameter model to prove that the staircases were the same. They were also able to prove empirically this result using the rectangle models by cutting and moving sections so that the stairs aligned. Moreover, a discussion was then made pos-

Table 34.2 Toilet paper tube models for Task 2

2D				2D and 3D	3D
Parallelogram model	Close to parallelogram model	Rectangle model	Other	Grey zone	Other
					
3	12	1	1	3	2

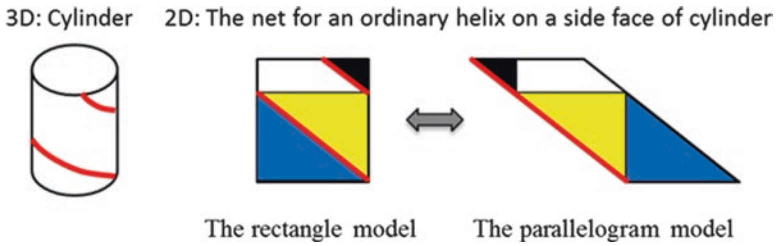


Fig. 34.3 Models to explain the same outcome (Saeki et al. 2016, p. 1748)

sible where students could argue why both the rectangle and parallelogram models produce the same result. The models displayed in Fig. 34.3 were used to assist in this explanation process.

34.5 Conclusions

There are several important findings related to the use of the Dual Modelling Cycle Framework. First, very few students were able to correctly complete either Task 1 or Task 2 by producing mathematically correct models. This result is different from the Japanese students who were able to draw on their findings from Task 1 to support their solution for Task 2 (see Kawakami et al. 2015). As we worked very hard to understand the models produced by the participating students, we developed new categories to allocate to student work. We believe that the students' previous study of 2D and 3D shapes has been influenced by work with orientation where the students have been required to visualise and draw the view of different shapes from the top, front, back and sides. This realisation lead to the reclassification of responses as representative of the new categorisation in our analysis protocol, the *orientation model* in the *grey zone* where students' responses seemed to incorporate some aspects of the 2D model and some aspects of the *orientation model*. We believe that this finding also supports the work of Stillman and Galbraith (1998) where they argue that Australian teaching of mathematics places a heavy emphasis on context. This emphasis on context was naturally continued by the students in this study where many elected to not only draw the net but also include features of the oil tank from different orientations, see for example Models C and D. This finding contrasts significantly with the Japanese approach to teaching mathematics where the focus is very much on the mathematics of the tasks with less focus on the context.

Second, when the students were intentionally switched to Task 2, as is the intent of the DMCF, again their prior experiences of orientation influenced their work. These cognitively demanding tasks (Lampert et al. 2013) resulted in teaching that focussed student attention on concrete models using toilet paper tubes where they successfully produced both the rectangle and parallelogram models. This approach captured every student's interest (Boaler and Staples 2008) and gave them the understanding and the confidence to return to Task 1 and respond to the question, "Were the lengths of the spiral stairs on these oil tanks the same or not?" The teacher's intentional switching back to Task 1, when the students had a fuller understanding of the two models to solve this task, resulted in their being able to provide evidence for their solution and make connections between the models in an orchestrated discussion as described by (Stein et al. 2008). The students were able to persuasively present their arguments that the staircases were the same length using evidence from their concrete models.

Third, we can confirm that the DMCF supports students who do not know how to solve an initial modelling task, but were able to advance their modelling of this task by modelling a similar but simpler task, Task 2. As a result of the students engaging with both tasks they developed a more enlightened mathematical understanding of an ordinary helix on the outside of a cylinder than they would have by doing only one of the two tasks. This approach to promote switching between Task 1 and Task 2 allowed students to solve Task 1, the *Oil Tank Task*.

The success experienced by students in this research by moving between Tasks 1 and 2 has led us to recommend the DMCF as a suitable mathematical modelling framework that should be introduced to Australian teachers as a way to address the diversity of modeller abilities and at the same time, realise the intent of the Australian Curriculum Mathematics.

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