Improved Stability and Stabilization Criteria for T-S Fuzzy Systems with Distributed Time-Delay

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Abstract. This paper investigates the stability and stabilization analysis problem of nonlinear systems with distributed time-delay. The T-S fuzzy model is employed to describe the nonlinear plant. When designing fuzzy controller, the novel imperfect premise matching method is adopted, which allows the fuzzy model and the fuzzy controller to use different premise membership functions and different number of rules. As a result, greater design flexibility can be obtained. By applying a new tighter integral inequality which involves information about the double integral of the system states, and introducing the information of membership functions, less conservative stability and stabilization conditions are derived. Finally, a numerical example is provided to clarify the effectiveness of the proposed approach.

Keywords: T-S fuzzy model · Distributed time-delay · Lyapunov stability theory · Imperfect premise matching

1 Introduction

Time-delay is a common phenomenon in practical control systems, which can deteriorate the system performance and cause instability. Therefore, the research for control systems with time-delay is crucial and has received considerable attention [1,2]. When analyzing the stability conditions of control systems with time-delay, a Lyapunov-Krasovskii functional (LKF) approach [3] is usually applied, which can denote the conditions in terms of linear matrix inequalities (LMIs). Though the LKF approach can handle the stability analysis problem for time-delay systems, the criteria derived from LKF method are conservative. To reduce conservatism, much research has been done: the free-weighting matrix [4] technique was introduced to obtain more tighter bound of the derivative of Lyapunov function; the wirtinger-based integral inequality [5] was developed to deal with integral term $\int_{t-h}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)$; especially, a new tighter integral inequality was proposed in [6], which yields less conservative stability conditions.

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Besides, the Takagi-Sugeno (T-S) fuzzy model [8] is normally used to describe fuzzy control systems. It can represent the dynamics of a complex nonlinear system as a weighted sum of some local linear models, which will facilitate the stability analysis. When the fuzzy controller is also denoted as a weighted sum of some linear sub-controllers, and is connected with the T-S fuzzy model in a closed loop, a Takagi-Sugeno fuzzy-model-based (TSFMB) control system is formed. When dealing with the stabilization analysis problem for TSFMB system, an efficient technique named parallel distribution compensation (PDC) [9] is usually adopted, which requires the fuzzy controller and the T-S fuzzy model have the same premise membership functions and the same number of rules. Such design can make the stabilization analysis easier. However, PDC method also bears some inevitable drawbacks such as limiting the design flexibility of the fuzzy controller and complicating the fuzzy controller structure unnecessarily in some cases. For the sake of solving these problems, some non-PDC design techniques were developed [7,8]. Among them, an effective methodology called imperfect premise matching method [8], which allows the fuzzy controller and the fuzzy model use different premise membership functions and different number of rules. Hence, this approach can avoid the limitations of PDC method.

Currently, the research of stability and stabilization analysis for TSFMB systems with time-delay has yet been thorough enough. First, most of literature applied PDC method to conduct stabilization analysis which would limit the design flexibility of fuzzy controller. Second, the existing stability and stabilization conditions can be further relaxed. This is because the bound of integral term $\int_{t-h}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)$ can be estimated more accurate, and the information of the membership functions has not been applied fully. Therefore, in this paper, we aim to investigate the improved stability and stabilization conditions for TSFMB systems with time-delay.

To achieve our goal, we will apply the imperfect premise matching method to design stable fuzzy controller. In order to relax the results, we will employ the novel integral inequality technique in the stability analysis, and take the information of the membership functions into account. The analyses results will be presented as theorems in terms of LMIs. Finally, a numerical example will be given to illustrate the advantages and superiority of the proposed approach.

2 Preliminaries

A TSFMB nonlinear control system with distributed time delay is considered.

Fuzzy Model: Construct a p-rule polynomial fuzzy model to represent the nonlinear system with time-delay:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} \omega_i(\mathbf{x}(t))\big(\mathbf{A}_i\mathbf{x}(t) + \mathbf{A}_{1i}\mathbf{x}(t-h) + \mathbf{A}_{2i}\int_{t-h}^{t} \mathbf{x}(s)ds + \mathbf{B}_i\mathbf{u}(t)\big), \quad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}^{n\times 1}$ denotes the vector of the system state; $\mathbf{A}_i \in \mathbb{R}^{n\times n}, \mathbf{A}_{1i} \in \mathbb{R}^{n\times n}, \mathbf{A}_{2i} \in \mathbb{R}^{n\times n}$ and $\mathbf{B}_i \in \mathbb{R}^{n\times m}$ represent the system matrices and the system input matrices; $\mathbf{u}(t) \in \mathbb{R}^{m\times 1}$ is system input; the time-delay h is a constant satisfying $h \in [h_{min}, h_{max}]$; $\omega_i(\mathbf{x}(t))$ stands for the normalized grade of membership, and satisfying: $\omega_i(\mathbf{x}(t)) \geq 0$ for all i, and $\sum_{i=1}^{p} \omega_i(\mathbf{x}(t)) = 1$.

Fuzzy Controller: Motivated by the imperfect premise matching method [8], we consider a c-rule polynomial fuzzy controller in this paper:

$$\mathbf{u}(t) = \sum_{j=1}^{c} m_j(\mathbf{x}(t))\mathbf{K}_j\mathbf{x}(t), \tag{2}$$

where $\mathbf{K}_j \in \mathbb{R}^{m\times n}$ represents the feedback gain of jth rule; $m_j(\mathbf{x}(t))$ stands for the normalized grade of membership, and satisfying: $m_j(\mathbf{x}(t)) \geq 0$ for all i, and $\sum_{j=1}^{c} m_j(\mathbf{x}(t)) = 1$.

According to (1) and (2), the closed-loop fuzzy control system can be easily acquired:

$$\begin{aligned}\dot{\mathbf{x}}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} \omega_i(\mathbf{x}(t))m_j(\mathbf{x}(t))\big(\mathbf{G}_{ij}\mathbf{x}(t) + \mathbf{A}_{1i}\mathbf{x}(t-h) + \mathbf{A}_{2i}\int_{t-h}^{t} \mathbf{x}(s)ds \\ + \mathbf{B}_i\mathbf{u}(t)\big),\end{aligned} \tag{3}$$

where $\mathbf{G}_{ij} = \mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j, \quad i = 1, 2, ..., p, \quad j = 1, 2, ..., c.$

Lemma 1 is given to facilitate the proof of the main results in the following sections.

Lemma 1 [6]. *It is assumed that $\mathbf{x}(t)$ is a differentiable function: $[\alpha, \beta] \to \mathbb{R}^n$. For $\mathbf{N}_1, \mathbf{N}_2, \mathbf{N}_3 \in \mathbb{R}^{4n\times n}$, and $\mathbf{R} \in \mathbb{R}^{n\times n} > 0$, the following inequality holds:*

$$-\int_{\alpha}^{\beta} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)ds \leq \xi^T \Omega \xi, \tag{4}$$

where

$$\begin{aligned}
&\Omega = \tau\Phi_2 + \Phi_3, \Phi_2 = \mathbf{N}_1\mathbf{R}^{-1}\mathbf{N}_1^T + \frac{1}{3}\mathbf{N}_2\mathbf{R}^{-1}\mathbf{N}_2^T + \frac{1}{5}\mathbf{N}_3 R^{-1}\mathbf{N}_3^T,\\
&\Phi_3 = Sym\{\mathbf{N}_1\Delta_1 + \mathbf{N}_2\Delta_2 + \mathbf{N}_3\Delta_3\}, \mathbf{e}_k = [\mathbf{0}_{n\times(k-1)n} \quad \mathbf{I}_n \ \mathbf{0}_{n\times(4-k)n}],\\
&k = 1, 2, 3, 4, \Delta_1 = \mathbf{e}_1 - \mathbf{e}_2, \Delta_2 = \mathbf{e}_1 + \mathbf{e}_2 - 2\mathbf{e}_3, \Delta_3 = \mathbf{e}_1 - \mathbf{e}_2 - 6\mathbf{e}_3 + 6\mathbf{e}_4,\\
&\xi = [\mathbf{x}^T(\beta) \quad \mathbf{x}^T(\alpha) \quad \frac{1}{\tau}\int_{\alpha}^{\beta} \mathbf{x}^T(s)ds \quad \frac{2}{\tau^2}\int_{\alpha}^{\beta}\int_{\alpha}^{s} \mathbf{x}^T(u)duds]^T, \quad \tau = \beta - \alpha.
\end{aligned}$$

In addition, to simplify the computational complexity, $\omega_i(\mathbf{x}(t))$ and $m_j(\mathbf{x}(t))$ will be denoted has ω_i and m_j, respectively in the following sections.

3 Stability Analysis

Firstly, the stability condition of TSFMB autonomous system, i.e., the system (1) with $\mathbf{u}(t) = \mathbf{0}$, will be investigated, which can be described by

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} \omega_i(\mathbf{A}_i\mathbf{x}(t) + \mathbf{A}_{1i}\mathbf{x}(t-h) + \mathbf{A}_{2i}\int_{t-h}^{t} \mathbf{x}(s)ds). \tag{5}$$

The inferred stability analysis results are summarized in the following theorems.

Theorem 1. *For TSFMB autonomous system (1) and a prescribed constant $h \in [h_{min}, h_{max}]$, if there exist positive definite matrices $\mathbf{P} = \mathbf{P}^T \in \mathbb{R}^{3n\times 3n}, \mathbf{R} = \mathbf{R}^T \in \mathbb{R}^{n\times n}$, $\mathbf{Q} = \mathbf{Q}^T \in \mathbb{R}^{n\times n}$, such that LMIs (6) are satisfied, then the system (1) is asymptotically stable.*

$$\begin{bmatrix} \mathbf{\Phi}_{1i} + \mathbf{\Phi}_3 + \mathbf{\Phi}_{4i} & \sqrt{h}\mathbf{N}_1 & \sqrt{h}\mathbf{N}_2 & \sqrt{h}\mathbf{N}_3 \\ * & -\mathbf{R} & \mathbf{0} & \mathbf{0} \\ * & * & -3\mathbf{R} & \mathbf{0} \\ * & * & * & -5\mathbf{R} \end{bmatrix} < 0, \tag{6}$$

where

$$\mathbf{\Phi}_{1i} = Sym\{\Delta_4^T \mathbf{P} \Delta_{5i}\} + \mathbf{e}_1^T \mathbf{Q} \mathbf{e}_1 - \mathbf{e}_2^T \mathbf{Q} \mathbf{e}_2 + h\mathbf{\Gamma}_i^T \mathbf{R} \mathbf{\Gamma}_i, \Delta_4 = \left[\mathbf{e}_1^T \ h\mathbf{e}_3^T \ \tfrac{h^2}{2}\mathbf{e}_4^T \right]^T,$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} & \mathbf{P}_{13} \\ * & \mathbf{P}_{22} & \mathbf{P}_{23} \\ * & * & \mathbf{P}_{33} \end{bmatrix}, \Delta_{5i} = \left[\mathbf{\Gamma}_i^T \ \mathbf{e}_1^T - \mathbf{e}_2^T \ h\mathbf{e}_3^T - h\mathbf{e}_2^T \right]^T, \mathbf{\Gamma}_i = [\mathbf{A}_i \ \mathbf{A}_{1i} \ h\mathbf{A}_{2i} \ \mathbf{0}]$$

$$\mathbf{\Phi}_{4i} = \mathbf{T}_i - \mathbf{F}_i + \sum_{r=1}^{p} \bar{h}_r \mathbf{F}_r - \sum_{k=1}^{p} \underline{h}_k \mathbf{T}_k, i = 1, 2, ..., p,$$

and $\Delta_1, \Delta_2, \Delta_3$ are defined in Lemma 1. Besides, in order to decrease the computational complexity, we will eliminate the free matrices $\mathbf{N}_1, \mathbf{N}_2, \mathbf{N}_3$ by assuming $\mathbf{N}_1 = \frac{1}{h}\left[-\mathbf{R}\ \mathbf{R}\ \mathbf{0}\ \mathbf{0}\right]^T$, $\mathbf{N}_2 = \frac{3}{h}\left[-\mathbf{R}\ -\mathbf{R}\ 2\mathbf{R}\ \mathbf{0}\right]^T$, $\mathbf{N}_3 = \frac{5}{h}\left[-\mathbf{R}\ \mathbf{R}\ 6\mathbf{R}\ -6\mathbf{R}\right]^T$.

Proof. To investigate the stability of the TSFMB autonomous system (1), the following Lyapunov-Krasovskii functional candidate is considered.

$$\begin{aligned} V(t) &= [\mathbf{x}^T(t) \quad \eta_1^T(t) \quad \eta_2^T(t)]\mathbf{P}[\mathbf{x}^T(t) \quad \eta_1^T(t) \quad \eta_2^T(t)]^T \\ &+ \int_{t-h}^{t} \mathbf{x}^T(s)\mathbf{Q}\mathbf{x}(s)ds + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)dsd\theta, \end{aligned} \tag{7}$$

where $\eta_1(t) = \int_{t-h}^{t} \mathbf{x}(s)ds, \quad \eta_2(t) = \int_{t-h}^{t}\int_{t-h}^{s} \mathbf{x}(u)duds.$

Therefore, the derivative of $V(t)$ can be presented as

$$\dot{V}(t) = \sum_{i=1}^{p} \omega_i \big(\zeta^T(t)\mathbf{\Phi}_{1i}\zeta(t) - \int_{t-h}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)ds\big), \tag{8}$$

where $\zeta(t) = \left[\mathbf{x}^T(t) \quad \mathbf{x}^T(t-h) \quad \frac{1}{h}\eta_1^T(t) \quad \frac{2}{h^2}\eta_2^T(t)\right]^T$, and $\mathbf{\Phi}_{1i}$ is defined in (6).

Moreover, according to Lemma 1, we can obtain

$$-\int_{t-h}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)ds \le \zeta^T(t)\left(\mathbf{\Phi}_2 + \mathbf{\Phi}_3\right)\zeta(t), \tag{9}$$

where Φ_2, Φ_3 are defined in (4). So we have

$$\dot{V}(t) \leq \sum_{i=1}^{p} \omega_i \zeta^T(t)(\mathbf{\Phi}_{1i} + \mathbf{\Phi}_2 + \mathbf{\Phi}_3)\zeta(t) = \sum_{i=1}^{p} \omega_i \zeta^T(t)\mathbf{\Phi}_i \zeta(t) \tag{10}$$

After some algebraic manipulations, we get

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{p} \omega_i \zeta^T(t)\mathbf{\Phi}_i\zeta(t) \leq \sum_{i=1}^{p} \omega_i \zeta^T(t)\mathbf{\Phi}_i\zeta(t) + \sum_{i=1}^{p}(\bar{\omega}_i - \omega_i)\zeta^T(t)\mathbf{F}_i\zeta(t) + \sum_{i=1}^{p} \\ &(\omega_i - \underline{\omega}_i)\zeta^T(t)\mathbf{T}_i\zeta(t) = \sum_{i=1}^{p} \omega_i \zeta^T(t)(\mathbf{\Phi}_i - \mathbf{F}_i + \mathbf{T}_i + \sum_{r=1}^{p} \bar{w}_r \mathbf{F}_r - \sum_{k=1}^{p} \underline{\omega}_k \mathbf{T}_k)\zeta(t),\end{aligned} \tag{11}$$

where $\underline{\omega}_i$ is the lower bound of ω_i, and $\bar{\omega}_i$ is the upper bound of ω_i, $\mathbf{F}_i$ and $\mathbf{T}_i$ are positive semi-definite matrics. Hence, if

$$\mathbf{\Phi}_i - \mathbf{F}_i + \mathbf{T}_i + \sum_{r=1}^{p} \bar{w}_r \mathbf{F}_r - \sum_{k=1}^{p} \underline{\omega}_k \mathbf{T}_k < \mathbf{0}, \tag{12}$$

then $\dot{V}(t) < \mathbf{0}$. By using Schur Complement theorem, the condition (12) is equivalent to condition (6). In other words, if condition (6) holds, the system (1) is asymptotically stable. Thus, the proof of Theorem 1 is accomplished.

Remark 1. It can be seen from the proof that a novel integral inequality (9) is adopted to deal with integral term $-\int_{t-h}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)ds$. The advantage of this integral inequality is that it is tighter than other similar integral inequalities, which can relax the results. Additionally, since the stability condition derived from it has a simpler structure, the implementation costs could be lowered.

4 Stabilization Analysis

Based on Theorem 1, we will mainly investigate how to design a fuzzy controller (2) to stabilize TSFMB control system (3) in this section.

Theorem 2. *For TSFMB control system (3) and the given constants σ, $t_i, i = 2, 3, .., 6$, $h \in [h_{min}, h_{max}]$, if there exist positive definite matrices $\bar{\mathbf{P}} = \bar{\mathbf{P}}^T \in \mathbb{R}^{3n\times 3n}$, $\bar{\mathbf{R}} = \bar{\mathbf{R}}^T \in \mathbb{R}^{n\times n}$, $\bar{\mathbf{Q}} = \bar{\mathbf{Q}}^T \in \mathbb{R}^{n\times n}$, and positive semi-definite matrices $\bar{\mathbf{F}}_i = \bar{\mathbf{F}}_i^T \in \mathbb{R}^{4n\times 4n}$, $\bar{\mathbf{T}}_i = \bar{\mathbf{T}}_i^T \in \mathbb{R}^{4n\times 4n}$, such that LMIs (13) are satisfied, then the system (3) is asymptotically stable.*

$$\begin{bmatrix} \Pi_1 & \sqrt{h}\bar{\mathbf{N}}_1 & \sqrt{h}\bar{\mathbf{N}}_2 & \sqrt{h}\bar{\mathbf{N}}_3 & \sqrt{h}\bar{\mathbf{\Gamma}}_{ij}^T \\ * & -\bar{\mathbf{R}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -3\bar{\mathbf{R}} & \mathbf{0} & \mathbf{0} \\ * & * & * & -5\bar{\mathbf{R}} & \mathbf{0} \\ * & * & * & * & \frac{1}{\sigma^2}\bar{\mathbf{R}} - \frac{2}{\sigma}\mathbf{X} \end{bmatrix} < 0, \tag{13}$$

where

$\mathit{\Pi}_1 = \bar{\mathbf{\Phi}}_{1ij} + \bar{\mathbf{\Phi}}_3 + \bar{\mathbf{\Phi}}_{4ij}, \bar{\mathbf{\Phi}}_{1ij} = Sym\{\mathit{\Delta}_4^T \bar{\mathbf{P}} \bar{\mathit{\Delta}}_{5ij}\} + \mathbf{e}_1^T \bar{\mathbf{Q}}_i \mathbf{e}_1 - \mathbf{e}_2^T \bar{\mathbf{Q}}_i \mathbf{e}_2, \bar{\mathbf{\Phi}}_2 = h\bar{\mathbf{N}}_1 \bar{\mathbf{R}}^{-1} \bar{\mathbf{N}}_1^T + \frac{h}{3} \bar{\mathbf{N}}_2 \bar{\mathbf{R}}^{-1} \bar{\mathbf{N}}_2^T + \frac{h}{5} \bar{\mathbf{N}}_3 \bar{\mathbf{R}}^{-1} \bar{\mathbf{N}}_3^T, \bar{\mathbf{\Phi}}_3 = Sym\{\bar{\mathbf{N}}_1 \mathit{\Delta}_1 + \bar{\mathbf{N}}_2 \mathit{\Delta}_2 + \bar{\mathbf{N}}_3 \mathit{\Delta}_3\}, \bar{\mathbf{\Phi}}_{4ij} = -\bar{\mathbf{F}}_{ij} + \bar{\mathbf{T}}_{ij} + \sum_{r=1}^{p} \sum_{s=1}^{c} \bar{h}_{rs} \bar{\mathbf{F}}_{rs} - \sum_{k=1}^{p} \sum_{l=1}^{c} \underline{h}_{kl} \mathbf{T}_{kl}.$

And the state feedback gain can be denoted as $\mathbf{K}_j = \bar{\mathbf{K}}_j \mathbf{X}^{-1}$. $i = 1, 2, ..., p, \quad j = 1, 2, ..., c.$

Proof. Substitute $\mathbf{A}_i$ for $\mathbf{G}_{ij} = \mathbf{A}_i + \mathbf{B}_i \mathbf{K}_j$, and by following the same line of analysis of Theorem 1, we can obtain

$$\dot{V}(t) = \sum_{i=1}^{p} \sum_{j=1}^{c} \omega_i m_j \big(\zeta^T(t) \mathbf{\Phi}_{1ij} \zeta(t) - \int_{t-h}^{t} \dot{x}^T(s) \mathbf{R} \dot{x}(s) ds\big). \tag{14}$$

Applying Lemma 1, we have

$$\dot{V}(t) \le \sum_{i=1}^{p} \sum_{j=1}^{c} \omega_i m_j \zeta^T(t) (\mathbf{\Phi}_{1ij} + \mathbf{\Phi}_2 + \mathbf{\Phi}_3) \zeta(t). \tag{15}$$

Since

$$\begin{aligned}&\zeta^T(t)\mathbf{\Phi}_{1ij}\zeta(t) = \zeta^T(t) Sym\{\mathbf{M}_{1ij}\}\zeta(t) + \zeta^T(t) Sym\{\mathbf{M}_2\}\zeta(t) + \zeta^T(t) Sym\{\\ &\mathbf{M}_{3ij}\}\zeta(t), \zeta^T(t)(\mathbf{\Phi}_2 + \mathbf{\Phi}_3)\zeta(t) = \zeta^T(t)\mathbf{M}_4\zeta(t) + \zeta^T(t)\mathbf{M}_5\zeta(t) + \zeta^T(t)\mathbf{M}_6\zeta(t),\end{aligned} \tag{16}$$

where

$$\mathbf{M}_{1ij} = \begin{bmatrix} \mathbf{P}_{11}\mathbf{G}_{ij} + \mathbf{P}_{12} & \mathbf{\Pi}_2 & h\mathbf{P}_{11}\mathbf{A}_{2i} + \mathbf{P}_{13}h & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ h\mathbf{P}_{12}^T\mathbf{G}_{ij} + h\mathbf{P}_{22} & \mathbf{\Pi}_3 & h^2\mathbf{P}_{12}^T\mathbf{A}_{2i} + h^2\mathbf{P}_{23} & \mathbf{0} \\ \frac{h^2}{2}\mathbf{P}_{13}^T\mathbf{G}_{ij} + \frac{h^2}{2}\mathbf{P}_{23}^T & \mathbf{\Pi}_4 & \frac{h^2}{2}\mathbf{P}_{13}^T\mathbf{A}_{2i} + \frac{h^2}{2}\mathbf{P}_{33} & \mathbf{0} \end{bmatrix}, \mathbf{M}_2 = \begin{bmatrix} \mathbf{Q} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{M}_{3ij} = \begin{bmatrix} \mathbf{G}_{ij}^T\mathbf{R}\mathbf{G}_{ij} & \mathbf{G}_{ij}^T\mathbf{R}\mathbf{A}_{1i} & \mathbf{G}_{ij}^T\mathbf{R}(h\mathbf{A}_{2i}) & \mathbf{0} \\ \mathbf{A}_{1i}^T\mathbf{R}\mathbf{G}_{ij} & \mathbf{A}_{1i}^T\mathbf{R}\mathbf{A}_{1i} & \mathbf{A}_{1i}^T\mathbf{R}(h\mathbf{A}_{2i}) & \mathbf{0} \\ (h\mathbf{A}_{2i}^T)\mathbf{R}\mathbf{G}_{ij} & (h\mathbf{A}_{2i}^T)\mathbf{R}\mathbf{A}_{1i} & (h\mathbf{A}_{2i}^T)\mathbf{R}(h\mathbf{A}_{2i}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \mathbf{M}_4 = \frac{1}{h}\begin{bmatrix} -\mathbf{R} & \mathbf{R} & \mathbf{0} & \mathbf{0} \\ \mathbf{R} & -\mathbf{R} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{M}_5 = \frac{3}{h}\begin{bmatrix} -\mathbf{R} & -\mathbf{R} & 2\mathbf{R} & \mathbf{0} \\ -\mathbf{R} & -\mathbf{R} & 2\mathbf{R} & \mathbf{0} \\ 2\mathbf{R} & 2\mathbf{R} & -4\mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \mathbf{M}_6 = \frac{5}{h}\begin{bmatrix} -\mathbf{R} & \mathbf{R} & 6\mathbf{R} & -6\mathbf{R} \\ \mathbf{R} & -\mathbf{R} & -6\mathbf{R} & 6\mathbf{R} \\ 6\mathbf{R} & -6\mathbf{R} & -36\mathbf{R} & 36\mathbf{R} \\ -6\mathbf{R} & 6\mathbf{R} & 36\mathbf{R} & -36\mathbf{R} \end{bmatrix}.$$

$\mathbf{\Pi}_2 = \mathbf{P}_{11}\mathbf{A}_{1i} - \mathbf{P}_{12} - h\mathbf{P}_{13}, \mathbf{\Pi}_3 = h\mathbf{P}_{12}^T\mathbf{A}_{1i} - h\mathbf{P}_{22} - h^2\mathbf{P}_{23},$

$\mathbf{\Pi}_4 = \frac{h^2}{2}\mathbf{P}_{13}^T\mathbf{A}_{1i} - \frac{h^2}{2}\mathbf{P}_{23}^T - \frac{h^3}{2}\mathbf{P}_{33},$

So we have $\zeta^T(t)(\mathbf{\Phi}_{1ij}+\mathbf{\Phi}_2+\mathbf{\Phi}_3)\zeta(t) = \zeta^T(t)(\mathbf{M}_{1ij}+\mathbf{M}_2+\mathbf{M}_{3ij}+\mathbf{M}_4+\mathbf{M}_5+\mathbf{M}_6)\zeta(t)$. Therefore, if

$$\mathbf{M}_{1ij}+\mathbf{M}_2+\mathbf{M}_{3ij}+\mathbf{M}_4+\mathbf{M}_5+\mathbf{M}_6 < \mathbf{0}, \tag{17}$$

we can obtain $\dot{V}(t) < 0$.

Define some new variables as

$$\begin{aligned}
&\mathbf{P}_{12} = t_2\mathbf{P}_{11}, \quad \mathbf{P}_{13} = t_3\mathbf{P}_{11}, \quad \mathbf{P}_{22} = t_4\mathbf{P}_{11}, \quad \mathbf{P}_{23} = t_5\mathbf{P}_{11}, \quad \mathbf{P}_{33} = t_6\mathbf{P}_{11},\\
&\mathbf{X} = \mathbf{P}_{11}^{-1}, \quad \bar{\mathbf{R}} = \mathbf{XRX}, \quad \bar{\mathbf{Q}} = \mathbf{XQX}, \quad \bar{\mathbf{K}}_j = \mathbf{K}_j\mathbf{X}, \quad \bar{\mathbf{G}}_{ij} = \mathbf{A}_i\mathbf{X}+\mathbf{B}_i\bar{\mathbf{K}}_j,\\
&\bar{\mathbf{A}}_{1i} = \mathbf{A}_{1i}\mathbf{X}, \quad \bar{\mathbf{A}}_{2i} = \mathbf{A}_{2i}\mathbf{X}, \quad \bar{\mathbf{B}}_i = \mathbf{B}_i\mathbf{X}, \quad \bar{\mathbf{N}}_1 = \frac{1}{h}\left[-\bar{\mathbf{R}}\ \bar{\mathbf{R}}\ \mathbf{0}\ \mathbf{0}\right]^T,\\
&\bar{\mathbf{N}}_2 = \frac{3}{h}\left[-\bar{\mathbf{R}}\ -\bar{\mathbf{R}}\ 2\bar{\mathbf{R}}\ \mathbf{0}\right]^T, \quad \bar{\mathbf{N}}_3 = \frac{5}{h}\left[-\bar{\mathbf{R}}\ \bar{\mathbf{R}}\ 6\bar{\mathbf{R}}\ -6\bar{\mathbf{R}}\right]^T,\\
&\bar{\mathbf{\Gamma}}_{ij} = [\bar{\mathbf{G}}_{ij}\ \bar{\mathbf{A}}_{1i}\ h\bar{\mathbf{A}}_{2i}\ \mathbf{0}], \quad \bar{\Delta}_{5ij} = \left[\bar{\mathbf{\Gamma}}_{ij}^T \quad \mathbf{X}(\mathbf{e}_1^T-\mathbf{e}_2^T) \quad \mathbf{X}(h\mathbf{e}_3^T-h\mathbf{e}_2^T)\right]^T.
\end{aligned} \tag{18}$$

Let Eq. (17) be pre-multiplied and post-multiplied by $diag\left[\mathbf{X}\ \mathbf{X}\ \mathbf{X}\ \mathbf{X}\right]$ and its transpose, then we can obtain

$$\begin{aligned}
&Sym\{\Delta_4^T\bar{\mathbf{P}}\bar{\Delta}_{5ij}\} + \mathbf{e}_1^T\bar{\mathbf{Q}}_i\mathbf{e}_1 - \mathbf{e}_2^T\bar{\mathbf{Q}}_i\mathbf{e}_2 + h\bar{\mathbf{N}}_1\bar{\mathbf{R}}^{-1}\bar{\mathbf{N}}_1^T + \frac{h}{3}\bar{\mathbf{N}}_2\bar{\mathbf{R}}^{-1}\bar{\mathbf{N}}_2^T\\
&+\frac{h}{5}\bar{\mathbf{N}}_3\bar{\mathbf{R}}^{-1}\bar{\mathbf{N}}_3^T + Sym\{\bar{\mathbf{N}}_1\Delta_1+\bar{\mathbf{N}}_2\Delta_2+\bar{\mathbf{N}}_3\Delta_3\} + h\mathbf{\Gamma}_{ij}^T\mathbf{R}\mathbf{\Gamma}_{ij} < \mathbf{0}.
\end{aligned} \tag{19}$$

So we have

$$\begin{aligned}
\dot{V}(t) &\le \sum_{i=1}^{p}\sum_{j=1}^{c}\zeta^T(t)\omega_i m_j\bar{\mathbf{\Phi}}_{ij}\zeta(t)\\
&= \sum_{i=1}^{p}\sum_{j=1}^{c}\omega_i m_j\zeta^T(t)\left(\bar{\mathbf{\Phi}}_{1ij}+\bar{\mathbf{\Phi}}_2+\bar{\mathbf{\Phi}}_3+h\mathbf{\Gamma}_{ij}^T\mathbf{R}\mathbf{\Gamma}_{ij}\right)\zeta(t) < \mathbf{0},
\end{aligned} \tag{20}$$

where $\bar{\mathbf{\Phi}}_{1ij}$, $\bar{\mathbf{\Phi}}_2$, $\bar{\mathbf{\Phi}}_3$, $i = 1,2,...,p$, $j = 1,2,...,c$ are defined in (13).

By denoting $\omega_i m_j$ as h_{ij}, $i = 1,2,...,p, j = 1,2,...c$, and using similar algebraic manipulations as in the proof of Theorem 1, we can get

$$\begin{aligned}
\dot{V}(t) &\le \sum_{i=1}^{p}\sum_{j=1}^{c}h_{ij}\zeta^T(t)\bar{\mathbf{\Phi}}_{ij}\zeta(t)\\
&\le \sum_{i=1}^{p}\sum_{j=1}^{c}h_{ij}\zeta^T(t)(\bar{\mathbf{\Phi}}_{ij}-\bar{\mathbf{F}}_{ij}+\bar{\mathbf{T}}_{ij}+\sum_{r=1}^{p}\sum_{s=1}^{c}\bar{h}_{rs}\bar{\mathbf{F}}_{rs}-\sum_{k=1}^{p}\sum_{l=1}^{c}\underline{h}_{kl}\mathbf{T}_{kl})\zeta(t),
\end{aligned} \tag{21}$$

where $\bar{h}_{ij} \ge h_{ij}$ is the upper bound of h_{ij}, $\underline{h}_{ij} \le h_{ij}$ is the lower bound of h_{ij}, $\bar{\mathbf{F}}_{ij} = \bar{\mathbf{F}}_{ij}^T \ge \mathbf{0}$, and $\bar{\mathbf{T}}_{ij} = \bar{\mathbf{T}}_{ij}^T \ge \mathbf{0}$.

So if

$$\bar{\mathbf{\Phi}}_{ij} - \bar{\mathbf{F}}_{ij} + \bar{\mathbf{T}}_{ij} + \sum_{r=1}^{p}\sum_{s=1}^{c} \bar{h}_{rs}\bar{\mathbf{F}}_{rs} - \sum_{k=1}^{p}\sum_{l=1}^{c} \underline{h}_{kl}\mathbf{T}_{kl} < \mathbf{0}, \tag{22}$$

we can obtain $\dot{V}(t) < 0$, which means the closed-loop TSFMB control system (3) is asymptotically stable.

Applying Schur Complement theorem, inequality (22) can be denoted as

$$\begin{bmatrix} \Pi_1 & \sqrt{h}\bar{\mathbf{N}}_1 & \sqrt{h}\bar{\mathbf{N}}_2 & \sqrt{h}\bar{\mathbf{N}}_3 & \sqrt{h}\bar{\mathbf{\Gamma}}_{ij}^T \\ * & -\bar{\mathbf{R}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -3\bar{\mathbf{R}} & \mathbf{0} & \mathbf{0} \\ * & * & * & -5\bar{\mathbf{R}} & \mathbf{0} \\ * & * & * & * & -\mathbf{R}^{-1} \end{bmatrix} < 0, \tag{23}$$

where $\Pi_1 = \bar{\mathbf{\Phi}}_{1ij} + \bar{\mathbf{\Phi}}_3 + \bar{\mathbf{\Phi}}_{4ij}, i = 1, 2, ..., p, j = 1, 2, ..., c.$

Moreover, as $\mathbf{R}$ is symmetric and positive definite matrix, for any scalar σ, the following inequality holds:

$$\left(\sigma\mathbf{R}^{-1} - \mathbf{X}\right)\mathbf{R}\left(\sigma\mathbf{R}^{-1} - \mathbf{X}\right) > \mathbf{0}. \tag{24}$$

Then we have

$$-\mathbf{R}^{-1} < -\frac{2}{\sigma}\mathbf{X} + \frac{1}{\sigma^2}\bar{\mathbf{R}}, \tag{25}$$

which means inequalities (23) is true, if LMIs (13) holds. Thus, the proof of Theorem 2 is completed.

5 Numerical Examples

In this section, a numerical example is given to demonstrate the effectiveness of the proposed methods.

Example 1. Consider a TSFMB autonomous system in the form of (1) with

$$\mathbf{A}_1 = \begin{bmatrix} -2.1 & 0.1 \\ -0.2 & -0.9 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}, \mathbf{A}_{11} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & 0.9 \end{bmatrix},$$

$$\mathbf{A}_{12} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}, \mathbf{A}_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{A}_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\omega_1(x_1(t)) = 1 - \frac{0.5}{1 + e^{-3-x_1(t)}}, \quad \omega_2(x_1(t)) = 1 - \omega_1(x_1(t)).$$

Using the stability conditions introduced in literature [1–3, 9, 10] and Theorem 1 of this paper, respectively to calculate the maximum allowable time-delays. The results are presented in the following table.

From Table 1, we can see that Theorem 1 of this paper can yield larger maximum allowable time-delays h than literature [1–3, 9, 10], which means the proposed method in this paper is less conservative than the ones in [1–3, 9, 10].

Table 1. The maximum allowable time-delays h for Example 1

Method	[3]	[1]	[2]	[9]	[10]	Theorem 2
h	3.37	4.28	4.61	4.64	5.58	7.62

Remark 2. There are two reasons for the less conservative results in Example 1. First, the introduction of the new integral inequality (4), which is tighter than other existing integral inequalities. Second, Theorem 1 of this paper takes the information of membership functions into consideration while the methods in other literature are membership functions independent.

6 Conclusion

This paper concerns the stability and stabilization analysis issue of TSFMB control systems with distributed time-delay. The imperfect premise matching methodology has been employed to design fuzzy controller, which allows the fuzzy controller to use different premise membership functions and different number of rules from the fuzzy model. Hence, greater design flexibility can be achieved. Besides, a tighter integral inequality has been applied, and the information of the membership functions has been introduced in the criteria as well. As a consequence, less conservative stability conditions and stabilization criteria have been developed. Finally, a numerical example has been provided to illustrate the effectiveness of the proposed approach.

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