Chapter 13
Optimal Multistage Defined-Benefit Pension Fund Management

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Abstract We present an asset-liability management (ALM) model designed to support optimal strategic planning by a defined benefit (DB) occupational pension fund (PF) manager. PF ALM problems are by nature long-term decision problems with stochastic elements affecting both assets and liabilities. Increasingly PFs operating in the second pillar of modern pension systems are subject to mark-to-market accounting standards and constrained to monitor their risk capital exposure over time. The ALM problem is formulated as a multi-stage stochastic program (MSP) with an underlying scenario tree structure in which decision stages are combined with non-decision annual stages aimed at mapping carefully the evolution of PF's liabilities. We present a case-study of an underfunded PF with an initial liquidity shortage and show how a dynamic policy, relying on a set of specific decision criteria, is able to gain a long-term equilibrium solvency condition over a 20 year horizon.

Keywords Pension fund management • Multistage stochastic programming • Scenario tree • Solvency ratio • Defined benefits

13.1 Introduction

Most pension systems in OECD countries rely on three pillars: a state-controlled security system which represents the fundamental instrument for welfare policies, a sector-specific complementary occupational pension pillar open to corporations' and sectors' employees and, finally, individual retirement contracts that can be

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agreed between pension funds, life insurers and individuals to provide deferred income during retirement. The first pillar is by definition public while the second and third pillars are mainly private. Third pillar's individual ALM problems and associated modeling and economic issues have attracted significant interest in recent years and find a natural formulation as long-term consumption-investment problems over a life-cycle (Medova et al., 2008; Consiglio et al., 2007; Consigli et al., 2012b; Konicz et al., 2015). In this chapter we analyse instead the key modeling and methodological issues needed to formulate and solve a second pillar occupational PF strategic asset allocation problem relying on a multistage stochastic program (MSP) with liabilities generated by a defined benefit (DB) scheme. ALM developments for pension funds have a relevant record starting from early works (Consigli and Dempster, 1998; Ziemba et al., 1998; Pflug and Świetanowski, 1999), then increasingly relying on real-world case-studies (Mulvey et al., 2006, 2007, 2008; Geyer and Ziemba, 2008) and again, more recently, exploring alternative modeling and optimization approaches (Aro and Pennanen, 2016; Pachamanova et al., 2016; Consigli et al., 2016). A focus on the differences between DB and defined contribution (DC) schemes can be traced in Dert (1998) and Consiglio et al. (2015).

PF ALM theory originally developed from the challenges associated with the modeling of liability streams typically extended over several decades. Those streams carry relevant risks to the PF manager. On the liability side, indeed, a DB PF manager faces mainly three relevant risk sources: inflation risk, interest rate risk and longevity risk (Aro and Pennanen, 2016; Consigli et al., 2016). In this chapter we consider a pension plan decumulation phase in which pensions are revaluated according to the previous year recorded Consumer Price Index (CPI) annual variation. The value of the pension fund liability, or DB obligation (DBO), is then computed as the discounted value of all expected pension payments and will depend on the evolution of the yield curve over the decision horizon. Such exposure is typically compensated for hedging purposes by fixed-income holdings in the asset portfolio. We assume that the PF manager, before any indexation, is informed by the actuarial division of future contributions and pension payments over a very long horizon. Longevity risk comes into the picture because upon determination of individuals' benefits through an annuity, the PF manager needs to assume a future life length that might underestimate passive members' actual future life duration (Aro and Pennanen, 2016; Shaw, 2007). Such phenomenon is attracting increasing interest by actuarial studies. In this chapter we assume that projected pension flows' estimates do already take into account such risk.

A PF manager primary concern is to preserve a sufficient funding condition: as liabilities evolve over time along random paths, the assets' market value is expected at least to match their value. The *funding ratio* (FR) is defined as the ratio of the asset portfolio to the DBO value. A FR close to 1 suggests proper funding of all future inflation-adjusted pension payments. A FR below 1 will indicate an under-funding condition. Notice that a FR below 1 doesn't necessarily imply a liquidity deficit; it does however describe a weak financial condition. The joint dependence on interest rates and inflation dynamics of assets and liabilities underlines the importance of a dynamic ALM approach to the problem and an effective risk assessment and monitoring (Pachamanova et al., 2016; Consigli et al., 2016).

We present in this chapter a novel methodological approach, inspired by Dempster et al. (2007), still in the stream of advanced applications of MSP with recourse, in which decision nodes are a subset of nodes along a scenario tree accommodating annual cash flows and obligations within what we refer to as intermediate nodes. We are able in this way to monitor accurately the cash condition of the PF and avoid anticipative portfolio rebalancing decisions. The key motivation of this work is related to the evidence that a careful representation and discrete formulation of a dynamic PF ALM model, including a liability-driven component and an innovative set of decision criteria, can consistently bring a maybe severely underfunded PF into a long-term funding equilibrium. This is what we believe will define our contribution in the first place. The model development and the definition of a case-study in cooperation with industry colleagues are also worth sharing.

The chapter evolves from Sect. 13.2 where we introduce the key elements of the ALM model and specify the scenario tree convention supporting the MSP formulation. The problem mathematical instance is introduced progressively in the first section and over the following sections to clarify the role played by each set of equations in the economy of a PF. In Sect. 13.3 the liability model is introduced. In Sect. 13.4 the risk exposure of the PF to market fluctuations is analyzed before focusing in Sect. 13.5 on funding conditions and long term PF solvency. In Sect. 13.6 we present a case study application and in Sect. 13.7 we conclude.

13.2 DB Pension Fund Management

We consider a company offering a complementary DB pension scheme to its employees: the PF collects contributions from the PF's sponsor, and marginally from the employees, and pays benefits. A DB plan ensures benefits based on the employee's remuneration at or near the retirement date. The adjustment of pension benefits to account for changes in costs and living standards is reflected in the fund's indexation rule. The level of indexation has an important impact on the income of retirees and the pension accrual of active members in pension plans.

Under a DB scheme, the PF manager faces several risk sources: core to this decision process is the need to keep the FR sufficiently high and monitor its evolution over time. The FR definition differs from the funding gap which is computed as the difference between liability and assets and coincides with the concept of net DBO. A negative net DBO thus coincides with a surplus condition of the fund.

A PF DBO is in general naturally affected by the ratio of active to passive members and a deteriorating population ratio in several economic sectors has been recorded in recent years. Furthermore survival probabilities have been increasing leading to a surge of longevity risk (Aro and Pennanen, 2016).

From a financial viewpoint, furthermore, since the 2008 global financial crisis, interest rates have materially decreased in most OECD economies, reducing the liquidity buffer generated historically by significant holdings of treasury bonds by PF managers and increasing their liabilities, ceteris paribus. Both phenomena motivated over the last decade or so an increasing role of less liquid and relatively risky portfolio strategies throughout the PF industry (OECD, 2011). In this chapter, we rely on an exogenous liability stream and focus on the definition of an optimal portfolio policy, when the population of active members is assumed not to increase (closed population model). The fund's contributions are generated by the employees, as a salary portion, and the employer, or sponsor of the pension fund. Further to such commitment, the sponsor has an obligation to fill up the PF resources in case of prolonged underfunding conditions. The PF wishes to minimize the injection into the fund of those unexpected and extraordinary contributions, comparable to undesirable recapitalization decisions. Throughout the chapter the PF manager represents the decision maker of the ALM problem.

The need of an effective portfolio policy over a long-term horizon is a strong motivation for the adoption of a dynamic approach: to preserve sufficient realism in the problem description we propose in this chapter a specific scenario tree structure, based on a distinction between decision nodes and so-called intermediate nodes, where pension payments are recorded. Such formulation makes the mapping of pension payments on an annual basis possible and allows the definition of an optimal investment strategy over a 20-year horizon with non-homogeneous time stages. The formulation of the PF ALM problem as a multistage stochastic program is shown to provide an effective way to manage the PF risk exposure and regain a stable funding condition at the end of the planning horizon.

We assume an extended investment universe including Treasury indices with different maturity, corporate indices, real estate investments, inflation-linked treasuries (TIPS) also spanning several maturities and equity. Even if indices are known to be almost constant-maturity financial benchmarks, they are here treated as investment opportunities carrying a time to maturity. Accordingly fixed income assets will generate both interests during the holding period and capital at current market prices when expiring.

The company's actuarial division is responsible for determining the expected evolution of the PF's liabilities, treated as exogenous to the definition of an optimal investment plan. In this analysis the population of active members is assumed closed and benefit cashflows are first inflation-adjusted and then discounted at a prevailing term structure of interest rates forward in time. As a result both net pension payments and the fund's DBO are considered exogenous and scenario dependent.

We adopt a simple multivariate Gaussian model to derive the random evolution of price dynamics and interest rates as well as dividend payments (see the estimated means, variances and covariances in appendix). The PF manager seeks an optimal policy based on a combination of decision criteria, including PF liquidity, duration-matching between assets and liabilities, risk-adjusted performance and sponsor's extraordinary contributions. Further at the end of the planning horizon the PF manager seeks a minimal, ideally null net DBO. A null net DBO, corresponding to a unit FR, is regarded as a Fund's equilibrium condition.

To exemplify a PF risky condition, assume for instance that a severe market shock will impact the PF asset portfolio causing a sudden decrease of its market value and to mitigate systemic risk, monetary authorities will decide a strong reduction of interest rates. In case of heavy mismatching between assets' and liabilities' duration, such intervention would add up to the market shock, inducing a further decrease of the FR and a deterioration of funding conditions. Such nightmare scenario has indeed occurred at several points of recent financial history and has deeply affected the pension industry, leading to more rigorous risk management approaches.

From a modeling viewpoint the MSP formulation leads to the definition of the optimal ALM strategy as a decision tree process describing the sequential interplay between decisions and random events. The beginning of the decision horizon coincides with the current time $t = 0$, while T will denote the end of the decision horizon. A reference period defined as 0^-, prior to 0, is introduced in the model to define an input portfolio from which the optimal time 0 portfolio will be determined. The extended time horizon is represented by $\widehat{\mathcal{T}} = \mathcal{T} \cup 0^-,\ t \in \widehat{\mathcal{T}}$. At the last stage, no investment decisions are allowed, while cash flows due to expiring bonds and income payments will be accounted for.

We denote with $T_d = \{1, 2, 3, 5, 10, 20\} \in \mathcal{T}$ a set of decision times expressed in years. Time 0 is associated with the so-called here and now decision (or H&N) and the subsequent stages have associated all scenario dependent decisions. In addition we consider $T_{int} = \mathcal{T} \backslash \{T_d\} = \{4, 6, 7, 8, 9, 11, \ldots, 19\}$ to denote a set of intermediate years; therefore $\mathcal{T} = T_{int} \cup T_d$. At intermediate times, income returns and the assets expiring provide random cash inflows to pay current pensions.

Random dynamics are modeled through discrete tree processes defined in an appropriate probability space $(\Omega, \mathcal{F}, \mathbb{P})$ (Dupačová et al., 2001; Dempster et al., 2011; Consigli et al., 2012a). Following the model notation in Consigli et al. (2012b), nodes along the tree, for $t \in \mathcal{T}$, are denoted by $n \in \mathcal{N}_t$; we distinguish between decision nodes $\mathcal{N}_t^d$ and intermediate nodes $\mathcal{N}_t^{int}$. Again $\mathcal{N}_t = \mathcal{N}_t^d \cup \mathcal{N}_t^{int}$. For $t = 0$ the root node (associated with the partition $\mathcal{N}_0 = \{\Omega, \emptyset\}$, corresponding to the entire probability space) is labeled $n = 0$. For $t > 0$ every $n \in \mathcal{N}_t$ has a unique ancestor n^- and, for $t < T$, a non-empty set of children nodes n^+. We indicate with $m \in \mathcal{C}_n$ the nodes in the sub-tree originating from node n. We denote with t_n the time associated with node n.

The set of all predecessors of node n: $n^-, n^{--}, \ldots, 0$ is denoted by $\mathcal{P}_n$. We distinguish between ancestor decision nodes $\mathcal{P}_n^d$, i.e. $n^{d-}, n^{d--}, \ldots, 0^-$, and ancestor intermediate nodes $\mathcal{P}_n^{int}$, i.e. $n^{int-}, n^{int--}, \ldots$, and we define $\widehat{\mathcal{P}} = \mathcal{P}_n^d \cup 0^-$.

The scenario tree conditional nodal structure defines the space $(\Omega, \mathcal{F}, \mathbb{P})$ for the problem. We define the probability distribution $\mathbb{P}$ on the leaf nodes ($n \in \mathcal{N}_T$) of the scenario tree so that $\sum_{n \in \mathcal{N}_T} p_n = 1$. One scenario is identified by a sequence of nodes from the root node to one of the leaf nodes, whose cardinality will coincide with the number of scenarios. Scenarios $S = N_T$ are sample paths originated from the root node defined at time 0 to the leaf nodes at T, determine the stochastic nature

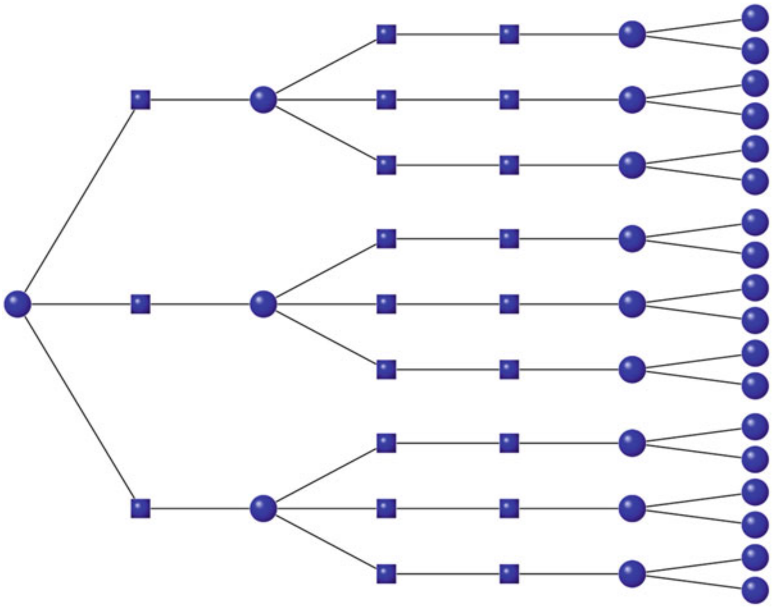


Fig. 13.1 An example of scenario tree with branching $3-(1)-3-(1)-(1)-2$

of the problem. As represented in Fig. 13.1, scenario branching will be allowed only at decision times (circle nodes) and not at intermediate times (squared nodes).

Accordingly we define the optimal investment policy, based on holding, buying and selling decisions along such tree. Let $X_n = \sum_i \sum_{h<n} x_{i,h,n}$ be the value of the investment portfolio in node n: i refers to the asset type and $h \in \mathcal{P}_n$ to a predecessor node of n where the investment originated. We consider four asset classes: I_1 for fixed-income and inflation-linked assets but corporates, I_2 for securitized and corporates, I_3 for equity and I_4 for real estate investments. The asset universe is defined by $I = I_1 \cup I_2 \cup I_3 \cup I_4$. In each class we have subsets of possible allocations for a total of 12 investment opportunities. The following decision variables are considered:

- $x^+_{i,n}$ *investment* (buying decision) in node n, of asset i (a maturity T_i is considered for fixed-income assets);
- $x^-_{i,h,n}$ *selling* decision in node n of asset i, that was bought in $h : (h \in \mathcal{P}^d_n) < n$; $t_n - t_h$ = holding period. In case of fixed-income assets, at the maturity T_i, the asset expires and it is assumed to generate a cash flow equal to the asset market value at maturity;
- $x^{exp}_{i,h,n}$ *maturity* (expiry) at node n, of asset i (fixed-income asset) that was bought in h, $(t_n - t_h = T_i - t_h)$, $n \in \mathcal{N}_t$;
- $x_{i,h,n}$ *holding* at node n, of asset i (with maturity T_i, for $i \in I_1$) that was bought in $h \in \mathcal{P}^d_n$;
- z_n *cash account* in node n.

The asset value evolution in each decision node is captured by the return generated by previous decisions and current buying and selling decisions. We denote with $\rho_{i,n}$ the price returns at node n. Therefore, the main equations describing the PF portfolio evolution and inventory balance constraints of the ALM model are for $t \in \mathcal{T}, n \in \mathcal{N}_t$:

$$x_{i,n,n} = x_{i,n^+} \tag{13.1}$$

$$x_{i,h,n} = x_{i,h,n^-}(1+\rho_{i,n}) - x^-_{i,h,n} - x^{exp}_{i,h,n} \tag{13.2}$$

$$x^{exp}_{i,h,n} = x_{i,h,n^-}(1+\rho_{i,n}) \;\; h < n,\; n-h = T_i \tag{13.3}$$

$$x_{i,n} = \sum_h x_{i,h,n} \tag{13.4}$$

$$X_n = \sum_i x_{i,n} \tag{13.5}$$

Consider the random process $\rho_{i,n}$: for given holdings x_{i,h,n^-} of asset i in the parent node n^-, we derive the clean price return in node n. The intermediate argument $h \in \mathcal{P}_n$, representing the buying stage, is needed as discussed below to estimate the investment and losses over the holding period $t_n - t_h$. Those returns will determine the optimal investment allocation after paying the pensions. Unlike buying and selling decisions which are allowed only for $n \in \mathcal{N}^d_t$, bonds expiries and asset cashflows (interests and dividends) are recorded annually for each $n \in \mathcal{N}^{int}_t$. At the end of the time horizon sellings and purchases are not allowed, while fixed-income assets may expire and in this way generate cash-inflows: for $n \in \mathcal{N}^d_T, i \in I$ we have $x^+_{i,T} = x^-_{i,h,T} = 0$.

The portfolio manager is concerned with the issue of collecting sufficient resources to pay regularly pension obligations. In the long-run however she/he will focus on the PF funding status, namely the ratio of her asset portfolio to the liability. The common exposure to interest rate movements, historically, has been managed primarily by controlling the asset-liability (A-L) duration gap. We focus next on the ALM model liability-driven components and then analyse the funding issue.

13.3 Liabilities, Liquidity and A-L Duration Matching

The PF obligation in node $n \in \mathcal{N}_t$ for each $t \in \mathcal{T}$ is Λ_n. We assume known a given stream of net nominal pension payments $\tilde{L}_t$ over a long horizon $t \leq \hat{T}$ where $\hat{T}$ in insurance mathematics might be as large as several decades and such stream is just specified as an average pension obligation: the evaluation of the current liability implies first an inflation adjustment resulting into a tree process for pension cashflows and then at each node in the tree the discounting of future pension payments based on evolving term structures of interest rates. For the sake of simplicity and without introducing formally a stochastic interest rate model, at each decision node the current yield curve is assumed to be generated by fitting a term structure from the Nelson-Siegel (N-S) family (Nelson and Siegel, 1987) on the set of interest rate benchmarks adopted in the asset return model. A 1-year inflation rate π_n is generated as described below by difference between a nominal and real interest rates, resulting into inflation-adjusted random liabilities L_n for all $n \in \mathcal{N}_t$, where $t \leq \hat{T}$:

$$L_n = \tilde{L}_n \prod_{m \in \mathcal{P}_n} (1 + \pi_m). \tag{13.6}$$

At the 20-year horizon, the value of the net DBO B_n at leaf nodes $n \in \mathcal{N}_T$ defines an *end-effect* capturing the PF terminal funding conditions which will reflect the time T-forward net present value of all pension payments beyond T. Such evaluation will be performed relying on the yield curves on each leaf node.

We have, for $t \in \widehat{\mathcal{T}}$, $n \in \mathcal{N}_t$, indicating with t_n the reference time of node n:

$$\Lambda_n = \sum_{m \in \mathcal{C}_n} p_m \frac{L_m}{(1 + r_{t_n,t_m})^{(t_m - t_n)}}. \tag{13.7}$$

In (13.7) r_{t_n,t_m} denotes the interest rate quoted at t_n for deposits expiring in t_m, and $\forall m$, p_m define the conditional probabilities associated with the subtree originating from node n. At each stage equally probable transitions are assumed.

Term structure of interest rates Following Eq. (13.7), the discounting at each node relies on the fitting at time 0 and then along all scenarios, of a benchmark interest rate curve from the N-S family whose evolution is assumed to depend on a set of interest rate benchmarks spanning from 3-months to above 10-years durations. The N-S parametric model at time 0 for rates with maturity t reads:

$$r(0,t) = \beta_0 + (\beta_1 + \beta_2) \cdot \frac{\beta_3}{t} \cdot \left(1 - e^{-\frac{t}{\beta_3}}\right) - \beta_2 \cdot e^{-\frac{t}{\beta_3}} \tag{13.8}$$

where $\beta_0, \beta_1, \beta_2$ and β_3 are parameters to be fitted on market curves adopting a splining procedure. Yield curve calibration is performed relying on the N-S model on both a nominal and real interest rate curve: their difference will determine a term structure of (annual) inflation rates. The fitting is based on the evolution of the 3-month EURIBOR and Treasuries or Inflation-linked Treasuries with different maturity buckets (see below the asset return generating process). The implied 1-year inflation rate is then used to derive the inflation-adjusted pension process in Eq. (13.6).

The PF obligation in current terms will increase if inflation increases pushing up future pension payments and will decrease if interest rates increase, due to a decreasing discount factor. The two forces may offset each other as when interest rates remain equal in real terms.

Dedicated and duration-matching portfolios The PF manager at every node in the scenario tree seeks an asset portfolio generating the cash flows required by the stream (13.6). Furthermore, to neutralize undesirable effects that may be generated by volatile interest rates the duration of the asset portfolio should over time be as close as possible to the duration of the liability portfolio. We capture in the model those hedging conditions by introducing explicitly a liquidity gap and what we will refer to as *ALM* risk variable in the objective function.

To minimize cash imbalances in the pension service, every year those are funded through current asset income sources (interests and dividends) and bonds' redemptions. The income generated by bonds and equities over the decision horizon is identified by $\sum_{h<n,h\in\mathcal{P}_n^d} x_{i,h,n^-}\cdot\xi_{i,n}$, where $\xi_{i,n}$ denote random income coefficients along the scenario tree. Accordingly either a cash surplus will emerge, if cash inflows exceed pension payments, or a deficit will emerge: in either cases the resulting surplus/deficit will be compounded until the next decision stage and included in the cash balance constraint. We use L_n^Z to denote cash imbalances in intermediate nodes, $\forall\ t\in\mathcal{T},\quad n\in\mathcal{N}_t^{int},\quad h\in\mathcal{P}_n^d$ and $i\in I_1\cup I_2$:

$$L_n^Z = \sum_{h<n,h\in\mathcal{P}_n^d} x_{i,h,n^-}\cdot\xi_{i,n} + \sum_{h<n,\ n-h=T_i} x_{i,h,n}^{exp} - L_n \tag{13.9}$$

For model accuracy we recall that, given the peculiar tree structure in Fig. 13.1, to preserve non-anticipativity, neither buying nor selling decisions are possible in intermediate stages. Equation (13.9) jointly with the cash balance constraint at decision stages clarify how a *dedicated* portfolio approach is captured in the model formulation. At time 0, the cash account is defined by the cash inflows due to selling decisions $x_{i,h,n}^-$ and outflows due to new investments $x_{i,n}^+$. Given an initial cash balance z_{0^-}, we have $z_0 = z_{0^-} + \sum_i x_{i,0^-,0}^- - \sum_i x_{i,0}^+$. Afterwards at each decision time the cash account will depend on pension payments L_n, on compounded cash imbalances $L_{h,n}^z$, on cash inflows generated by the investment portfolio and on potential *sponsor*'s contributions Φ^u. The latter, as clarified below, are regarded as last-resort, costly liquidity injections to fund temporary liquidity deficits.

$$\begin{aligned} z_n = {} & z_{n^-}(1+\zeta_n) + \Phi_n^u + \\ & -L_n + \sum_{n^{d-}<h<n} L_{h,n}^z(1+\zeta_{h,n}) + \\ & +\sum_i \sum_{h<n,h\in\widehat{\mathcal{P}}} x_{i,h,n}^- - \sum_i x_{i,n}^+ \\ & +\sum_i \sum_{h<n,\ n-h=T_i} x_{i,h,n}^{exp} + \sum_{i\in I\setminus I_1,I_2} \sum_{n^{d-}<m<n,\ n-m\geq T_i} x_{i,m,n}^{exp}(1+\zeta_{h,n}) \\ & +\Pi_n^{1,INV} \end{aligned} \tag{13.10}$$

The quantity $\Pi_n^{1,INV}$ represents interest and dividend income and it is computed in the model as:

$$\Pi_n^{1,INV} = \sum_{i \in I} \sum_{h<n, h \in \mathcal{P}_n^d} \sum_{n^{d-}<m<n, m \in \mathcal{P}_n^{int}} \left(x_{i,n^{d-}} \cdot (1+\rho_{i,m}) - x_{i,h,m}^{exp} \right) \cdot \xi_{i,n} + + \sum_{i \in I} \sum_{h<n, h \in \mathcal{P}_n^d} x_{i,h,n^{d-}} \cdot \xi_{i,n}. \tag{13.11}$$

In Eq. (13.11) we compute at decision nodes $\mathcal{P}_n^d$ the income generated by holdings at the parent decision node $x_{i,h,n^{d-}}$ and that generated by holdings that didn't expire and were held during preceding intermediate stages.

Along each scenario, consistent with the assumed tree structure, cash surpluses and deficits will be passed forward to the following stage together with the accrual interest. Very low positive interest rates ζ_n^+ and penalty negative interest rates ζ_n^- will force the investment manager to minimize cash holdings over time. The cash surplus at the end of the horizon is part of company terminal wealth. In the adopted problem instance we do now allow cash account deficits over the horizon. The PF sponsor will always intervene in case of shortages.

Further to holding a portfolio generating sufficient cash to face current liabilities, th PF manager wishes to minimize exposure to interest rate fluctuations by maintaining a sufficiently narrow A-L duration gap. The *ALM Risk* in node n as generated by an A-L duration gap in the parent node is denoted by K_n^1 to indicate that it is the primary variable contributing to the definition of the PF interest rate risk exposure:

$$K_n^1 = dr^+ \cdot (t_j - t_{j-1}) \cdot (\Delta_n^x - \Delta_n^\Lambda)_+ - dr^- \cdot (t_j - t_{j-1}) \cdot (\Delta_n^\Lambda - \Delta_n^x)_+ \tag{13.12}$$

where $dr^{+/-}$ are positive and negative interest rate changes and $\Delta_n^x - \Delta_n^\Lambda$ defines the difference between asset and liability durations. Only the linear first-order interest rate sensitivity on assets and liabilities is considered here. The ALM, or interest rate risk will increase if an exogenous positive interest rate shock occurs when asset duration exceeds liability duration, while it will decrease in the opposite case.

Summarizing, we handle the two risk sources – liquidity and interest rate risk – by introducing in the objective function the sum of *liquidity gap* and ALM risk:

$$\Psi_n = \Omega_n + K_n^1 + \Psi_{n^-} \tag{13.13}$$

where $\Psi_{t_0} = 0$, K_n^1 is the ALM risk, while the liquidity gap Ω_n is defined as:

$$\Omega_n = L_n - \Pi_n^{1,INV} - \sum_{h, t_{n^{d-}} < h < t_n,} L_h^Z (1 + \zeta_h) - \sum_{h<n,\ n-h=T_i} x_{i,h,n}^{exp}. \tag{13.14}$$

The PF manager aims at minimizing Ψ_n over the planning horizon. In addition she is expected to control the portfolio market risk exposure: K_n^1 reflects the interest rate exposure, while K_n^m will reflect the exposure of the asset portfolio to extreme market losses. By extending the Sharpe ratio concept to tail risk, the PF manager is assumed in the long-run to seek a strategy such that the portfolio return per unit K_n^m risk is maximised.

13.4 Risk Capital and Risk-Adjusted Performance

By dynamically rebalancing the portfolio the PF manager will continuously revise the risk exposure and seek a higher risk-adjusted return. We assume a rather simple, highly conservative, constant correlation model to clarify how such exposure is kept under control over the decision horizon. Indeed it can be shown that under assumptions of perfect positive correlation between assets' risk factors, following Consigli and Moriggia (2014), not only are we considering a worst-case scenario from a financial viewpoint but the resulting MSP will be linear:

$$K_n^f = K_n^1 + K_n^m + K_{n-}^f \tag{13.15}$$

where $K_{t_0}^f = 0$, K_n^1 is the ALM risk, while K_n^m evaluates the linear market risk exposure as:

$$K_n^m = \sum_i \sum_{h<n, h \in \mathcal{P}_n} x_{i,h,n} \cdot k_i \cdot (t_n - t_{n-}). \tag{13.16}$$

The asset-specific coefficients k_i describe the extreme returns-at-risk associated with holdings in asset i over a unit period (say 1 year). These risk coefficients, focusing on tail returns, may be determined either through an appropriate statistical approach or by introducing regulatory-based risk charges. The latter approach is taken here with k_i expressing the 99% returns at risk associated with different asset classes. Such approximate simple risk estimation is assumed to be roughly consistent with the return generating processes for price and income returns, needed by the model instantiation, as shown next. The aggregate tail risk exposure of the pension fund is defined by the sum of the financial and the actuarial risk capital $K_n = K_n^f + K_n^l$, where K^l denotes an estimate of possible extreme losses generated by liability dynamics, due to unexpected longevity records or unprecedented lump-sum pension requests. This is assumed to be defined as a (small, say 5%) percentage of the PF obligation: $K_n^l = \phi^l \cdot \Lambda_n$. In the definition of the optimal strategy the PF manager is assumed to seek a return per unit tail risk target based on:

$$Z_n = \frac{\Pi_n^{cum}}{K_n^f}. \tag{13.17}$$

The portfolio's return per unit tail risk is defined as a ratio between the total profit generated by the investment (including the variation of unexpected gain and losses) and the investment risk exposure. To avoid handling a nonconvex variable within the optimization problem, rather than maximizing the ratio we maximize the difference $Z_n = \Pi_n^{cum} - K_n^f$. The investment risk capital K_n^f at each stage is computed assuming perfectly positively correlated asset classes and then deriving from the optimal portfolio generated by the solution the dynamics of the risk capital. We define the total portfolio return by cumulating period price returns, upon selling decisions, and unrealized gain and losses from the holding portfolio.

$$\Pi_n^{cum} = \Pi_{n-}^{cum} + \Pi_n^{INV} + [UGL_n - UGL_{0^-}] \tag{13.18}$$

where $\Pi_n^{INV} = \Pi_n^{1,INV} + G_n$ is the sum of portfolio income and trading profits. The unrealized gain and losses UGL are defined by $UGL_n = \sum_i \sum_{h<n, h\in\mathcal{P}_n} x_{i,h,n} \cdot \chi_{i,h,n}$ with $\chi_{i,h,n}$ to denote an unrealized gain-loss coefficient in node n per unit investment in asset i in node h. Realized gain and losses G_n are instead determined by clean price variations upon selling and upon assets redemptions:

$$G_n = \sum_i \sum_{h<n, h\in\mathcal{P}_n^d} \left(x_{i,h,n}^- + x_{i,h,n}^{exp}\right) \cdot g_{i,h,n} \tag{13.19}$$

The coefficients $g_{i,h,n}$ quantify unit gains-losses upon sellings assets i in node n which was bought in node h. Both realized and unrealized gains depend on the outcome of a return process for given rebalancing decision: at the beginning of each stage an asset allocation must be selected whose outcome at the end of the stage will depend on realized returns. We summarize the adopted Gaussian return model before discussing the objective function and the dynamic risk-reward tradeoff considered in this study.

Return generating processes Assets' random return coefficients are derived by simulation along the scenario tree. From a methodological viewpoint we apply first a simple Monte Carlo method to determine the risk factors evolution over a simulation horizon, then assume a given tree process and generate the returns' nodal realizations by sampling consistently with the conditional tree structure. We are not applying any specific sampling method (Consigli et al., 2012a; Dempster et al., 2011; Dupačová et al., 2001) but just focusing on a simple scenario generation approach to evaluate the ALM model and the PF induced funding policy. For each node n in the tree $\rho_{i,n}$ is the price return of asset i in node n, $\xi_{i,n}$ is the income return and ζ_n indicates the cash account return in node n. Each return type is assumed to be associated with a specific benchmark $V_{i,n}^j$ where $j = 1, 2, 3$ to distinguish price from income and money market returns respectively, while i refers to the asset class. We have $v_{i,n}^j := \frac{V_{i,n}^j - V_{i,n-}^j}{V_{i,n-}^j}$ and

$$v_n^j = \mu^j \Delta t_{n-} + \Sigma^j \Delta W_{t_{n-}}. \tag{13.20}$$

In (13.20) v_n^j is a return vector whose dimension coincides with the asset universe cardinality, with mean μ^j and variance-covariance matrix Σ^j. Δt_{n-} denotes the time increment between node $n-$ and n while $\Delta W_{t_{n-}}$ denotes a white noise random vector. We have $v^j \sim N(\mu^j, \Sigma^j)$. The set of coefficients actually adopted in the case study can be found in the appendix. The following investment opportunities are considered: for $i = 0$, $j = 3$: the EURIBOR 3 months; for $j = 1, 2$, $i = 1, 2, 3, 4, 5$: the Treasury indices for maturity buckets 1–3 years, 3–5 years, 5–7 years, 7–10 years and 10+ years, respectively; for $i = 6$: the Securitised Bond index; for $i = 7$: the Corporate Investment Grade index; for $i = 8$: the Corporate High Yield index; for $i = 9$: the Real Estate Indirect index;for $i = 10$: the Public equity index; for $i = 11, 12$: the Treasury Inflation Protected Securities (TIPS) indices for maturity buckets 3–5 years and 10+ years, respectively. Treasury indices and TIPS are used respectively to infer annual interest rates for the nominal and real yield curves of the N-S model in Eq. (13.8).

We consider 4 asset classes: I_1 includes all fixed-income and inflation linked assets, I_2 the securitised asset and the corporate, I_3 the public equity investment and I_4 the real estate.

Each asset i is characterized by a return process and in addition by a tail risk coefficient k_i. From above, leaving aside the 3-month interest rate, all other assets have associated both a clean price return process $\rho_{i,n}$ and an income process $\xi_{i,n}$. We indicate with $\rho_i \sim N(\mu^i, \sigma^i)$ the return i marginal distribution, in what follows we assume that the regulatory-based risk coefficient k_i is determined from Solvency II regulatory estimates by an updating based on historical estimates. As such they will just provide a maybe conservative estimate of the 99% return-at-risk associated with ρ_i. The following tail risk coefficients are considered in this study: $k_i = \{2.5\%, 4\%, 5\%, 4\%, 7\%, 20\%, 39\%, 25\%, 4\%, 3\%\}$ to identify potential 1-year price return tail losses associated with Treasuries (1–3, 3–5, above 5 years), Securitized bonds, Investment- and Speculative-grade corporates, equity, real estate and short- and long-term TIPS, respectively.

13.5 Funding Conditions and ALM Optimization

The ALM problem is formulated as a MSP recourse problem (Birge and Louveaux, 1997; Consigli and Dempster, 1998) over six stages. The optimal root node decision is taken at time $t_0 = 0$ and, from a practical viewpoint, represents the key implementable decision. Recourse decisions occur at t_1, t_2, t_3, t_4 and t_5 which represent respectively 1st, 2nd, 3rd, 5th and 10th year. At t_6, the $20 - year$ horizon, no buying or selling decisions are allowed but assets expiries and income leverages from the holding portfolio.

The PF manager has a primary interest to run the pension plan smoothly, match all the funds' liabilities and allow retired employees to keep a sufficiently good living standard during retirement. Of major concern to PF managers is any condition of liquidity shortage that may result into extrafunding from the fund's sponsor. In practice this is regarded as a last-resort highly undesirable option. The ALM problem objective function combines those elements to yield jointly a good hedging strategy and the generation of risk-adjusted extra returns. Through the decision horizon the portfolio manager will monitor carefully the net DBO $B_n = \Lambda_n - X_n$ defined by the difference between the present value of the accrued pension obligation Λ_n and the fair value of plan assets X_n in all nodes n of the scenario tree.

The net DBO is directly related to the FR $\Gamma_n = \frac{X_n}{\Lambda_n}$. These two quantities represent the key drivers of the PF long-term policy. A PF underfunding condition is expressed by a FR below 1, above 1 for an overfunding condition. Regulators pay now-a-day increasing attention to avoid any undesirable risk-taking by PF managers and specifically structural underfunding conditions. In the case study we show that the adopted ALM model formulation proves effective to drive a PF from an under to an overfunding condition: it is of interest to study how such outcome is associated with a given risk control strategy.

The objective function is based on a convex combination of the liquidity gap plus the ALM risk ($\Psi_n = Y_{1,n}$), the return on plan assets per unit tail risk ($Z_n = Y_{2,n}$), the plan sponsors' unexpected contributions ($\Phi_n^u = Y_{3,n}$), and the net defined benefit obligation (DBO) ($B_n = Y_{4,n}$):

$$\min_{x \in X} \left[\sum_{j=1,2,3,4} \lambda_j \mathbb{E} \left(\widetilde{Y_j} - Y_{j,n} | F_j \right) \right] \tag{13.21}$$

with $\sum_{j=1,2,3,4} \lambda_j = 1$.

In (13.21) further to Y_j, $\widetilde{Y_j}$ identify the exogenous j targets, and $\mathbb{E}(Y_{j,n}|\mathcal{F}_j)$ denotes conditional expectation with respect to information sets $\mathcal{F}_j$ associated with target $\widetilde{Y_j}$.

The optimal decision sequence x^* of the problem is defined over the decision stages $x \in X, x := \{x_t\}_{t \in T_d}$ only and will not include intermediate nodes, used instead to manage annual pension payments. By including the above set of targets, the PF management seeks a good trade-off between a dedication and duration-matching strategy, as reflected in $Y_{1,n}$, a goal risk-adjusted return $Y_{2,n}$, a long-term funding surplus $Y_{4,n}$ and avoid or minimize sponsor's unexpected contributions $Y_{3,n}$.

The ALM model develops from a canonical liability-driven investment approach to incorporate a risk-adjusted return variable and an extended set of constraints. Inflation and interest rate risk play a central role in determining liability dynamics. Interest rates and market risk factors affect instead the asset portfolio.

Overall the ALM model risk-reward trade-off considers the following risk sources, whose control is key to the maintenance of a sufficient funding status.

- Liquidity risk: annually the asset portfolio generates cash-inflows that jointly with incoming pension contributions are used to pay current pensions. In case of positive asset-to-passive PF members ratio, contributions may very well be sufficient to fund current pensions, but otherwise a liquidity gap will emerge, defined by the difference between net pension payments and portfolio income.
- Inflation risk: DB occupational PFs' liabilities are typically (even if not always) inflation-adjusted and this is a primary risk source affecting PF liabilities, jointly with interest rate risk.
- Interest rate risk: PF liabilities carry a payment term structure. The DBO joint dependence on interest rates and inflation leads to exposure to so-called real duration mismatching. Accordingly the PF manager will hedge such exposure by compensating liability real duration on the asset side. Real duration may be considered as first-order sensitivity of assets and liability values to changes of real interest rates.
- Market (tail) risk: Increasing volatility in financial markets facilitated the penetration of advanced risk management practice in the historically traditional PF industry. Specifically in the case of occupational PFs within financial and insurance conglomerates, underfunding risk has an impact on the sponsor's financial equilibrium. The PF manager, consistently with Solvency II regulations, seeks also an optimal control of extreme market risks: this is based in the ALM model on the introduction of a set of risk coefficients under an assumption of perfect positive correlation between risk factors.
- Longevity risk: In case of underestimated passive members survival probabilities, upon conversion at retirement time of the pension credit into an annuity may in the long-run weaken the PF solvency resulting into longer than expected pension payments. We assume that such risk is not affecting the PF and already accounted for in the liability estimation.

A negative PF scenario may result from joint negatively correlated shocks to assets and liabilities resulting into a sudden increase of liabilities matched by a decrease of asset values. In presence of poor duration matching and excessive exposure to market risk on the asset side, a shock in the equity market is often associated with easing conditions in the money market and increasing liability values.

A poor reaction to market risk is sometimes induced by portfolio policy constraints, including: lower and upper bounds on asset positions, a turnover constraint limiting portfolio revisions whatever the market condition, and liquidity constraints. These are considered in the ALM model instance and here summarized.

For $t \in \mathcal{T}$, $n \in \mathcal{N}_t^d$, $h \in \mathcal{P}_n^d$, and $i \in I$ we have:

$$l_i X_n \leq x_{i,n} \leq u_i X_n \tag{13.22}$$

where l_i and u_i define the percentage lower and upper bounds on holdings in asset i with respect to the current portfolio position.

For $t \in \mathcal{T}$, $n \in \mathcal{N}_t^d$, $h \in \mathcal{P}_n^d$, and $i \in I$, a maximum turnover constraint:

$$\sum_{i \in I} \sum_{h<n} x_{i,h,n}^{-} + \sum_{i \in I} x_{i,n}^{+} \leq \vartheta \left[\sum_{i \in I} x_{i,n-} \cdot (1+\rho_{i,n}) \right] \tag{13.23}$$

where ϑ is a predefined turnover coefficient. In our analysis the turnover constraint implies that the volume of the investments and disinvestments, at each node n, must be less than or equal to a certain percentage of the portfolio value before any rebalancing.

Finally a liquidity bound:

$$\sum_{i \in I} \left(X_{i,n} \cdot \widehat{\ell}_i \right) \geq X_n \cdot \widehat{\ell} \tag{13.24}$$

where $\widehat{\ell}$ is a 50% input coefficient.

After considering the set of assets' policy constraints, we summarize the key elements of the ALM model and their rationale from an economic viewpoint:

1. A distinctive modeling feature is represented by the introduction of intermediate nodes between subsequent decision nodes in the scenario tree, where however, the non-anticipativity of investment decisions is preserved and we may accommodate annual frequency of pension payments and liability evaluation.
2. The model integrates dedication and AL duration matching with the introduction of a risk-adjusted return measure and a net DBO target: both determine a strong incentive to the PF manager to maximize compounded portfolio returns and at the same time minimize the exposure to financial risk.
3. The definition of intermediate stages allows a more accurate mapping of liquidity conditions and by doing so a sufficiently safe estimation of the sponsor's extraordinary contributions, if any.
4. A perfect positive correlation between assets' returns is assumed in the estimation of the investment portfolio risk exposure, leading to a conservative risk capital estimation (thus forcing the statistical estimates used to generate assets' returns). The investment universe is agreed with experts from the insurance world and includes Treasuries, TIPS, corporates, equity and real estate.
5. Fixed income benchmarks are treated in the model as carrying a maturity and generating price as well as income returns. Real estate as well as TIPS investments, sometimes referred to as *hard* assets play historically a central role in the PF's inflation hedging strategies.

13.6 Case Study: A 20 Year Pension Fund ALM Problem

In this final section we analyse a case problem focusing directly on the conditions for a DB pension fund to recover a positive funding surplus from an initial FR severely below 1. To do this we solve the optimization problem under operational assumptions and after ranking scenarios in terms of net DBO evolution we consider first the FR evolution across all scenarios, then a strategy employed to achieve a FR closed to or greater than 1 specifically under a worst-case-scenario.

Initial conditions include a discounted value of pension payments (from year 1 to the far future) Λ_0 estimated at EUR 115 million (mln), a fair value of plan assets A_0 equal to EUR 100 mln and expected net pension payments in the first year of EUR 35 mln. Accordingly at time 0 the fund has a FR around 87%. The initial portfolio X_{0^-} is assumed to be well-diversified across the investment universe. The PF liquidity condition is also delicate since an annual shortage of approximately EUR 31 mln can be anticipated assuming a portfolio income return of 4% in the first year. Under this condition the PF manager wishes to recover over the forthcoming 20 years a funding surplus. We assume a close fund in which members can only decrease over time. As pensions are paid, the PF liability will decrease: inflation scenarios and longevity will however induce a relatively slow decreasing path. The case problem develops from an input expected stream of future annual pension payments, provided directly by the actuarial division of an occupational PF, by generating inflation-adjusted pension flows and then at each node by discounting future cash-flows using nodal specific term structure of interest rates. All data in this case-study are modified and rescaled for confidentiality reasons but agreed with industry colleagues and representative of real-world business conditions.

The optimal investment policy is constrained by a set of upper and lower bounds defined with respect to the nodal portfolio values. No short positions are allowed over the planning horizon. Treasury and TIPS are unconstrained, as well as corporate fixed income investments. The equity holdings as well as real estate holdings cannot exceed 30% of the portfolio and a 30% turnover constraint with a minimum 30% liquidity bound are assumed.

The following (decision) tree is considered in Table 13.1. The current implementable decision, corresponding to the root node, is set to January 1, 2015.

Table 13.1 Pension fund case study time and space discretization

Decision stage	1	2	3	4	5	6	7
Stage distribution (years)	0	1	2	3	5	10	20
Branching degree	10	4	2	2	2	2	
Scenarios		10	40	80	160	320	640

Between two decision stages, maybe distant in time, we consider annual intermediate times at which income returns and expiring assets are used to pay annual pension payments. Any liquidity surplus or shortage at intermediate stages is compounded to the next decision stage.

The objective function (13.21) of the PF ALM problem considers a trade-off between different goals through the coefficients $\lambda_1 = 0.2$, $\lambda_2 = 0.2$ and $\lambda_3 = 0.4$ $\lambda_4 = 0.2$. We assume that the pension fund's management will revise its strategy so as to minimize the shortfall from given targets of (1) liquidity gap plus ALM risk, (2) return per unit tail risk, (3) plan sponsors' unexpected contributions and (4) net DB. The following targets are assumed: EUR 7 mln for (1), a 2% per annum for (2), 0 for (3) and a EUR 4 mln of funding surplus.

Rebalancing decisions can be taken at decision stages from time 0 up to the beginning of the last stage; no decisions are allowed at the end of planning horizon. The optimal decision sequence does not include intermediate decisions. Nevertheless, pension payments and assets' returns will affect the revenues and asset-liability streams.

The results are generated through a set of modules combining Matlab R2011b as the main development tool, GAMS 23.2 as the model generator and solution algorithms interface and Excel 2010 as the input and output data collector running under a Windows 10 operating system with 8 GB of RAM and a dual core with four logical processors. The MSP is generated through GAMS and the CPLEX dual simplex algorithm is used to solve it. With 640 scenarios this is a very large-scale optimization problem with 1.344.995 rows, 1.464.345 columns and 7.659.404 nonzero coefficients, which is generated in roughly 16 min and solved in 5:29 (minutes:seconds) CPU time.

Consider along a specific representative scenario, the evolution of the net DBO and the FR in Fig. 13.2. The net DBO over the first years is positive reflecting a negative PF condition and accordingly the FR is deeply below 1, then between years 15 and 20 the portfolio strategy leads to a surplus at the horizon with a negative net DBO and a FR above 1.

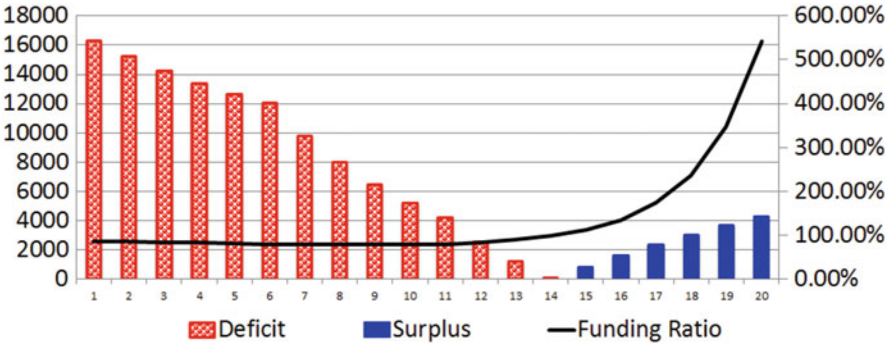


Fig. 13.2 Funding ratio (right Y axis) and net DBO (left Y axis, thousand EUR) mean scenario

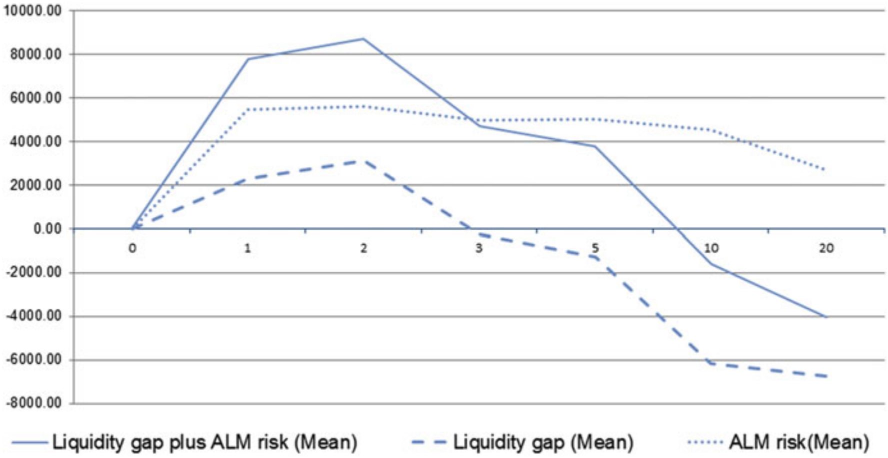


Fig. 13.3 Liquidity gap and ALM risk (thousand EUR)

A condition of underfunding, FR below 1, due to assets versus liabilities cashflows, may be associated with a liquidity surplus in the short term. Here however along the same representative scenario a condition of liquidity shortage emerges with a positive exposure to interest rate risk. Indeed the liquidity gap is positive over the first 3 years but then it starts decreasing until a liquidity surplus emerges. The exposure to interest rate risk also remains low as shown in Fig. 13.3 to witness the effectiveness of the joint risk control pursed by the optimal strategy over the planning horizon.

We see below that despite such negative initial conditions the PF will recover a robust funding condition: such outcome is primarily induced by decreasing liabilities and an effective disinvestment strategy leading to increasing risk-adjusted investment profit and terminal FR above 1 across all scenarios, but at the cost of a significant reduction of the asset portfolio. Indeed even in the worst case scenario, a decreasing liability scenario, due to outgoing pension fund members and relevant lump-sum payments in the first stages, is funded through income returns and asset sellings without the need of sponsor's intervention.

In this case study, we concentrate primarily on the FR evolution over time and across all the scenarios. The plots in Fig. 13.2 refer as indicated to an average scenario: this is identified, after the problem's solution, by ranking scenarios with respect to the evolution of the net DBO over the entire planning horizon. Out of 640 scenarios, the mean case of all is identified by considering the scenario that at each stage remains closer in the Euclidean norm to the mean.

The chance to recover a positive funding condition depends, for given exogenous liability scenarios, entirely on the investment strategy. Portfolio revisions take into account the need to preserve a sufficient liquidity buffer and minimize sponsor's interventions. We show that indeed these goals are attained over the 20 years in all scenarios.

13.6.1 Evolution of Funding Conditions

We present in Fig. 13.4 the evolution of the net DBO across time and scenarios: at the end of the first stage we show net DBO values in each of 10 nodes in decreasing value from left to right. At the end of the second the associated descending 40 nodes and so forth until the horizon. Red colour implies positive net DBO, thus liabilities exceeding asset values, blue colour is instead associated with a negative net DBO with the asset portfolio now exceeding the corresponding nodal DBO estimates.

At the end of the 10th year, already, only a limited but large portion of the scenarios carry a negative though rather high net DBO: such evidence is of extreme interest to the PF manager because under that condition all regulatory and long-term financial and risk constraints are satisfied. At the 20 year horizon across all 640 scenarios the net DBO is negative: in each leaf node the terminal value of the asset portfolio exceeds the discounted value of future pension payments. Net DBO leaf nodal values thus depend jointly on the terminal investment portfolio and for given future inflation scenarios on the prevailing yield curve. The EUR 4 mln funding surplus is achieved in all scenarios as shown in Stage 6.

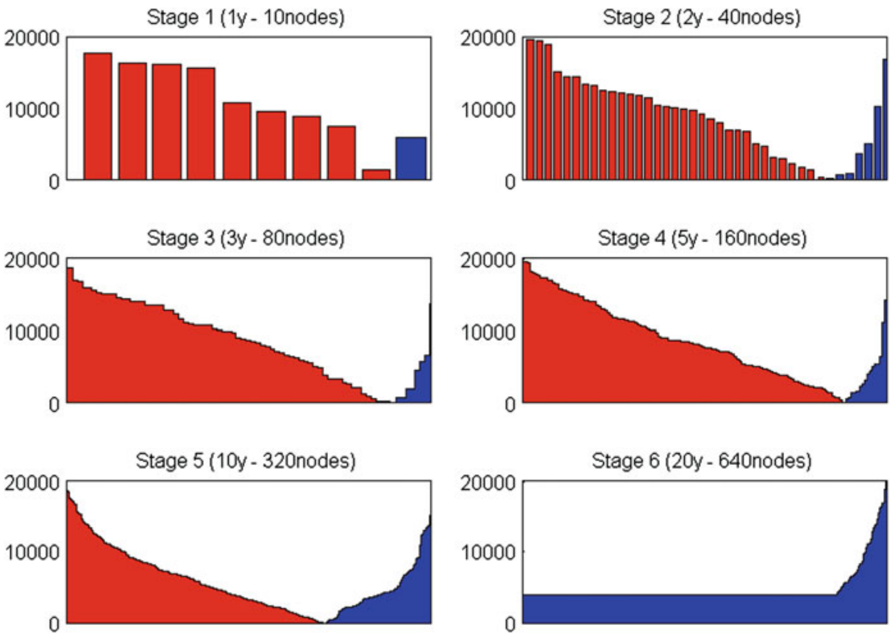


Fig. 13.4 Net DBO (Y axis, thousand EUR) at stage nodes (X axis)

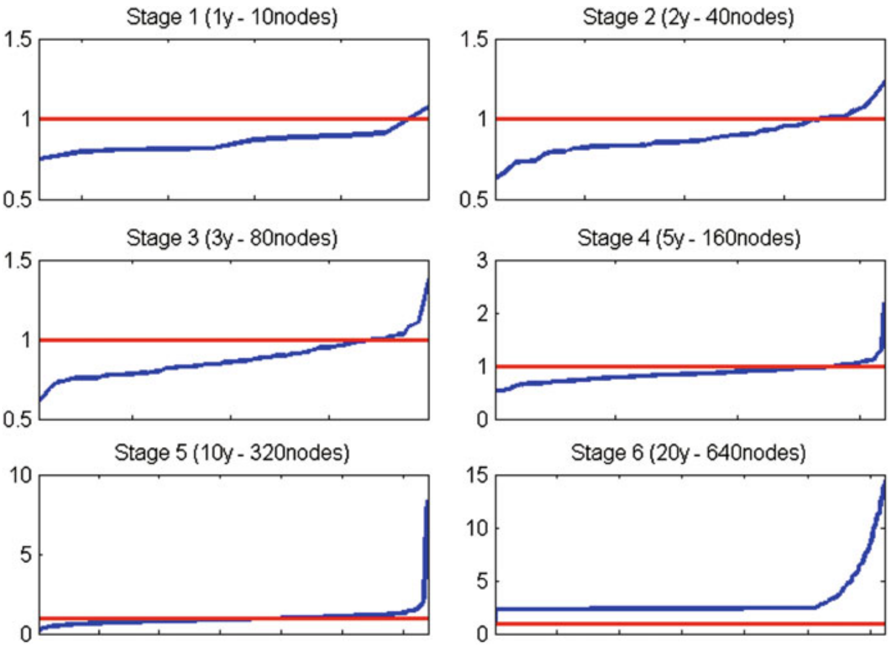


Fig. 13.5 Funding ratio (Y axis) at stage nodes (X axis)

The evolution of the FR can be analyzed under the same graphical structure. In Fig. 13.5 we show the corresponding stage-by-stage FR nodal values: the constant red line in each plot indicates an equilibrium condition: from the first to the last stage across all scenarios the FR moves from below to above 1.

The evidence reported in Figs. 13.4 and 13.5 highlights the achievement of a funding surplus across all scenarios. The ALM solution refers to ex-ante information and it supports the claim that under the given initial conditions and input scenarios, the problem solution leads to full recovery of a funding surplus consistently with the PF managerial goals. Such surplus is achieved satisfying risk capital and policy constraints.

We provide in Fig. 13.6 a final set of evidence on targets' achievements over the 20 years. We indicate in light colour the percentage of scenarios at each relevant stage where the goals were achieved or even overachieved and in dark those scenarios for which that target wasn't achieved. At the 1 year horizon a positive liquidity gap and ALM risk persist in all scenarios and from the given initial shortage condition the PF recovers a good liquidity status after 10 years. Nevertheless, as shown on the top right plot, all liquidity deficits are funded avoiding any sponsor's

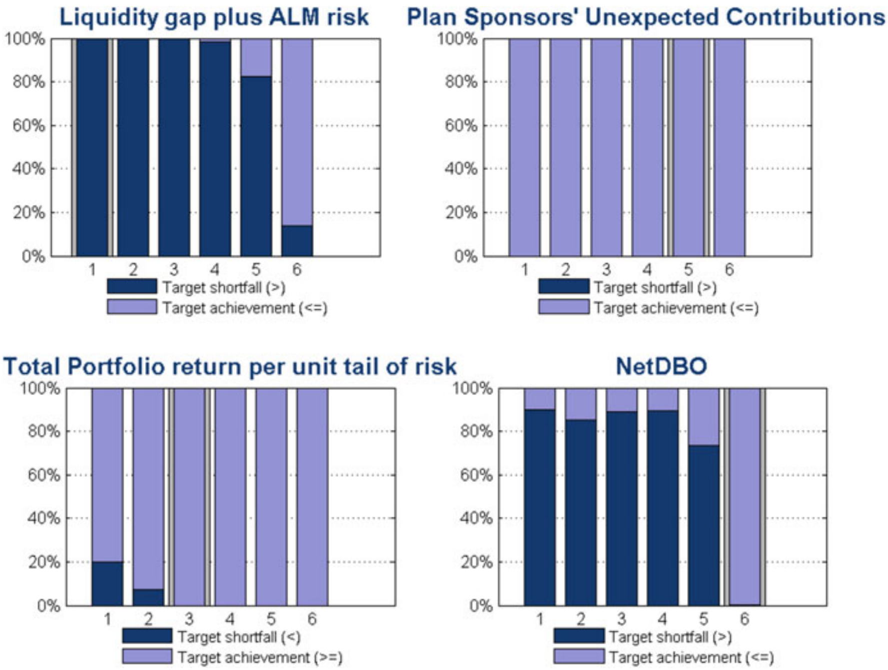
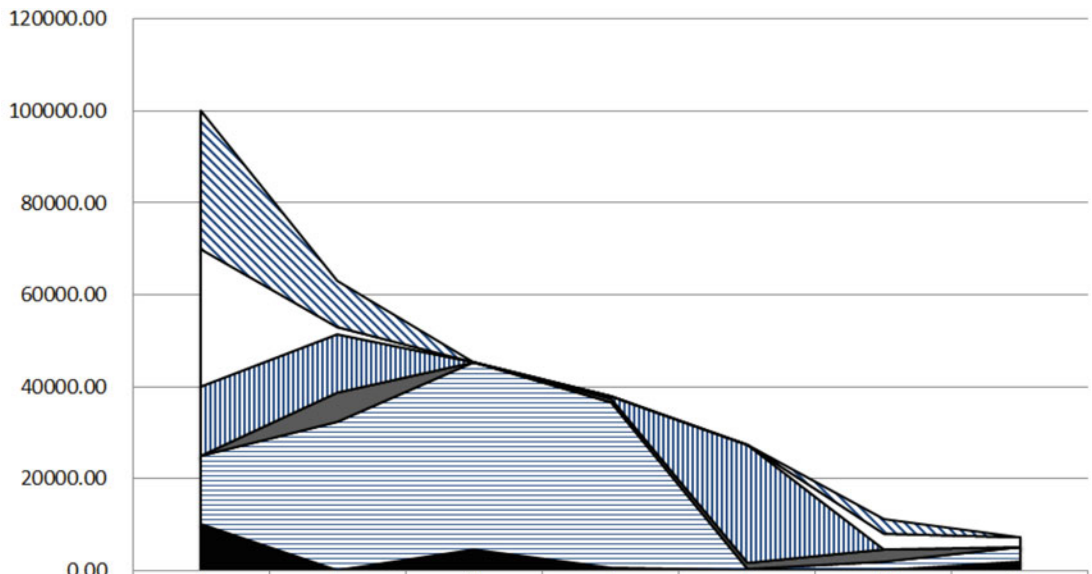


Fig. 13.6 Targets achievement

contributions which remain equal to 0 throughout the decision horizon and through assets sellings (see evidence in Fig. 13.7). The 2% risk-adjusted return target is achieved in all scenarios starting from the 3-year horizon. Finally (bottom right plot), as already discussed, progressively over time the PF recovers a long-term funding equilibrium condition.

Two relevant issues arise: which key elements drive the PF into such long-term equilibrium condition? And to which extent is such outcome due to the proposed modeling and optimization approach? Relatively in this latter case, to other approaches currently used in the industry.

As for the first question, we analyze in Sect. 13.6.2 which portfolio policy, under the worst possible DB scenario, leads to a full recovery of funding conditions over the two decades. We argue that the introduced targets' combination and a large investment universe lead, within a dynamic formulation, to an efficient dynamic portfolio diversification together with a very effective risk control. Extensive computational experiments, furthermore, have shown that such outcome may have been difficult to achieve without including in the objective function the net DBO and the risk-adjusted return goals. Indeed, the collected evidence supports the claim that a duration-based short-term hedging goal is consistent with a medium- to long-term more aggressive investment policy. A tight liquidity management of the PF thanks to the intermediate stages, also played across most experiments a relevant role.



	0	1	2	3	5	10	20
Real Estate	30000.00	10083.20	0.00	0.00	0.00	3381.32	0.00
Public Equity	30000.00	1415.19	0.00	0.00	0.00	3381.32	2130.83
Corporates	14945.23	12727.97	0.00	729.78	25883.10	0.00	0.00
Securitized	0.00	6450.65	0.00	723.35	1227.39	2602.14	0.00
TIPS	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Treasuries	15054.77	32285.54	40678.17	36072.25	295.26	1906.29	3010.14
Cash	10000.00	0.00	4518.69	506.62	0.00	0.00	1961.79

Fig. 13.7 Optimal dynamic allocation worst scenario (Y axis, thousand EUR)

As for the second question, the benefits carried by a dynamic optimization approach are undoubtable in presence of long-term liability streams. Widely in the European and U.S. PF's industry, and even earlier in the U.K. (Franzen et al., 2007) investment managers started moving over the last two decades towards the adoption of dynamic modelling approaches and long-term scenario analysis. Either based on Monte Carlo methods often matched by policy rules optimization (Mulvey et al., 2008) or on stochastic optimization (Dert, 1998; Consigli and Dempster, 1998) techniques. From a recent survey (Senders, 2010) a majority of European PF managers is now-a-days concerned with an accurate and statistically sound liability projection based on internal risk models and increasingly with an effective estimation of both asset and liability risk exposure estimation. The approach here reported, however, is the first application of a multi-criteria objective function over such long-term horizon with annual liability cash-flow matching.

13.6.2 Worst Case Scenario Analysis

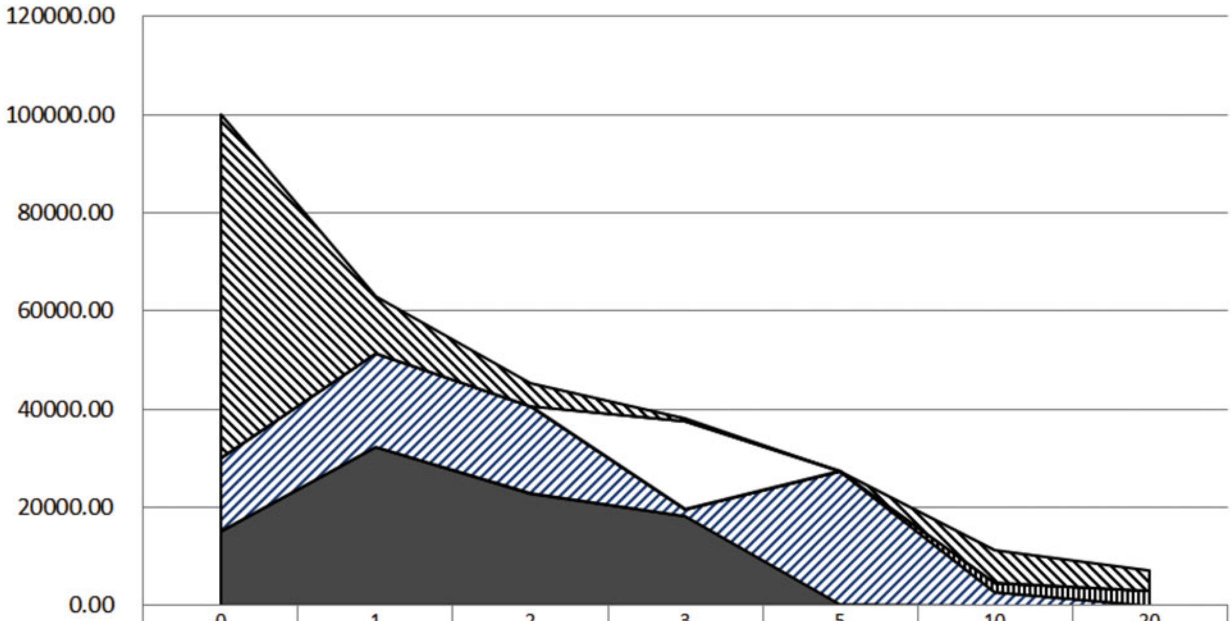
We move from a mainly quantitative analysis to a more qualitative one by analyzing the adopted optimal investment strategy along a specific, particularly negative, FR scenario.

In Fig. 13.2 we have shown the evolution of liquidity and funding conditions along a specific average scenario: that represented a mean-case-scenario from the perspective of the net DBO dynamics after solution. Indeed under that scenario the FR was around 80% for many years then decreased to less than 70%, an already warning FR, and finally it recovered to almost 120% after 15 years and kept increasing until the horizon.

Not only under the average scenario but along all scenarios the asset portfolio value decreases from the initial 100 mln value over the planning horizon. In the following we analyze the optimal portfolio strategy under worst-case FR conditions, where however the terminal funding surplus is also achieved.

The optimal H&N solution is well-diversified at time 0 with, however, equity and real estate at their upper bounds. As shown in Fig. 13.7 over the first 3–4 years the optimal strategy concentrates on treasuries and fixed-income investments and then increasingly on real estate and equity beyond the 5 year horizon. Such strategy is consistent with the achievement of a minimal shortfall with respect to liquidity gap and ALM risk in the short term and then increasingly in search of price gains and returns. Real assets seem to be contingently invested depending on the evolution of the risk-bearing capacity of the PF.

In Fig. 13.8 we see that the short-term duration matching goal and the medium term risk-adjusted performance goal rely primarily on a portfolio including assets with no expiry (equity and real estate) plus 1–5 and 5–10 fixed income maturities. At the 20 year horizon the portfolio includes only 10–20y Treasuries and equity. At that stage the DBO along such scenario is very low and the current portfolio composition is the result of buying and selling decisions taken at the beginning



	0	1	2	3	5	10	20
no expire	70000.00	11498.38	4518.69	506.62	0.00	6762.64	4092.63
20-30	0.00	0.00	0.00	17886.81	0.00	0.00	0.00
10-20	0.00	0.00	0.00	0.00	0.00	1906.29	3010.14
5-10	14945.23	19178.63	17922.99	1453.12	27110.48	2602.14	0.00
1-5	15054.77	32285.54	22745.18	18185.45	295.26	0.00	0.00

Fig. 13.8 Optimal dynamic allocation time to maturity worst scenario (Y axis, thousands EUR)

of the last stage. Extraordinary sponsor's contributions are systematically avoided by selling assets, preserving the asset portfolio duration, and compounding portfolio profits to achieve a FR above 1: the introduction of a EUR -4 mln net DBO target at the 20 year horizon proves very effective even if at that point very limited resources are left. A negative net DBO at the decision horizon provides a positive *end-effect* value: from the ALM model formulation, such value does reflect the terminal ability of the PF to fund all future liabilities.

Figures 13.7 and 13.8 provide summary information on the portfolio strategy employed by the PF under a negative net DBO scenario, resulting nevertheless into a slightly positive funding surplus at T. Under any other scenario the final net DBO value improves. At the same time at the horizon, no extra resources are requested to the sponsor and the risk-adjusted return target is achieved. The latter depends directly on the assumptions on the correlation matrix introduced in the risk capital model: namely that all assets' underlying risk factors are perfectly correlated. We will provide in a separate study a range of relevant results for the cases of market or regulatory based correlation matrices.

All the analyses have been conducted relying on a simple multivariate normal asset returns model and a set of exogenous liabilities generated by the actuarial division of an occupational PF.

13.7 Conclusion

In this chapter we have presented the key elements of an ALM model formulated so as to effectively incorporate several relevant PF short-, medium- and long-term features and be consistent with recently introduced regulatory (e.g. Solvency II) and industry standards.

From a modeling viewpoint the combination along the scenario tree of decision nodes – where buying and selling decisions are allowed in the face of residual uncertainty – with intermediate nodes – from which no multiple branching originates and where pension payments and portfolio income are required to match–, allows a more flexible and effective portfolio and liquidity hedging policy without leading to a curse-of-dimensionality problem.

The adopted combination of liquidity and interest rate risk hedging in the short run, a sufficient risk-adjusted return in the medium term and long-term funding surplus goals, with the explicit inclusion of sponsor's contributions in the model proves very effective to recover a funding equilibrium for a DB pension fund. This framework is indeed at the grounds of an ongoing development of ALM methods and related tools for PF management.

Acknowledgements This research includes the formulation of a pension fund ALM problem reflecting a real-world case-study developed in cooperation between Allianz Investment Management and University of Bergamo. The presented numerical evidence has been modified and rescaled for confidentiality reasons but it does reflect actual operational conditions and the presented results remain valid.

Appendix

In Tables 13.2 and 13.3 we show the estimated statistical parameters adopted to generate correlated quarterly returns through a Monte Carlo simulation (Glasserman, 2003; Consigli et al., 2012a) for each benchmark i and node n. Once the return scenarios are aggregated in tree form, they are passed to an algebraic language deterministic model generator, to produce the stochastic program deterministic equivalent instance (Consigli and Dempster, 1998). The returns' statistics have been estimated on an historical window composed of 63 observations starting from the first quarter of 1999 till the third quarter of 2014.

Table 13.2 Average annual price (ρ) and income (ξ) returns of the entire asset universe. These parameters are estimated as the historical mean value of time series from January 1, 1999 to December 31, 2014. Data are in percentage

	ρ	ξ
EURIBOR 3m	2.41	
Treasury (T) 1–3y	−0.78	4.25
Treasury (T) 3–5y	0.13	4.38
Treasury (T) 5–7y	0.75	4.56
Treasury (T) 7–10y	1.35	4.28
Treasury (T) 10+y	1.73	4.87
Secururitized (S)	0.70	4.16
Corporate Investment Grade (IG)	0.23	4.78
Corporate High Yield (HY)	−0.93	8.05
Real Estate (R-E)	6.72	3.24
Equity (E)	6.00	3.67
TIPS 3–5y	1.53	1.74
TIPS 10+y	3.29	2.52

Table 13.3 Estimated variance-covariance matrix of the price annual returns with historical data from January 1, 1999 to December 31, 2014 of the asset universe: EURIBOR 3m (E-3m), Treasury (T), Securitized (Sec), Corporate Investment Grade (IG) and High Yield (HY), Real Estate (R-E), Equity (Eq) and TIPS

	E-3m	T 1–3y	T 3–5y	T 5–7y	T 7–10y	T 10+y	Sec	IG	HY	R-E	Eq	TIPS 3–5y	TIPS 10 + y
E-3m	0.0002	0.5467	−0.0001	−0.0002	−0.0002	−0.0005	−0.0003	−0.0005	−0.0016	−0.0015	−0.0015	0.0001	−0.0004
T 1–3y	0.5467	0.0010	0.0018	0.0023	0.0026	0.0029	0.0016	0.0010	−0.0014	−0.0021	−0.0033	0.0009	0.0012
T 3–5y	−0.0001	0.0018	0.0036	0.0049	0.0059	0.0073	0.0036	0.0024	−0.0019	−0.0018	−0.0054	0.0018	0.0039
T 5–7y	−0.0002	0.0023	0.0049	0.0069	0.0086	0.0111	0.0052	0.0035	−0.0021	−0.0011	−0.0061	0.0025	0.0067
T 7–10y	−0.0002	0.0026	0.0059	0.0086	0.0111	0.0148	0.0065	0.0044	−0.0027	0.0002	−0.0066	0.0030	0.0093
T 10+y	−0.0005	0.0029	0.0073	0.0111	0.0148	0.0221	0.0087	0.0062	−0.0018	0.0049	−0.0028	0.0031	0.0145
Sec	−0.0003	0.0016	0.0036	0.0052	0.0065	0.0087	0.0046	0.0036	0.0031	0.0034	0.0013	0.0022	0.0068
IG	−0.0005	0.0010	0.0024	0.0035	0.0044	0.0062	0.0036	0.0043	0.0110	0.0073	0.0065	0.0022	0.0072
HY	−0.0016	−0.0014	−0.0019	−0.0021	−0.0027	−0.0018	0.0031	0.0110	0.0717	0.0456	0.0595	0.0040	0.0172
R-E	−0.0015	−0.0021	−0.0018	−0.0011	0.0002	0.0049	0.0034	0.0073	0.0456	0.0653	0.0614	0.0012	0.0171
Eq	−0.0015	−0.0033	−0.0054	−0.0061	−0.0066	−0.0028	0.0013	0.0065	0.0595	0.0614	0.0939	0.0001	0.0151
TIPS 3–5y	0.0001	0.0009	0.0018	0.0025	0.0030	0.0031	0.0022	0.0022	0.0040	0.0012	0.0001	0.0027	0.0051
TIPS 10+y	−0.0004	0.0012	0.0039	0.0067	0.0093	0.0145	0.0068	0.0072	0.0172	0.0171	0.0151	0.0051	0.0218

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