# Fuzzy Spatio-Temporal Querying the PostgreSQL/PostGIS Database for Multiple Criteria Decision Making

**Renata Ďuračiová and Jana Faixová Chalachanová**

**Abstract** Multiple criteria decision making usually means selection of the best objects or their parameters which best meet conditions or criteria. Human decision making often involves also uncertain and vague criteria. The terms as a short distance, a high building or long time are commonly used in human speech. Nevertheless, it is not necessary to describe them exactly. In case of decision making based on lot of data and multiple criteria, data are usually stored in databases and a query language is used for handling and querying them. In this case, the criteria should be defined in a formal computer language. The standard for querying data in relational databases is the Structured Query Language (SQL). All objects and phenomena, and everything people do, is experience in space and time. Space and time, therefore, should be used as a framework for querying and reasoning about information stored in database systems. Databases, in which spatial and temporal data types are included and functions for their handling are supported, are called the spatio-temporal databases. To spatio-temporal query a database, an extension of standard SQL to support spatial and temporal data is needed. Uncertain spatio-temporal queries are not yet standardized and they are a current topic of research. For uncertain queries creation and expression of uncertain decision criteria, development of new methods and techniques is required. One of the most widely used approach to model, analyse and process uncertain and vague data is fuzzy set theory. Therefore, in this paper, we propose a new way of application fuzzy set theory to spatio-temporal querying. For the case study realisation, the open source PostgreSQL database system extended by the PostGIS we have used. The implementation of the proposed principles of fuzzy set theory to spatio-temporal querying databases can bring an opportunity for the efficient decision making based on multiple uncertain criteria.

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## Introduction

Modelling of uncertainty of spatio-temporal data is not a new topic of research, but approaches and tools for creation of uncertain spatio-temporal queries to databases are still not sufficiently supported. For data querying and manipulation in relational and object-relational databases, the Structured Query Language (SQL) is standard (International Organization for Standardization (ISO) standard ISO 9075-1:2011 Information technology—Database languages - SQL also known as ISO SQL:2011 Standard). ISO SQL:2011 Standard includes, inter alia, support for temporal data types. The spatial data types and standard functions for spatial data analysis are included in both the Open Geospatial Consortium (OGC) specification Simple Feature Access (SFA) and standard ISO 19125 (ISO 19125-1:2004 Geographic information—Simple feature access—Part 1: Common architecture) and ISO 19125-2:2004 Geographic information—Simple feature access—Part 2: SQL option). The handling of uncertain data in relational and object-relational databases is not yet standardized, and is presented only as the subject of research in several papers. The some concepts and first results of uncertain data modelling and querying are presented, for instance, in papers (Buckles and Petry 1982; Kacprzcyk et al. 2010; Hudec 2009; Caha et al. 2014b; Ďuračiová 2014; Carniel et al. 2016). Those papers introduce possibilities to extend SQL for the processing of uncertain data and vague queries in databases, but they mostly do not include spatio-temporal data. Therefore, the aim of this paper is to introduce a simple way for uncertain spatio-temporal data querying to object-relational database, which can be useful in multiple criteria decision making (MCDM) based on uncertain thematic, spatial or temporal criteria.

For uncertainty modelling, fuzzy set theory (Zadeh 1965) we use. Much work has been focused on an application of the fuzzy set approaches to modelling of spatial or temporal data uncertainty (Burrough and Frank 1996; Bosc et al. 2005). Fuzzy sets have been used, for example, for the definition of fuzzy spatial data types (Schneider 1999; Jiang and Eastman 2000; Schneider 2008; Carniel et al. 2016); fuzzy spatial topology (Tang et al. 2006); and also fuzzy spatial decision making (Malczewski 1999; Morris and Jankowski 2005; Petry et al. 2005; Changa et al. 2008; Caha et al. 2014a). In spatio-temporal data processing, fuzzy techniques have been used to measure uncertainties about geographical data and decision rules (Robinson 1988), or to represent imprecise concepts (Malczewski 1999). Soft temporal data have been described, for example, in Christakos et al. (2001), Deng et al. (2008). Fuzzy classes, domains for imprecise vague data, and fuzzy equality operators are defined in Cuevas et al. (2008), semantic interpretation of fuzzy spatial queries is described in Wang (1994, 2000), Cuevas et al. (2008), and some theoretical aspects of flexible querying spatio-temporal data are presented in De Caluwe et al. (2004a, b), Sözer et al. (2008), Emrich et al. (2012). Furthermore, several approaches dedicated to modelling and handling spatio-temporal data uncertainty have been developed (De Caluwe et al. 2004a), but their practical applications are still not sufficient.

Therefore, in this paper, we introduce a new approach to data querying to object-relational database, based on uncertain spatial and temporal criteria. For data storing and handling, we use the most widely used open source database system PostgreSQL extended with the PostGIS spatial data support. The result is a simple way for formulation of uncertain spatio-temporal queries without having to install special programs and other software tools to the PostgreSQL/PostGIS database system.

## Materials and Methods

Everything people do, is experience in space and time. Space and time, therefore, constitute conditions for the existence of all objects and phenomena. As such they are used as a framework for querying and reasoning about information kept in databases (De Caluwe et al. 2004b), which can be used as the basis for MCDM. State of each object in space and time is given by its thematic attribute values, which can be represented as the third dimension of decision making (Fig. 1 left). All these dimensions may be inherently affected by uncertainty (Fig. 1 right).

Therefore, for fuzzy spatio-temporal querying to a database, we need to define multiple criteria SQL queries to relational databases, but also extend common possibilities of SQL to support handling of temporal data, spatial data, and also modelling of uncertain decision criteria.

### *Multiple Criteria Queries to Relational Databases*

Spatio-temporal MCDM usually means selection of the best location, appropriate objects or their parameters with the best fulfilment of conditions for a future action.

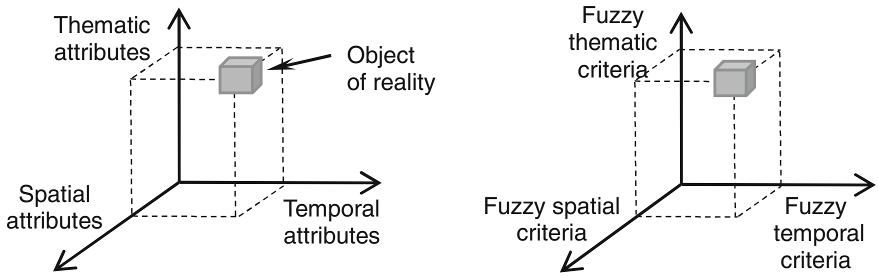



**Fig. 1** Object of reality in the space time (*left*) and spatio-temporal decision criteria (*right*)

For MCDM based on data that are stored in relational or object-relational databases, SQL is commonly used.

The basic syntax of command SELECT, which is used for data selection from a database, is as follows:

```
SELECT List_of_attributes
FROM List_of_tables
WHERE Conditions;
```

If the criteria for decision making are given, SQL query contains usually their aggregation realised by logical operator AND (Fig. 2).

The SQL query is then defined as follows:

```
SELECT List_of_attributes
FROM List_of_tables
WHERE c_1 AND c_2 AND … AND c_n;
```

where ***c_1***, ***c_2***, … ***c_n*** are criteria defined for MCDM.

## *Storing and Querying Data in Spatio-Temporal Databases*

A temporal database is a database that includes querying and handling data involving time and supports a temporal data model and a temporal version of SQL (ISO 9075:2011). Temporal table have usually two fields added, *valid_from* and *valid_to* (Fig. 3). There are three basic temporal data types implemented in object-relational databases: DATE, TIME, and TIMESTAMP. The duration of a period of time can be represented by a value of an interval type (ISO 9075-1:2011). There are also two classes of intervals defined in ISO SQL:2011 Standard: year-month intervals, and day-time intervals. Basic syntax of temporal query in accordance with ISO SQL:2011 Standard is:

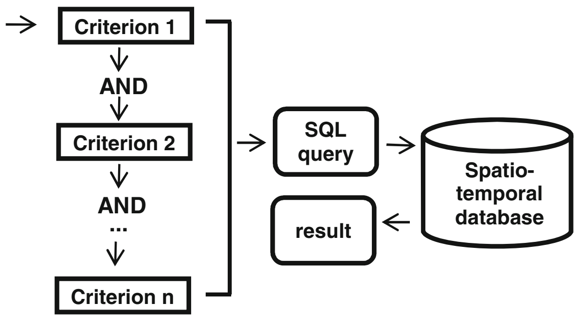



**Fig. 2** Aggregation of criteria in multiple criteria decision making

```
SELECT List_of_attributes
FROM List_of_tables
AS OF SYSTEM TIME Datetime
WHERE Conditions
```

A spatial database is a database that is optimized to store and query spatial data in spatial tables. The spatial table is a table with spatial (geometry or geography) column (the_geom). This column can be added to conventional tables stored in object-relational databases. Basic spatial data types and spatial functions for querying and analysis of spatial data stored in object-relational databases are included in standard ISO 19125 or OGC specification SFA. One of the basic spatial function for MCDM is ST_Distance, which is very useful for decision making based on spatial properties of objects. Simple spatial SQL query using function ST_Distance can be created, for example, as follows:

```
SELECT List_of_attributes
FROM Table 1 T1, Table 2 T2
WHERE ST_Distance(T1.the_geom,T2.the_geom) [< |<= |=| >=| >] k;
```

where the_geom is spatial attribute (e.g. data type POINT or POLYGON) and k is a constant.

Then the basic structure of conventional spatio-temporal table can include some conventional thematic attributes, two temporal attributes and the spatial attribute (Fig. 3).

## *Modelling of Uncertain Queries Using Fuzzy Set Theory*

Two people can use a different vocabulary or use the same words in different ways. That means that also definitions of criteria or query in common language is very

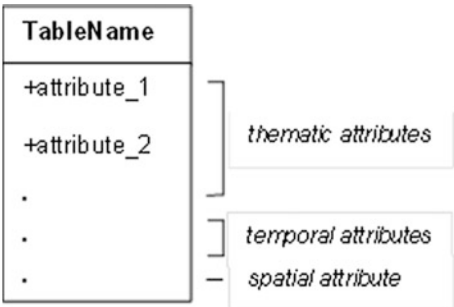



**Fig. 3** The basic structure of spatio-temporal table

often affected by uncertainty. This type of semantic uncertainty can be very well treated by fuzzy set theory (Zadeh 1965; Shekhar and Xiong 2008). It is also important to note that fuzzy set theory does not work with the concepts of randomness and probability, but expresses uncertainty in terms of fuzziness or vagueness (Kosko 1990).

Fuzzy set theory was first introduced by Zadeh (1965): "Let $X$ be a space of points (objects), with a generic element of $X$ denoted by $x$. Thus, $X = \{x\}$. A fuzzy set (class) $A$ in $X$ is characterized by a membership (characteristic) function $\mu_A(x)$ which associates with each point in $X$ a real number in the interval [0,1], with the value of $\mu_A(x)$ at $x$ representing the "grade of membership" of $x$ in $A$."

Because of simple implementation, the linear membership functions are the most widely used. For example, the most known trapezoidal membership function $\mu_{A'}(x)$ of the fuzzy set $A' = \{(x,\ \mu_A(x));\ x \in X\}$ is defined as follows:

$$\mu_{A'}(x) = \begin{cases} 0 & \text{if } x < a, \\ \frac{x-a}{b-a} & \text{if } a \le x < b, \\ 1 & \text{if } b \le x \le c, \\ \frac{d-x}{d-c} & \text{if } c < x \le d, \\ 0 & x > d \end{cases} \tag{1}$$

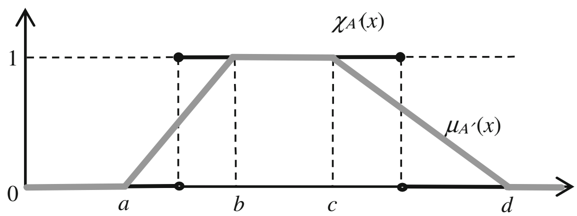
The trapezoidal membership function $\mu_{A'}(x)$ is shown in Fig. 4, where $\chi_A(x)$ represents the characteristic function of the crisp set $A$:

$$\chi_A(x) = \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{if } x \notin A. \end{cases} \tag{2}$$

In general, the characteristic function takes a value of either 1 or 0 (Fig. 4), while a range of values of fuzzy membership function is interval [0,1] (higher values of fuzzy membership function indicates higher degrees of set membership) (Klir and Yuan 1995).

Other commonly used membership functions include the triangular membership function, which is the special case of the trapezoidal function for $c = d$, the S-shaped membership function, L-shaped membership function, etc. (Ross 2010). Especially for a more reliable description of uncertainty in measurement of spatial objects (Kreinovich et al. 1992), the Gaussian membership function

**Fig. 4** The crisp set ($A$) versus the trapezoidal fuzzy set ($A'$)



$$\mu_A(x) = e^{\frac{-(x-c)^2}{2\sigma^2}} \tag{3}$$

can be used ($\sigma^2$ is the variance and $c$ the expected value).

The logics used to build the expressions, which make up the core of the queries, can be the traditional binary (Boolean) logic, a ternary logic (Kleene's logic), a four value logic (Belnap's logic), multivalue logic, or general one (fuzzy logic) (De Caluwe et al. 2004b). There are many ways of extending the basic crisp set operations to fuzzy sets (Schneider 1999) and Boolean logic to fuzzy logic. In general, the operations of fuzzy conjunction (AND), fuzzy disjunction (OR), and fuzzy complement (NOT) are, in accordance with basic concept of fuzzy set theory, represented as generalizations of crisp ones.

If we comply with the definitions presented in Zadeh (1965) and *A'* and *B'* are fuzzy sets, then

$$A' \cap B' = \{(x, \mu_{A' \cap B'}(x)) | x \in X \wedge \mu_{A' \cap B'}(x) = \min(\mu_{A'}(x), \mu_{B'}(x))\} \tag{4}$$

$$A' \cup B' = \{(x, \mu_{A' \cup B'}(x)) | x \in X \wedge \mu_{A' \cup B'}(x) = \max(\mu_{A'}(x), \mu_{B'}(x))\} \tag{5}$$

$$\neg A' = \{(x, \mu_{\neg A'}(x)) | x \in X \wedge \mu_{\neg A'}(x) = 1 - (\mu_{A'}(x))\} \tag{6}$$

In fuzzy set theory and fuzzy logic, intersection (conjunction) and union (disjunction) are generally expressed by triangular norms (t-norms) and triangular conorms (t-conorms), respectively (Grabisch 2009).

## *Creation of Multiple Criteria Fuzzy Spatio-Temporal SQL Queries*

Based on the syntax of the SELECT command in SQL and basic principles of fuzzy set theory, the fuzzy SQL query for MCDM could be defined in the following way:

```
SELECT List_of_attributes
FROM List_of_tables
WHERE fuzzy_c_1 Fuzzy_AND fuzzy_c_2 Fuzzy_AND … Fuzzy_AND fuzzy_c_n;
```

where ***fuzzy_c_1***, ***fuzzy_c_2***,… ***fuzzy_c_n*** are uncertain criteria and Fuzzy_AND is fuzzy conjuction.

To express the fulfilment of i-th fuzzy_c_i criterion, for example, by the trapezoidal membership function (1), we can use SQL command CASE (conditional expression):

```
(CASE
  WHEN x>=a AND x<b THEN (x-a)/b-a)
  WHEN x>=b AND x<=c THEN 1
  WHEN x>=c AND x<d THEN (d-x)/d-c)
  ELSE 0 END) AS mi_i
```

where `x` is variable value of the fuzzy set $A'$, `a`, `b`, `c`, `d` are parameters of the trapezoidal membership function, and `mi_i` is a degree of membership of a feature in the fuzzy set $A'$.

As an effective alternative, the degree of membership in the fuzzy set $A'$ can be expressed using the functions LEAST (returns the least value in a list of expressions) and GREATEST (returns the greatest value in a list of expressions), if they are implemented in the used database management system (DBMS) (Ďuračiová 2014):

```
(LEAST(GREATEST(LEAST(((x-a)/(b-a)),((d-x)/(d-c))),0),1)) AS mi_i
```

This function can be also used for both spatial and temporal criteria modelling.

As a fuzzy logical operator AND, standard conjunction (also known as the minimum t-norm) (3) we use:

```
(LEAST(mi_1,mi_2,… mi_n))
```

## *Implementation of the Basic Principles of Multiple Criteria Fuzzy Spatio-Temporal SQL Queries into the PostgreSQL/PostGIS Database System*

In this paper, we propose a simple way of implementation of fuzzy set theory into SQL querying by defining several new fuzzy functions. For implementation of the fuzzy spatial functions and fuzzy spatial queries realisation, we use the open source PostgreSQL/PostGIS database system. First, we suggest to define the following basic fuzzy membership functions, which can be used for universal processing of fuzzy thematic, spatial, and temporal criteria:

```
Fuzzy_Tpz(a,b,c,d,x) – the trapezoidal fuzzy membership function,
Fuzzy_Trg(a,b,d,x) – the triangular fuzzy membership function,
Fuzzy_S(a,b,x) – the S-shape fuzzy membership function,
Fuzzy_L(c,d,x) – the L-shape fuzzy membership function,
Fuzzy_Gs(c,sig,x) – the Gaussian fuzzy membership function.
```

Using procedural language PL/pgSQL in the PostgreSQL DBMS, the `Fuzzy_Tpz` function can be written in the following way:

```
CREATE OR REPLACE FUNCTION public."Fuzzy_Tpz"(
    a real,
    b real,
    c real,
    d real,
    x real)
  RETURNS real AS
$BODY$BEGIN
    RETURN (MIN(MAX(MIN(((x-a)/(b-a)),((d-x)/(d-c))),0),1));
END;$BODY$
  LANGUAGE plpgsql VOLATILE
  COST 100;
ALTER FUNCTION public."Fuzzy_Tpz"(real, real, real, real, real)
  OWNER TO postgres;
```

function `Fuzzy_Gs is` can be created as follows:

```
CREATE OR REPLACE FUNCTION public."Fuzzy_Gs"(
    c real,
    sig real,
    x real)
  RETURNS real AS
 $BODY$BEGIN
    RETURN (exp((-(x - c)^2)/(2 * sig^2)));
END;$BODY$
  LANGUAGE plpgsql VOLATILE
  COST 100;
ALTER FUNCTION public."Fuzzy_Gs"(real, real, real)
  OWNER TO postgres;
COMMENT ON FUNCTION public."Fuzzy_Gs"
(real, real, real) IS 'Gaussian fuzzy
membership function';
```

where `sig` is the variance and `c` the expected value.

All other types of fuzzy membership functions (L-shaped, S-shaped, and triangular) can be represented in a similar manner.

The above functions can be used directly in MCDM, but also in the definition of the more specific spatial and temporal fuzzy functions. For expression of fuzzy spatial criteria in multiple criteria queries, for instance, the following basic fuzzy spatial function can be created and subsequently used (Fig. 5):

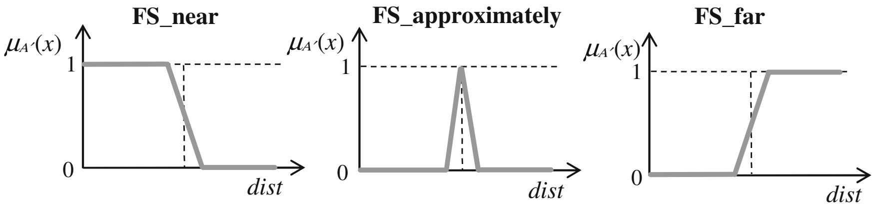



**Fig. 5** Fuzzy spatial database functions: fuzzy spatial near (*left*), fuzzy spatial approximately (*middle*), fuzzy spatial far (*right*)

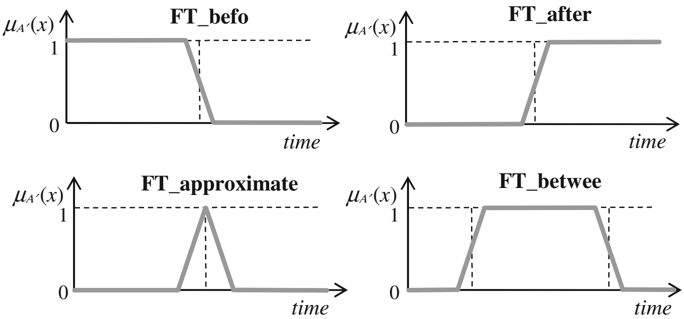



**Fig. 6** Fuzzy temporal database functions: fuzzy temporal before (*top left*), fuzzy temporal after (*top right*), fuzzy temporal approximately (*bottom left*), fuzzy temporal between (*bottom right*)

- `FS_near(c,d,dist)`—fuzzy spatial near,
- `FS_approximately(a,b,d,dist)`—fuzzy spatial approximately,
- `FS_far(a,b,dist)`—fuzzy spatial far,

where `a`, `b`, `c`, `d` are parameters of the piecewise linear fuzzy membership function and `dist` is a distance of the selected spatial object from the spatial object given by decision criteria. To compute the distance in the fuzzy spatial criteria, we apply the spatial function ST_Distance().

To create the SQL queries with uncertain temporal criteria, we propose to implement and use the special fuzzy temporal functions such as (Fig. 6):

- `FT_before(c,d,time)`—fuzzy temporal before,
- `FT_after(a,b,time)`—fuzzy temporal after,
- `FT_approximately(a,b,d,time)`—fuzzy temporal approximately,
- `FT_between(a,b,c,d,time)`—fuzzy temporal between,

where time is a temporal information expressed as number.

For processing fuzzy temporal criteria, we use, for example, the temporal attribute `valid_from` with DATETIME data type and the temporal function `age (TIMESTAMP)` implemented in the PostgreSQL DBMS or the computed attribute `today - valid_from`. For extraction subfield count of years from interval, we can use the extract function: `EXTRACT(`***`field`*** `FROM` ***`source`***`)`. The extract function returns values of type double precision, which can be used as argument in the fuzzy temporal functions.

All the above functions can be implemented in the database system in the above mentioned form and consequently used to determine the degree of membership to the corresponding fuzzy set. In this way, we can express, for example, uncertain temporal or spatial criteria such as: "*far from the city centre*", "*about 500 m from the bus station*", "*about a year ago*", "*approximately in June*", etc.

It is important to note, that all thematic, spatial and temporal criteria could be expressed by the basic fuzzy function such as `Fuzzy_Tpz`, `Fuzzy_Trg`, `Fuzzy_L`, `Fuzzy_S`, and `Fuzzy_Gs`, which are defined above. For example, the `Fuzzy_L` function, which is designed for modelling the L-membership function, can be used for expression the `FS_near` and the `FT_before` functions; the `Fuzzy_S` function can be applied in definition of the functions `FS_far` and `FT_after`; the triangular `Fuzzy_Trg` function can be used as the `FS_approximately` function for spatial criteria or `FT_approximately` function for temporal criteria; and the `Fuzzy_Tpz` function, which is designed for modelling the fuzzy trapezoidal membership function, can be used for the modelling of the `FT_between` function.

Due to the completeness of the solution, it is also necessary to create functions for fuzzy aggregation operators. For example, the standard fuzzy logical operator AND, which is the most important operator in the process of MCDM, can be implemented as follows:

```
CREATE OR REPLACE FUNCTION public."Fuzzy_AND_Min" (
  m1 real,
  m2 real)
 RETURNS real AS
$BODY$BEGIN
  RETURN least(m1,m2);
END;$BODY$
  LANGUAGE plpgsql VOLATILE
 COST 100;
ALTER FUNCTION "Fuzzy_AND_Min"(real, real)
 OWNER TO postgres;
```

To implement other t-norms and t-conorms, the function body can be created, for example, according to Ďuračiová (2014). Another way is to use the procedural language PL/pgSQL to define the function body. For example, the function body of the drastic t-norm `Fuzzy_AND_Drs` can be written as follows:

```
BEGIN
   IF m1 = 1 THEN RETURN m2;
   ELSIF m2 = 1 THEN RETURN m1;
   ELSE RETURN 0;
   END IF;
END;
```

If the above described functions are implemented in a database system, they can help to formulate most of the fuzzy spatio-temporal queries much more efficiently. If needed, the function body of any other less known operation, such as the nilpotent minimum:

$$T_{nM}(a,b) = \begin{cases} \min(a,b) & \text{if} \quad a+b>1. \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

can be whenever created. For example, based on the above formula, the new `Fuzzy_AND_MNi` function can be written in PL/pgSQL as follows:

```
BEGIN
   IF m1 + m2 > 1 THEN RETURN least(m1,m2);
   ELSE RETURN 0;
   END IF;
END;
```

## Case Study

As a case study, spatio-temporal MCDM based on the ZB*GIS*® (the fundamental database for GIS in Slovakia) data we present. The database schema of the ZB*GIS*® is defined in accordance to the FACC DIGEST (Digital Geographic Information Exchange Standard) and the task is to find all large parking areas that are close to the sports field and were actualized approximately two years ago. Then the following fuzzy spatio-temporal query to object-relational database based on the ZB*GIS*® can be used as an example:

*Q: Select all large parking areas (from table Parking_areas) that are close to sports field (stored in the table Sports_field) and were actualized approximately two years ago.*

This spatio-temporal query includes three fuzzy terms: *large parking area*, *close to sport field* and *approximately two years ago*, which can be modelled using fuzzy sets. It is important to note that each fuzzy term can be expressed by the specific fuzzy membership functions dependent on the context of its use. For example, the above fuzzy terms can be modelled by the following fuzzy sets:

The fuzzy set $A$ ("*Large parking areas*"):

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < 600, \\ \frac{x-600}{400} & \text{if } 600 \le x \le 1000, \\ 1 & \text{if } x > 1000. \end{cases} \tag{8}$$

The fuzzy set $D$ ("*Parking areas close to the sports field*"):

$$\mu_D(x) = \begin{cases} 1 & \text{if } x < 150, \\ \frac{500-x}{350} & \text{if } 150 \le x \le 500, \\ 0 & \text{if } x > 500. \end{cases} \tag{9}$$

The fuzzy set $T$ ("*Parking areas actualised approximately two years ago*"):

$$\mu_T(x) = \begin{cases} 0 & \text{if } x < 18, \\ \frac{x-18}{4} & \text{if } 18 \le x < 22, \\ 1 & \text{if } 22 \le x \le 26, \\ \frac{30-x}{4} & \text{if } 26 < x \le 30, \\ 0 & x > 30 \end{cases} \tag{10}$$

Parameters of the fuzzy membership functions $A$, $D$, and $T$ are in square meters, meters, and in months, respectively. In solution that we suggest in this paper, the fuzzy membership function $A$ can be represented by the `Fuzzy_S` function using the spatial `ST_area` implemented in the PostgreSQL/PostGIS database system. The fuzzy membership functions $D$ and $T$ can be simply expressed by the `FS_near` function and the `FT_approximately` function, respectively.

Therefore, if we have two tables `Parking_areas(id, type, valid_from, valid_to, the_geom)` and `Sports_field(id, use, valid_from, valid_to, the_geom)`, the SQL command for the spatio-temporal query using fuzzy functions can be written as follows:

```
SELECT
 id,
 ST_Area(the_geom) AS a, "Fuzzy_S"(600,1000,a) AS mi_a,
 ST_Distance(P.the_geom,S.the_geom) AS d, "FS_near"(150,500,
d) AS mi_d,
 time AS t, "FT_approximately"(18,22,26,30,t) AS mi_t,
 "Fuzzy_AND_Min"(mi_a,mi_d,mi_t) AS mi
FROM Parking_Areas AS P, Sports_field AS S
WHERE mi > 0;
```

The result of the fuzzy spatio-temporal query can be created as a spatial table and can then be visualized in GIS software environment (e.g. QGIS or ArcGIS). The visualization of the result of the above query is presented in Fig. 7.

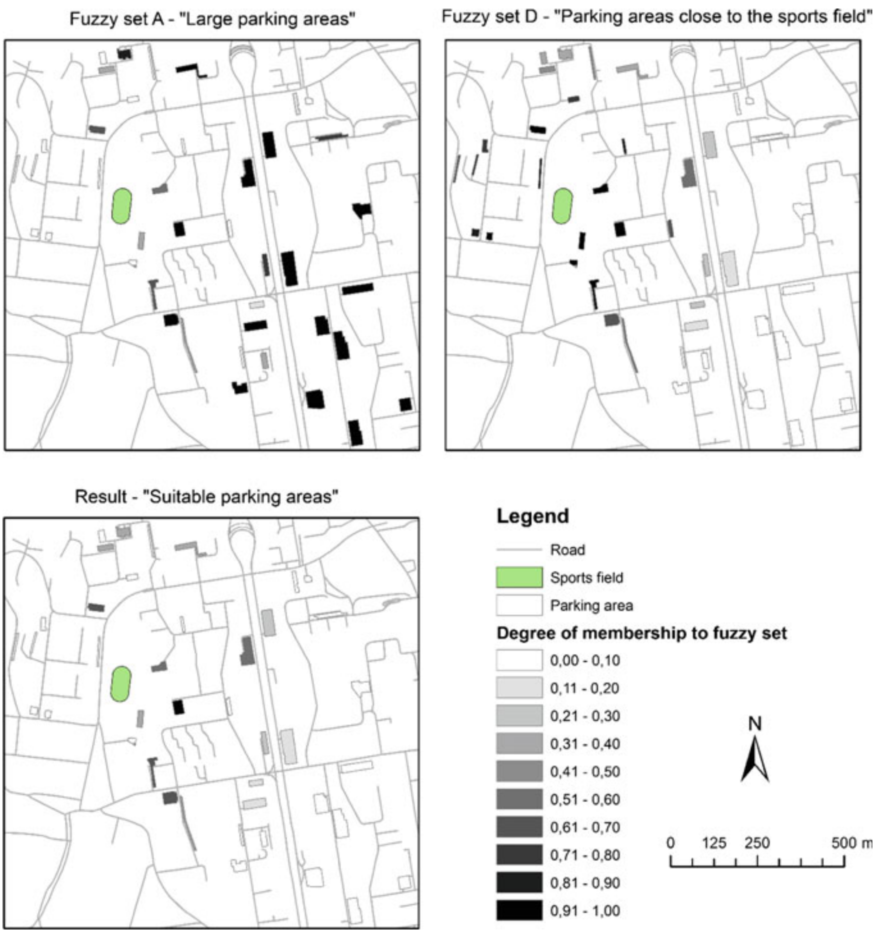



**Fig. 7** The visualization of the result of the fuzzy spatio-temporal query: degree of membership to the fuzzy set A (*top left*), degree of membership to the fuzzy set D (*top right*), degree of membership to the result fuzzy set "suitable parking areas" (*bottom left*)

## Conclusions and Discussion

The result of this paper is a proposal for implementation of the basic principles of fuzzy set theory into spatio-temporal querying object-relational databases. Fuzzy spatio-temporal querying enables both natural and effective data querying in the process of MCDM. The approach proposed in this paper brings an appropriate solution for spatial decision making based on the data stored in a database system. We have introduced the implementation of the fuzzy membership functions and the fuzzy logical operators AND into the PostgreSQL database system with the PostGIS extension. All implemented fuzzy spatial and fuzzy temporal functions can be used in common form of the SQL queries using the SELECT command. The main advantages of our proposal includes:

- spatial and temporal data handling,
- fast and simple implementation in open source software environment,
- independence from other software tools (it does not require installation and use of other special tools),
- applicability in solving various types of tasks, which use the process of MCDM based on uncertain thematic, spatial and temporal criteria.

The further improvement and the development of the proposed approach to uncertain spatio-temporal querying can focus on an additional development and implementation of other principles of fuzzy set theory into database systems. Then can be created a complex set of tools for fuzzy spatio-temporal data querying and analysis based on uncertain criteria.

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