# Chapter 11 Flipping Coins and Coding Turtles

# The Evolution of M&S in the Social Sciences

#### David C. Earnest and Erika Frydenlund

Abstract Nearly four decades ago, Thomas Schelling used coins and a checkerboard to simulate how simple social rules could produce stark neighborhood segregation. That early social science model marked the beginning of a movement to incorporate simulation into social science that continues to gain momentum today. Using political science and international studies as a frame of reference, this chapter explores the incomplete permeation of simulation into the statistical and qualitative research toolkits of those pursuing social inquiry. We begin by chronicling the development of several key advancements in modeling social systems, including formal modeling such as game theory, the adoption of statistical and computerbased modeling, and the advancement of computational social sciences using evolutionary computation and other dynamic modeling paradigms. Then, we discuss how and why simulation remains at the periphery of social science research methodologies. We compare a classic Prisoner's Dilemma model to one designed using an agent-based simulation approach to illustrate the population ecology of emergent strategies. The chapter concludes with a discussion on the ways simulation of social systems would have to evolve to have more impact on the field of social sciences.

Keywords Computational social science · Social simulation · History · Emergence · Empiricism · Behavioralism · Game theory · Computer-based simulation · System dynamics · Complex systems theory · Complexity · Sugarscape · Prisoner's dilemma · Social structures · Agent memory · Social science  $\cdot$  Validation  $\cdot$  Verification  $\cdot$  Participatory methods  $\cdot$  Model docking

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#### 11.1 Introduction

In his 1979 book Micromotives and Macrobehavior, social scientist Thomas Schelling introduces a rudimentary but important model of residential segregation. Using a checkerboard whose squares he randomly filled with pennies and dimes, Schelling ([1978\)](#page-21-0) moved each coin according to a simple rule: a penny or quarter would move to an available adjacent square if it was "unhappy"—that is, if the coin was a minority among the coins on the neighboring eight squares. Of course, as each coin moved to a new square, it may change the calculus of its neighbor's happiness, illustrating the interdependent choices of all the coins on the board. Nearly forty years old now, Schelling's pennies and dimes represent an early example of the synergy between game theory and agent-based modeling, albeit in a manual rather than computational form. By varying the threshold of a coin's unhappiness (for example, a coin is unhappy if fewer than 25% of adjacent squares are filled with like coins), Schelling's model produced an astonishing result. Even in a "society" of coins that would tolerate being in the minority in their own neighborhood, stark patterns of segregation emerged after a few rounds of movement of the coins. Now considered a classic, Shelling's segregation model illustrated an important point of social theory: individual rational choices (seeking like neighbors) may produce a collective outcome that no one intends (segregation). More generally, the model illustrates the perils of the ecological inference problem: attributing motives to individuals based on observation of macro-social dynamics. Beyond these important points of social theory, however, Schelling's simple model also can help us think about how quantitative and simulation methodologies have contributed to the social sciences.

In this chapter, we examine the role of modeling and simulation methodologies in the social sciences. Starting with a brief history of the social sciences, we trace the origins of contemporary methods to the emergence of the study of societies as a scientific endeavor in the nineteenth century to the twentieth century innovations in social statistics, game theory, and computational methods. This history documents the considerable contributions of formal, empirical and mathematical modeling to social theory in a variety of disciplines. To illustrate the emerging convergence of various simulation methodologies including game theory, agent-based modeling (ABM) and evolutionary computation, we provide a simulation of a classic social choice problem: the prisoner's dilemma. Our model illustrates how simulation may elaborate traditional games of social choice by extending the game to multiple players; by examining the roles of iteration and learning; and by experimentally varying the social structure in which players interact. Although merely illustrative, we argue the ABM demonstrates how modern computational methods help social scientists produce models of social phenomena that are richer, more generalizable, and more tractable than previous formal and statistical methods of social analysis. We conclude the chapter with some informed speculation about the future of modeling and simulation in the social sciences.

# 11.2 History of Quantitative Methods in the Social Sciences

Since their emergence as scientific fields in the 19th century, the social sciences have adopted various methods of quantitative analysis varying from systematic empirical inquiry in the nineteenth century to today's computational social simulation. As in other scientific fields, the social sciences often have benefitted from simultaneous innovations in complementary methodological fields, first statistics and then more recently low-cost computation. In this section, we provide a brief overview of this history by focusing on three broad paradigms of quantitative social scientific research: empiricism and behavioralism; mathematics and game theory; and computer-based simulation.

#### 11.2.1 Empiricism and Behavioralism

The social sciences first emerged in the 19th century when founders of the field of sociology, principally French scholar Émile Durkheim, proposed that observers of social life should bring to their subjects the methods of science (Durkheim and Lukes [1982](#page-19-0)). Durkheim advocated for an inductive approach to building social theory based on systematic observation and inquiry. The founders of social science argued that regularities exist in social life that researchers can discover, observe and measure. As in the natural and physical sciences, early social scientists espoused a rigor of methodology that would permit not only the verification or falsification of social theories but also the replication of results. Their emphasis on non-normative social sciences—theorization, observation and inference free from the value judgments of scholars—represented an important disjuncture from social theory's origins in the humanities. Although social philosophy and history remain important (normative) fields of inquiry in modern social scientific disciplines, the predominance of positivist epistemologies in sociology, anthropology, political science, economics and psychology attests to the enduring influence of the scientific method on social research.

The earliest quantitative methods in the social sciences were large-sample surveys, the data from which scholars would create and/or validate social theories. Durkheim's Suicide [\(1897](#page-19-0)) is an early example of this inductive, data-driven approach. Using data on suicides from different police districts, Durkheim hypothesized that stronger social "control" (norms and values) among Catholic communities explained their lower suicide rates when compared to Protestant communities. Although Durkheim's study subsequently faced considerable criticism, it typifies early quantitative methods in social science: it used systematically collected data to argue that properties of societies (social control) rather than individual or psychological factors can explain observed differences in societies. In this respect, early survey research proposed a social structural ontology that one can

observe and measure; that exists external to the individual; and that is fundamentally different than the simple aggregation of individual preferences or choices.

In the mid-twentieth century, however, some social scientists began to question the structural theories of social research. Originating in political science, behavioralism accepted the positivist methods of social research including verifiability, systematic measurement, replicability and non-normative theory. However, behavioralists proposed that social inquiry should re-focus on individuals rather than social practices and institutions. By examining how people process information, make decisions, and communicate with each other, behavioralism distinguished itself from the structural emphases that characterized the social sciences in the first half of the twentieth century. In addition to the discipline of political science, psychology also adopted behavioral methods. More generally, the behavioral sciences differ from the social sciences in their greater emphasis on observation of, measurement of, and theorization based on individuals rather than the properties of social groups. Although behavioralism and social structuralism differed in ontologies, theories, and hypotheses, both benefitted from innovations that marked the emergence of statistics as a discipline distinct from mathematics. In particular, sociologist Paul Lazarsfeld, who founded Columbia University's Bureau of Applied Social Research, pioneered the statistical analysis of survey data and latent class analysis (Clogg [1992](#page-19-0)). In general, during the mid-twentieth century social scientists increasingly combined the systematized empirical methods of early social research with the inferential methods of statistics (Blalock [1974](#page-18-0)).

## 11.2.2 Game Theory

Just as behavioralism was a reaction to the structuralism of early social research, so was game theory a reaction to the inductive empiricism of most social sciences. Game theory is a set of formal (mathematical) methods for understanding bargaining, conflict, and cooperation among rational actors who have interdependent "payoffs" or rewards for their choices. Rather than constructing social theory through induction based on empirical observation of regularities in social actors, game theory proposes to deduce social behaviors from formal representations of actor choices, incentives, and rewards. Game theorists propose that social choices involve uncertain outcomes ("gambles"); interdependent rewards (payoffs); and variable amounts of uncertainty about these choices. By representing gambles, payoffs, and information in mathematical equations, game theory proposes that social scientists can deduce likely choices of individuals and, by extension, the prospects for social cooperation or conflict.

Most historians of the social sciences date the origins of modern game theory to discussions in the 1940s between mathematicians Oskar Morgenstern and John von Neumann, both of whom worked at Princeton University's Institute for Advanced Studies. O'Rand ([1992](#page-20-0)) argues that the historical context helps understand both the motivation for a deductive social science, and its evolution from an obscure branch

to a widely practiced methodology in the social sciences. During and following World War II, federally sponsored research in the United States emphasized problems of war and peace as well as industrialization and scientific management (which found expression as operations research). In addition, tumult in Europe led to the migration of a considerable number of scientists to the United States, including Morgenstern (an Austrian who was visiting Princeton when Nazi Germany invaded Austria) and von Neumann (a Hungarian who left a position in Germany for Princeton in 1930). In 1944, von Neumann and Morgenstern published The Theory of Games and Economic Behavior which introduced the concept of expected utility, or the subjective valuation that a rational actor attributes to a choice characterized by uncertainty (Von Neumann and Morgenstern [1944](#page-21-0)). Von Neumann–Morgenstern utility theory remains the foundation of contemporary game theory. Although scientists at the Institute for Advanced Study engaged in productive exchange and collaboration, O'Rand argues that the relatively insular community (and another at the University of Michigan) shared ideas and innovations through informal and social communications more than through scholarly publishing (O'Rand [1992](#page-20-0)). For this reason, adoption of game theory in the social sciences proceeded relatively slowly, only finding a broader audience in the late 1950s.

Among the challenges of early game theory was how to model players' knowledge about a game's structure of payoffs and its history of play. Although early models of games assumed that players would make simultaneous choices, the innovation of sequential play games required modelers to explicate whether players know the history of prior plays (what they call "perfect" information) or the strategies and payoffs available to other players ("complete" information) (Gibbons [1992\)](#page-19-0). Another Hungarian-born mathematician, John Harsanyi, made substantial contributions to the study of games of incomplete information (Harsanyi [1967\)](#page-19-0). Together with von Neumann and Morgenstern, Harsanyi's contributions form the foundation of modern game theory. Their deductive methods allow researchers to model "static" games of simultaneous play; dynamic games of sequential play; with perfect, imperfect, complete and incomplete information.

Whereas in the 1950s game theory remained the province of mathematically inclined social scientists, two more recent works contributed to the broad adoption of the methodology in all social scientific disciplines. The first was Thomas Schelling's *The Strategy of Conflict* ([1960\)](#page-20-0), which relaxed the assumption of von Neumann–Morgenstern utility theory to hypothesize that irrational actors and credible threats to cheat could alter equilibrium solutions to games. In this respect, Schelling brought to game theorists important discussions about player motives including fear, honor, and myopic perception. The other important contribution arises from a series of studies conducted by political scientist Robert Axelrod on the prisoner's dilemma, one of the canonical static games of complete information in which players face strong incentives to eschew cooperation ("defect" in the argot of game theory). Axelrod's ([1980\)](#page-18-0) research, including a tournament in which he invited fellow scientists to propose optimal strategies for the prisoner's dilemma, culminated in the publication of The Evolution of Cooperation ([1984](#page-17-0)), which illustrated that repeated plays of the prisoner's dilemma allows players to learn cooperative strategies that improve their long-term payoffs. This finding has informed not only a vast subsequent literature on how social institutions produce cooperation but has also informed policy-making in diverse fields.

O'Rand notes that while at the Institute for Advanced Study, Morgenstern was "a pariah among the traditional economists on the faculty at Princeton" (O'Rand [1992:](#page-20-0) 184–85). In the half decade since its wide adoption in the social sciences, several game theorists have received the Nobel Prize in Economics including Harsanyi, John Nash and Reinhard Selten (in 1994); Robert Lucas (in 1995); and Schelling (in 2005). The recognition of these scholars attests to the widespread influence of game theory in the social sciences today. Along with statistics, it remains a foundational methodology in most graduate curricula in the social sciences.

#### 11.3 Computer-Based Simulation

In the 1990s, computer-based simulation for social science research questions began to grow in popularity as a research method. The 1990s ended with the establishment of the Journal of Artificial Societies and Social Simulation (JASSS) in 1998. This move paved the way for broader acceptance of M&S in the social sciences by serving as a platform for interdisciplinary research. Two of the main modeling paradigms, system dynamics and agent-based modeling, are discussed below with primary emphasis on ABM.

#### 11.3.1 System Dynamics

Economists paved the way for system dynamics modeling in the social sciences since the 1970s, with computational models giving way to system dynamics (SD) models of global socio-political and economic interactions (Chadwick [2000\)](#page-18-0). The International Futures model, begun in the 1980s, links data on countries grouped by region and evaluates factors such as economies, demographics, and food and energy systems for policy analysis (Hughes [1999](#page-19-0)). In that era of advancing computer-based modeling technologies, anthropological research utilizing system dynamics approaches also appeared (Picardi [1976\)](#page-20-0), but with less influence than the economic models. Early on, some even proposed that SD models could serve as learning tools for articulating processes informed by qualitative data in the social sciences (Wolstenholme and Coyle [1983\)](#page-21-0) and the study of history (Bianchi [1989\)](#page-18-0). The purposes of these models, as of the late 1990s, was largely to plan or conceptualize complex processes rather than to predict or test hypotheses (Chadwick [2000\)](#page-18-0).

Starting in the early 2000s, more social phenomena become the subject of SD models. van Dijkum et al. [\(2002](#page-21-0)) introduced a system dynamics model to look at individual learning and fatigue based on survey data for health psychology research. They contend that since social scientists are familiar with evaluating phenomena using cause and effects models, and system dynamics models provide a natural way to translate these traditional research approaches into mathematical models that can accommodate both quantitative and qualitative data. Continuing with the movement toward modeling socio-economic phenomena, more contemporary models include SD approaches to understanding diffusion of democracy (Sandberg [2011\)](#page-20-0), land use among agrarian societies (Rasmussen et al. [2012\)](#page-20-0), housing markets (Ouml et al. [2014](#page-20-0)), and refugee migration (Vernon-Bido et al. [2017\)](#page-21-0). System dynamics, while still not widely used by many social scientists, is a fruitful area for modeling complex processes.

### 11.3.2 Agent-Based Modeling

Agent-based modeling (ABM), however, is particularly well-suited to the field of social sciences and has experience wider acceptance than system dynamics paradigms. The accessibility of object-oriented programming languages and user-friendly environments has created a community of researchers—students and faculty alike—who embrace ABM as a method for exploring the physical and social sciences (Lerner et al. [2010](#page-19-0)). From Schelling's [\(1978\)](#page-21-0) model of housing segregation based originally on a physical checkboard model and later in a computer algorithm, the vast variety of agent-based models is reflected in modern repositories like OpenABM (<https://www.openabm.org>) and the NetLogo Models Library and Modeling Commons [\(http://modelingcommonns.org](http://modelingcommonns.org)).

Axelrod [\(1986](#page-18-0)) led the forefront of the social science ABM movement when he simulated the emergence of behavioral norms and firmly grounding ABM in the fields of sociology and political science. In the 1990s, Epstein and Axtell [\(1996](#page-19-0)) developed the Sugarscape model where simple behavior rules about agents consuming resources resulted in emergence of behaviors representing those that evolve in society. There were even models constructed to inform policy and practice in real-world settings in the early years of social science ABM (Doran [2001\)](#page-19-0). Social scientists began steadily to contribute to the growing field of ABM as they were drawn to the ability to construct heterogeneous, autonomous agents in boundedly rational space, rather than reconstruct social realities based on traditional mathematical or statistical models (Gilbert and Troitzsch [2005;](#page-19-0) Epstein [2006;](#page-19-0) Gilbert [2008;](#page-19-0) Macy and Willer [2002\)](#page-20-0).

Throughout the decades that followed this initial work, social scientists constructed agent-based models to explore and test theories on identity and norm creation (Lustick [2000;](#page-20-0) Rousseau and van der Veen [2005](#page-20-0)), ethnic violence and conflict (Bhavnani and Backer [2000](#page-18-0); Miodownik and Cartrite [2010;](#page-20-0) Yamamoto [2015\)](#page-21-0), and political processes (Lustick [2011](#page-20-0); Epstein [2013](#page-19-0); Bhavnani [2003](#page-18-0); Castro [1999\)](#page-18-0), among countless other social phenomena. Agar [\(1997a](#page-18-0)) went on to propose that ABM presents an opportunity to move beyond the dichotomy of deductive versus inductive approaches. That argument continues on in the development of ABM for educational purposes as a way to more rigorously analyze physical and social phenomena (Jacobson and Wilensky [2006](#page-19-0); Wilensky and Resnick [1999](#page-21-0)).

### 11.4 Complex Systems Theory

Various modeling paradigms have begun to infiltrate the traditional statistical and qualitative methods of social sciences, but Complex Systems Theory unified many of these pursuits across disciplines. Weaver [\(1948](#page-21-0)) proposed that mathematical and statistical advances, while important, were not yet powerful enough to capture the complexity of many physical and social systems. He proposed a move toward exploring what cannot be quantified by traditional scientific methods, thus opening the door for complex systems thinking. In a modeling context, Wolfram [\(1985](#page-21-0)) used cellular automata models to demonstrate regularities that underlie complexity. He began using computer simulations and experiments to test the boundaries of complexity, try to understand where simple rules result in complex outcomes, and witness the edges of chaos. Scholars adapted these ideas to other fields, including international relations where the concepts are applied to the complex adaptive systems of political processes (Cederman [1997;](#page-18-0) Bousquet and Geyer [2011;](#page-18-0) Earnest [2015\)](#page-19-0).

Relative to computational social sciences, complex systems have certain characteristics in common: simple components/agents, nonlinear interactions, emergent behavior (often through learning and evolution), and no centralized control entity (Mitchell [2009\)](#page-20-0). These attributes align well with the defining features of agent-based models, specifically heterogeneity, repeated and localized interactions of autonomous agents, and bounded rationality where agents have limited global knowledge about the system in which they exist (Epstein [2006\)](#page-19-0). These fundamental attributes of ABM lend themselves to generating complex social phenomena from the bottom up by endowing agents with attributes and algorithms for decision-making and learning, and then observing the macroscopic-level phenomena that arise from repeated interactions. Epstein ([2006\)](#page-19-0) summarizes, "The agent-based approach invites the interpretation of society as a distributed computational device, and in turn the interpretation of social dynamics as a type of computation". ABM, then, opened up a new avenue for social science research that has yet to fully reach the mainstream.

Two distinctive schools of agent-based modeling emerged in the 1990s, which one might call the North American and European traditions. Reflecting the approach of Epstein and Axtell's [\(1996](#page-19-0)) seminal "Sugarscape" model, the North American school views agent-based models as abstract and general representations of a complex system. There is no real-world referent for the Sugarscape, for example. Much as game theorists rely on deductive methods to model interaction, agent-based modelers in this tradition deduce agent rules and interactions from extant theory. Only after model construction and experimentation would the modeler seek to compare simulation results with empirical data. One advantage of the North American school is that modelers can investigate social phenomena for which empirical data is costly, scarce, difficult to obtain for ethical reasons, or from historically rare events. For example, researchers have used ABMs to study dynamics of secessionist movements (Lustick et al. [2004\)](#page-19-0) or insurgencies (Bennett [2008\)](#page-18-0). The North American school of ABM reflects the earliest abstract agentbased modeling developed at the Santa Fe Institute; Epstein and Axtell's initial collaboration and then Axtell's work at George Mason University; and a cluster of researchers at the University of Michigan including Agar's work on The Evolution of Cooperation ([1997b\)](#page-18-0), John Holland's pioneering work on genetic algorithms, and political scientist Scott Page.

By contrast, a European tradition emerged around this same time primarily in Paris and Manchester, UK. In contrast to the North American tradition, the European school embraced "evidence-driven" modeling. As the name suggests, the European school views empirical measurement and data-gathering as antecedent to model construction. Rather than an ABM serving as an abstract generalization of a social phenomenon, modelers in the European tradition view ABMs as representations of real-world social systems and problems. Modelers in this tradition often engage with stakeholders or subjects to understand their perceptions, decisionmaking, and strategies when confronted with a social choice. The companion modeling approach developed by Barreteau et al. [\(2003a\)](#page-18-0) exemplifies this engagement with human subjects: the approach not only consults with stakeholders but involves them in an iterative process of model construction, feedback, and validation. Variously referred to as evidence-driven modeling or participatory modeling, this approach's emphasis on fidelity to real-world social systems provides modelers with ABMs that enjoy strong micro- and macro-level validity (i.e., at the level of both agents and emergent phenomena). Typical of this school's approach are studies that have used evidence-driven ABMs to study the management of common pool resources (e.g., Le Page and Bommel [2005\)](#page-19-0). Whereas the North American tradition sacrifices some level of validity for generality, the European school prefers validity to generalization. Examples of the European school include the work of Bousquet [\(2005](#page-18-0)), Geller and Scott Moss ([2008\)](#page-19-0), and Barreteau ([2003b,](#page-18-0) [2007](#page-18-0)).

Although the European and North American schools of modeling originated with distinctive approaches, today modelers productively borrow from both traditions such that the distinction is blurred. Neither tradition is a substitute for the other; they are complementary methods, the choice of which depends upon the modeler's need for validity, generalization, and empirical data.

### <span id="page-9-0"></span>11.5 Prisoner's Dilemma Example

Among the most studied problems of collective action, the prisoner's dilemma offers a useful metaphor for understanding challenges to social cooperation when utility-maximizing actors cannot enforce agreements and are concerned about the distribution of gains (Table 11.1). Prisoner's Dilemma is a game theoretic model involving two players attempting to cooperate to receive a lighter sentence. The premise is that two people are arrested for a crime, but the scant evidence requires a confession. Each player cannot know what the other will do. The sentencing works as follows:

- If both players choose to stay silent, they will both receive a reduced sentence, say one year each.
- If both players implicate the other, they will each receive two years in jail.
- If one prisoner implicates the other, but the other remains silent, the accuser gets off scott free while the other gets three years.

Cooperation would yield the best collective result, when both stay silent (for an aggregate payoff of two years). However, since the players cannot communicate about their choices, they are motivated to implicate each other and thus each receive two years sentencing (for an aggregate payoff of four years; see Table 11.1 for payoff matrix).

In its original two-player, single-play form—what game theorists call a static game—its very parsimony limits its application to a broad range of real-world cooperation problems. The two-player game tells us little about how large groups may cooperate, whether multinational firms seeking to set technical standards or civil society groups seeking to mobilize supporters in support of a common cause.

Early studies of the prisoner's dilemma, particularly Schelling's ([1980\)](#page-20-0) seminal The Strategy of Conflict, noted the imperative of extending the game to a multiplayer and iterated form. The challenge of multiplayer games is that, once extended beyond the simple two-player structure, the modeler must make theoretically informed assumptions about the structure of social relations among the many players. In this context, social "structure" refers to the organization of relations among actors, including direct or face-to-face interactions (a spatial assumption) or indirect interactions such as among firms competing in a market (a network assumption). Such assumptions must include how actors communicate preferences among each other, and whether they play simultaneously with all other players or a series of sequential two-player games. One scholar calls this the problem of



"context preservation", which requires some mechanism to preserve the social neighborhood or temporal context of the players (Power [2009\)](#page-20-0). Studies in multiplayer prisoner's dilemma have found that outcomes of the game are highly sensitive to different assumptions about the spatial or networked structure of iterated player interactions (Skyrms and Pemantle [2009;](#page-21-0) Matsushima and Ikegami [1998\)](#page-20-0).

Contemporary modelers have adopted two strategies for representing social structure in the multiplayer prisoner's dilemma. One is to use cellular automata to govern the structure of interaction among players: an actor would play the game only with its Moore neighbors (those represented spatially as above, below, left and right of the player's cell) (Nowak and May [1992](#page-20-0); Akimov and Soutchanski [1994\)](#page-18-0). The other is to use explicit representations of the physical space of game play—for example, using geographic information science (GIS) data about real-world locations of social interaction (Power [2009](#page-20-0))—or adopting social network structures, either theoretical or empirical, to examine how network properties affect the pro-spects of cooperation (Earnest [2015](#page-19-0)). One advantage of the networked representation of game context is that the modeler may allow social structure to evolve endogenously in a multiplayer game. For example, players may learn to play repeatedly with "trusted" players while ignoring others, effectively rewiring the social network to find the most cooperative partners.

Because differences in social networks may profoundly affect the prospects for cooperation in the multiplayer prisoner's dilemma, we build an agent-based model that allows us to vary experimentally the network structure in which players interact. Although the formalism of social network analysis and its mathematical cousin graph theory may be daunting, the simple intuition these methods capture is that rarely do actors know everyone else in a social system, a so-called "all-to-all" network. It is much more likely for an individual to have a few close relations, who in turn have a circle of social relations that only partially overlaps with the first individual. Studies of multiplayer prisoner's dilemma may use commonly studied network structures such as a fully connected (all-to-all), small world, or scale-free network. Prior studies have found that network structure affects the prospects for cooperation. One study found that cooperation emerges if the average number of a player's neighbors (the player's degree) exceeds the ratio of the benefits to costs of action (Ohtsuki et al. [2006\)](#page-20-0). Another finds that scale-free networks greatly enhance the prospects for cooperation in a variety of multiplayer games (Santos and Pacheco [2005\)](#page-20-0).

Independent of a game's social structures, the number of players likely will affect the prospects for cooperation in prisoner's dilemma. Interesting, extant theory provides contradictory expectations concerning the effect of the number of players on cooperation. Classic collective action theory suggests that as the number of actors grows, problems of "free riding" (consuming the benefits of cooperation without contributing to collective efforts) overwhelm the incentives for cooperation (Olson [2009](#page-20-0)). However, some studies have found that in larger groups, players may be more likely to play Pareto-improving strategies because they are less concerned about the consequences of inequitable payoffs (Snidal [1991](#page-21-0); Pahre [1994](#page-20-0); Kahler [1992\)](#page-19-0). Similarly, an empirical study has found that larger groups of human subjects tend to produce a more equitable and altruistic distribution of the gains from cooperation. Larger groups may make it easier for the group to detect and sanction free riders (Liebrand [1984](#page-19-0)).

Prior research suggests, then, that in multiplayer prisoner's dilemma, the prospects of cooperation depend on both the number of players of the game, and the social structure in which they interact. To test these expectations, our simulation experimentally varies both the number of game players and the social network in which they play the prisoner's dilemma game.

#### 11.5.1 The Simulation

We constructed an agent-based model of the multiplayer prisoner's dilemma. It allows the researcher to choose the number of players, from the classic two-person game to 20 players. To explore the effect of social structure on cooperation, the model arrays players in a circle from which it constructs social networks. It incorporates four well-known social network structures: the fully connected, "all-to-all" network; a small world network (Watts and Strogatz [1998](#page-21-0)); a scale-free network (Barabási and Albert [1999](#page-18-0)); and a nearest neighbor network (i.e., the player interacts only with players to the left and right in the circle). Because prior research has found that repeated plays of the game (iteration) produce cooperative strategies (Axelrod [2006](#page-18-0)), the simulation endows each agent with a memory of previous plays of the game, which the model varies experimentally from one to five previous plays.

The game proceeds as follows: a randomly chosen agent randomly picks one partner from its social network with whom to play the PD game. Each pair plays a choice and receives a cardinal payoff, as represented in Table [11.1](#page-9-0). Because agents randomly choose a network neighbor, the likelihood of a given agent repeatedly playing the game with the same counterparty varies inversely with the number of players. After all players have played this pairwise game once, the process repeats for a limited number of iterations that the modeler determines. Table [11.2](#page-12-0) presents the pseudocode for the model. Strictly speaking, the simulation adopts an approach that has a series of simultaneous and parallel two-player games; a true multiplayer game would be one in which all agents play against each other simultaneously.

To model how players learn and adapted in iterated games, we implement a genetic algorithm developed by Axelrod ([1987\)](#page-18-0), as described by Mitchell [\(2009](#page-20-0)). In brief, the model encodes agent strategies as a random bit string of length  $2 \text{ m}^2$ , with m = memory of past plays. At initiation agents receive a set of strategies and play each strategy once (a "generation" of the algorithm). For each strategy, an agent plays the choice (1 = cooperate,  $0 =$  defect) at a bit position that corresponds to the pattern of previous rounds of play. After all agents have played a round, the algorithm implements a fitness-proportionate selection routine; a crossover procedure that exchanges among selected strategies one of every four bits on average; and a mutation strategy that flips each bit with  $p = 0.001$ . Our experiment has the agents play 50 strategies per generation, for 100 generations.

Initialization	Create N negotiator agents and distribute them in a circle				
	Endow them with a memory of length M				
	Seed initial memory set with random bit string of length M				
	Endow with a network of other negotiators				
	Network types: fully connected, nearest neighbor, small world, scale-free				
Execution	Loop for 20 rounds of play:				
	Each negotiator agent:				
	Randomly select one neighbor to play				
	Checks memory of game play				
	Convert memory bit string m from binary to decimal format $=$ history h				
	Play choice x from position h in the strategy				
	Record partner's choice y				
	Receive payoff for outcome x, y for the game				
	Add partner's choice y to the end of memory bit string				
	End Loop				
Genetic	Initialization:				
algorithm	Endow negotiator agents with a set of 40 strategies				
	One strategy = bit string of length 2 m <sup>2</sup> with $p(i = 1) = 0.5$				
	40 strategies $= 1$ generation				
	Loop for 40 generations:				
	Record the mean Hamming distance of negotiators' strategies				
	Record the percentage of cooperation plays in all negotiators' strategies				
	Record the mean total payoffs of negotiators				
	At the end of each generation:				
	Select 20 strategies, using either pairwise or fitness-proportionate rule				
	With $p = 0.25$ , crossover two selected strategies at a randomly selected bit				
	With $p = 0.001$ , flip each bit of a selected strategy				
	Add 20 strategies of length 2 m <sup>2</sup> with $p(i = 1) = 0.5$				
	Execute the game play				
	End Loop				

<span id="page-12-0"></span>Table 11.2 Pseudocode for the Multiplayer Prisoner's Dilemma

# 11.5.2 Expectations and Findings

Prior experiments with two-player prisoner's dilemma give us two expectations for the simulation. First, iteration of the game is more likely to produce cooperation. Because iteration implies players remember past plays of the game, in our implementation we expect that agents with longer memories should produce more cooperation than those with shorter memories. Second, prior studies have found that a strategy known as "tit-for-tat" is an optimal solution for the prisoner's dilemma. This strategy is a simple one: players will reciprocate the choice of their partner in the previous round. In our implementation, the ABM measures outcomes of the game as a bit string of length 2 m, with odd-numbered positions recording the agent's choice, and the even-numbered positions recording the choice of the agent's randomly chosen network partner. Given this implementation, the emergence of tit-for-tat will appear as alternating ones and zeroes in the outcome bitstring, e.g., "10101010" for a game in which agents remember the previous four rounds of play. Because there are well-known variants of tit-for-tat (e.g., tit-for-two-tats,

two-tits-for-tat) we expect to find a variety of strategies with broad patterns of reciprocation of cooperation and defection.

Our experiment varied three model parameters. To test the hypothesized effect of player numbers on cooperation, we experimentally vary the number of players from two (which is the classic game) to 4, 6, 8, 10, and 20. Because we expect iteration to improve prospects for cooperation, we experimentally vary the length of players' memory from one previous round of play to 2, 3, 4 and 5 previous rounds. Finally, to examine the effect of social structure on the prospects for cooperation, we have agents play the PD game in four different social structures: fully connected; small world; scale-free; and nearest neighbor networks. The experiment thus produces 120 runs, though within each run the genetic algorithm has players learning over the course of 100 generations. We measure player strategies and outcomes only at the end of each of the 120 runs. Because we measure each player in each run, the experiment gives us 100,000 observations of players' strategies at the end of each run.

Table [11.3](#page-14-0) reports the most frequent outcomes of the game for each value of the memory parameter. The reported results are consistent with two theoretical expectations. First, as players' memories grow longer, the frequency of the pure defection outcome (all zeroes indicate both players are choosing defect as their strategy) declines. As expected, at  $m = 1$  pure defection is the most frequent outcome of the simulation, accounting for more than half of games played in the final generation. At  $m = 4$ , pure defection occurs in only about three percent of the games in the final generation. Pure defection is not even among the top ten most frequent outcomes when  $m = 5$ . These results suggest that when playing iterated games with a greater number of plays, the genetic algorithm allows players to "learn" or evolve strategies that produce more cooperation and avoid the trap of pure defection. Second, the results reported in Table [11.2](#page-12-0) illustrate the emergence of pure tit-for-tat strategies as agents play longer iterated games. When  $m = 2$ , tit-for-tat ("1010" and "0101") occurs as the sixth- and seventh most frequent outcomes. Tit-for-tat is the second- and third most frequent outcomes when  $m = 4$ , and the first- and third most frequent outcomes when  $m = 5$ . Although tit-for-tat may represent a smaller percentage of the outcomes as  $m$  grows, this is because the universe of possible outcomes grows by 2  $m^2$ . Given the very large number of possible outcomes, the relative preference of tit-for-tat as a very frequent outcome suggests that agents evolve cooperative strategies over the course of the simulation.

To examine whether the number of players and network structure affects the likelihood of cooperation, we regressed several model parameters on the average player score in the final round of each generation. Although players' scores might theoretically be higher if they play a constant defect strategy against "suckers" that play regular cooperative strategies, the genetic algorithm suggests this is unlikely as players learn not to play the sucker strategy. Because Table [11.3](#page-14-0) found increasingly frequent tit-for-tat cooperative strategies, we can reasonably assume that higher player scores in the final generation associate with cooperative strategies. The results reported in Table [11.4](#page-14-0) are consistent with theoretical expectations. As expected, as the number of players increases, agents tend to earn higher scores. Players' memory has a strong and positive effect, again suggesting that iteration

$Memory = 1$		Memory $= 2$		$Memory = 3$		$Memory = 4$		$Memory = 5$	
Outcome	Pct.	Outcome	Pct.	Outcome	Pct.	Outcome	Pct.	Outcome	Pct.
00	56.91	0000	19.86	000000	4.13	00000000	2.92	1010101010	0.41
10	18.32	0010	8.93	001000	2.81	01010101	1.15	1100000111	0.40
01	18.26	1000	8.68	000010	2.46	10101010	1.04	0101010101	0.36
-11	6.51	0100	7.91	010101	2.44	00001001	0.56	1100001011	0.36
		0001	7.51	000100	2.43	10110000	0.53	0001111010	0.19
		0101	6.90	101010	2.30	10111100	0.52	1111010111	0.18
		1010	6.57	100010	2.23	00010100	0.51	0010000100	0.17
		1001	4.98	101000	2.19	00100000	0.50	0011000100	0.17
		0110	4.45	100000	2.12	10000001	0.50	0010110000	0.17
		1100	4.06	000001	2.10	10001010	0.50	10000100111	0.17

<span id="page-14-0"></span>Table 11.3 Most frequent outcomes of the game, by memory m parameter. Tit-for-tat outcomes highlighted by outcome and strategy (Outcome) and percentage in the population (Pct.)

Table 11.4 Regression results, with robust standard errors

					$N =$	100000	
	F(6, 99993) 12353.58						
					Prob > F	$\theta$	
					R-squared	0.4257	
	Adj R-squared						
					Root MSE	7.3319	
	Coef.	Robust std.	$\boldsymbol{t}$	P > t	$9.5\%$	Interval	
		err.			conf.		
No. of players	0.253	0.004	56.60	< 0.001	0.244	0.262	
Memory (m)	3.861	0.017	227.80	< 0.001	3.828	3.894	
Small World dummy	$-3.857$	0.067	$-57.43$	< 0.001	$-3.989$	$-3.725$	
Scale-free dummy	$-3.382$	0.070	$-48.63$	< 0.001	$-3.518$	$-3.246$	
Nearest Neighbor	$-7.507$	0.065	115.12	< 0.001	$-7.635$	$-7.379$	
dummy							
Generation of the	$-0.017$	0.001	$-21.59$	< 0.01	$-0.019$	$-0.016$	
<b>GA</b>							
Constant	82.192	0.116	707.97	< 0.001	81.965	82.420	

leads to cooperative strategies and higher scores. To assess the effect of social network structure, we treat the fully connected ("all-to-all") network as the ideal baseline: all agents play against every other agent with a uniform probability. The results in Table 11.3 indicate that different network structures have a significant effect on players' cooperation, in this case a negative effect when compared to the baseline of a fully connected social network. The nearest neighbor network is the least advantageous, with players scoring an average of seven and a half points lower in game ceteris paribus. By contrast, players in scale-free networks score significantly higher than players in the small world or nearest neighbor network structures  $(t = 32.14, p < 0.001)$ .

The consistency of these results with theoretical expectations illustrates how researchers may use simulation methodologies to build upon formal models and prior empirical scholarship. Because formal models of social choice problems do not permit easy extrapolation to the multiplayers games more typical in the social realm, computer-based methodologies allow researchers to expand these formal models and to test their sensitivity to assumptions about the social structure of players. Our use of a genetic algorithm and network theory illustrates how multiple formal and computational methods may inform analyses of social problems. This integration of formal and computational methods is, in our view, an emerging paradigm that promises the theoretical breakthroughs that characterized the emergence of empirical large-sample studies in the late nineteenth century and then game theory in the mid-twentieth century.

#### 11.6 Model Validation and Verification Challenges

Despite the power that modeling and simulation brings to the study of social sciences, major challenges remain before it will likely receive widespread acceptance as a research method. Social science, and political science in particular, relies upon humanist approaches that foreground agency of actors at the local, regional, or international levels. This is not entirely at odds with the modeling and simulation approaches presented here, but for many, it is philosophically complex to reduce social interactions and behaviors to simple algorithms. The critical realist theory-based, post-positivist views that dominate much of the social sciences see theory as revisable and the scientific method, while worthy of pursuit, as a flawed process. Much of modeling and simulation as a discipline, as well as complexity theory, seeks positivist truths and theories about the generalizable underlying causal mechanisms that shape our world. Agar [\(2004](#page-17-0)) offers this thought, "[Complex Adaptive System] offers an ironic combination of the poststructural and the scientific, a framework that accepts the heresy of researcher influence but then deals with it in a systematic fashion". When simulating physical systems, modeling and simulation can achieve a relatively complete picture of the system in question. For social systems, however, models are often drastically simplified and can only capture a fraction of the potential causal mechanisms driving human behavior and interactions. Though other quantitative and qualitative methods may suffer from weaknesses, the inability of M&S to capture the full range of a social context is often seen as a flaw in the methodology.

M&S also requires a large amount of empirical data to calibrate and validate models. The data-intensive requirements of most models are problematic for social sciences, depending on the type of research question (Borero and Squazzoni [2005;](#page-18-0) Janssen and Ostrom [2006](#page-19-0)). In most social sciences, sample sizes are small compared to datasets available for physical sciences. Many social data are also contextualized and temporally limited, for example, an ethnographic study over ten years of an isolated ethnic group. The data required to construct models and

simulations of social systems thus are often not available. Given ABM's relative integration with social science, some scholars have advanced methods for tying these models to qualitative data. Malerba et al. [\(2001](#page-20-0)), for instance, propose an evolutionary economic model derived from historical analysis. Ethnographic data (Agar [2005\)](#page-17-0), including through proposed Grounded Theory approaches (Neumann [2015;](#page-20-0) Dilaver [2015\)](#page-19-0) have found their way into the evolution of ABM for social sciences, though not without critiques about the roles of this type of data or results (Agar [2003](#page-17-0), [2005](#page-17-0); Yang and Gilbert [2008\)](#page-21-0). Even participatory methods common to qualitative studies have emerged as 'companion modeling' practices, where subject matter experts play a role in developing, executing, and verifying the simulation (Gurung et al. [2006](#page-19-0); Polhill et al. [2010\)](#page-20-0).

Due to the data challenges facing social science modelers, validation of these simulations also proves problematic (Ormerod and Rosewell [2009\)](#page-20-0). Anthropologists have looked toward comparing models of similar phenomena to validate high-fidelity models (Kuznar [2006](#page-19-0)). This often is performed as a kind of 'model docking' where models of similar phenomena are tested to determine if they can produce similar results (Axtell et al. [1996](#page-18-0)). While not validating against real-world data, this comparison method provides some means for checking that the model is accurately reflecting the real-world phenomena as the researchers intended. To achieve a level of verification, Miller [\(1998](#page-20-0)) proposes using algorithms such as genetic algorithms or simulated annealing to search the parameter space of models in an attempt to "break" them or find combinations of parameters that produce unexpected results. This can serve as a method of verifying that the model is performing without overt errors (debugging), but also provide insight into particular combinations of parameters that produce emergence of macro-level effects that were unintended at the outset of the model.

These challenges are at the forefront of innovation in computational social science research. The primary goal, as with any method, is to design a study and utilize the computational tool in order to answer the research question or objectives at hand. Rather than focusing on models as predictive tools, Epstein [\(2008](#page-19-0)) proposes that we consider the multitude of reasons for developing models of social phenomena. Among those of particular interest to social scientists are to explain phenomena, provide a framework for systematically thinking about problems and dynamics, testing accepted theories, and grounding policy development and discussion (Epstein [2008](#page-19-0)).

#### 11.7 Conclusions

Traditional quantitative and qualitative methods dominate the social sciences, with statistics and ethnography at the forefront. While scientists understand that these methods have limitations in terms of over-generalization by the former and ungeneralizable small sample sizes by the latter, it is difficult to break away from familiar analytical tools. As this chapter demonstrates, great strides have been made <span id="page-17-0"></span>to develop methods to ground models in qualitative and limited quantitative datasets, as well as validate models in restricted data environments. Modeling and simulation, particularly ABM, holds great promise for becoming a fundamental tool in many social scientists' research toolkit as it evolves to incorporate multiple types of data and theories and programming environments become increasingly user-friendly. The advancement of the state of the art will require flexibility in the discipline of M&S as well as the social sciences. For M&S, modeling paradigms and stringent expectations for traditional forms of calibration, validation, and verification must bend to meet the non-predictive purposes of many social science models. Social scientists must also adjust their approach to research, thinking outside the bounds of traditional statistical or qualitative methods to explore how M&S can further our understanding of social phenomena.

#### Review Questions

- 1. What are some of the differences between social sciences and the natural and physical sciences? What are the implications of these differences for modeling and simulation in the social sciences?
- 2. Who were the important innovators in empirical social scientific methods? Game theory? Agent-based modeling?
- 3. What is behavioralism? How does it differ from earlier paradigms of empirical social research?
- 4. What is the prisoner's dilemma? What does it tell us about the relationship between individual choices and collective action?
- 5. What are the differences in assumptions and methods of the North American and European schools of agent-based modeling for the social sciences?
- 6. Methodological paradigms in the social sciences often depend upon technical innovations in scientific research. What are some examples of these technical innovations and their contributions to modeling in the social sciences?
- 7. What lessons might the social sciences learn from modeling and simulation in other disciplines? Conversely, what lessons might disciplines such as engineering learn from social scientists?

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