Shapes of Design: Traditional Geometry, Symmetry and Representation

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Abstract Geometry, traditional arts and design universally contribute both to the formation of ideas that to the creation of form by establishing a cultural contract between art and science. The introduction of digital design media has been imposing new and practical ways to rediscover knowledge or culture in representation and design from traditional arts. The subject of geometric tiling of the Euclidean plane is always a contemporary challenge by combining creative roots of artistic and scientific research. From the ornamental motifs of the Alhambra in Granada and the Mezquita of Cordoba, geometry, shape and colour allow designers to experience a wide range of meanings, ranging from the practical and technical sphere, to the spiritual and sacred one.

Keywords Traditional arts · Geometry · Symmetry · Design pattern · CAD

1 Introduction

Science and art form the cultural foundations of a designer, because they share common roots, the first addresses the public experience, universal, objective, quantitative, unitary, and its language is precise, rational, made of ideas and concepts. The second looks at the experience rather private, particular, subjective, qualitative, and its language is ambiguous, emotional, made of images and stories [1]. The connection between the two cultural spheres is in mathematics and geometry which generate languages and representations (Fig. 1).

Among the Greek philosophers, Plato, illustrates his aesthetic thought that was the language of mathematics. The Philebus is a dialogue written in the final stage of its production (366-365 BC), in which the philosopher attributes to the master the leading role: discussing with Philebus and Protarco, Socrates seeks the "*true Good*"

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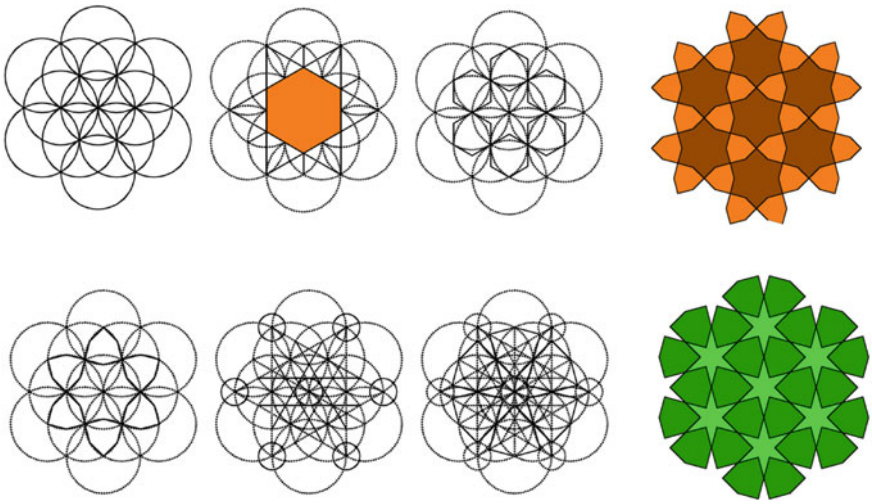

Fig. 1 Geometric construction starting with the star-and-hexagon design

that can guarantee a happy life, starting with the possibility—later denied—that it coincides with the pleasure.

In the dialogues between the philosophers we explore the concerns of beauty of form that is matched to the shape of finite, those geometric ones achieved by the same antique tools, a compass, a straight edge and a T-square. These dissertations open the way to the concepts of proportion, harmony, beauty and truth through memory and spiritual approach to life the art forms [2].

"*Art holds out the promise of inner wholeness*" wrote Alain de Botton [3] but perhaps the deepest argumentation on how arts serve the spirit came more than a century earlier, in 1910, when Russian painter and art theorist Wassily Kandinsky published Concerning the Spiritual in Art—an exploration of the human and most original reasons for performing art, the "*internal necessity*" that move artists to create as a spiritual impulse and audiences to admire art as a spiritual hunger.

Kandinsky, who was greatly influenced by Goethe's theory of the emotional effect of colour and who was himself synesthetic, highlights the powerful psychic effect of colour, and its interactions, in the comprehensive spiritual experience of art [4].

Today the practice of the traditional arts is the starting point of generation of designers bringing their soul more close to the creative process based upon "*universal spiritual truths*" (Fig. 2).

The fundamental principle that has to be pinpointed is that tradition is a continual renewal when scholars, designers start their apprentice. The book of the universe which we learned about from Galileo is written in the language of mathematics and its characters are triangles, circles and other geometric figures. The Italian scientist was responsible for the birth of the modern research method, hypothetical and experimental that aims to formulate a scientific law, in the years when the ideas of Platonism had spread again throughout Europe and in Italy and

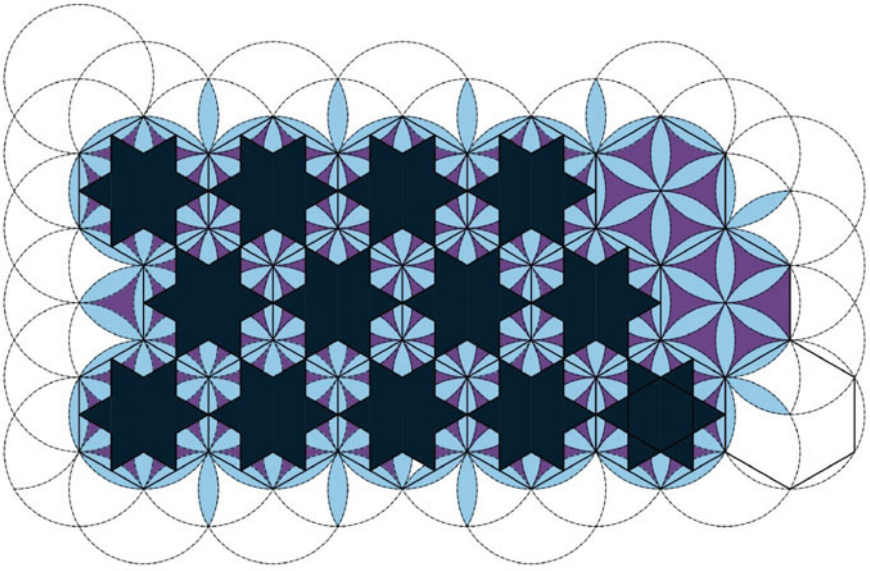

Fig. 2 Geometric pattern found in the Ibn Tulun Mosque in Cairo (879 C.E.)

probably also for this reason the symbols of mathematics are identified by him with geometric entities and not with numbers. Essentially, it was the contribution of Galileo to the scientific language, in particular, in the writings of Galileo many words are taken from the common language and undergo a process of assigning new and specific meaning (a form, then, of semantic neologism) [5].

Geometric design and the use of digital systems of representation demonstrate the validity of Galileo Galilei statement that nature is compared to a book; it is written in a language whose letters are polygons and circles, and using as pens the classical tools of Euclidean geometry, ruler and compass, following the universal tradition that goes from the early Greek geometers to the expressions of modern design.

The Platonic vision settled the fundamental research about "*ideas*", what we currently assume as shapes and forms; the unique way to develop meanings is to research the right geometry and, in this way, return to investigate about common roots, formulas and ratios coming from math.

This is the reason of the continuous success of the golden ratio in arts and design; the same common numerical and irrational code that it was conceptualized and built in form of a temple, a painting, a building type and a music instrument. A universal truth and knowledge that is often described as the essence of classical architecture where proportions have the meaning of equality of ratios and a number becomes "*divine*" according to Luca Pacioli [6].

Experiencing traditional arts as performing skills, designers participate in the regeneration of knowledge about the geometric universe (Fig. 3).

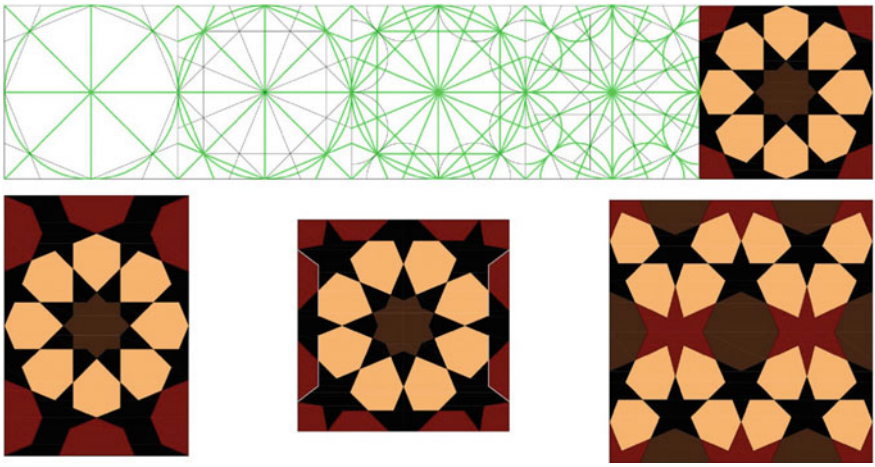

Fig. 3 Geometric rosette generation

The study aims to improve the awareness among young designers that form, pattern and colour as manifested in the various branches of the traditional arts and design, have a deeper meaning not simply pleasing to the senses, but playing a strategic role transferring both philosophical and technical contents. For example the relations between geometry and illusory design that was conceived and expressed in a multiple forms of representation: optical corrections, perspectival frescoes, perspective-relief and scenography (Fig. 4).

For centuries practitioners used always the same simple tools to design and build their concepts, from sacred architecture to dwellings and objects.

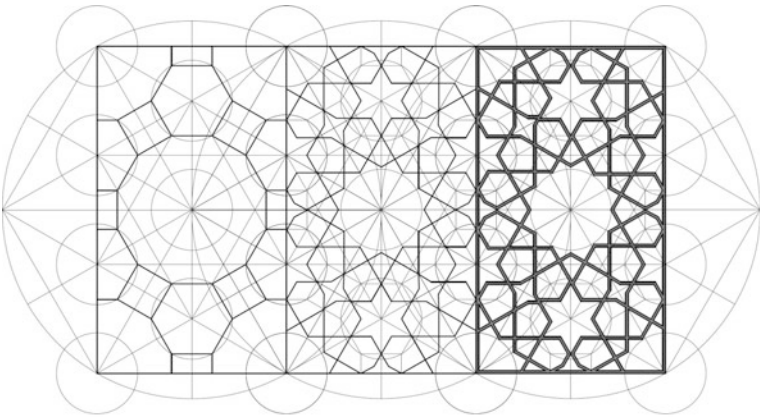

Fig. 4 Semiregular tiling of regular dodecagons, regular hexagons and squares

Whether a traditional art is representational (iconic) or non-representational (aniconic) it will always be based on the principles of ordered space. All civilisations have acknowledged that geometry is fundamental to the cosmic order, or as Plato explained, "*...geometry is the knowledge of the eternally existent*" (Republic, Book 7, 527b). The School teaches geometry, not only as an objective language informing the traditional arts of the world, but also as an essentially sacred language. Students learn that patterns of traditional art reflect nature and are underpinned by the same geometry that is the basis of the natural world. Thus geometry is seen as a reflection of a universal order, as was taught by the ancient Greeks and recognised by the great Arab architects and scientists, as well as the cathedral builders of the Middle Age.

Geometry intervenes in the creative and conceptual process in three forms: as a language, as a representation or as structure.

Symmetry, in the words of Hermann Weyl, both in the broadest sense and in specialized fields, it is an idea that has guided the man through the centuries to the understanding and the creation of order, beauty and perfection [7].

2 The Illusion of Infinity

Among the challenges that have always engaged mathematicians and creatives is the regular division of space that has found a special contribution from Escher that studied and drawn formal and aesthetic solutions influenced by his discoveries of Islamic patterns in Granada, Sevilla and Cordoba. It is the theory of tessellations, a graphic process that tiles a plane in a finite number of small elements obeying to geometric rules related to the concept of symmetry. This idea has inspired the research of meanings and forms combined with multiple functions, involving artists, scientists and philosophers. This is the universal necessity of representing balance and harmony through the language of the form that includes a deep mathematical theory that describes a phenomenon. First templates of tessellations were found in some Sumerians buildings (about 4000 BC) as primitive geometric wall decorations built as patterns of clay tiles. The first documented studies about tessellations are referenced to Johannes Kepler when, in 1619, he published his Harmonices Mundi (Harmony of the Worlds); he presented regular and semiregular tessellation to be assumed as covering of a planar surface with regular polygons. In 1891, the Russian crystallographer Yevgraf Fyodorov proved that every periodic tiling of the plane features one of seventeen different groups of isometries, surprisingly used by Egiptians in the Valley of the Kings and by Arabs in the Alhambra's decoration. Fyodorov's work, in the field of geology, marked the beginning of the mathematical study of tessellations. Other prominent contributors include Shubnikov and Belov (1951); and Heinrich Heesch and Otto Kienzle (1963), but also Rosen (1995) that studied the modern mathematical theory of symmetry and its deep influence in science branche (Figs. 5, 6).

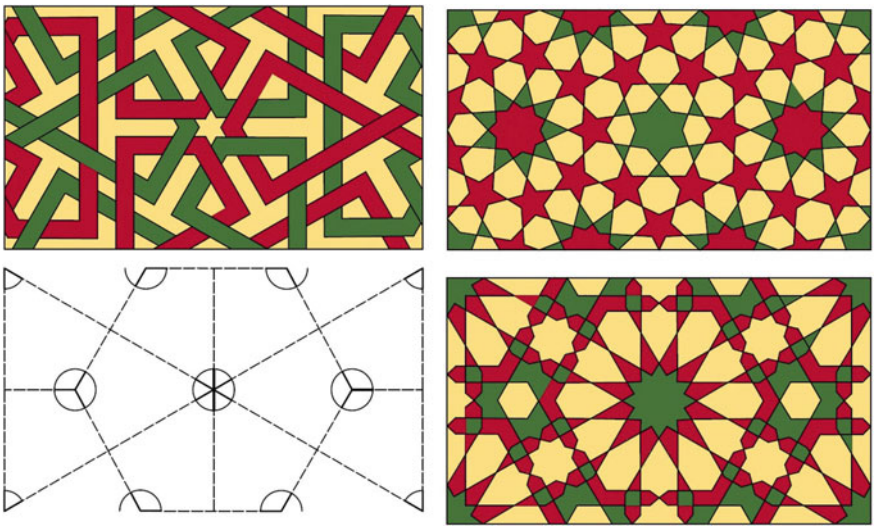

Fig. 5 Three-fold permutations: hexagonal grid (or equilateral triangle) generated patterns

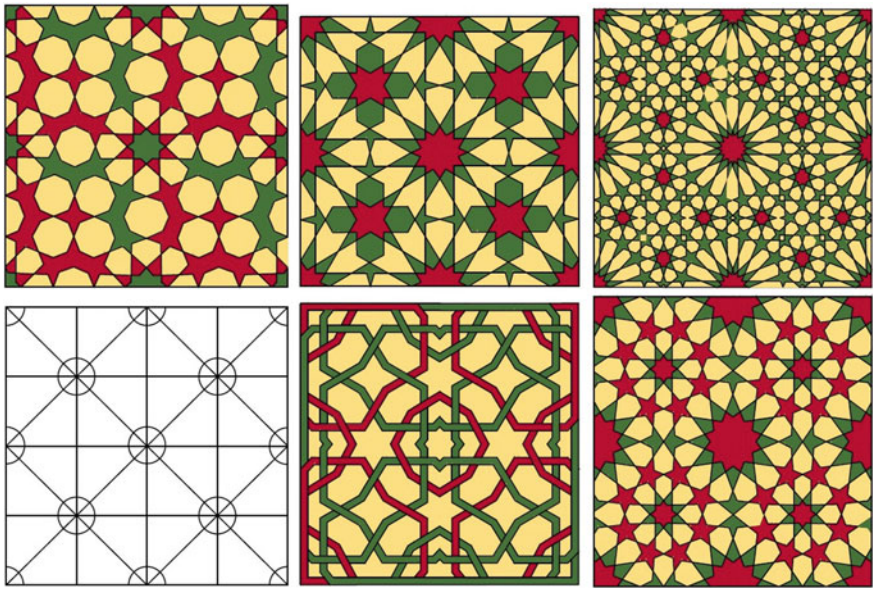

Fig. 6 Four-fold permutations: square grid generated patterns

Unfolding geometrical elements from the starting concept of unity, it starts the creative process and the repetition of patterns where the loop can continue indefinitely; Sutton describes this solution as "*the perfect visual solution to calling to*

mind the idea of infinity, and hence the Infinite, without any pretence of being able to truly capture such an enigmatic concept visually".

Transforming shapes into psychological and emotional effects reminds us the concept of optical illusions, well known since the early designers steps and that have appealed to the mind of spectators throughout history, and have had great impact when combined with real architectural elements or spaces. Illusionary methods in design have been used by artists and architects since antiquity, but from Renaissance they were scientifically analysed with the invention of perspective and later in the baroque with the discover of anamorphosis, and their integration with arts: the architectural perspective known as quadratura and the illusory painted architecture of Andrea Pozzo in Rome.

Forms of tessellations and their use as artworks play, according to Spiliotis, the specific illusion of "*plane dematerialisation*" that Mediterranean and Eastern cultures have exhibited as "*patterned optical illusions*" on floors of tiles or woodwork which seem to deconstruct the flat floor plane and create additional illusory ones; templates have been found in a number of countries including Japan, China, India, Persia, and Italy.

The theoretical foundation is that of invariance under isometric transformations of the Euclidean plane (and space): named isometry when the bijective transformation does not alter the distance between two points. Isometries of the plane belong to four main classes, reflections, rotations, translations and glide reflections, establishing measurement tasks. Substituting figures with numbers we introduce algebraic operations defining "*composition laws*", as transactions between numbers, which allow to obtain the algebraic structure of a "*group*" and then of a "*group of symmetries*".

A periodic tiling produces a repeating pattern: some templates include regular tilings with regular polygonal tiles all of the same shape, and semi-regular tilings with regular tiles of more than one shape and with every corner identically arranged [8].

A plane symmetry group is a mathematical classification of a two-dimensional repetitive pattern, based on the symmetries in the pattern. Such patterns occur frequently in architecture and decorative art. The patterns formed by periodic tilings can be categorized into 17 wallpaper groups. Wallpaper groups are two-dimensional symmetry group, intermediate in complexity between the simpler frieze groups and the three-dimensional crystallographic groups (also called space groups).

Among the works of Escher we find himself curiously displaying the principle of psychological perception of symmetrical element that from the alteration of the geometric symmetry allows just to appreciate its value; this principle concerns a hierarchical sequence of elements or groups of symmetry found in natural forms or design solutions that allows to highlight their beauty or harmony.

According to Escher the rules of symmetry and harmonic balance are conceived, reproduced and broken building hidden meanings and introducing an additional degree of freedom: colour becomes the aesthetic key of altered symmetry [9].

Sometimes the colour of a tile is understood as part of the tiling, at other times arbitrary colours may be applied later. When discussing a tiling that is displayed in colours, to avoid ambiguity one needs to specify whether the colours are part of the

tiling or just part of its illustration. This affects whether tiles with the same shape but different colours are considered identical, which in turn affects questions of symmetry. The four colour theorem states that for every tessellation of a normal Euclidean plane, with a set of four available colours, each tile can be coloured in one colour such that no tiles of equal colour meet at a curve of positive length. The colouring guaranteed by the four-colour theorem will not in general respect the symmetries of the tessellation. To produce a colouring which does, it is necessary to treat the colours as part of the tessellation.

Finite groups of symmetry belong to the geometry of rosettes, templates of radial symmetry, special works of art embodying issues of symbolism and fascinating beauty; in the rosettes geometry all rotations must have the same centre according to a principle that Weyl attributed to Leonardo who also dealt of symmetry in architecture.

However one of the moments of maximum splendour of radial symmetry belongs to the Islamic art practice [10]. The prohibition of representing holy figures led the figurative Islamic culture to move towards geometric patterns [11]. From Middle Ages another artistic representation of radial symmetry can be seen in the cosmatesque floors, a style of geometric decorative inlay stonework typical of the architecture of Italian Medieval churches and derived from the Byzantine Empire templates dealing with the geometry of circles and squares.

This geometric tradition finds multiple expressions in the design of "*mandala*", (in Sanskrit maṇnala means circle), that is a spiritual and ritual symbol in Hinduism and Buddhism, representing the Universe but also the best image to visually frame the infinite into the finite.

A variant of the Mandala are the "*kolam*", drawings made at the entrance of the house by Indian women with auspicious function.

Both mandala that kolam can be drawn starting from intersections of circles and squares and then stained freely. It is an exercise very suitable to approach both geometry and harmony of shapes and colours which is free from figurative representation [12].

3 Conclusion

The study of traditional arts and geometric principles that from time to time assign functional, symbolic, philosophical and spiritual meanings still represents a living challenge.

Hidden geometries and their degrees of complexity intervene in the structure, in the language and in the representation of design shapes, guiding the mature and advanced creative process.

The research about symmetry holds a special place in the generative processes that now constitute a new challenge for designers when digital systems of representation are introduced.

This contribution is a simple step towards a return to the study of geometry as a multimedia environment that today the use of digital technologies in the design and also generative software can renovate delivering innovative forms of mind.

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