The Harmonic Balance Method for Temporally Periodic Free Surface Flows

Inno Gatin, Gregor Cvijetić, Vuko Vukčević and Hrvoje Jasak

Abstract The Harmonic Balance Method for temporally periodic, non-linear, turbulent, free surface flows is presented in this work. The method transforms a periodic transient problem into a set of coupled steady-state problems, increasing the efficiency of calculation. The methodology is primarily targeted to efficient simulations related to wave–structure interaction in naval and offshore hydrodynamics. The method is validated on a 2D periodic free surface flow over a ramp test case and a 3D ship wave diffraction test case.

1 Introduction

Transient flows in marine hydrodynamics are often periodic, e.g. due to ocean waves (wave propagation and diffraction, seakeeping of a ship) and rotating propellers. Such flows often have a well-defined base frequency: the wave frequency or rotational frequency of the propeller. In fully non-linear, two-phase state-of-the-art CFD algorithms, such flows are almost exclusively resolved in the time domain [7, 8]. Transient simulations usually require a large number of periods in order to achieve a harmonically steady (purely oscillatory) solution. Due to its spectral decomposition, the Harmonic Balance Method (HBM) allows us to efficiently model flow effects up to a specified order, without performing a fully transient simulation. Hence, a substantial performance improvement is expected, with an almost negligible decrease in accuracy for flows with a well-defined base frequency. Due to the steady-state mathematical formulation of the HBM, the authors believe that the method is highly

I. Gatin (✉) · G. Cvijetić · V. Vukčević · H. Jasak
University of Zagreb, Zagreb, Croatia
e-mail: innogatin@gmail.com

G. Cvijetić
e-mail: gregor.cvijetic@gmail.com

V. Vukčević
e-mail: vuko.vukcevic@fsb.hr

H. Jasak
e-mail: hrvoje.jasak@fsb.hr

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suitable for adjoint optimisation regarding seakeeping of ships in the ship-building industry. This suitability has been recently confirmed by Huang and Ekici [6], who developed an adjoint shape optimisation tool based on the HBM for turbomachinery applications.

The HBM [5] was originally developed to tackle periodic single-phase turbomachinery flows in an efficient way. This paper presents an extension of the single-phase HBM [2, 4] to two-phase free surface flows, comparing the results and computational efficiency with a transient simulation. The implementation is carried out in a second-order accurate, polyhedral Finite Volume framework developed within foam-extend, a community-driven fork of the OpenFOAM® software.

2 Harmonic Balance Method

In the HBM, a transient governing equation set is replaced with a specified number of coupled steady-state problems, each represented by an equation for a unique time instant. The method simulates a periodic flow by evaluating the temporal derivative via spectral decomposition, yielding a flow solution at discrete instants in time simultaneously. Multi-mode transformation from a transient to a set of coupled steady-state problems is achieved through a Fourier transform, assuming temporally periodic flow. The accuracy of the model is controlled by a specified number of harmonics to allow for the efficient capturing of higher order flow effects. Generally, the specified number of harmonics n, yields solutions at $2n + 1$ discrete time instants.

2.1 Mathematical Model

A general field variable ϕ is expanded in a truncated Fourier series with a known base frequency ω:

$$\phi(t) = \Phi_0 + \sum_{i=1}^{N} \left(\Phi_{S_i} \sin(i\omega t) + \Phi_{C_i} \cos(i\omega t) \right), \tag{1}$$

where Φ is the general field variable in the frequency domain, while indices S_i and C_i indicate the sine and cosine coefficients, respectively. Equation 1 can be presented using a Fourier transformation matrix $\underline{\underline{E}}$: $\underline{\phi} = \underline{\underline{E}}\,\underline{\Phi}$, where $\underline{\phi}$ denotes the vector of ϕ at discrete time instants. The general transport equation reads as

$$\frac{\partial \phi}{\partial t} + \mathcal{R} = 0, \tag{2}$$

where $\mathcal{R}$ stands for convection, diffusion and sink/source terms. The field transformed using Eq. 1 is then inserted into Eq. 2, and corresponding terms are equated, yielding $2n + 1$ equations:

$$\omega \underline{\underline{A}}\, \underline{\Phi} + \underline{R} = \underline{0}, \tag{3}$$

where $\underline{\underline{A}}$ represents the $2n + 1$ by $2n + 1$ coupling matrix stemming from the Fourier transformation, while $\underline{\Phi}$ and $\underline{R}$ are the vectors of field variables and convection, diffusion and sink/source terms in the frequency domain, respectively. The time-spectral approach is based on the equations obtained by transforming Eq. 3 into the time domain using Discrete Fourier Transform (DFT) matrix $\underline{\underline{E}}$:

$$\omega \underline{\underline{E}}^{-1}\, \underline{\underline{A}}\, \underline{\underline{E}}\, \underline{\phi} + \underline{\mathcal{R}} = \underline{0}. \tag{4}$$

Equation 4 presents a set of $2n + 1$ equations coupled with the analytically transformed temporal term via spectral decomposition. Each equation represents one discrete time instant in one period of oscillation corresponding to ω.

In the present study, HBM is applied to Navier–Stokes equations and the Level Set interface capturing Eq. [9], yielding a coupled set of two-phase flow equations for discrete instants of time within one period. In addition, SWENSE decomposition [9] is used to facilitate incident wave propagation. The reader is directed to [3, 4] for further details.

2.2 *Coupling of Steady-State Equations*

The term $\omega \underline{\underline{E}}^{-1}\, \underline{\underline{A}}\, \underline{\underline{E}}\, \underline{\phi}$ presents a source term that couples the steady-state equations. The coupling can either be resolved in an explicit or implicit manner. In this work, the coupling is resolved implicitly by solving the equations simultaneously in one block system. The block system contains the block matrix and block vectors, where each entry presents a vector of size $2n + 1$. The implicit approach enhances the stability of the calculation and enables low mean velocities with respect to the magnitude of oscillation. This is important for the naval hydrodynamic application, since wave-related flows often have low or zero mean velocities. The explicit approach is also used for purposes of comparison.

3 Test Cases

In this section, two test cases are shown: a 2D simulation of a periodic flow over a ramp and a 3D wave diffraction of a DTMB ship simulation. The results are compared with transient simulations to validate the method.

3.1 2D Ramp Test Case

A simple 2D test case is devised to validate the method for periodic free surface flows. A periodically changing inlet velocity is prescribed that enforces a periodic variation of the free surface throughout the domain. The simulation geometry can be seen in Fig. 1. The inlet velocity is determined as $U_{inlet} = [6, 0, 0] + [1, 0, 0] \sin(2\pi t/T)$, where $T = 0.5$ s stands for the prescribed period of oscillation. Figure 2 shows the initial condition with the calm free surface. The free surface elevation is measured 0.5 m from the outlet boundary. 13,000 cells are used in both the transient and HBM simulations, while 200 time steps per period are used in the transient simulation. Simulations using 1–8 harmonics are performed to asses the sensitivity of the solution on spectral resolution. In this case, the oscillation of velocity is small compared to the mean velocity, hence the explicit approach for resolving the source coupling can be used. For comparison, both the explicit and implicit methodologies are used.

Figure 3 shows the dynamic pressure and velocity field in the discrete time instants for the simulation with 2 harmonics. The comparison of the free surface elevation from the HBM simulations using different numbers of harmonics with the transient simulation is shown in Table 1, where η_a stands for the free surface elevation amplitudes, with indices 0 and 1 indicating zero- and first-order harmonic amplitudes, respectively. ε is the relative difference of the transient result and the HBM method, $\varepsilon = \left(\eta_{a,t} - \eta_{a,hb}\right)/\eta_{a,t}$; here indices t and hb present the transient and HBM results, respectively. The difference decreases with the increase in the number of harmonics, reducing to -0.2 % for the mean and -2.1 % for the first order.

Table 2 shows the comparison of required computational time for the explicit and implicit HBM simulation and the transient simulation. The explicit HBM simulation is more than ten times faster than the transient simulation. The implicit simulation is more than three times slower than the explicit HBM simulation, however, it is still 2.7 times faster than the transient simulation. The decrease in performance between the explicit and implicit approaches is caused by the high cost of solving the block system of equations. It should be noted that two harmonics were resolved in these simulations, and that using more than two harmonics would deteriorate the increase in speed. However, the motivation behind applying HBM to the field of naval hydrodynamics is to provide a trade-off between accuracy and performance by choosing the number of harmonics accordingly, rather than increasing the accuracy of the existing methods.

3.2 DTMB Wave Diffraction Test Case

Wave diffraction against a static DTMB ship model [1] is simulated using the implicit HBM and transient approaches. Regular waves are imposed, while induced longitudinal and vertical forces acting on the hull are measured and compared. The model is $L = 3.05$ m long with a velocity of $U = 1.52$ m/s corresponding to the Froude

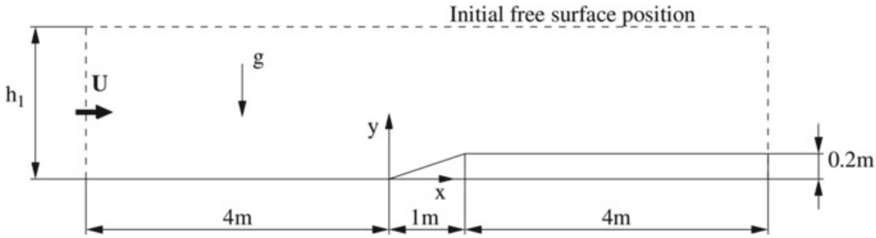


Fig. 1 2D ramp test case geometry

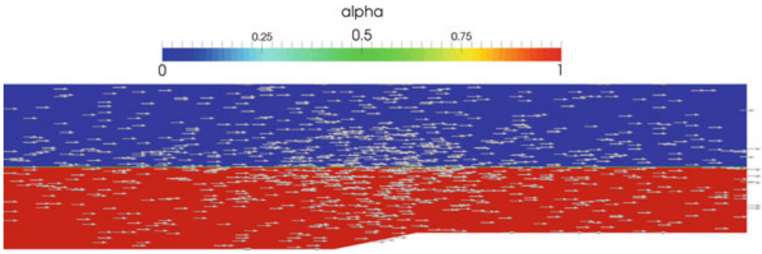
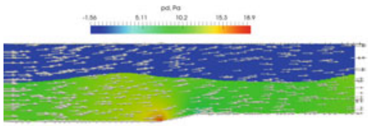
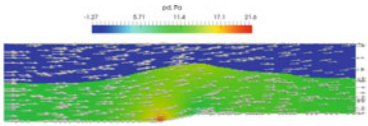


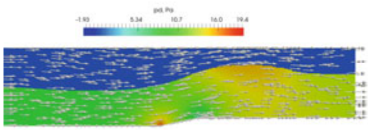
Fig. 2 Initial free surface and velocity in the ramp test case



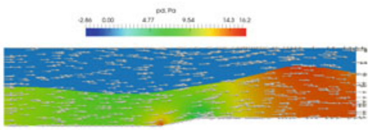
(a) $t = T/5$,



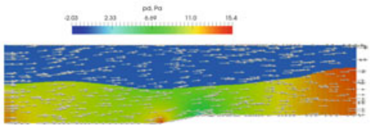
(b) $t = 2T/5$,



(c) $t = 3T/5$,



(d) $t = 4T/5$,



(e) $t = T$.

Fig. 3 Dynamic pressure and velocity distribution in the HBM simulation in discrete time instants

Table 1 Comparison of HBM and transient simulation results for the ramp test case

No. Harmonics	$\eta_{a,0}$ (m)	$\eta_{a,1}$ (m)	ε_0 (%)	ε_1 (%)
1	1.29035	0.179472	−1.6	−15.0
2	1.28485	0.186762	−1.2	−19.6
3	1.26487	0.175538	0.4	−12.5
4	1.27065	0.163556	−0.1	−4.8
5	1.27084	0.164377	−0.1	−5.3
6	1.27240	0.161346	−0.2	−3.4
7	1.27210	0.159447	−0.2	−2.1
8	1.27218	0.159308	−0.2	−2.1

Table 2 Comparison of computational time between the HBM and transient simulation for the ramp test case

Simulation type	CPU time (s)	Acceleration
Transient (10 periods)	5067	1
Explicit HB (2 harmonics)	488	10.4
Implicit HB (2 harmonics)	1851	2.7

number $F_r = 0.28$. The waves are $H = 0.036$ m high with a period of $T = 1.09$ s and wavelength $\lambda = 4.57$ m. Two harmonics are used for the HBM simulations, while 200 time steps per period are used in the transient simulation. 521,000 cells mesh is used in both the simulations. The wave-induced velocity is significant with respect to the ship model's velocity, therefore, implicit treatment of the HBM source terms must be used [4] in order to ensure numerical stability.

Figure 4 shows the convergence of the mean (zero) and first-order longitudinal forces and the first order of the vertical force, where N_{Iter} denotes the number of iterations. It can be seen that the forces converge smoothly. The mean of the vertical force is excluded, since it has a very large absolute value.

Figure 5 shows the comparison of the free surface elevation in the transient with the HBM simulation, where good correspondence can be observed. The colour scale represents the elevation of the free surface. The forces calculated on the hull of the model in the HBM and transient simulations are compared in Table 3, where $\varepsilon = (F_t - F_{hb})/F_t$ is given in percentages. Indices x and z denote the axis of force direction, while 0 and 1 denote zero- and first-order harmonic amplitudes. The difference is smaller than ≈10% for all items, the smallest difference being for the mean of vertical force $F_{z,0}$ (−0.11%), and the largest for the mean of the longitudinal force $F_{x,0}$ (−10.2%).

The comparison of the required computational time in the two simulations is shown in Table 4. The increase in speed is similar to that in the ramp test case with implicit coupling.

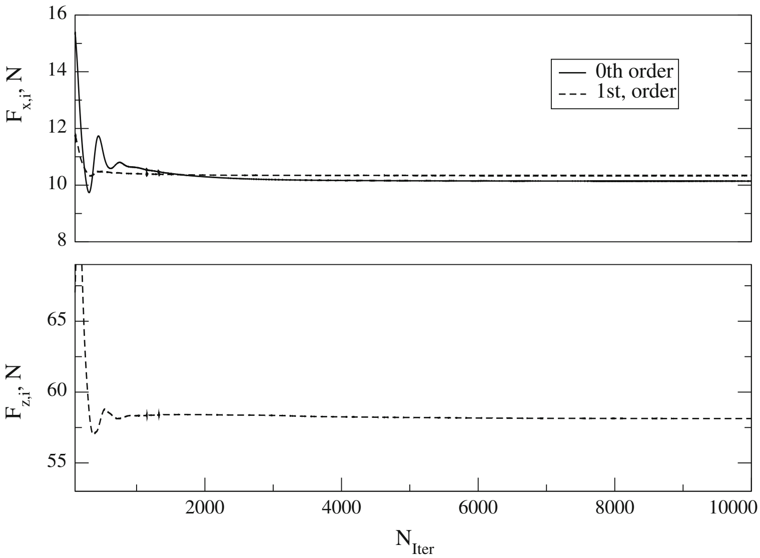
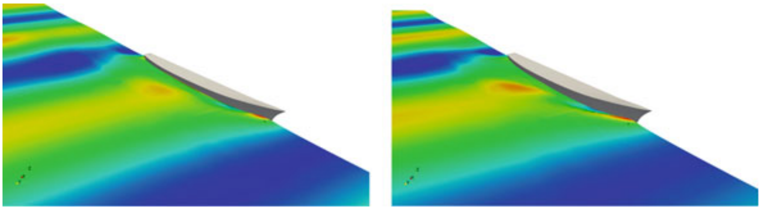


Fig. 4 Convergence of longitudinal and vertical forces acting on the DTMB hull in the HBM simulation



(a) HBM simulation, (b) Transient simulation.

Fig. 5 Free surface elevation in the HBM and transient wave diffraction simulations

Table 3 Comparison of diffraction forces in the HBM and transient simulations

Item	Transient	Harmonic balance	ε (%)
$F_{x,0}$, N	9.20	10.14	−10.2
$F_{x,1}$, N	10.70	10.34	3.36
$F_{z,0}$, N	784.88	785.72	−0.11
$F_{z,1}$, N	62.63	58.14	7.17

Table 4 Comparison of computational time between the HBM and transient simulations for the wave diffraction test case

Simulation type	CPU time (h)	Acceleration
Transient (20 periods)	18.86	1
Implicit HB (2 harmonics)	8.6	2.2

4 Conclusion

A Harmonic Balance Method applied to two-phase flows is presented in this paper with the application in the field of marine hydrodynamics. The method transforms transient periodic flows into a set of coupled steady-state problems, accelerating the calculations.

Two test cases are presented to validate the method: a 2D periodic two-phase flow over a ramp, and a ship model wave diffraction case in 3D. Both cases showed good agreement with the transient simulations with lower necessary computational time. The transient ramp test case simulation took ten times more computational time than the explicit HBM simulation, and 2.7 times more than the implicit HBM. The wave diffraction test case was simulated using an implicit HBM for reasons of numerical stability, with an acceleration by a factor of 2.2.

The implicit HBM is applicable for wave-related problems in naval hydrodynamic, however, larger computational savings were anticipated by the authors. The implicit treatment of coupling source terms exerts higher computational demands, reducing the efficiency of the method. Future efforts will be directed towards enhancing the efficiency of the implicit approach to achieve larger savings in computational resources.

Nonetheless, the method presents an attractive alternative for transient periodic flows with a free surface. Moreover, the steady-state formulation enables automatic optimisation techniques such as adjoint shape optimisation, presenting a new opportunity to optimise ships for added wave resistance in the future.

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