Actual Issues of Design and Production of Advanced Worm Gears

S. Lagutin, A. Sandler and E. Gudov

Abstract The advanced synthesis of a worm gear in real production should involve not only its geometrical and strength analysis according to the assigned performance characteristics. It must also provide compensation of inevitable manufacture errors, power and temperature deformations by localization of the bearing contact at the assigned area of the tooth flank. This paper considers the methods of solving this problem for both high-loaded worm gears and gearboxes of metallurgical equipment and precision gears of metal-cutting machine-tools.

Keywords Worm gears · Hob · Localization of bearing contact · Longitudinal and profile modification

1 Introduction

For all types of worm gear, including double-enveloping and spiroid ones, the gear ratio exceeds the ratio of the pitch diameters of the worm and gearwheel by a multiple factor. It allows for replacing a multi-stage gearbox with one compact-sized gear. That is why these gears are commonly applied in advanced mechanisms, in particular, in gearboxes for general engineering, screw-down mechanisms, roller conveyor drives for rolling mills, in kinematic chains of various machine-tools including those for gear-machining, etc.

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However, compact sizes provided by a mismatch of pitch surfaces of the worm and gearwheels with the gear axoids intrinsically causes a high sliding speed at contact and, therefore, significant friction losses, accelerated wear, necessity of applying antifriction materials, and low load-carrying capacity and efficiency, as compared to other types of gear. These drawbacks are especially vital at the initial stage of gearbox operation when inevitable manufacture and assembly errors, as well as power and temperature deformations, are shifting the bearing contact to one of the tooth edges during operation, thus increasing the contact stresses and deteriorating the formation of the lubrication layer.

These circumstances should be considered at the stage of design and compensated for in production. That is why the gearing should be designed as that approximated through the deliberate introduction of the parabolic function of errors of rotation angle for the driven element during meshing of one tooth pair. In practice, such a function is implemented by introducing longitudinal and profile modification of one or both conjugated surfaces, that is, their deliberate deviations from theoretically mutually enveloping ones.

The theoretical fundamentals of the local synthesis of general type worm gears are stated in [1–4]. Issues of computer-aided simulation of the localized contact of cylindrical worm gears are considered in [8, 14]. Different manufacturing methods of providing the localized contact, accounting for the functional purpose of the gear, are described in [5, 10–12, 15]. Long-term experience in the design and production of heavy-loaded worm gears at the Electrostal plant of heavy engineering (Moscow region) and of precision worm gears at Russian machine-tool building plants is summarized in the monograph [13].

The present paper proposes the English version of the most urgent chapters of this monograph.

2 The Functional Purpose of Worm Gears and an Integrated Approach to Their Design

Worm gears are divided into two types, according to their functional purpose: index worm gears and power wormgears.

Index worm gears are used in the kinematic chains of various machine tools. In gear-machining equipment that operates by the generating method, as well as in advanced machining centers with rotary tables, they are intended to transmit continuous rotation. These gears are manufactured for precision, as a rule, within degrees of accuracy of the 3rd–5th, according to the Russian Standard GOST 3675-81. Parameters of the kinematic accuracy for such gears have to be calculated according to the required accuracy rating of the machine-tool, or the allowable error of angular rotation of the working table. The smoothness and norms of contact are consequently assigned for the worm, the wheel and the gear as a whole. The same accuracy is required for gears of single rotation that are used in division mechanisms of machine-tools for profile gear grinding, the sharpening of the front surfaces of hobs, etc. The rotary tables for coordinate boring and milling machine-tools are made with index worm gears of 6th–7th degrees of accuracy. In all cases, a unified set of validation that determines the quality of worm gears is the area of bearing contact with its regulation of the tooth height and width.

Power worm gears are intended for transmitting the torque and power of the drive to actuators. They are manufactured, as a rule, with the accuracy provided by the available manufacturing equipment, and the quality of their production is also controlled according to norms of contact. The main parameters of design are the overall dimensions of the space assigned for the gearbox, load-carrying capacity (torque at the output shaft), and efficiency. It should be considered in design that under other equal conditions, the gears of the 6th–7th degrees of accuracy surpass the gears of the 10th degree by 1.5 times, according to their load-carrying capacity. Smoothness of operation, reducing the wear and increasing the lifetime of gear operation, is of great importance for power gears. In most loaded drives, it is reasonable to apply globoid (double-enveloping) gears.

The essence of designer and technological synthesis of the worm gear under real production conditions is as follows:

1. To assign the tasks to the designer and technologist within development of both working drawings of the gear and the set of technological documentation: the order of manufacturing processes, working drawings of hobs, tool layouts for machining of the gear, and sharpening and relieving for producing the generating worm of the hob.
2. According to the functional purpose of the gear, it is necessary to determine the tolerances for center distances in the assembled gear and machine-tool gearing, and hob parameters (pitch diameter, profile and lead angles of the generating worm).
3. For each step of generating the surfaces of threads for operating and generating worms, it is necessary to determine such layouts for grinding wheel dressing and mounting of its axis ultimately to provide the contact localization for active flanks within the assigned meshing zone.

3 Accuracy Standards and Contact Localization in Worm Gears

It is historically common for production of worm gears to make identical operating and machine-tool settings, that is, to make the shape and position of surfaces of the generating worm coincide with those of the operating one. And the strict conjugation of the operational gearing can be provided theoretically. This property is applied for production of precision gears.

However, inevitable manufacture and assembly errors during the initial period of gear operation cause a shift of the bearing contact to one of the tooth edges, thus abruptly increasing the contact stresses and deteriorating the conditions of formation of the lubrication layer. In this connection, the effective means of increasing the quality of gears is localization of the initial contact at the assigned area of the gearwheel tooth flank, thus eliminating the necessity for long-time running-in.

The technique of manufacture of worm gears with localized contact is based on modification of the parameters of the generating worm, which not only implies the difference in dimensions and parameters of the hob with respect to the operating worm, but adjusts the machine-tool settings for cutting the worm gearwheel.

Strictly speaking, the functionality of localization should not be determined according to the tooth profile or length, but with respect to the nominal line of the instantaneous contact. Localization along this line does not change the instantenous gear ratio, and it prevents the outrun of the bearing contact to tooth edges, completely or partially compensating for the negative influence of errors of the assembly, power and heat deformations of the casing.

Localization in the direction normal to the line of instantenous contact causes discontinuity of the instantenous gear ratio during the meshing of one tooth pair, but it smoothens the edge impacts within tooth re-conjugation, thus allowing for compensation of the pitch errors of gear-cutting and tooth deformation under load.

In gears with a different character of contact line, the profile and longitudinal modifications cause unequivalent effects. In double-enveloping worm gears, the contact lines make up the angle close to 90° with the vector of the worm's tangential velocity. That is why the profile modification mainly reduces the sensitivity to assembly errors and the longitudinal one smoothens the discontinuity of the gear ratio. Similar effects are present in gears with cylindrical worms of ZT type.

In worm gears with ruled **(the term "ruled worm" was introduced long ago and it is widely used in scientific literature—a comment, not to be included into the text)**, in particular, Archimedes worms, the contact lines are stretched along the tooth and the longitudinal modification of the tooth flank compensates for the errors of relative position of worm and gearwheel axes. The influence of pitch and profile errors is reduced through modification of the tooth flanks in the direction perpendicular to the contact lines, that is, through profile localization.

3.1 Basics of Profile Localization Analysis

The profile localization of contact in gears with cylindrical **ruled** (or close to them) worms is provided by deliberate deviation of the upper and lower parts of the tooth profile "into" the tooth solid, that is, by creation of the reduced concavity for the profile of a hob-generating surface with respect to the worm thread profile.

The pointed profiles do not coincide organically, since they are generated in different production operations: for the operating worm, it is the grinding of the thread flank; for the generating worm, it is the sharpening of the front surface and relieving of the flanks of the hob.

The task of designers is to choose the most efficient basic thread profile for the operating and generating worms and to regulate the value of the reduced concavity of the pointed profiles, as applied to a specific degree of gear accuracy. The technologist has to develop the methods for implementation of the desired localization.

Smoothness of operation for a worm gear depends on a combination of errors of the axial pitch of the worm threads f_{Pxr} and of the tangential pitch of the wheel teeth f_{Ptr}.

Negative values of f_{Pxr} and positive values of f_{Ptr} reduce the actual tooth pitch, that is, the distance along the line of action between the points of contact of the driving pair of teeth and the subsequent pair. Entry into the mesh will be accompanied here by a "rigid" impact of the thread addendum and tooth dedendum. The reverse combination of pitch errors causes a "soft" impact at re-conjugation, that is, the contact of the upper edge of the worm thread with the tooth dedendum outside the conventional line of action. This phenomenon is less risky, but it should be eliminated.

Taking into account the axial angle α of the thread profile, the total influence of errors f_{Pxr} and f_{Ptr} on deviation of the base pitch is determined by the square-law summation of their allowable values with high probability

$$f_P = \left(f_{Pt}^2 + f_{Px}^2\right)^{0.5} \cos\alpha. \tag{1}$$

The expression (1) determines the maximally sufficient value (the arrow) f_{max} of the reduced concavity f of axial profiles of the generating and operating worms. At the same time, since the value f affects the cyclic error of the tooth mesh frequency f_{zz0r} in a gear, under the most unfavorable combinations of errors (f_{Ptr} + f_{Pxr}), it should be related to the tolerance f_{zz0} for the cyclic error according to the inequality

$$(|f_{Ptr}| + |f_{Pxr}|)\cos\alpha - f \le f_{zz0}. \tag{2}$$

Hence, by identifying the pitch errors with the tolerances, let us determine the minimum required value of f_{min} when producing gears with the localized contact:

$$f_{min} = (|f_{Pt}| + |f_{Px}|)\cos\alpha - f_{zz0}. \tag{3}$$

In worm gears with localized contact, the parameter of operation smoothness f_{hsr} (the distance between the active flank of the worm thread and the coaxial generating surface of the hob applied to finish the machining of gearwheel teeth) becomes irrelevant, since it is replaced by the value f of the profile localization.

The authors found the relation between values of f_{max} and f_{min} determined by expressions (1) and (3) with tolerances f_{Pt}, f_{Px}, f_{zz0}, f_{hs} for different degrees of accuracy, according to the Russian standard GOST 3675-81. It is stated that the required value of the reduced concavity f should correspond to the range

$$f_{\mathrm{hs}} \leq f \leq 2.0 f_{\mathrm{hs}}. \tag{4}$$

The reduced concavity of the hob-generating surface with respect to the worm thread profile is formed due to the concavity of the front surface of teeth obtained during sharpening (Fig. 1) and the concavity of the profile of the relieved tooth flanks of the hob with respect to the profile of the ground worm thread (Fig. 2).

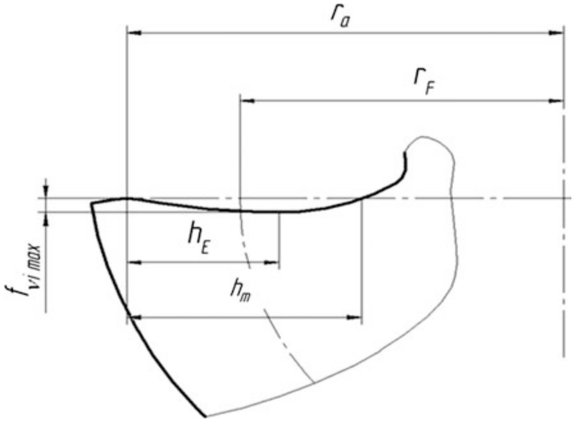

Fig. 1 Profile of the generating helical front surface of the hob teeth

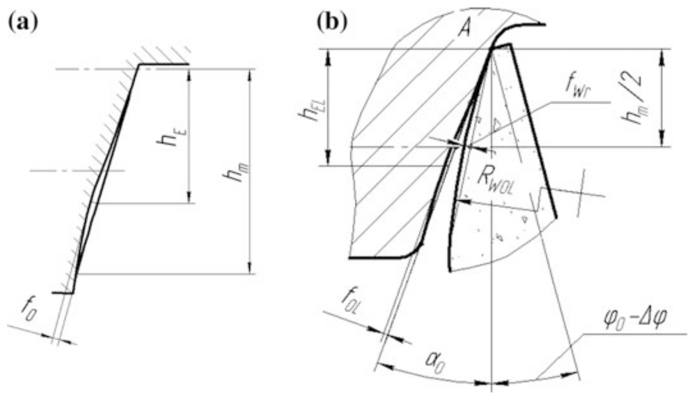

Fig. 2 Profiles of relieved surfaces of the hob teeth in the case of localized contact

Figure 1 presents the front surface profile of the hob with the following parameters:

m is the axial (or normal) module of the hob;
r_F is the radius of the pitch cylinder of the generating worm;
r_a is the radius of the outer cylinder of the active profile of the hob tooth;
h_m = 2 m is the height of the active profile of the hob tooth;
$f_{vi\ max}$ is the maximum deviation of the generatrix from the radial straight line;
$h_E = 2h_m/3$ is the distance from the boundary of the active segment of the profile to the point of maximum concavity of the generating line of the front surface.

Figure 2a shows the profile of the relieved surface for which the helix angle λ_r is greater than the helix angle λ for the thread of the generating surface (the right flank of the tooth of the right-hand hob). In this case, the distance from the internal cylinder of the active flank of the thread to the maximum value of the concavity is $h_E = 2h_m/3$. In Fig. 2b, the helix angle λ_r of the shank is less than the angle λ (the left flank of the tooth of the right-hand hob), in this case, $h_{EL} \approx 0.5774h_m$.

The concavity of the profile of flanks for the hob teeth is provided by adjusting the set-up parameters for the radial-axial relief [11]. In this case, different adjusting methods are used for two flanks of the tooth, which is why coordinates of maximums for the concavity of profiles of the front and relieved surfaces do not coincide and are separated by distance: ≈0.67 m for one flank of the tooth and ≈0.49 m for the other flank.

The influence of both components on the value of the reduced concavity of the profile of the generating surface for the hob should be considered when producing worm gears with the 3rd–5th degrees of accuracy.

The influence of the deviation f_{vi} of the generating line for the front surface of teeth on the axial profile of the generating surface will be related here with the radial relief angle $\lambda_b = |\lambda_r - \lambda|$ by the expression

$$f(f_{vi}) = f_{vi} \tan \lambda_b \tag{5}$$

For the flank of the hob teeth where the angle $\lambda_r > \lambda$, the optimal value is $\lambda_b \approx 5°$, and for the opposite flank, it is $\lambda_b \approx 7°$.

For worm gears of the 6th and lower quality degrees of accuracy, the required value of the profile localization is provided only as a result of relieving the hob teeth, since the increase in the allowable value of the concavity of the profile of the generating surface leads to an essential decrease in the influence of the concavity of the generating surface for the front flank of the tooth.

Analysis and implementation of manufacturing parameters of certain processes (grinding of worm threads, sharpening and relieving of the hob teeth, including the techniques for analysis of set-up parameters for worm-grinding, sharpening and relieving machines) are stated in detail in [13].

3.2 Alternative of Profile Modification

The profile localization of the contact in a worm gear requires the science-intense analysis of machine-tool settings and grinding wheel profiling at grinding operations for helical and relieved surfaces of the worm and the hob. In this relation, the authors have developed and tested an alternative method for providing the smoothness of worm gear operation based on a combination of tolerances for manufacture of the worm and the hob.

Parameters of smoothness include the parameter of identity f_{hs}, pitch errors f_{px} of the worm threads and f_{pt} of the gearwheel teeth, and allowable deviations of the profile f_{f1} of the worm thread and f_{f2} of the gearwheel tooth. The mutual relation of these parameters should be considered when designing both worms and hobs. In particular, it is necessary to consider the influence of the difference in angles of the worm and hob profiles on the combination of base pitches.

In [9], it was shown that in order to provide smoothness of gear operation, the base pitch of the driving element should be greater than that of the driven one. As applied to worm gears, this means that the base pitch of the worm p_b should be greater than the base pitch of the gearwheel, that is, the gearwheel should be cut by the hob with the pitch $p_{b0} < p_b$.

Sources of the error f_{pbr} of the base pitch of the worm are the error f_{pxr} of its axial pitch p_x and deviation of the angle $\Delta\alpha_1$ of the thread profile.

$$f_{pbr} = f_{pxr}\cos\alpha_1 - \pi m \sin\alpha_1 \Delta\alpha_1. \tag{6}$$

With a worm as the driving element of the gear, the increase in the base pitch is inadmissible, that is, the condition $f_{pbr} \geq 0$ should be fulfilled. It follows from (6) that the following condition should be fulfilled for this purpose:

$$\Delta\alpha_1 \leq f_{pxr}/(\pi m \tan\alpha_1). \tag{7}$$

That is, in order to compensate for the negative value of the error of the base pitch f_{pxr} of the worm, it is necessary to assign the tolerance $\delta\alpha_1$ for the pressure angle of its thread only as a negative deviation and to determine it through the expression

$$\delta\alpha_1 \leq -f_{px}/(\pi m \tan\alpha_1). \tag{8}$$

The sign of equality in expression (8) determines the upper value of the tolerance $\delta\alpha_1^{\max} = -f_{px}/(\pi m \tan\alpha_1)$ for the pressure angle of the worm thread.

The lower value of the tolerance $\delta\alpha_1^{min}$ is determined by the tolerance range for the worm thread profile f_{f1} provided by standards. Hence:

$$\delta\alpha_1^{\min} = -f_{\mathrm{px}}/(\pi m \tan\alpha_1) - f_{\mathrm{f1}}\cos\alpha_1/2m. \tag{9}$$

Positive values of errors f_{ptr} of the tangential pitch of the gearwheel and f_{pxr0} of the axial pitch of the hob increase the base pitch of the gearwheel. It is also increased while decreasing the pressure angle of the generating worm during re-sharpening of the hob. The influence of these factors should be compensated for by decreasing the base pitch of the hob through assigning positive deviations (both the upper and the lower) for the angle α_0 of the profile of relieved flanks of teeth.

When designing a hob, the upper value of this tolerance $\delta\alpha_0^{\max}$ should be calculated according to the formula

$$\delta\alpha_0^{\max} = \left(f_{\mathrm{px}} + f_{\mathrm{pt}}\right)/(\pi m \tan\alpha_1) + f_{\mathrm{f2}}\cos\alpha_1/(3m), \tag{10}$$

where f_{px} is the positive part of the tolerance for the pitch error of the generating worm taken to be equal to the pitch tolerance of the worm, and f_{pt} is the positive part of the tolerance for the tangential pitch of the gearwheel.

Formula (10) implies that the 1/3 part of the tolerance f_{f2} for the gearwheel tooth profile should be related to the error appearing due to faceting.

The first component of expression (10) determines the minimum value of the angle α_0 necessary to provide the smoothness of meshing. It corresponds to the deviation of the pressure angle for a completely re-sharpened hob.

$$\delta\alpha_{0c}^{\min} = \left(f_{\mathrm{px}} + f_{\mathrm{pt}}\right)/(\pi m \tan\alpha_1). \tag{11}$$

The second component of expression (10) determines the sum of the tolerances for manufacture and for variation of the angle α_0 during re-sharpening.

It is reasonable to distribute the value f_{f2} cos $\alpha_1/3m$ between the pointed tolerances for manufacture and re-sharpening in equal portions. Then, ***when manufacturing a new hob***, the upper limit of the tolerance for the angle α_0 should be:

$$\delta\alpha_0^{\max} = \left(f_{\mathrm{px}} + f_{\mathrm{pt}}\right)/(\pi m \tan\alpha_1) + f_{\mathrm{f2}}\cos\alpha_1/6m. \tag{12}$$

The allowable number for re-sharpening is calculated through the remaining value of the tolerance $\delta\alpha_{0i} = f_{\mathrm{f2}}$ cos $\alpha_1/6m$. Thus, when designing the worms and hobs, the necessary clearance Z_z in the meshing during the instance of teeth re-conjugation is stated to provide the smoothness of gear operation—causing the base pitch for the driving element to exceed the base pitch of the driven one:

$$Z_z = \left(2f_{\mathrm{px}} + f_{\mathrm{pt}}\right)\cos\alpha_1. \tag{13}$$

If the pressure angle of the generating surface is assigned for the applied hob tolerances for deviations of the pressure angle of the worm thread, it should be analyzed according to (8) and (9), and after that, the profile of the helical surface of the worm thread should be corrected.

Therefore, in addition to the requirements stated in the Russian standard GOST 3675-81, it is necessary to regulate the following parameters of the worm and the hob:

- Upper and lower allowable deviations of the pressure angle of the operating worm, which should be negative deviations with respect to the nominal value;
- Upper and lower allowable deviations of the pressure angle of lateral flanks of teeth during manufacturing and re-sharpening of the hob, with both deviations being positive with respect to the nominal value.

3.3 Analysis of Parameters and Technique of Longitudinal Localization

The theoretical lines of instantenous contact in a completely conjugated worm gear with a cylindrical, in particular, Archimedes worm are stretched along the gear-wheel tooth and over its whole width. In real gears, the total bearing contact should be concentrated in the middle part of the tooth, so that the assembly errors will not shift the contact to the boundaries (edges) of the gear rim. Modification of the gearwheel tooth flank along the tooth length (called longitudinal localization) is aimed at compensation for these errors.

The most common methods for longitudinal localization of the contact for worm gears of the 6th–12th degrees of accuracy is the application of hobs having an increased diameter [5, 12–15]. And it is important to assign such a diameter of the hob and manufacturing tolerances that allow for compensating the manufacturing errors of gearbox parts and for obtaining the gear with a specific degree of accuracy and the required quality parameters.

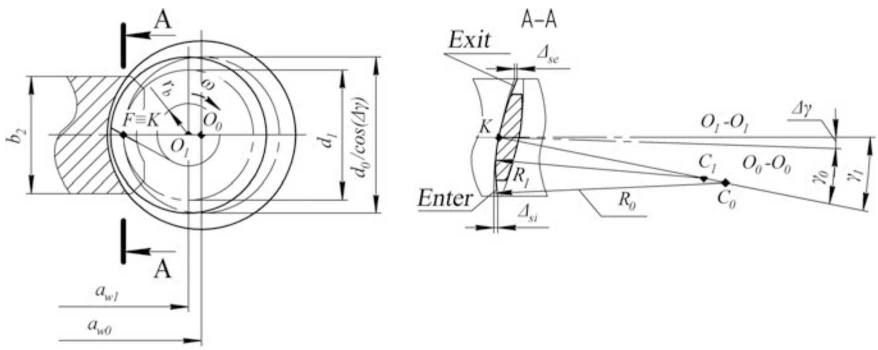

Fig. 3 The combined scheme of the operating and machine-tool meshing for the *centre* of the bearing contact arranged on the *middle* plane of the gear

3.3.1 Machine-Tool Meshing for Application of Hobs with the Increased Diameters

Figure 3 shows the combined scheme of meshing of the orthogonal worm gear and the machining of gearwheel teeth by a hob with an increased diameter. The diameter d_0 of the pitch cylinder of the hob is greater than the diameter d_1 of the pitch cylinder of the worm, however, these cylinders make contact with each other at the pitch point F. Let us consider the version of the arrangement of the nominal centre of the bearing contact (the point K) on the middle plane of the gear.

The left side of this figure shows the section of the gear by a plane perpendicular to the worm axis and passing through the gearwheel axis. The section A–A is drawn tangentially to the pitch cylinder of the worm (and the hob) through the straight line parallel to the gearwheel axis at the distance equal to the radius $r_2 = 0.5m_x z_2$ of the pitch cylinder of the gearwheel.

This straight line is the axis of meshing that possesses a number of properties [4]:

(1) In the case of linear, as well as localized, contact, all contact normal lines of the gear intersect this axis;
(2) In a completely conjugated gear, this axis definitely lies on the surface of the meshing, which is a set of lines of instantenous contact;
(3) In a gear with longitudinally localized contact, the points of instantenous contact of active flanks generate the line of meshing in a fixed space, the line intersecting the pointed axis by necessity.

Due to these properties, when synthesizing a gear with longitudinal localization, it is necessary to choose the centre K of the bearing contact exactly on this axis: either on the middle plane of the gear by aligning it with the point F, or by shifting it with respect to F towards the worm leaving the meshing.

To synthesize such a gear, it is necessary and sufficient to rotate the axis of the hob so that its generating surface could contact the active flank of the worm exactly at this point, that is, normal lines to two surfaces should coincide at the point K.

The normal line to any helical surface is skewed with its axis at a distance equal to the radius r_b of the base cylinder of the equivalent involute worm:

$$r_b = 0.5m_{x1}z_1/(\tan^2\alpha_{x1} + \tan^2\gamma_1)^{0.5} \tag{14}$$

That is why the straight line passing through the point K tangential to the circle with the radius r_b represents a projection of the common normal line at the design point. Simultaneously, this tangent line shows the direction of displacement of the centre of the bearing contact along the tooth flank at worm rotation.

In the picture on the right side of Fig. 3, the axis of the worm O_1–O_1 is arranged horizontally, and the axis of the hob O_0–O_0 is rotated with respect to it at an angle $\Delta\gamma$. The shaded area is the section of the worm thread by the plane A–A tangential to the pitch cylinder of the worm. This section is in contact with the section of the thread of the hob-generating surface by the same plane at the design point K.

The straight line KC_0 represents the common normal line to longitudinal profiles of threads. This line makes up the angle γ_1 (equal to the pitch lead angle of the worm) with the worm axis O_1–O_1 and the angle γ_0 (equal to the pitch lead angle of the hob-generating surface) with the projection of the hob axis. Points C_1 and C_0 represent the centers of curvature of the considered sections of the worm and the hob, and the distances $C_1K = R_1$ and $C_0K = R_0$ are the radii of curvature of the pointed sections. Values R_1, R_0 are determined by the expressions

$$R_1 = 0.5d_1/(\tan\alpha_{x1}\cos^3\gamma_1), R_0 = 0.5d_0/(\tan\alpha_{x0}\cos^3\gamma_0) \tag{15}$$

where α_{x1}, α_{x0} are axial pressure angles of the worm thread and the generating surface of the hob, respectively.

In order to arrange the design point K on the pitch cylinder of the worm at the middle plane of the gear, the hob axis should be rotated towards increasing the tooth helix angle at an angle $\Delta\gamma$ equal to

$$\Delta\gamma = \gamma_1 - \gamma_0. \tag{16}$$

When the worm is rotating in the direction shown by the arrow ω, the active part is that side of the profile which passes through the point K; and the "*Enter*" and "*Exit*" points of the worm out of the meshing are shown on the right side of Fig. 3.

The arrow of the convexity (curvature) δ_1 of the surface of the worm thread along the gearwheel tooth in the considered section is determined to be

$$\delta_1 = b_2^2/(8R_1\cos^2\gamma_1)\,. \tag{17}$$

As for the generating surface of the hob having an increased diameter due to the increase of the curvature radius of the thread by the value $\Delta R = R_0 - R_1$, the arrow of the convexity in the same section will be decreased by the value

$$\Delta\delta = -b_2^2\Delta R/(8R_1^2\cos^2\gamma_1), \tag{18}$$

where the minus sign shows that deviations of $\Delta\delta$ and ΔR have opposite signs.

3.3.2 Compensation for the Influence of Errors of Assembly and Machine-Tool Meshing

Standards for the accuracy of the worm gears with non-adjustable arrangement of axes, including the Russian standard GOST 3675-81, assign the allowable values for the following errors: deviations of the center distance f_{ar} in the gear (worm – worm gearwheel) and f_{acr} at machining (hob - gearwheel); deviations of the cross angle $f_{\sum r}$ in the gear and $f_{\sum cr}$ at machining; shifts of the middle plane of the gearwheel f_{xr} in the gear and f_{xcr} at machining.

The technique for calculating the increase in the pitch diameter of the hob with respect to the operating worm necessary to compensate each of the pointed pairs of errors is given below; and limitations imposed on this parameter by the assigned value of the bearing contact along the gearwheel width are considered.

The error of the gear center distance depends on the accuracy of producing the bores for the worm and gearwheel shaft supports in the casing of the gearbox. The error of the machine-tool center distance is determined by the accuracy of the gear-milling machine-tool, by the manufacture accuracy of the master worm with the thickness of its thread, considering the assigned backlash in the gear, and by tolerances for the center distance adjusted according to the master worm.

That is why, at equal pitch diameters d_1 of the operating worm and d_0 of the generating worm, the actual values of center distances in the gear a_{w1} and in machine-tool meshing a_{w0} differ from each other. Let us determine the influence of this difference on the formation of contact along the helical surface of the thread.

As is known, one of the properties for the helical surface is the increase of its curvature and helix angle with decrease of the current radius. Therefore, the decrease in a_{w0} with respect to a_{w1} is the most risky at formation of the bearing contact. In this case, the generating worm forms a more concave tooth surface than the surface enveloped by the thread of the operating worm with the axis shifted from the gearwheel axis at a greater distance. The initial contact of active flanks is valid for that face end of the gearwheel where the worm thread enters the meshing.

To make the curvature of the helical surface of the generating worm less than the curvature of the thread of the operating worm under the most unfavorable combination of errors, it is necessary to increase the pitch diameter d_0 of the hob and to provide the nominal of the center distance a_{w0} equal to

$$a_{w0} = a_{w1} + 0.5(d_0 - d_1). \tag{19}$$

For a gear of the assigned degree of accuracy, the maximum value of the error of the center distance does not exceed the sum of absolute magnitudes of allowable values of components:

$$f_a + f_{ac} = 1.75 f_a. \tag{20}$$

Having assigned the value $(a_{w0} - a_{w1}) = 1.75 f_a$, we obtain the required increment Δd_{0a} of the diameter d_0 of the hob with respect to the diameter d_1 of the worm:

$$\Delta d_{0a} = (d_0 - d_1) = 3.5 f_a. \tag{21}$$

The error $f_{\sum r}$ of the cross angle of the gear occurs when manufacturing the gearbox casing due to deviation from the perpendicularity of the axes of the bores for the worm and gearwheel supports (or the assigned value of the angle between them). According to the Russian standard GOST 3675-81, the symmetrical tolerance for this deviation is assigned by the linear value $f_{\sum}$ at the width b_2 of the gearwheel rim. The error $f_{\sum cr}$ of the cross angle at machining occurs at the gear-milling machine-tool, first, due to mounting errors of the hob axis, and second, due to non-perpendicularity of the axis of the mounting bore of the gearwheel and its mounting face end. The standard assigns the symmetrical (±) tolerance for deviation of the cross angle at machining $f_{\sum c} = 0.75 f_{\sum}$ for these two components.

The gearbox casing, the gearwheel and the hob are produced by machine-tools with differently-directed errors. It is impossible to control the mutual compensation of these errors. Values of $f_{\sum}$ stated by the Russian standard do not provide the contact concentration at the middle of the gearwheel tooth. For the 6th–12th degrees of accuracy, the absolute values $f_{\sum}$ vary from 9 to 160 μm, essentially exceeding the thickness (≤ 5 μm) of the paint layer fixing the location and length of the bearing contact at the inspection and measuring machine.

That is why, practically speaking, the only means of compensation of the total error of the cross angle for stable localization of the contact is the longitudinal modification of the gearwheel teeth through application of hobs having the increased diameter of the generating surface. In order to prevent the shift of the bearing contact to the face edges of the teeth, the arrow of concavity for the surface of the generating worm along the tooth length should be decreased by the value Δδ.

By equating Δδ to the absolute value of the tolerance $\left| f_{\sum} \right|$, and accounting for (18), let us determine the value of the necessary increase in the diameter d_0 of the hob with respect to the diameter d_1 of the worm:

$$\Delta d_{0\sum} = (d_0 - d_1) = 4 f_{\sum} d_1^2 / (b_2^2 \tan \alpha_{x1} \cos \gamma_1). \tag{22}$$

The value $f_{\sum}$ in formula (22) surpasses the influence of angular errors of mounting the gearwheel and the hob axis over gear-cutting operations.

The shift f_{xr} of the middle plane of the worm gearwheel in the gear is limited by the symmetrical tolerance $\pm f_x$ and it is formed by design dimension chains of the worm gearbox. The shift f_{xcr} of the middle plane of the gearwheel at machining is limited by the tolerance $\pm f_{xc} = 0.75 f_x$ and it is determined by a manufacturing dimension chain for the setting of the gear-milling machine-tool. Methods for the valid control of these parameters at manufacturing have not yet been developed.

In this connection, values of tolerances $\pm f_x$ and corresponding values of $\pm f_{xc}$ given in the Russian standard GOST 3675-81 are applied as allowable errors of the master link in calculation of the pointed dimension chains.

The shift of the middle plane is controlled according to the dimension of the total bearing contact given in the same standard.

In the practice of single or low-batch production, the quality of this parameter is achieved through application of compensators in the layout of the gearwheel shaft assembly. Compensators are bushings or rings, their length being machined by measurements. One of compensators “helps” the gearwheel approach the position at which the required bearing contact along the tooth length is formed. The second compensator closes the axial play of the trust bearing of the gearwheel shaft.

For series production of large gears having the 6th–12th degrees of accuracy, for compensation of errors f_{xr} and f_{xcr}, it is also reasonable to implement the increase in the diameter of the hob with respect to the worm diameter on the value Δd_{0x}:

$$\Delta d_{0x} = 2f_x d_1/(K_b b_2), \tag{23}$$

where K_b is the design factor of the relative length of the bearing contact.

Since tolerances for errors f_{xr} and f_{xcr} are symmetrical, the value f_x in (23) should be taken to be equal to the absolute value of the tolerance $|f_x|$. Values of the factor K_b, according to the Russian standard GOST 3675-81 for different degrees of accuracy, are given in Table 1. When calculating the value Δd_{0x}, the greatest value of K_b shown in the table should be used.

When applying the hob with the increased diameter, it is necessary to provide the equality of base pitches (or basic modules $m_{b1} = m_{b0}$) of threads of the operating and generating worms by correction of the worm thread parameters. For Archimedes gears, the equality of axial modules $m_{x1} = m_{x0}$ should be provided at the same time. In this case, the values of axial pressure angles and pitch lead angles of threads are related by the equation

$$\tan^2\alpha_{x1} + \tan^2\gamma_1 = \tan^2\alpha_{x0} + \tan^2\gamma_0. \tag{24}$$

Methods for correction of the worm (or hob) axial profile providing this equality are stated in greater detail in Sect. 4.

When choosing the hob diameter, it is necessary to consider the value of the backlash Z_b between active surfaces at the boundary of the bearing contact. This backlash should not exceed the thickness of the paint layer fixing the bearing contact at the control and measuring machine. By assigning the boundary of the bearing contact at the distance ($K_b b_2/2$) from the middle plane of the gearwheel, let us determine the value Z_b at the worm pitch cylinder, accounting for the hob parameters:

$$Z_b = 0.25(K_b b_2)^2[(\sin\alpha_{x1}\cos^3\gamma_1)/d_1 - (\sin\alpha_{x0}\cos^3\gamma_0)/d_0]. \tag{25}$$

Having assigned the ultimate value of the clearance $Z_b \leq 5$ mcm, let us determine, from (25), the maximum allowable increase Δd_{0m} in the pitch diameter of the hob d_0 with respect to the worm diameter d_1:

$$\Delta d_{0m} = d_0 - d_1 = 4Z_b d_1^2/(K_b^2 b_2^2 \sin\alpha_{x1}\cos^3\gamma_1), \tag{26}$$

where the least value of the factor K_b (shown in Table 1) should be taken.

Table 1 Design factors K_b of the length of the bearing contact

Degree of accuracy	6, 7	8, 9	10	11, 12
Factor K_b	0.54–0.6	0.43–0.5	0.34–0.4	0.2–0.3

Analysis of the design values of the necessary increase Δd_{0a}, $\Delta d_{0\sum}$, Δd_{0x} in the pitch diameter d_0 of the hob depending on the required degree of accuracy (6th–12th) of gear production [12] showed the following:

1. The increase Δd_0 in the pitch diameter of the hob relative to the operating worm should be considered according to the condition of obtaining the minimum necessary dimension of the length of the bearing contact for the maximum backlash at its boundary (no more than 0.005 mm).
2. For gears of the 9th–10th degrees of accuracy, the allowable values of $[f_a]$ and $[f_{ac}]$ should be stiffened, as compared to the Russian standard GOST 3675-81, proportionally to the allowable value of the increase Δd_{0m} in the hob diameter:

$$[f_a] = \Delta d_{0m}/3.5. \tag{27}$$

3. For gears of the 9th–12th degrees of accuracy assembled without compensators, the tolerances f_x for the shift of the middle plane should be stiffened in manufacturing dimension chains according to the condition

$$[f_x] = \Delta d_{0m} K_b b_2/(2d_1). \tag{28}$$

3.3.3 Features of Contact Localization in Precision Gears

Precision worm gears are mainly applied in kinematic chains of gear-machining machine-tools. The accuracy of such gears should be two degrees higher than the accuracy of the machined parts and it should correspond to the 3rd–5th degrees of accuracy. The edge contact is inadmissible, so both longitudinal and profile localizations are necessary.

The finishing machining of the precision gear is implemented by master machine-tools that do not allow for varying the angle of the hob axis setting. That is why the pitch diameter of the finishing hob should be equal to the pitch diameter of the operating worm. Under such conditions, the contact localization along the tooth length is made by an additional machining of some segments of tooth flanks about 10–15% of the face width from both of its sides. This eliminates the possibility of edge contact near the gearwheel face ends.

Moreover, in order to provide longitudinal localization, the nominal of the machine-tool center distance at gear machining is taken to be equal to $a_{w0} = a_w + f_{ac}$, and when manufacturing the gear casing, only the negative tolerance f_a is assigned for the center distance a_w of bores for the worm and gearwheel supports.

The profile localization of the bearing contact is provided in a similar fashion, as was described in Sect. 3.1.

4 The Principle of Priority of Gear-Cutting Tools in the Individual Production of Power Gears

The need to manufacture single samples of worm gears, including those for spare parts for the existing equipment, initiated the requirement for designers to widen the range of machined products without widening the range of available tools.

When there is no hob with parameters identical to the parameters of the design drawing, in order to obtain the conjugated gear, it is necessary to follow the principle of priority of the gear-cutting tool. This principle implies the selection of a hob that has a close (greater in priority) diameter and the correction of parameters of the operating worm. The correction of the worm module and/or the pressure angle should provide the equality of base pitches at the design point. The calculation of the hob-setting parameters should provide the longitudinal localization.

Experience shows that proper selection of the hob from the available range and obtainment of the localized contact in a gear is a task that is solvable in practice. First of all, such a hob should be selected from the list of available ones that has the base pitch p_{b0} maximally close to the base pitch p_{b1} of the worm. Here, one can consider both hobs with the Archimedes generating surface intended for Archimedes wormgears and hobs according to the Russian standard GOST 9324-80 "Finishing single-thread hobs for cylindrical gears with the involute profile".

4.1 Selection of Hob Parameters

The initial information needed for the analysis is the assigned parameters of the gear. As for the gears with the Archimedes worms most common in practice, these parameters are: the axial module m_{x1}, the number of threads z_1, the pitch diameter of the worm d_1 and the pressure angle α_{x1} at the axial section of the worm.

The base pitch of the Archimedes worm is determined by the expression

$$p_{b1} = \pi m_{x1} \cos\gamma_1 \cos\alpha_{n1}, \tag{29}$$

where α_{n1} = atan (tan α_{x1} cos γ_1) is the pressure angle of the worm at the normal section, and γ_1 = atan ($m_{x1}z_1/d_1$) is the pitch lead angle of the thread.

The main parameters of the hob are: the number of threads z_0 and the pitch diameter d_0; the module m_{x0} and the pressure angle α_{x0} at the axial section for Archimedes hobs; the module m_{n0} and the pressure angle α_{n0} at the normal section for involute hobs, the pitch helix angle γ_0:

$$\gamma_0 = \mathrm{atan}(m_{x0}z_0/d_0) = \mathrm{asin}(m_{n0}z_0/d_0). \tag{30}$$

For both types of hobs, the parameters of the axial and normal sections are mutually related by the known expressions

$$m_{n0} = m_{x0} \cos\gamma_0, \quad \tan\alpha_{n0} = \tan\alpha_{x0}\cos\gamma_0. \tag{31}$$

For the Archimedes hob, the base pitch is determined as

$$p_{b0} = \pi m_{x0}\cos\gamma_0\cos\alpha_{n0}. \tag{32}$$

The base pitch of the hob for cutting involute cylindrical gearwheels is not related to the hob diameter and the pitch lead angle of the thread:

$$p_{b0} = \pi m_{n0}\cos\alpha_{n0}. \tag{33}$$

In any case, in order to provide the conjugacy of elements of the obtained gear, the following condition should be strictly fulfilled:

$$p_{b0} = p_{b1}. \tag{34}$$

4.2 Worm Correction in the Application of Archimedes Hobs

The requirement of equality for the base pitches of the Archimedes worm and the Archimedes hob means that the following condition should be strictly fulfilled:

$$m_{x0}\cos\gamma_0\cos\alpha_{n0} = m_{x1}\cos\gamma_1\cos\alpha_{n1}. \tag{35}$$

We consider the parameters of the hob $m_{x0,}$ γ_0 and α_{x0} as being the assigned ones, and the parameters of the worm m_{x1} and α_{x1} can be varied within certain limits, keeping its pitch diameter and the helix angle γ_1 of the thread. The axial profile m_{x1} of the worm can be kept equal to the initial one, or it should be varied discretely based on setting possibilities of the worm grinding machine-tool. That is why the angle of the worm profile should be considered to be the main, continuously varied parameter. This angle α_{n1} is determined from (35) at the normal section

$$\cos\alpha_{n1} = m_{x0}\cos\alpha_{n0}\cos\gamma_0/(m_{x1}\cos\gamma_1). \tag{36}$$

Then, the axial pressure angle α_{x1} is determined by the formula

$$\tan\alpha_{x1} = \tan\alpha_{n1}/\cos\gamma_1. \tag{37}$$

Strictly speaking, condition (37) determines the conjugacy of the active flanks of the worm thread and the gearwheel teeth only within the differential vicinity of the design point of contact. In order to provide the conjugacy of these surfaces along the entire height of the thread, the worm should be made quasi-Archimedes and its axial profile should be outlined along the convex curve described by the expression

$$x(r) = \int_{r_{p1}} \tan \alpha_x(r) \partial r, \tag{38}$$

where r is the current radius at the active thread profile, which varies within the range from r_{p1} to r_{a1};

$r_{p1} = a_w - 0.5 d_{a2} \cos(\alpha_{a2} - \alpha_{x1})$ is the radius of the lower boundary point of the active profile; and $r_{a1} = d_{a1}/2$ is the outer radius of the worm.

The current value of the axial pressure angle α_x is determined from (37) for the current value r and the corresponding lead angles of the worm threads $\gamma_1(r) = \text{atan}\,(m_{x1}z_1/2r)$ and the hob $\gamma_0(r) = \text{atan}[m_{x0}z_0/(2r - d_1 + d_0)]$.

The axial profile of the modified worm is shown in Fig. 4. Its curvature at the current point M is determined by the expression

$$K(r) = \frac{\partial}{\partial r}[\alpha_x(r)] \cos \alpha_x(r). \tag{39}$$

The required axial profile can be approximated by the arc of a circle, parabola or ellipse, depending on the type of contour follower applied at the worm grinding machine-tool. Its arrow of the convexity f_n is determined by the following expression with sufficient accuracy for practical purposes:

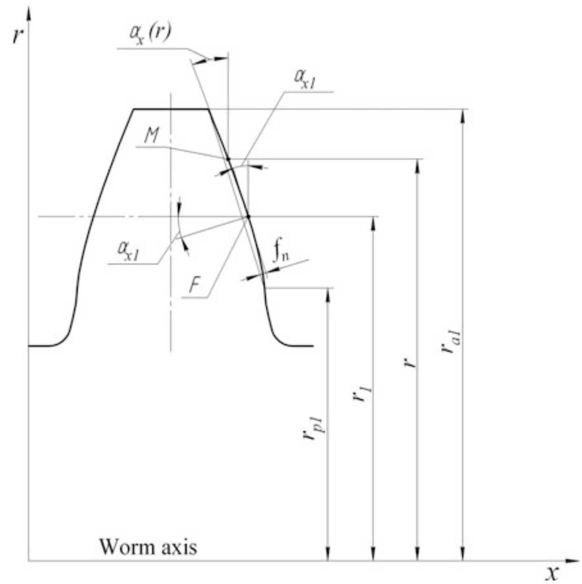

Fig. 4 The axial profile of the thread of the modified worm

$$f_{\mathrm{n}} \geq m_{\mathrm{x}}^2 K(r_1) / \left(2\cos^2\alpha_{\mathrm{x}1}\right), \tag{40}$$

where $K(r_1)$ is the curvature calculated according to (39) for the radius $r = r_1$.

The sign > in this expression means that the bearing contact in the gear will be localized along the height of the tooth, thus allowing for smoothening of the negative influence of errors of the pitch and profile on the smoothness of gear operation.

Note that the required convexity of the axial profile is, as a rule, less than that of the organic error of grinding by the conical grinding wheel.

In the case when the axial modules of the worm $m_{\mathrm{x}1}$ and the hob $m_{\mathrm{x}0}$ are equal to each other, the expression (36) is simplified to

$$\cos\alpha_{\mathrm{n}1} = \cos\alpha_{\mathrm{n}0}\cos\gamma_0 / \cos\gamma_1, \tag{41}$$

and the angle $\alpha_{\mathrm{x}1}$ is directly related to the angles $\alpha_{\mathrm{x}0}$, γ_1 and γ_0 by the formula

$$\tan\alpha_{x1} = \sqrt{\tan^2\alpha_{x0} - \tan^2\gamma_1 + \tan^2\gamma_0}, \tag{42}$$

and, revealing the value of the derivative $d[\alpha_{\mathrm{x}}(r)]/dr$ at the design point, formula (40) can be reduced to the following:

$$f_{\mathrm{n}} \geq m_{\mathrm{x}}\cos^2\alpha_{\mathrm{x}1}\left(\tan^3\gamma_1 - \tan^3\gamma_0\right) / \left(z_1\sin\alpha_{\mathrm{x}1}\right). \tag{43}$$

If the hob pitch diameter d_0 exceeds the diameter of the worm d_1 by 1–2 modules, then for a single-thread gear, the axial pressure angle calculated by formula (42) turns out to be less than the standard one $\alpha_{\mathrm{x}1}$ = 20° by 0.5°–2°. In this case, the necessary arrow f_{n} of the profile convexity is within the tolerance f_{f} according to the 8th degree of accuracy and is provided by application of a conical grinding wheel without its special dressing.

Application of hobs having increased diameters automatically provides the longitudinal localization of contact along the face width and reduces the gear sensitivity to manufacture and assembly errors.

In a case of production necessity, the hobs with diameter d_0 less than the worm diameter d_1 can be used for cutting the gearwheel teeth. All the formulas indicated above for correction of the worm profile remain valid here. However, in order to avoid edge contact, the gearwheel teeth should be machined in three transitions. This version of the machining technique is described in detail in [13].

4.3 Application of Involute Gear-Cutting Hobs

If the tool shop of the enterprise does not have Archimedes hobs with parameters rather close to the worm, then hobs specified by the standard GOST 9324-80 and intended for machining the involute spur and helical gearwheels can be used for cutting the worm gearwheel teeth. The calculation technique in this case is similar to that stated above, differing from it in only two circumstances.

First, since for these hobs, the normal module m_{n0} is standardized instead of the axis module m_{x0}, the condition for equality of base pitches is reduced to

$$m_{n0} \cos \alpha_{n0} = m_{x1} \cos \gamma_1 \cos \alpha_{n1}. \tag{44}$$

Hence, the required angle of the worm profile is determined according to formula (37) and does not depend on the lead angle γ_0 of the hob thread, which affects only the setting parameters of the hob axis at gear-cutting.

Second, strictly speaking, in this case, the operating worm of the gear can be the involute one, having the radius of the base cylinder

$$r_{b1} = 0.5 z_1 m_{x1} / (\tan^2\alpha_{x1} + \tan^2\gamma_1)^{1/2}. \tag{45}$$

For single-thread gears, such a version is even more convenient in production, since it is provided by the worm grinding with the conical wheel at parallel axes of the wheel and worm with the theoretical accuracy.

The axial profile of the involute worm is determined from the function of the current radius r by integral (39) or directly by the equation

$$x(r) = 0.5 m_x z_1 [\tan\theta(r) - \theta(r)], \tag{46}$$

where $\theta(r) = \mathrm{acos}(r_{b1}/r)$ is the current pressure angle of the face involute.

Having differentiated Eq. (46), the variable pressure angle of the axial section of the thread will be determined by the following expression:

$$\alpha_x(r) = \mathrm{atan}[\tan\gamma_1(r) \tan\theta(r)]. \tag{47}$$

The curvature of the axial profile can be determined by substitution of $\alpha_x(r)$ into expression (39). At the design point located on the pitch cylinder of the worm, this expression is reduced, after transformations, to

$$\mathrm{K}(r_1) = \tan^2\gamma_1 \cos^3\alpha_{x1} / (r_1 \tan\alpha_{x1}). \tag{48}$$

The required arrow of the convexity of the axial profile of the thread f_n is determined by expression (41), with sufficient accuracy for practical analysis.

For single-thread gears, the normal module of the hob m_{n0} can be chosen to be equal to the axial module of the worm $m_{x1.}$ The deviation of the worm axial profile from the straight line is rather insignificant, and condition (44) is fulfilled through correction of the thread profile according to the expression

$$\cos\alpha_{n1} = \cos\alpha_{n0}/\cos\gamma_1. \tag{49}$$

Example 1 Gear parameters are $z_1 = 1$, $z_2 = 42$, $a_w = 120$, $m_{x1} = 4$, $d_1 = 72$, $\gamma_1 = 3°11'$. Parameters of the hob are $\alpha_{n0} = 20°$, $m_{n0} = 4$, $z_0 = 1$, $d_0 = 78{,}6$, $\gamma_0 = 2°55'$.

The corrected axial angle of the worm profile is $\alpha_{x1} = 19°47'$, and the required arrow of convexity is $f_n = 0.002$ mm, that is, it is negligibly low. According to expression (16), the angle $\Delta\gamma$ of the turn of the hob axis is $\Delta\gamma = 0°16'$.

For the manufacturing of multi-thread worm gears, the hob module m_{n0} has to be selected to be less than the worm axial module m_{x1}. In this case, the profile curvature described by Eq. (48) can be sufficiently significant and it has to be realized by means of a dressing device of the worm grinding machine-tool.

Example 2 The worm has the parameters $m_{x1} = 10$, $z_1 = 4$, $d_1 = 80$, $\gamma_1 = 26°34'$, and the base module $m_{b1} = 8.505$. The most suitable involute hob has $m_{n0} = 9$, $z_0 = 1$, $d_0 = 114.5$, $\gamma_0 = 4°30'$ and $m_{b0} = 8.457$. Being corrected according to (44), the worm pressure angle becomes $\alpha_{x1} = 19°47'$. The maximum deviation of the profile (46) from a straight line comes to $f_n = 0.68$. To provide a practically sufficient accuracy, such a profile may be replaced by a circular arc with the radius R = 76.0.

The angular correction of the worm according to Eq. (44) is also applied when manufacturing the gears with the Archimedes worm and the involute cylindrical gearwheel that must perform a helical motion along its axis [6]. These gears are applied, for example, in adjustment mechanisms of mill rollers of screw rolling. They minimize the number of links and the amount of clearances in the kinematic chain mechanism, thereby improving the accuracy of rolled products.

4.4 Design of Duplex Worm Gears

The reversible worm gears (both power and index ones) need to meet the requirement of the backlash-free meshing, and this backlash-free character should be maintained in regard to the wearing-out of the gearwheel teeth. In these gears, the worm has a different axial pitch along the left flank of the thread p_L than the pitch along the right flank, and correspondingly, the axial modules m_{xL} and m_{xR} are also different. This provides the thread thickness variable along the worm axis. As the gearwheel teeth are worn out, the worm is periodically shifted in the axial direction and a thicker segment of the thread is introduced into meshing, thus eliminating the backlash. Such types of gear are called Duplex.

In Fig. 5, such a gear is shown at the middle section and at the section by the plane *A–A* tangent to the pitch cylinder of the worm.

The difference between the right and left pitches $\Delta p_x = p_L - p_R$ is usually taken within the range (0.1–0.2) of the average module. In previously known layouts of such gears, the axial pressure angle at both flanks of the thread was equal to the standard $\alpha_{x1} = 20°$. Cutting of the gearwheel teeth was implemented either with a

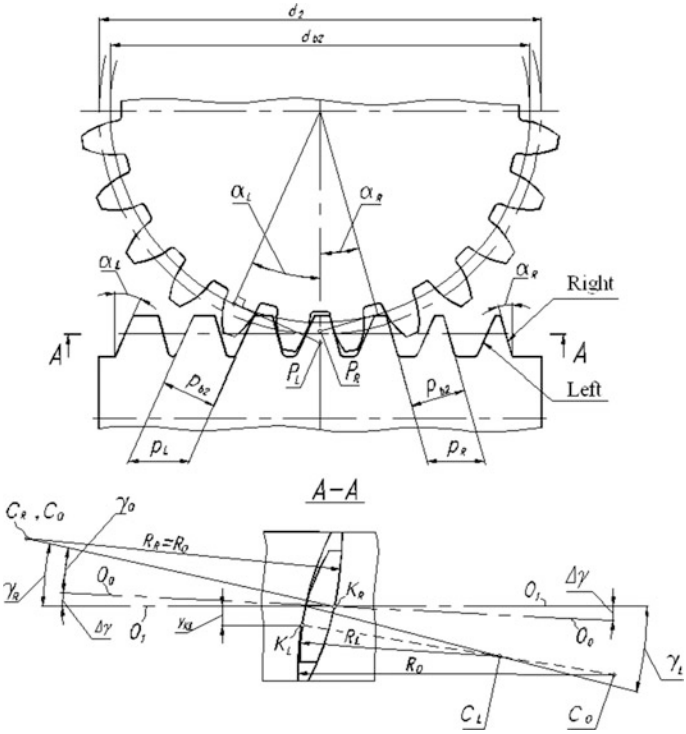

Fig. 5 Duplex worm gears

special hob with a generating surface identical to the active flank of the worm threads, or by two cuts of a single-cutter flying blade with different tangential feeds for each flank of the tooth. Neither method is particularly manufacturable. Moreover, tooth profiles are produced asymmetrically, and the profile corresponding to the greater module is undercut.

In order to eliminate these drawbacks, it is proposed that gearwheel teeth be cut with an Archimedes hob that has an increased diameter as compared to the worm diameter; and one which has a standard value of the module m_{x0}, axial pressure angle α_{x0} and their corresponding meshing module m_{b0}. Equality of base pitches for each of the flanks of the thread is provided by correction of axial pressure angles of the worm according to the condition

$$m_{bL} = m_{bR} = m_{b0}. \tag{50}$$

Let us consider the consequence of calculating the parameters of the worm and hob-setting through a specific example with the following initial data:

For the gear : $a_{\mathrm{w}} = 165, z_1 = 2, z_2 = 24, d_1 = 90, m_{\mathrm{x\,cp}} = 10.0; m_{\mathrm{xL}} = 10.4; m_{\mathrm{xR}} = 9.86.$
For the hob : $m_{\mathrm{x0M}} = 10.0; d_0 = 120.0; z_0 = 2, \alpha_{\mathrm{x0}} = 20^\circ, m_{\mathrm{b0}} = 9.2838, \gamma_0 = 9^\circ 28'.$

According to this initial information, we consequently determine:
Axial pitches for the left and right profiles of the worm:

$$p_{\mathrm{xL}} = \pi m_{\mathrm{xL}} = 32.672\,\mathrm{mm}, p_{\mathrm{xR}} = \pi\, m_{\mathrm{xR}} = 30.976\,\mathrm{mm}.$$

The difference between axial pitches of the left and right profiles:

$$\Delta p_{\mathrm{x}} = p_{\mathrm{xL}} - p_{\mathrm{xR}} = 1.696\ \mathrm{mm}.$$

Pitch helix angles of the worm thread:

$$\gamma_{\mathrm{L}} = \mathrm{atan}\ (m_{\mathrm{xL}} z_1 / d_1) = 13^\circ 00' 48'', \mathrm{similarly}\ \gamma_{\mathrm{R}} = 12^\circ 21' 32''.$$

Normal angles of the worm thread profile:

$$\alpha_{\mathrm{nL}} = \mathrm{acos}[m_{\mathrm{b0}} / (m_{\mathrm{xL}} \cos\ \gamma_{\mathrm{L}})] = 23^\circ 37' 24'', \mathrm{similarly}\ \alpha_{\mathrm{nR}} = 15^\circ 26' 36''.$$

Axial angles of the worm thread profile:

$$\alpha_{\mathrm{xL}} = \mathrm{atan}(\tan \alpha_{\mathrm{nL}} / \cos \gamma_{\mathrm{L}}) = 24^\circ 10' 32'',\ \ \mathrm{similarly}\ \alpha_{\mathrm{xR}} = 15^\circ 47' 30''.$$

If the obtained values for α_{xL} or α_{xR} go out of range (15°–25°), initial values for m_{xL} and m_{xR} should be corrected, keeping, if possible, the assigned difference of Δp_{x}, and the calculation is repeated.

After this, we determine the radii of curvature of the section of the thread by the plane tangent to its pitch cylinder:

$$R_{\mathrm{R}} = 0.5 d_1 / (\tan \alpha_{\mathrm{xR}} \cos^3 \gamma_{\mathrm{R}}) = 170.7, R_{\mathrm{L}} = 0.5 d_1 / \ (\tan \alpha_{\mathrm{xL}} \cos^3 \gamma_{\mathrm{L}})\ = 108.4.$$

And the greater of them (the first one in our case) is compared to the radius of curvature of the similar section of the hob thread:

$$R_0 = 0.5 d_0 / (\tan \alpha_{\mathrm{x0}} \cos^3 \gamma_0) = 171.8.$$

The condition $R_0 > R_{\mathrm{R}}$ means that the longitudinal localization of the bearing contact is provided in the gear. If this condition is not fulfilled, it is necessary to choose the hob of greater diameter and repeat the calculation. At the other flank of the thread, the value of $\mathrm{R_L}$ will obviously be less than $\mathrm{R_0}$.

The machine-tool center distance at gear-cutting is:

$$a_0 = a_{\mathrm{w}} + 0.5(\mathrm{d}_0 - \mathrm{d}_1) = 180\mathrm{mm}.$$

The hob axis should be turned towards the increase of the tooth inclination by the angle $\Delta\gamma = \gamma_{\mathrm{R}} - \gamma_0 = 2°53'47''$. Then, the design contact point on the right flank of the tooth will be located on the middle plane of the gear and the bearing contact will be spread practically over the whole width of the gearwheel. On the left flank of the tooth, the bearing contact is localized in the longitudinal direction and its center is shifted by the value $Y_{\mathrm{K}L}$ towards the worm entering the meshing:

$$Y_{\mathrm{K}L} = -R_0 R_{\mathrm{L}}[\sin(\Delta\gamma + \gamma_0) - \sin\gamma_{\mathrm{L}}]/(R_0 - R_{\mathrm{L}}) = 3.24\ \mathrm{mm}.$$

The proposed method of machining eliminates the necessity of manufacturing the special tool for gear-machining of gears with the variable thickness of the worm thread, and it increases the manufacturing efficiency of the gear-cutting process.

Experience showed the viability of the method for synthesis of worm gears with the localized contact based on the priority of the gear-cutting tool.

5 Modification of Double-Enveloping Worm Gears

Globoid, or double-enveloping, worm gears have a higher load capacity than the single-enveloping gears with a cylindrical worm. This advantage is especially appreciable for center distances exceeding 200 mm [7, 13]. That is why double-enveloping gears are commonly applied in power drives, in particular, rolling mills, in spite of manufacturing difficulties.

Experience in the production of double-enveloping worm gears showed that a number of specific manufacturing features should be stated in the technical documentation within their design. These are, in particular, parameters of the machine-tool meshing and setting of cutters when machining the modified worm.

As is known, in the classical double-enveloping Cone worm gear, the active flank of the worm threads is generated through rotation of the straightline generatrix tangent to the profile circle of the gearwheel. In such a gear, the initial bearing contact is a narrow strip located across the gearwheel tooth near its middle plane. During the running-in, the wear of surfaces takes place both for the gearwheel tooth and the worm thread. The run-in area is formed on the tooth flank, and gradually spreads over a considerable part of its surface.

Wear of the worm thread is also non-uniform. The maximum wear takes place at the input segment of the thread. The wear becomes minimal at the middle part of the thread and increases again when approaching the thread output. Such a running-in continues over 150–200 h, after which the geometry of the active flanks of threads and teeth is stabilized and the velocity of wear multiply decreases [7].

Change of the thread geometry during wearing-out is called *natural* modification. It is achieved by a long-term and expensive running-in process. If the

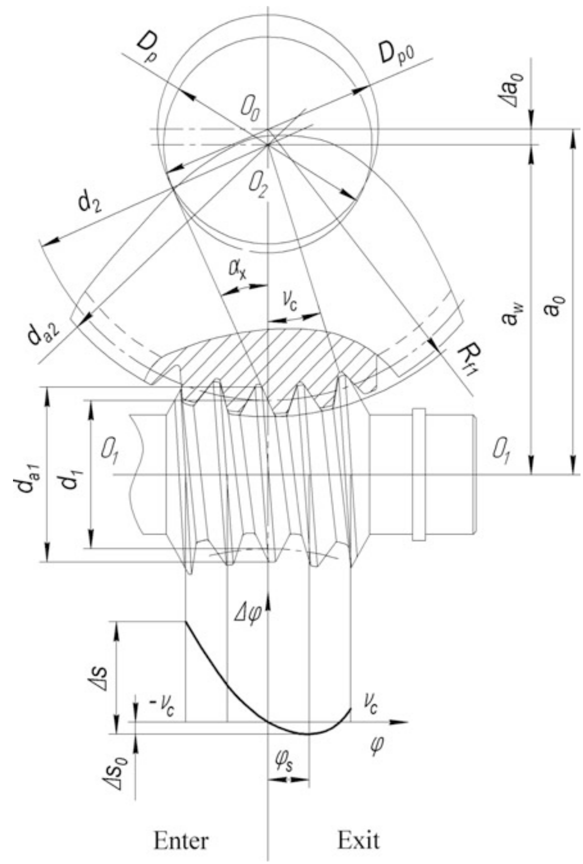

Fig. 6 Double-enveloping worm gear modified by the *AU* method

deviation close to its natural modification is provided at cutting of the worm thread, the time of running-in is multiply reduced and the gear can operate under maximum load from the very beginning.

Various methods for machine-tool modification of the double-enveloping worm thread during its cutting are known. The most common of them is the method in which the modification law that is closest to the assigned one is provided by a simultaneous deviation of the center distance and the gear ratio from nominal values to greater ones. Since both flanks of the thread in this case are modified at one machine-tool setting, it is called the "*AU* double-flank correction-free method."

Figure 6 shows the schemes of the machine-tool meshing for cutting the double-enveloping worm by the *AU* method and the curve line of the modification law, with its maximum value at the input ΔS and extreme value ΔS_0 at the point $\varphi = \varphi_s$.

When calculating modification by the *AU* method, the following parameters are determined: the tooth number of the generating gearwheel z_{20}, the increase in the machine-tool center distance at worm cutting $\Delta a_0 = a_0 - a_w$, the pitch diameter of the generating gearwheel d_0, and the diameter of its profile circle D_{p0}.

The modification law (in radians) is described depending on the angle of rotation φ of the rectilinear generatrix for the thread by the following expression:

$$\Delta(\varphi) = \mathrm{asin}[\sin\alpha_x + (2\Delta a_0/d_2)\cdot(\sin\alpha_x - \sin(\alpha_x + \varphi - K_u\varphi))] + K_u\varphi - \alpha_x \quad (51)$$

Here, $K_u = (z_{20} - z_2)/z_{20}$ is the factor of variation of the machine-tool gear ratio.

The advanced technique for the development of modified double-enveloping worm gears is considered in detail in [7, 13]. It comprises analysis of parameters for worm modification, elements of designing two- or four-teeth flying cutters for generating gearwheel teeth, and methods for evaluation of the load-carrying capacity and efficiency of these gears.

6 Conclusions

Any real worm gears have to be designed as approximated ones in order to compensate for inevitable manufacturing errors, as well as strength and heat deformations.

For single-enveloping worm gears, especially those with an Archimedean worm, the profile localization of the bearing contact allows for reducing the drive sensitivity to pitch errors of its elements. For profile localization, the designer should define and specify the equivalent concavity of profiles of the active and generating worms. An alternative method is also proposed to ensure the smooth operation of the gears through the appointment of mutually-related tolerances for pressure angles and pitches of the worm and the hob manufacturing of worm wheel teeth.

The assembly errors are compensated for and the edge contact is eliminated in such gears by the longitudinal localization of the bearing contact at the middle of the tooth width. The simplest, though not the only, method of longitudinal localization is to use worm hobs with increased diameter. It is shown that, in this case, worm profile correction is needed, along with proposed methods for its calculation.

In double-enveloping worm gears, the impact of the longitudinal and profile localization of the contact on the drive sensitivity to assembly and pitch errors is directly opposite. The method for machine-tool modification of the double-enveloping worm thread is considered.

In the individual production of power worm gears, it is proposed that production be guided by the principle of the priority of a tooth-cutting tool. In particular, this principle is used in the design of Duplex gears.

When longitudinal and/or profile contact localization is applied, it is necessary to specify the size and location of the bearing contact in the design documentation, taking into account the functional purpose of worm gears and the requirements for the accuracy and load-carrying capacity of the gear drive.

The problem of the synthesis of real worm gears with localized contact is as inexhaustible as the problem of the synthesis of spiral bevel and hypoid gears.

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