# Load Distribution in Meshing of Planetary Gearwheels and Its Influence on the Technical and Economic Performance of the Mechanism

**F. Plekhanov, V. Goldfarb and E. Vychuzhanina**

**Abstract** The paper presents a method for determination of the laws of load distribution in meshing of gearwheels for a multi-satellite planetary gear. The influence of layout features of the mechanism, its parameters and manufacture errors on compliance of elements, factors of non-uniform load distribution and technical and economic performance is established.

**Keywords** Planetary gears · Compliance · Load distribution

## 1 Introduction

Multi-satellite planetary gears are commonly used in mechanical engineering due to their capacity for a number of types of distinctive technical and economic performance: high load carrying capacity at small overall dimensions and mass, low friction power losses, satisfactory vibroacoustic characteristics [1–3]. These gears are most effectively applied in aerospace engineering, transport and hoisting machines, robotics, mechatronic systems, etc.; essentially, any field in which the above-mentioned features prevail in the choice of the type of mechanical drive.

In order to eliminate excessive links and equalize the load in meshing of planetary gearwheels, satellites are usually arranged on spherical bearings, with the sun pinion being placed on the gear clutch. However, such a layout allows for the complete elimination of excessive links only in the case of a three-satellite version. In practice, planetary gears with a greater number of satellites are applied. Inevitable manufacture errors cause non-uniform load distribution in power flows, even in the presence of "floating" and self-aligning elements. In order to provide the workability of a drive in a case of its limited radial dimension, it is necessary to mount each satellite on two bearings, or to produce a multi-row gear [4] which causes non-uniform load distribution along tooth length or rows of satellites. Strain

F. Plekhanov (✉) · V. Goldfarb · E. Vychuzhanina
Kalashnikov Izhevsk State University, Izhevsk, Russia
e-mail: fplehanov@list.ru



V. Goldfarb et al. (eds.), *Advanced Gear Engineering*,
Mechanisms and Machine Science 51, DOI 10.1007/978-3-319-60399-5_6

of individual elements of the mechanism partially compensates for errors in its manufacture which can be used in load equalizing in meshing of gears, increasing the load carrying capacity of the drive and reducing its overall dimensions and mass. In this connection, it is important to establish the influence of manufacture error of a gear, its layout features and the compliance of elements in the technical and economic performance of the drive.

## 2 Distribution of Load and Bending Stresses by the Length of Gearwheel Teeth

In the presence of excessive links obstructing the self-alignment of satellites, manufacture errors of a planetary gear and deformation of its sun pinion cause the non-uniform distribution of load and bending stresses at the tooth root along its length (Fig. 1). In this case, factors of initial (not accounting for tooth running-in ability) non-uniform distribution of load and bending stresses differ from each other.

Laws of variation of the unit load $W(x)$ and the bending moment $M(x)$ caused by action of the load and its corresponding normal stresses at the root of a straight tooth of the gearwheel can be established according to equations of displacement compatibility written with consideration of tooth torsion relative to the longitudinal axis and its distortion due to deformation of gear elements (the equations are written for the direction of initial mismatch unfavorable in regard to load distribution and when teeth come into contact in their middle part) (Fig. 2):

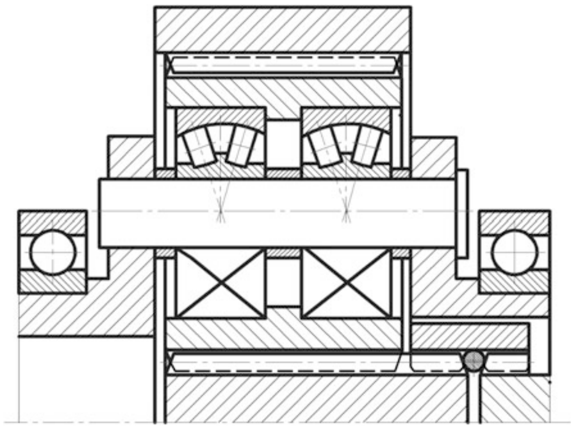


**Fig. 1** Planetary gear with non-self-aligning satellites

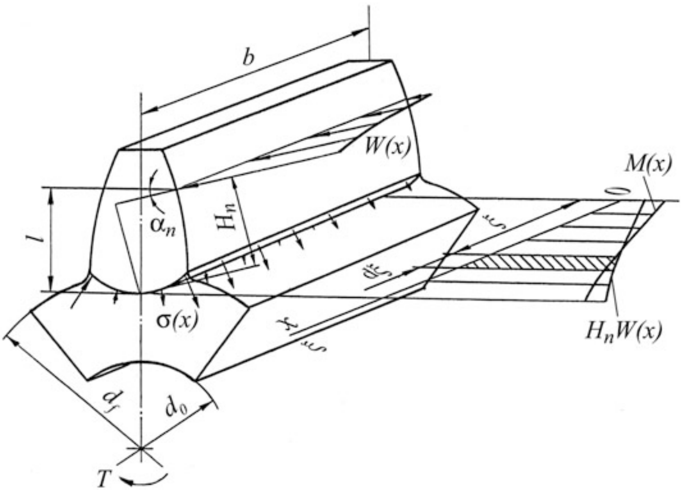



**Fig. 2** Element of the sun pinion and diagrams of loads acting on it

$$
\begin{aligned}
0.5x[\gamma-(\gamma_0+\gamma_H)\cos\alpha_W]+0.5\varphi(x)r_b-\frac{W(x)-W(0)}{C_W} \\
=\frac{H_n}{I_K G}\int_0^x [M(x)-H_nW(\xi)](x-\xi)d\xi,
\end{aligned} \quad (1)
$$

$$
0.5x[\gamma-(\gamma_0+\gamma_H)\cos\alpha_W]+0.5\varphi(x)r_b-\frac{W(x)-W(0)}{C_W}=\frac{M(x)-M(0)}{H_nC_M} \quad (2)
$$

where $\varphi(x)$ is the angle of sun pinion torsion at an arbitrary cross-section with respect to the face end,

$$
\varphi(x)=\frac{r_b n_W}{I_P G}\int_0^x W(\xi)(x-\xi)d\xi,
$$

$\gamma$ is the angle of the initial mismatch of teeth in the plane of meshing, $\gamma_0$ is the angle of misalignment caused by deformation of the axis and satellite supports, $\gamma_H$ is the angle of misalignment of the satellite axis caused by deformation of the carrier,$\alpha_W$ is the pressure angle, $r_b$ is the base radius of the sun pinion, $H_n$ is the arm of a force acting on the tooth with respect to the centre of its bending, $G$ is the shear modulus, $I_K$ is the moment of inertia of the lateral section of the tooth (determined according to the approximated relation written for a rectangular section with the width equal to tooth thickness in its middle part and the height equal to its height: $I_K = 0.19HS^3$, where $H = 2.25m$, $S = 0.5\pi m$, $m$ is the module of meshing), $n_W$ is the number of power flows, $I_P$ is the polar moment of inertia of the cross-section for the sun pinion, $C_M$ is the specific rigidity of the tooth when it is deformed under the action of the bending moment (determined with regard to compliance of a part of the rim adjoining the teeth), and $C_W$ is the specific rigidity of the pinion tooth under its contact deformation, shear and compression deformation,

$$\frac{1}{C_W} = \frac{1}{C_t} - \frac{1}{C_M},$$

where $\frac{1}{C_t}$ is the total specific compliance of the tooth with regard to one half of the contact compliance of the meshing.

Components of rigidity and its reverse value (compliance) are determined through methods of construction mechanics, elasticity theory and experimentalism [1, 5, 6].

In order to obtain analytical relations, let us solve the stated task in parts, having first considered the laws of distribution of the load and bending stresses for rigid elements of the gear, only taking into account the initial mismatch of teeth and their compliance. In this case, expressions (1) and (2) can be represented as a Volterra integral equation

$$W(x) = \lambda^2 \int_0^x W(\xi)(x - \xi)d\xi + F(x), \tag{3}$$

where

$$F(x) = W(0) + 0.5\gamma x C_W + 0.5(\lambda x)^2 \left[0.5\gamma C_t \left(\frac{b}{2} - \frac{x}{3}\right) - W\right],$$
$$\lambda^2 = \frac{H_n^2 (C_W + C_M)}{GI_K}.$$

Let us write Eq. (3) in operator form:

$$L[W(x)] = \omega(p) = \frac{p^2 f(p)}{p^2 - \lambda^2}. \tag{4}$$

Turning to the original equation, and transforming the obtained expression with regard to statics, we obtain

$$W(x) = W + \frac{\gamma b C_\Sigma}{2}\left[\frac{2x}{b} - 1 + \frac{2C_W}{\lambda b C_M}\left(sh\lambda x + \frac{1 - ch\lambda b}{sh\lambda b} ch\lambda x\right)\right], \tag{5}$$

$$\frac{M(x)}{H_n} = W + \frac{\gamma b C_\Sigma}{2}\left[\frac{2x}{b} - 1 - \frac{2}{\lambda b}\left(sh\lambda x + \frac{1 - ch\lambda b}{sh\lambda b} ch\lambda x\right)\right]. \tag{6}$$

Here, $W$ is the average unit load in the meshing; $b$ is the tooth length equal to the working width of the rim $b_W$; and $C_\Sigma$ is the specific rigidity of the meshing in the absence of a mismatch ($C_\Sigma = 0.5C_t = 0.075E$ [1], $E$ being the Young modulus).

When the load is applied to the tooth at the middle part of its height, the following approximated values of components of Eqs. (5) and (6) will be obtained: $C_W/C_M = 0.5$, $\lambda = 1/m$. Then, the maximum values of the unit load and the moment and corresponding initial factors of non-uniformity are equal to

$$K_W = \frac{W(b)}{W} = 1 + \frac{\gamma b}{2W} C_H(b) = 1 + \frac{\gamma b}{2W} C_\Sigma \left[1 + \frac{m\left(e^{b/m} - 1\right)}{b\left(e^{b/m} + 1\right)}\right], \tag{7}$$

$$K_m = \frac{W(b)}{WH_n} = 1 + \frac{\gamma b}{2W} C_F(b) = 1 + \frac{\gamma b}{2W} C_\Sigma \left[1 - \frac{2m\left(e^{b/m} - 1\right)}{b\left(e^{b/m} + 1\right)}\right]. \tag{8}$$

Figure 3 shows the diagrams of dependences of $W(X) = W(x/b)/W$ and $M(X) = M(x/b)/H_n W$ on the parameter $X = x/b$ for a relative angle of the initial mismatch $\Gamma = E\gamma b/W = 20$ and tooth length $b = 18m$. Figure 4 shows the same dependences for $b = 5m$. Figure 5 presents the dependences of $K_W$ and $K_m$ on $\Gamma$.

The most unfavorable case with regard to load distribution along the length of the gearwheel teeth is the case when angles of initial mismatch $\gamma$ of the teeth at external and internal meshing have opposite signs, and the carrier is fixed. Then, equations for determining the unit loads and bending moment can be presented as follows:

$$W(x) = W(0) + f(x) C_H(b), \tag{9}$$

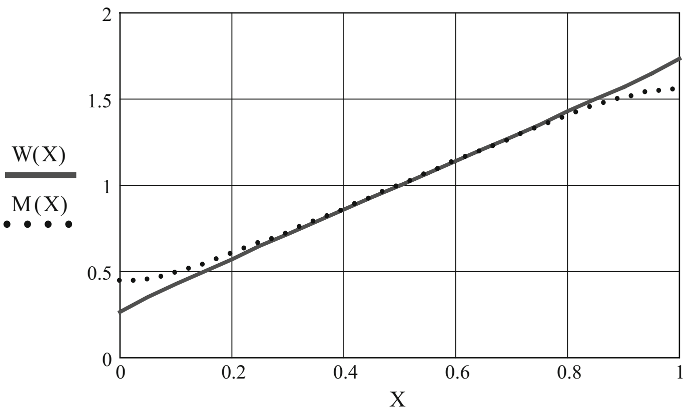



**Fig. 3** Distribution of relative load and bending stresses along the gearwheel tooth length for fixed elements of the gear, Γ = 20, $b$ = 18$m$

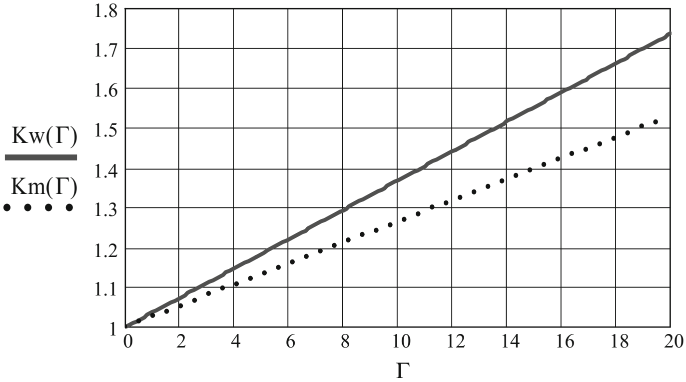



**Fig. 4** Distribution of the relative load and stresses along the length of the gearwheel tooth for fixed elements of the gear

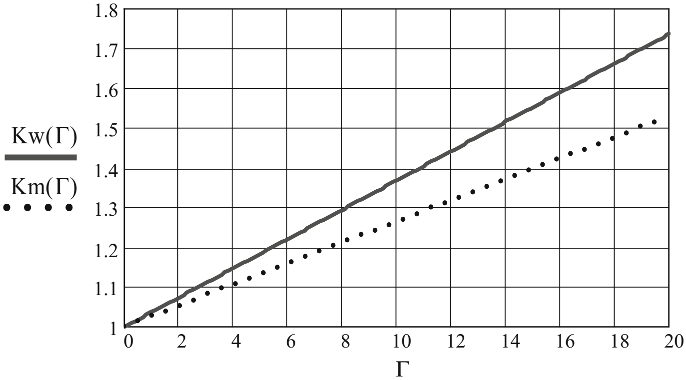



**Fig. 5** Factors of non-uniform distribution of load and bending stresses of the tooth versus the relative angle of initial mismatch for fixed elements of the gear and $b$ = 18$m$

$$\frac{M(x)}{H_n} = \frac{M(0)}{H_n} + f(x)C_F(b), \tag{10}$$

where the function of mismatch of the teeth with consideration of torsion of the sun pinion when the torque is transmitted from its face end is

$$f(x) = \gamma x + \frac{r_b^2 n_w}{I_p G} \int_n^x W(\xi)(x - \xi)d\xi. \tag{11}$$

Let us determine the unit load *W(x)* at external meshing of the gear by substituting Eq. (11) for Eq. (9) and differentiating the obtained expression twice:

**Fig. 6** Factor of non-uniform load distribution along the tooth length for a sun pinion versus *G* and *B*. *Subscripts 1* for Γ = 0; *2* for Γ = 10; *3* for Γ = 20

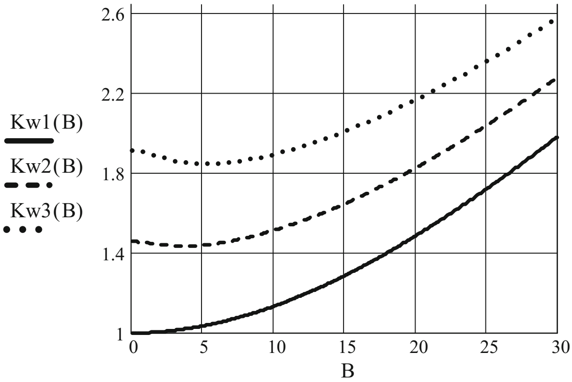
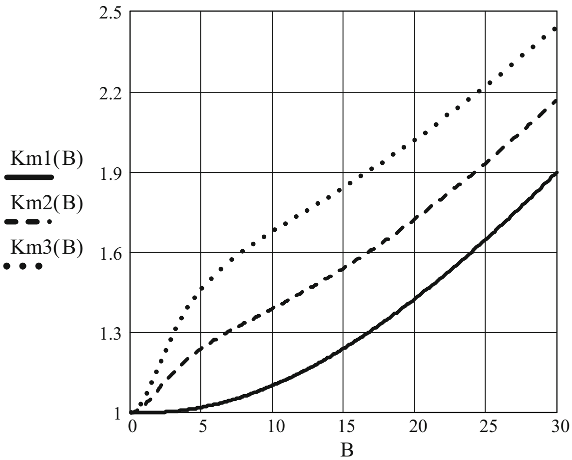



**Fig. 7** Factor of non-uniform stress distribution along the tooth length for a sun pinion versus Γ and *B*. *Subscripts 1* for Γ = 0; *2* for Γ = 10; *3* for Γ = 20




$$W''(x) - \frac{r_b^2 n_W C_H(b)}{I_p G} = W''(x) - \mu^2 W(x) = 0. \tag{12}$$

Taking into account the equations $\int_0^b W(x)dx = Wb$ and $W'(0) = \gamma C_H(b)$, the solution to this equation will be as follows:

$$W(x) = \frac{\gamma}{\mu} C_H(b) sh\mu x + \left[ W\mu b - \frac{\gamma C_H(b)}{\mu}(ch\mu b - 1)\right] \frac{ch\mu x}{sh\mu b}. \tag{13}$$

Then, applying Eqs. (9), (10) and the equations of statics, we determine

$$\frac{M(x)}{H_n} = W + \frac{C_F(b)}{C_H(b)}[W(x) - W]. \tag{14}$$

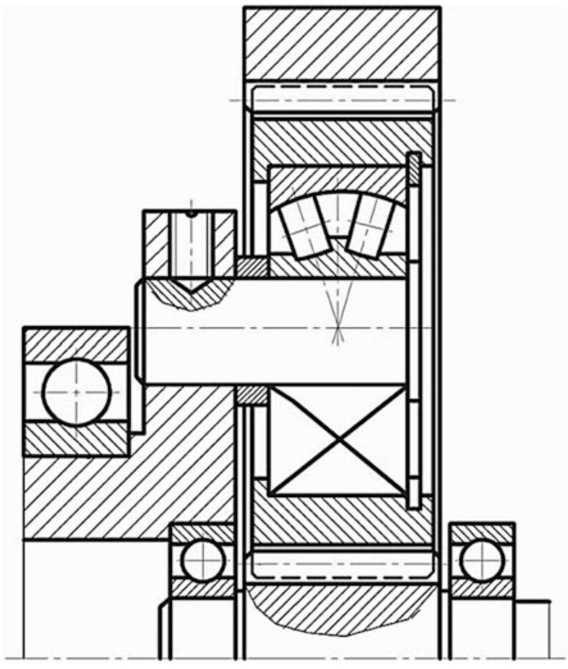
Figures 6 and 7 represent diagrams of dependences of non-uniformity factors $K_W$, $K_m$ on $G$ and $B = b/m$ for a fixed layout of the carrier and the number of teeth of the sun pinion $z = 18$, $n_W = 3$, $d_0 = 0$. Diagrams are plotted according to expressions (13) and (14) and allow for evaluating the pointed strength performances of a multi-satellite planetary gear.

## 3 Influence of the Compliance of Planetary Gear Elements on Load Distribution Among Satellites

One of the simplest and most efficient layouts of a multi-satellite planetary gear is the gear comprising axes as cantilevers in the carrier jaw, the axes containing spherical bearings and two central gearwheels (Fig. 8). Compliance of cantilever axes and bearings of satellites provides load equalization in meshing of gearwheels, and at corresponding parameters, it can lead to close to uniform load distribution, even in the absence of mechanisms of self-alignment for elements of a planetary gear.

In this connection, it is important to determine the compliance of the axis and the parts in contact with it and to establish the level of its influence on the value of the factor of non-uniform load distribution in meshing of gearwheels.

**Fig. 8** Planetary gear with cantilever axes of self-aligning satellites



Accounting for distortion of the satellite axis under the action of the force applied to it and increased compliance of the inner ring of the bearing at its face ends, the load on the axis from the bearing side can be presented by the equation (Fig. 9):

$$q(x) = q_m \sin\left(\frac{\pi x}{l}\right) = \frac{\pi P}{2l} \sin\left(\frac{\pi x}{l}\right), \tag{15}$$

where $q_m$ is the maximum value of the unit load.

Then, deflection of a steel axis in the middle part of the area of contact with the bearing ring caused by action of the bending moment and lateral force and determined by known Mohr's formulas will be

$$y_0 = \frac{Pl^3}{IE}\left(\frac{5}{96} - \frac{0.5\pi - 1}{2\pi^3}\right) + \frac{1.11Pl}{SG}\left(0.25 + \frac{0.5}{\pi}\right), \tag{16}$$

where $d$ is the diameter of the satellite axis, $I$ is the axial moment of inertia of its cross-section, $S$ is the area of the lateral cross-section of the axis, and $E$ and $G$ are elasticity moduli of the 1st and 2nd types, respectively.

Let us represent the equation of the deformed axis in the area of its contact with the carrier jaw as follows (see Fig. 9):

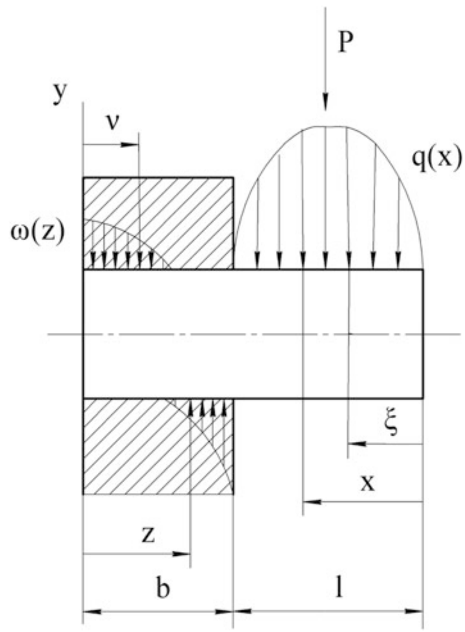



**Fig. 9** Loading scheme for cantilever axis of the satellite and the carrier jaw

$$\frac{1}{C_H}\frac{d^2\omega(z)}{dz^2} = \frac{d^2y(z)}{dz^2} = \frac{M(z)}{IE} + 1.11\frac{\omega(z)}{SG}. \tag{17}$$

Here, $C_H$ is the specific contact rigidity of the joint determined experimentally ($C_H \cong E/1.2$ [4]), $M(z) = -\int_0^z \omega(v)(z-v)dv$.

After double differentiation of the equality (17), we obtain

$$\frac{d^4\omega(z)}{dz^4} - \frac{1.11C_H}{SG}\frac{d^2\omega(z)}{dz^2} + \frac{C_H}{IE}\omega(z) = 0, \tag{18}$$

hence,

$$\begin{aligned}\omega(z) &= C_1 sh(\alpha z/b)\sin(\beta z/b) + C_2 ch(\alpha z/b)\sin(\beta z/b) \\ &\quad + C_3 sh(\alpha z/b)\cos(\beta z/b) + C_4 ch(\alpha z/b)\cos(\beta z/b),\end{aligned} \tag{19}$$

$$\alpha = b\sqrt{4}\frac{C_H}{IE}\cos\left[0.5\ \arccos\left(\frac{\sqrt{IEC_H}}{1.8FG}\right)\right]$$

$$\beta = b\sqrt{4}\frac{C_H}{IE}\sin\left[0.5\ \arccos\left(\frac{\sqrt{IEC_H}}{1.8FG}\right)\right]$$

In order to determine constants of integration $C_1 \div C_4$, we apply the following boundary conditions and equations of statics:

$$\int_0^b \omega(z)dz = -P; \int_0^b \omega(z)(b-z)dz = 0.5Pl;$$

$$\text{for } z = 0\ \frac{d^2\omega(z)}{dz^2} = \frac{1.11C_H}{SG}\omega(z);$$

$$\text{for } z = b\ \frac{d^2\omega(z)}{dz^2} = \frac{-0.5C_H Pl}{IE} + \frac{1.11C_H}{SG}\omega(z).$$

Displacement of the axis in the area of the bearing mounting caused by compliance of the “axis—carrier jaw” contact is

$$\begin{aligned}y_h &= \frac{1.2}{E}\left[\omega(z) + \frac{l}{2}\frac{d\omega(z)}{dz}\right]_{Z=b} = \frac{1.2}{E}\{sh\alpha\ \sin\beta[C_1 + 0.5\frac{l}{b}(C_2\alpha \\ &\quad - C_3\beta)] + ch\alpha\ \cos\beta[C_4 + 0.5\frac{l}{b}(C_2\beta + C_3\alpha)] + sh\alpha\ \cos\beta[C_3 \\ &\quad + 0.5\frac{l}{b}(C_1\beta + C_4\alpha)] + ch\alpha\ \sin\beta[C_2 + 0.5\frac{l}{b}(C_1\alpha - C_4\beta)]\}\end{aligned} \tag{20}$$

Correctness of analytical determination of the axis compliance accounting for the deformability of the carrier jaw is proved by SolidWorks finite-element analysis of the mode of deformation of the “satellite axis—carrier jaw” contact. Figure 10 shows the mode of deformation of the carrier jaw for $P = 10000\,N$, $d = 20$ mm, $b = 10$ mm.

In order to determine the factor of non-uniformity of load distribution for satellites, we apply equations of displacement compatibility accounting for the deformation of axes and parts in contact with them (carrier jaw, rolling bearings) along with gear manufacture and assembly errors:

$$\left.\begin{array}{c} F_1 = \frac{P_1}{2\cos\alpha_W} = b_W C_\Sigma[\varepsilon - \delta_1 - (y_1 + y_{B1})\cos\alpha_W], \\ \ldots, \\ F_i = \frac{P_i}{2\cos\alpha_W} = b_W C_\Sigma[\varepsilon - \delta_i - (y_i + y_{Bi})\cos\alpha_W], \\ \ldots, \\ F_N = \frac{P_N}{2\cos\alpha_W} = b_W C_\Sigma[\varepsilon - \delta_N - (y_N + y_{BN})\cos\alpha_W], \\ \sum\limits_{i+1}^{N} F_i = b_W \sum\limits_{i=1}^{N} W_i = NWb_W, \end{array}\right\} \quad (21)$$

where $F_i$ is the normal force in meshing of the $i$th satellite with central gearwheels; $P_i$ is the load acting on the $i$th axis from the side of the bearing ring; $W_i$ is the unit load in meshing of the $i$th satellite with the stationary gearwheel and pinion; $W$ is the average unit load in meshing of satellites with sun pinions; $N$ is the number of satellites; $\alpha_W$ is the pressure angle (this system of equations corresponds to the equality of angles for internal and external meshing); $C_\Sigma$ is the specific rigidity of a single-pair meshing ($C_\Sigma = 0.075E$); $\delta_i$ is the initial mismatch of teeth (clearance) in meshing of the $i$th satellite with gearwheels caused by errors of tangential

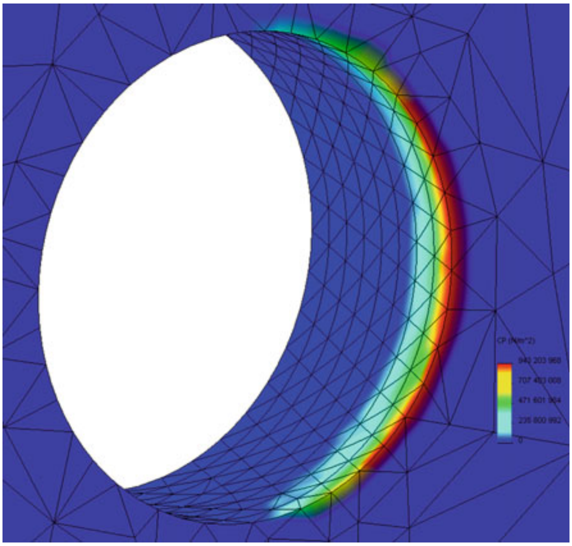


**Fig. 10** Computer-aided model of the carrier jaw

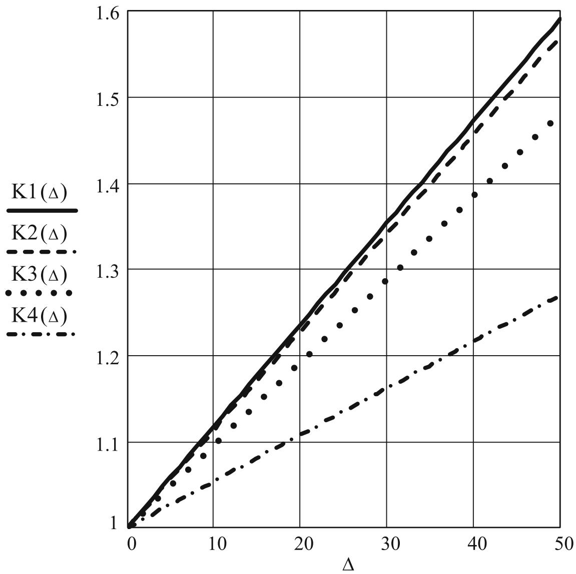
arrangement of the axes and pitch of teeth (the clearance in the internal meshing is equal to the clearance in the external one); $b_W$ is the working face width of the satellite; $y_i$ is the displacement of the $i$th axis at the area of bearing mounting caused by the axis bending, deformation of the carrier jaw and contact deformation of the "axis—bearing ring" contact ($y_i = y_{0i} + y_{hi} + 0.5\pi q_i/C_H$, where $q_i$ is the average unit load acting on the axis of the $i$th satellite); $\varepsilon = const$; and $y_{Bi}$ is the displacement of the $i$th satellite caused by the compliance of the rolling bearing (accounting for variation of the bearing compliance within a wide range depending on the load; let us assign its approximate value as $y_{Bi} \cong \frac{5q_i}{E}$).

The initial mismatch of teeth in meshing of gearwheels with the most loaded satellite is absent ($\delta_1 = 0$). Then, knowing the values $\delta_i$, corresponding to the degree of accuracy of gear manufacture, we use Eq. (21) and determine the values $F_i, \varepsilon$ and the factor on non-uniform load distribution for power flows $K = F_{\max}/F = W_{\max}/W$.

Figure 11 represents the diagram of dependence of the factor of non-uniform load distribution for satellites of the gear with excessive links (see Fig. 8) on $\Delta = \delta E/W = b_W \delta E/F$ and $\overline{b} = b/l$ for $N = 3$, $\alpha_W = 20^0$, $l = d$, $b_W = l$, $\delta_2 = \delta_3 = \delta$ (the most unfavorable case with regard to load distribution).

According to the analysis performed and the diagrams plotted thereafter, the compliance of cantilever axes leads to considerable reduction of non-uniform load distribution for satellites. But for a small ratio of the carrier jaw thickness to the axis diameter and cantilever length, it is possible to go beyond allowable values (see Fig. 10) of stresses in the area of the "axis—carrier jaw" contact. It is also feasible to reduce the negative influence of manufacture errors for planetary gears on load distribution in meshing of gearwheels by applying satellites with complaint rims;

**Fig. 11** Factor of non-uniform load distribution for three satellites versus the value of relative initial mismatch of teeth. *Subscripts 1* for $\overline{b} = 2$; *2* for $\overline{b} = 1.5$; *3* for $\overline{b} = 1$; *4* for $\overline{b} = 0.6$



however, the bending strength of this rim can become the limiting factor for the load-carrying capacity of the gear and negatively influence its lifetime.

As for load distribution in meshing of gearwheels and the load-carrying capacity of a mechanical drive, the planetary gear with the "floating" sun pinion, the fixed rigid carrier and single-rim self-aligning satellites mounted on spherical bearings (Fig. 12) is the most efficient. The compliance of this double-support axis of the satellite is determined according to the design scheme shown in Fig. 13, where $b$ is the width of the carrier jaw, $l$ is the half width of the bearing ring for the satellite, and $L$ is the half length of the span between the jaws of the carrier.

The unit load in the area of contact of the satellite axis and carrier jaw $\omega(z)$ is determined through Euler's method according to the expression (19), and in the area of bearing mounting on the axis, according to the formula

$$q(x) = C_5 sh(\alpha x)\sin(\beta x) + C_6 ch(\alpha x)\cos(\beta x). \tag{22}$$

Constants of integration $C_1 \div C_6$, which are components of Eqs. (19) and (22), are determined according to statics equations and boundary conditions:

(1) $\int_0^l q(x)dx = 0.5P = F\cos\alpha_W$;

(2) $\int_0^b \omega(z)dz = 0.5P$;

(3) for $z = b$ $M(z) = 0$, and according to the equation of the deflected axis

$$\omega^{//}(b) = 1.11\frac{C_H\omega(b)}{SG};$$

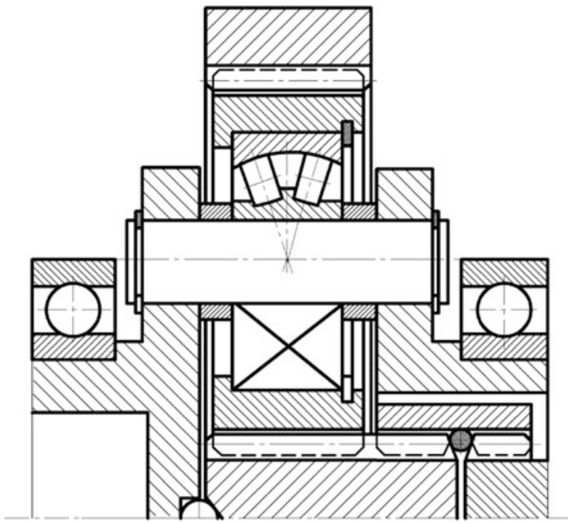


Fig. 12 Planetary gear with self-aligning satellites and a "floating" sun pinion

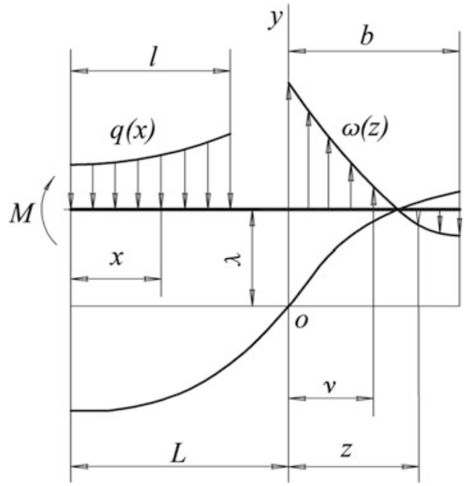



**Fig. 13** Design scheme with double-support axis of the satellite

(4) for $z = 0$ $M(z) = M(0) = \int_0^b \omega(z) z dz = IE\left[1.11 \frac{\omega(0)}{SG} - \frac{\omega^{//}(0)}{C_H}\right]$;

(5) bending moments at segments for $z = 0$ and $x = l$ are inter-connected by the equation

$$M(l) = IE\left[\frac{q^{//}(l)}{C_H} - 1.11\frac{q(l)}{SG}\right] = 0.5P(L-l) + M(0);$$

(6) inter-relation between angular deformations of segments of the axis is as follows:

$$-\frac{q^{/}(l)}{C_H} = \frac{\omega^{/}(0)}{C_H} + \frac{M(0)}{IE}(L-l) + 1.11\frac{0.5P}{SG} + \frac{0.5P(L-l)^2}{2IE}.$$

Tangential displacement of the satellite caused by deformation of its axis and the contacting jaws of the carrier and bearing ring is

$$y = \frac{\omega(0)}{C_H} + \frac{q(l)}{C_H} + \frac{q^{/}(l)}{C_H}(L-l) + \frac{M(0)}{2IE}(L-l)^2 + \frac{0.5P}{3IE}(L-l)^3. \quad (23)$$

The factor of non-uniform load distribution for satellites is determined by solving the system of Eq. (19), according to the displacement of the $i$th satellite $y_i$ and above-assigned values of apposition of bearing rings $y_{Bi}$ obtained from (23).

Results of this analysis are represented as diagrams of dependences $K = F_{\max}/F = W_{\max}/W$ on the relative error $\Theta = b_W \delta E/F$ and the ratio of the half length of the slot between the carrier jaws to the diameter of the satellite axis $L/d$

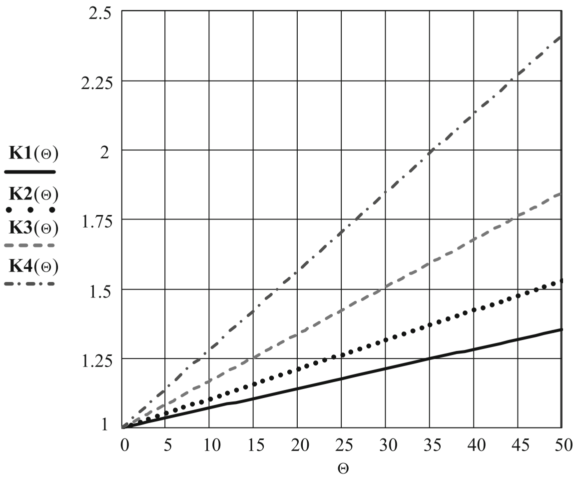



**Fig. 14** Factor of non-uniform load distribution for power flows versus the relative error for $N = 5$. *Subscripts 1* for $L/d = 1$; *2* for $L/d = 0.7$; *3* for fixed axes and carrier; *4* for fixed axes, carrier and bearings

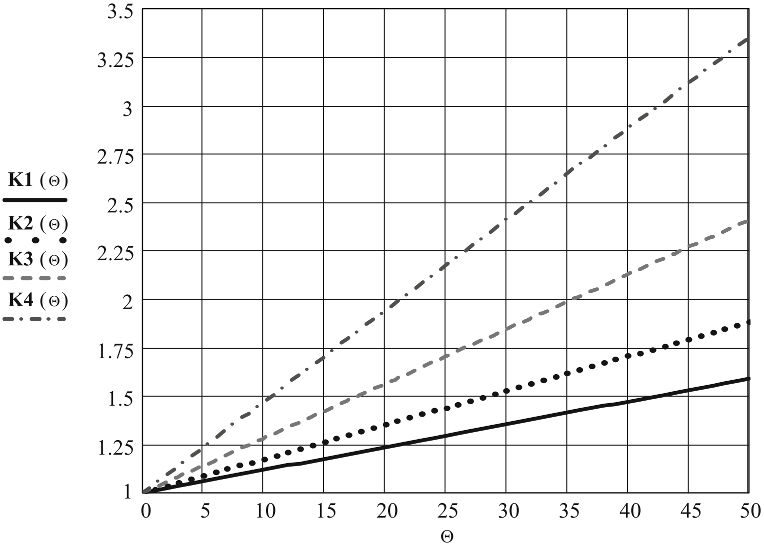



**Fig. 15** Factor of non-uniform load distribution for power flows versus relative error for $N = 6$. *Subscripts 1* for $L/d = 1$; *2* for $L/d = 0.7$; *3* for fixed axes and the carrier; *4* for fixed axes, the carrier and bearings

for $0.5 \le b/d \le 0.7$; $l/d = 0.5$; $b_w/L = 1.8$ and different numbers of satellites $N$ (Figs. 14, 15 and 16).

The diagrams correspond to the ratio between deviations of satellite axes from a theoretically accurate position that is unfavorable for a gear with a “floating” sun pinion: for $N = 5$ $\delta_1 = \delta_2 = \delta$, $\delta_3 = \delta_4 = \delta_5 = 0$; for $N = 6$ $\delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta$, $\delta_5 = \delta_6 = 0$; $N = 7$ $\delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta$, $\delta_5 = \delta_6 =$

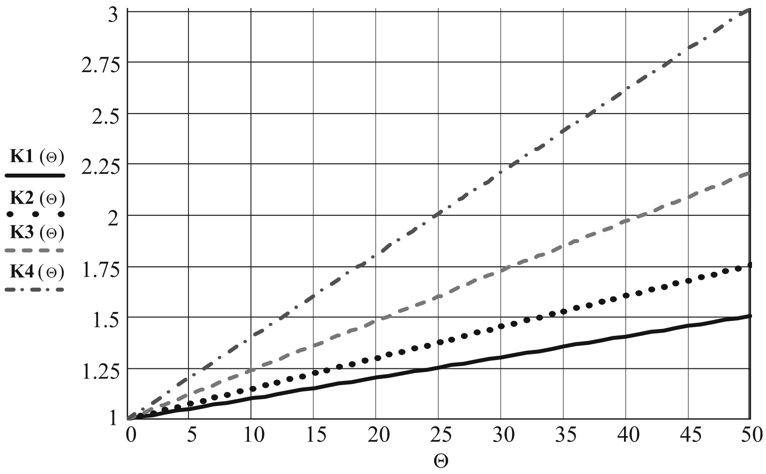



**Fig. 16** Factor of non-uniform load distribution for power flows versus relative error for $N = 7$. *Subscripts 1* for $L/d = 1$; *2* for $L/d = 0.7$; *3* for fixed axes and the carrier; *4* for fixed axes, the carrier and bearings

$\delta_7 = 0$ (for an odd number of satellites, the clearances in three of them are eliminated by self-alignment of the sun pinion even at a low load).

An investigation conducted according to the above given method shows that, in case of a "floating" sun pinion and unfavorable deviation of satellite axes from a theoretically precise position, the load carrying capacity of a five-satellite layout does not differ, practically speaking, from this parameter of a gear with six satellites, since the increase in the load carrying capacity due to an increase in the number of power flows is compensated for by the possible increase in the factor of non-uniform load distribution in meshing of gears. The load carrying capacity of a seven-satellite gear in the presence of its manufacturing errors exceeds the pointed technical and economic parameters for five- and six-satellite layouts by approximately 20–30%, which should be considered when analyzing planetary gears.

## 4 Load Distribution Among Satellite Rows

In the case of a gear with a limited radial dimension, it is reasonable to make each satellite with two or more self-aligning gearwheels that are not rigidly connected to each other and that are fixed on cantilever axes (Fig. 17). In this layout, there is a non-uniformity of load distribution for rows of satellites caused by both gear manufacturing errors and torsion of the sun pinion under the action of a torque applied to it.

The factor of non-uniform load distribution for rims of a double-row satellite is determined by solution of a system of equations:

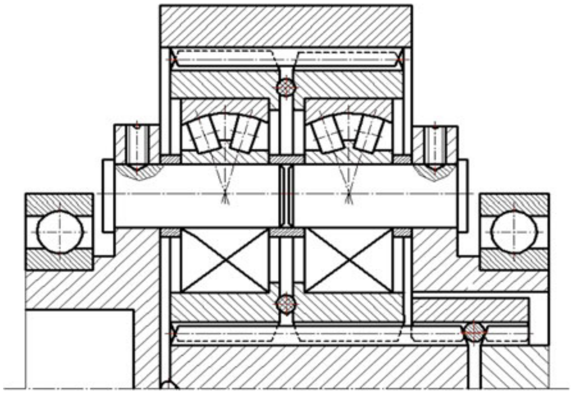


**Fig. 17** Planetary gear with double-row satellites and a "floating" sun pinion

$$\left.\begin{aligned} \frac{q_1 l}{2\cos\alpha_W} &= b_W C_\Sigma[\varepsilon - (y_1 + y_{B1})\cos\alpha_W], \\ \frac{q_2 l}{2\cos\alpha_W} &= b_W C_\Sigma\left[\varepsilon - \delta - y_\varphi - (y_2 + y_{B2})\cos\alpha_W\right], \\ q_1 + q_2 &= 2q = 4W\ \cos\alpha_W. \end{aligned}\right\} \tag{24}$$

Here, $\varepsilon = const$; $b_W$ is the face width of one rim of a satellite; $\delta$ are clearances between the teeth of the second rim of the satellite and central gearwheels (clearances in both meshes are identical) at tight contact of the teeth in meshing of the first satellite with central gearwheels; $q_1$ and $q_2$ are unit loads acting on the axes of the satellite rims from the perspective of the side of the bearings; $q$ is the average unit load; $l$ is the length of the cantilever part of the axis (see Fig. 9); $y_{B1}$ and $y_{B2}$ are displacements of the satellite rims caused by compliance of the rolling bearings; $y_1$ and $y_2$ are displacements of the satellite rims caused by deflection of the axes and contact deformation in areas of their contact with carrier jaws and bearing rings ($y_{Bi}$ and $y_i$ are determined in a manner similar to that of a gear with single-row satellites, as shown in Figs. 8 and 9); and $y_\varphi$ is the half difference of tooth displacements for a sun pinion in its lateral cross-sections by symmetry planes for satellite rims due to torsion:

$$y_\varphi = 0.5\varphi r_b = \frac{0.5 r_b b_W^2}{I_p G}(0.875t_2 + 0.125t_1) \tag{25}$$

where the unit moment of pinion torsion at sections of its contact with the $i$ th rim of the satellite is $t_i = \frac{0.5Nq_i r_b}{\cos\alpha_W}$, $r_b$ is the pinion base radius, $I_p$ is the polar moment of inertia of the pinion cross-section, $\varphi$ is the difference between angles of torsion for cross-sections, and $N$ is the number of rims of satellites in one row.

Equations are written for the case when the most loaded rim (the first one) of a satellite is located from the side of torque input to the sun pinion (an unfavorable case), and the load is distributed uniformly along the width of separate self-aligning rims of the satellite.

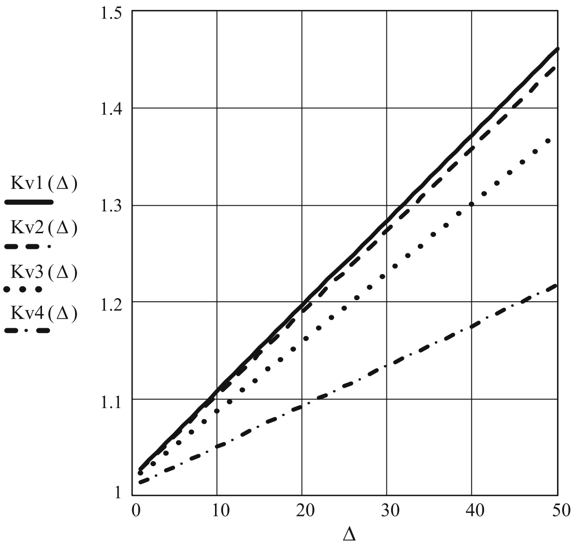



**Fig. 18** Factor of non-uniform load distribution among rims of a double-rim satellite versus Δ for $\overline{b}_a = 1$. *Subscripts 1* for $\overline{b} = 2$; *2* for $\overline{b} = 1.5$; *3* for $\overline{b} = 1$; *4* for $\overline{b} = 0.6$

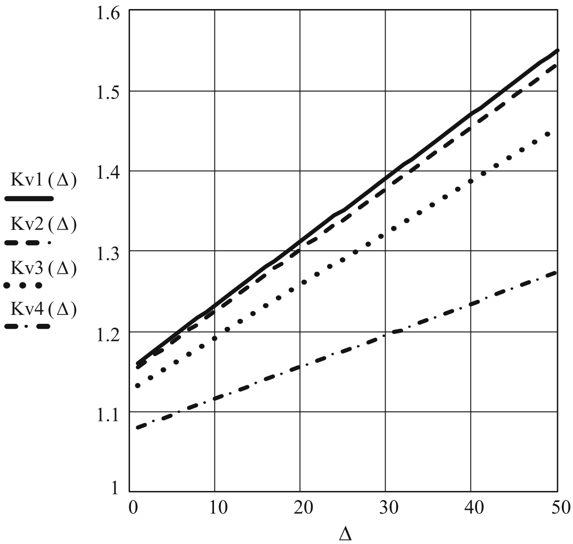



**Fig. 19** Factor of non-uniform load distribution among rims of a double-rim satellite versus Δ for $\overline{b}_a = 3$. *Subscripts 1* for $\overline{b} = 2$; *2* for $\overline{b} = 1.5$; *3* for $\overline{b} = 1$; *4* for $\overline{b} = 0.6$

Figures 18 and 19 show diagrams of the factor of non-uniform load distribution among rims of a double-row satellite $K_V = q_1/q$ depending on $\Delta = \delta E/W$ at a number of satellites in a row $N = 3$, $\alpha_W = 20^0$, $\overline{l} = l/d = 1$, $\overline{b}_W = b_W/l = 1$ and different values of relative thickness of the carrier jaw $\overline{b} = b/d$ and the relative width of a sun pinion $\overline{b}_a = 2b_W/d_a$ ($d_a$ is the diameter of the pitch circumference of the sun pinion).

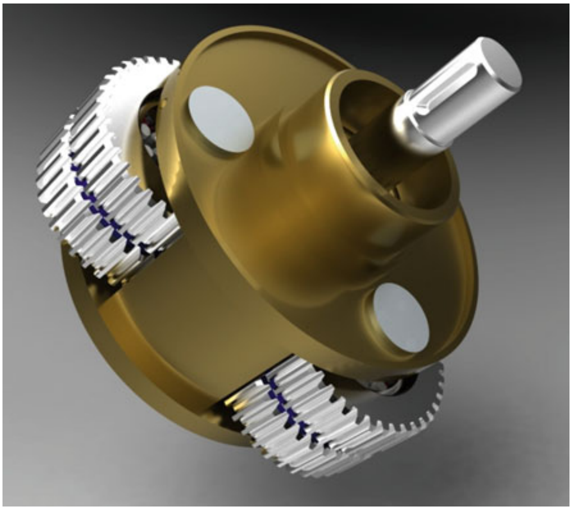


**Fig. 20** Assembly unit of satellites for a planetary gear with double-row satellites

Figure 20 shows the assembly unit of satellites for a planetary gear with a limited radial radius. Each satellite is constructed of two self-aligning gearwheels mounted on cantilever axes.

## 5 Technical and Economic Parameters of Multi-Satellite Planetary Gears

The important technical and economic parameters of a gear are its efficiency and the relation of its mass to the moment at the output shaft. As numerous research works have shown [1], including experimental ones (Fig. 21), the first of the pointed parameters is rather high (about 95%), and the second one is determined by analyzing the contact strength for external gears that limits the load carrying capacity of the drive, and accounting for the gear ratio $i$, it can be found according to the following approximated relation:

$$M = \frac{0.5\pi\rho KE\left[1 + N(0.5i - 1)^2 + 4k_b(i - 1) + k_h(i - 1)^2\right]}{(\sigma_{HP}/0.418)^2 N(0.5i - 1)\sin\alpha_W \cos\alpha_W}\,\text{kg/Nm}, \quad (26)$$

where $\rho$ is the density of the gearwheel material; $K$ is the factor considering the non-uniform load distribution among satellites; $\sigma_{HP}$ is the allowable contact stress of the teeth; $k_b$ is the factor that considers the thickness of the rim of the fixed gearwheel and the gear casing; and $k_h$ is the factor that considers the thickness of the carrier jaws and gear caps.

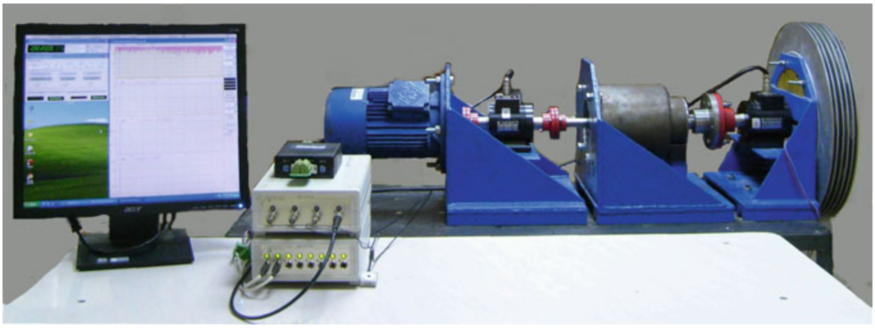


**Fig. 21** Experimental apparatus for testing coaxial gears

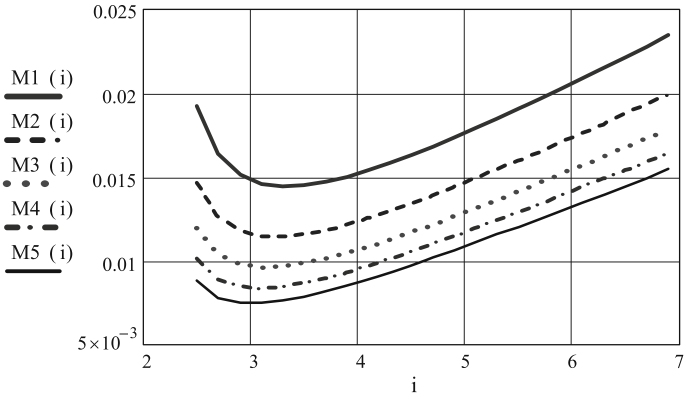



**Fig. 22** Relative mass of a gear versus gear ratio and the number of satellites in the absence of manufacturing errors. *Subscripts 1* for $N = 3$; *2* for $N = 4$; *3* for $N = 5$; *4* for $N = 6$; *5* for $N = 7$

Figure 22 shows a diagram of the parameter M depending on the gear ratio and the number of gear satellites at uniform load distribution in meshing of gearwheels, $k_b = 0.2$; $k_h = 0.6$; the material of gear parts is steel, $\sigma_{HP} = 600$ MPa. As we see, the minimum value of the specific mass is $i \cong 3.5$. When the number of satellites is increased, under other equal conditions, the mass-dimensional parameters are improved. The most common configurations in practice are three-satellite layouts or layouts with an odd number of satellites, since the load is distributed less uniformly in the case of an even number of satellites.

The diagram in Fig. 23 is made for a gear with self-aligning satellites and a “floating” sun pinion at $L/d = 0.7$ (see Figs. 12 and 13); values of the non-uniformity factor $K$ for number of satellites $N = 5$ and $N = 7$ correspond to the value of a relative error $\Delta = 30$, for $N = 3$ the value $K = 1$.

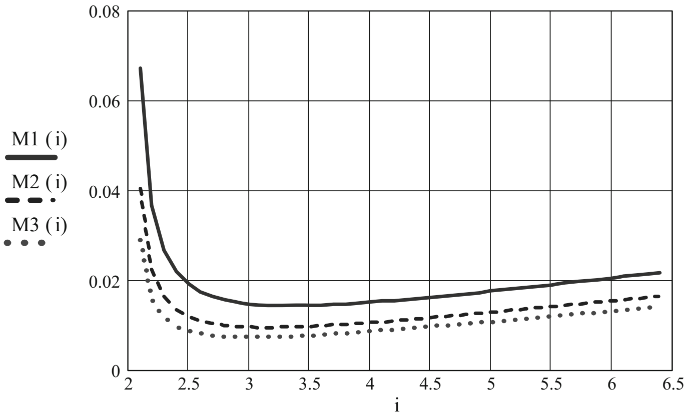



**Fig. 23** Relative mass of a gear versus gear ratio and the number of satellites for a gear with a "floating" sun pinion. *Subscripts 1* for $N = 3$; *2* for $N = 5$; *3* for $N = 7$

The relations shown allow for selecting rational values of parameters of a planetary gear providing close to uniform load distribution in meshing of gearwheels and, therefore, a high load carrying capacity of a mechanism at good mass-dimensional parameters and high efficiency.

## References

1. Kudryavtsev, V.N., Kirdyashev, Yu.N., Ginsburgm, E.G. In: Mashinostroyeniye, L. (ed.) Planetary Gears. Reference book (1977) (in Russian)
2. Plekhanov, F.I., Goldfarb, V.I.: Rational Designs of Planetary Transmissions, Geometry of Gearing and Strength Parameters (Series: Theory and Practice of Gearing and Transmissions, Mechanisms and Machine Science), pp. 285–300. Springer (2016)
3. Sondkar, P., Kahraman, A.: A Dynamic Model of a Double-Helical Planetary Gear Set. Mech. Mach. Theory **70**, 157–174 (2013)
4. Patent 2581222, RF, MPK F16H 1/32. Multi-row planetary gear / F.I. Plekhanov, A.D. Plekhanov—Publ. 27.10.2015, N30
5. Airapetov, E.L., Genkin, M.D. In: Nauka, M. (ed.) Deformability of Planetary Mechanisms (1973) (in Russian)
6. Plekhanov, F.I.: Investigation of mode of deformation of "satellite axis—carrier jaw" contact for a planetary gear. Izvestiya Vuzov, Mashinostroyeniye **2**, 36–41 (2015). (in Russian)