Geometric Pitch Configurations—Basic Primitives of the Mathematical Models for the Synthesis of Hyperboloid Gear Drives

V. Abadjiev and E. Abadjieva

Abstract The pitch configurations (circles and surfaces) are the basic primitives, upon which the mathematical models for synthesis of spatial gears with crossed axes of rotation are worked out. These mathematical models are created after the approach to synthesis based on one common point of contact between the operating tooth surfaces of the mating gears, this point being, at the same time, a common point of the pitch configurations. This point is called a pitch contact point. When the pitch circles and surfaces are in a static position, they are treated as geometric characteristics of the designed gears, and determine not only the basic parameters of their structure but also the dimensions of the gears' blanks. If the pitch configurations are put in a rotation according to a given law of motions transformation, then the dimensions and the mutual position of the configurations serve to define the dimensions and the longitudinal and profile geometry of the tooth surfaces contacting at the pitch point. The study deals with the synthesis of geometric pitch configurations for two main cases of three-link hyperboloid gears with externally mating gears: with normal (traditional) orientation of the gears and with inverse (opposite of the traditional) orientation of the gears.

Keywords Mathematical modeling · Synthesis · Hyperboloid gears · Pitch configurations

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1 Introduction

The mathematical models oriented to the synthesis of spatial gears transforming rotations between crossed axes (hyperboloid gear sets) ensure, in principle, a possibility for their multi-parametrical and multi-criteria optimization. This is explained by the existence of a large number of free parameters which take part in the description of the rotations transformation process. This fact creates possibilities for achieving desired technological and exploitation characteristics of the considered transmissions in their synthesis by looking for suitable combinations of the free parameters. Some of these characteristics are: use of universal and simple equipment for the manufacture of gears; high reliability and durability; low vibration activity and noiselessness; high accuracy of realization of the motions transformation law; high hydrodynamic loading capacity, etc. As a rule, the positive technological and exploitation qualities of the spatial gear mechanisms result from the higher requirements needed to obtain gears' specific kinematical characteristics. Everything mentioned up to now determines, to a great extent, the kinematical character of the chosen approach to the synthesis and the kinematical character of the created models [1–5].

The successful introduction of spatial gearings with new kinematical and strength characteristics in technics depends directly on the creation of adequate mathematical models for synthesis in accordance with the motion transformation processes described by them.

The global structure of each mathematical model for synthesis of a three-link spatial gear mechanism is determined by [6]:

- the purpose the gear-pair is designed for, from a viewpoint of the defined law of motion transformation;
- the geometry and the character of the conjugation of the tooth surfaces (i.e., whether the tooth surfaces contact at a point or along a line);
- the technological reasons for a choice of the instrumental surfaces' geometry and of the kinematics of the technological process by which the active tooth surfaces of the gear set are generated.

The performed and published researches [6–12] determine the authors' opinion about the types of mathematical models that are suitable for the synthesis of hyperboloid gears. Below, we will summarize the basic specific characteristics of one of these models.

The mathematical model for synthesis upon a pitch contact point is based on the assumption that the necessary quality characteristics (that define concrete exploitation and technological requirements to the active tooth surfaces) are guaranteed only at one concrete point P of the active tooth surfaces Σ_1 and Σ_2, and in its close vicinity (see Fig. 1) [9]. This model can be applied for synthesis of spatial gears with both point and linear contact. According to it, the common contact point

P of the conjugate tooth surfaces Σ_1 and Σ_2 is the common point of the circles $H_i^c (i = 1, 2)$ that are called *a pair of pitch circles* $(H_1^c{:}H_2^c)$. The point P is called *a pitch contact point*, the plane T_m, including the tangents to $H_i^c (i = 1, 2)$ at the point P is *a pitch plane*, and $m - m$ is the *pitch normal* to T_m at the point P. The mutual position of these two circles in the fixed space in the case of traditional constructions of hyperboloid gears with externally meshed tooth surfaces is illustrated in Fig. 1. The diameters $d_i (i = 1, 2)$ of $H_i^c (i = 1, 2)$, together with the parameters $a_i, \theta_i, \delta_i (i = 1, 2), \delta$ and a_w, define their mutual position in the fixed space [in the coordinate frames $S_i(O_i, x_i, y_i, z_i), (i = 1, 2)$]. These parameters are related to the definition of the longitudinal and profile orientations of the active tooth surfaces $\Sigma_i (i = 1, 2)$ at the pitch contact point. The pair of rotation surfaces, including the pair of pitch circles whose common normal at P is the straight-line $m - m$, are an analogue of $H_i^c (i = 1, 2)$. These surfaces are called *pitch surfaces.* The pair of circles $(H_1^c{:}H_2^c)$ is directly related with the evaluation of the pitch and of the tooth module of the designed gear set. The parameters $d_i, \delta_i (i = 1, 2)$ define the dimensions of the reference coaxial rotation surfaces, i.e., the blank proportions of the gears depend on them. The above parameters are used when the mounting dimensions of the synthesized gear-set are calculated.

Thus, the mathematical model for synthesis based on a pitch contact point ensures the solution of two basic problems:

- synthesis of the pitch circles/pitch surfaces;
- synthesis of the active tooth surfaces.

The necessary, and preliminarily-defined, geometric characteristics of the synthesized gear set in a close vicinity of the pitch contact point are found by solving these two problems together.

In conclusion, it should be pointed out that the approach to the synthesis of spatial gears described here is based on the following kinematical condition: *The relative velocity vector* $\overline{V}_{12}$ *at the pitch contact point P has to lie both in the pitch plane* T_m *and in the common tangent plane of the tooth surfaces* Σ_1 *and* Σ_2 *contacting at P, this vector being oriented along the common tangent to the longitudinal lines of the active tooth surfaces* $\Sigma_i (i = 1, 2)$.

Therefore, the considered approach to a basic synthesis upon a pitch contact point gives a possibility for the mathematical model and the algorithm (worked out on the model) to have a universal structure for all types of hyperboloid gears. The algorithm can be developed and become an algorithm for an optimizing synthesis. This is achieved through construction of criteria for control of the quality of meshing in the vicinity of the pitch contact point, the criteria taking into account the specifics of the geometry and of the technology of different hyperboloid gears in an adequate way.

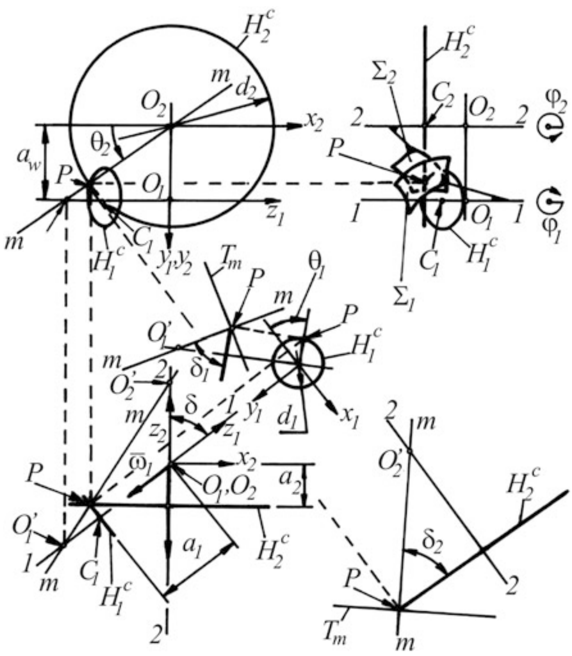

Fig. 1 Geometric and kinematic interpretation of the mathematical model for synthesis based on a pitch contact point: $H_i^c(i = 1, 2)$ are pitch circles; T_m is a pitch plane; $m - m$ is a pitch normal to T_m at the point P; $\Sigma_i(i = 1, 2)$ are tooth surfaces contacting at the point P

2 Pitch Configurations: Essence and Definition

Here, we study the pitch configurations that are basic elements of the mathematical models for synthesis of hyperboloid gears. These elements treat one actual, but still disputable (in terms of content and terminology), part of the meshing theory.

In the theory of spatial gearing, the terms “primary surfaces” and “pitch surfaces” have been used at the same time [13–16]. In most cases, these terms (primary and pitch surfaces) have been used for one and the same surfaces.

Professor F. Litvin gave the following definition of primary surfaces [13]:

“The primary surfaces H_1 and H_2 firmly connected with the movable links of the mechanism are called primary ones if the following conditions are fulfilled: (a) the rotation axis of the primary surface coincides with the rotation axis of the movable link; (b) the surfaces H_1 and H_2 tangent at a given point P of the fixed space, and the velocity of the relative motion of the links 1 and 2 at P lies on the common tangent to the helical lines of the surfaces H_1, H_2 and Q (author’s note: Q is a family of coaxial cylinders and the vector (helical) lines of the vector field of the relative motion velocity $\overline{V}^{(12)}$ are situated on them). The second requirement means that they have a common normal at the chosen point, and the velocity vector of the relative motion $\overline{V}^{(12)}$ lies in the common tangent plane of H_1 and H_2. … If $i_{12} =$ constant the primary surfaces could be arbitrary surfaces of revolution only if: (a) the axis of rotation $i - i$ of H_i is an axis of rotation of i-th link; (b) $\overline{V}^{(12)}$ lies in the common tangent plane of H_1 and H_2. For i_{12} = constant for the primary surfaces could be chosen random surfaces of rotation if: (a) the axis of rotation of $H_i\, i - i$ is the axis of rotation of the i-th link; (b) the common tangent planet to H_1 and H_2 contains $\overline{V}^{(12)}$.

In the theory of gearing, without any restriction to consider small parts of H_1 and H_2 in the vicinity of P, these surfaces are defined as a whole: they have a form of cylinders or cones depending on the vector $\overline{\varpi}^{(i)}$ being parallel to the tangent plane or not (author' note: $\overline{\varpi}^{(i)}$ is the vector of rotation of the movable link i). Such treatment is possible but not obligatory: H_1 and H_2 could be other surfaces of revolution if they have a common tangent plane at the point P. For the practice, it is convenient as primary surfaces to be chosen cylinders (for worms, cylindrical gears) or cones (for conical and hypoid gears).

In order to avoid any misunderstanding it is necessary to note the following principles:

(a) In the most common case the primary surfaces can not be identified with the axoids; such identification is possible only in the case of gear-pairs with parallel or intersecting axis, and is not permitted for gear sets with crossed axes of rotations; (b) the tooth surfaces Σ_1 and Σ_2 do not coincide with the primary surfaces. Although Σ_1 and Σ_2 tangent at the point P, the normal vectors $\overline{e}^{(\Sigma_i)}$ and $\overline{e}^{(H_i)}$ have different directions. The common tangent plane of Σ_1 and Σ_2 at the point P does not coincide with the common tangent plane of H_1 and H_2 but $\overline{V}^{(12)}$ belongs to each of them; (c) the condition of the simultaneous tangent of H_1, H_2 and Q at the point P is possible but not obligatory…".

Later, Litvin (see [14, 16]) called the primary surfaces "operating pitch surfaces" in accordance with their practical application in the design of spatial gears with crossed axes. They differ from the axoids of the movable links [15]:

"The operating pitch surfaces represents: (i) two cylinders for a worm-gear and helical gears with crossed axes and (ii) two cones for a hypoid gear drives. The chosen surfaces that are called in the technical literature "operating pitch surfaces" must satisfy the following requirements:

(i) The axes of cylinders (cones) have to form the same crossed angle and be at the same shortest distance as for the designed gears.
(ii) The cylinders (cones) must be in tangency at the middle point of contact of the surfaces of the gears to be designed.
(iii) The relative sliding velocity $\overline{V}_{12}$ at point P of tangency of the cylinders (cones) must lie in the plane that is tangent to the cylinders (cones) and $\overline{V}_{12}$ must be directed along the common tangent to the helices of the gears to be designed. The term "helix" is a conventional one. Actually, we have to consider a spatial curve that belongs to the operating cylinders (cones) and represents the line of intersection of the gear tooth surfaces with the operating cylinders (cones). For the case of a helical gear, a cylinder worm, this line of intersection is indeed a helix. For the case of spatial bevel gears and hypoid gears,

the line of intersection is a spatial curve that differs from a helix and might be represented with complicated equations.

(iv) The tangent point P of operating pitch cylinders (cones) will be simultaneously the point of tangency of gear tooth surfaces if the surfaces have a common normal $n-n$ at P and $n-n$ is perpendicular to $\overline{V}_{12}$ …"

In [17], W. Nelson treated the pitch surfaces for one concrete type of spatial gears—the Spiroid®[1] ones. There, he used the terms "primary pitch cone" (the coaxial cone limiting the tips of the Spiroid pinion threads) and "pitch surface" (an envelope of the primary pitch cone in its relative motion with respect to the axis of the second movable link of the Spiroid gear). On the common line of contact of both pitch surfaces, he looked for that pitch contact point which had determined the most suitable spatial curve used as a longitudinal line of the synthesized tooth surfaces of the Spiroid pinion. This is an approach to the choice of such surfaces that does not differ from those already considered.

From the survey conducted, it is established that the pitch configurations have influence when they define such basic characteristics of the gear-pair as: the structure and the geometry of the gear set, the longitudinal and profile orientations of the active tooth surfaces of the gears, pitch value, the tooth module, the strength loading of the gears, the shafts and bearings of the gear mechanism, the efficiency coefficient, etc. All said illustrates the great significance of the development of this scientific field of the theory of gearing from scientific, applied and methodological viewpoints. The authors of the present paper have given up a part of their researches to the mentioned topics in relation to the solution of problems connected with the synthesis and design of spatial gears [6–12]. First of all, our researches have been oriented to the precision of the content of the basic terms*: pitch circles* and *pitch surfaces*. The exact definition of these notions gives us the possibility to precise the applied mathematical models for synthesis of spatial gears on the one hand, and, on the other hand, ensures possibilities for new ideas referring to the creation of hyperboloid gears with new qualities, and new applications in technics, respectively. It is natural that each study in this field will be effective when it leads to creation of an adequate mathematical model, describing the status of the pitch configurations in the process of the spatial transformation of rotations.

Analyzing illustrations in Figs. 1 and 2, and commented upon above, we can conclude that:

- If the law of transformation of rotations $i_{12} = \omega_1/\omega_2 =$ constant between fixed crossed axes $1-1$ and $2-2$ (the shortest distance between them being $a_w =$ constant and the angle between them—$\delta = \angle(\overline{\omega}_1, \overline{\omega}_2) =$ constant) is given, and if the position of a point P (treated as a point of contact of conjugate tooth surfaces Σ_1 and Σ_2) in the fixed space is known, then the diameters and the mutual position of the circles $H_i^c (i = 1,2)$ are completely and uniquely determined. The circumferential velocity vectors $\overline{V}_i (i = 1,2)$ of the common point P,

[1]Spiroid and Helicon are trademarks registered by the Illinois Tool Works, Chicago, Ill.

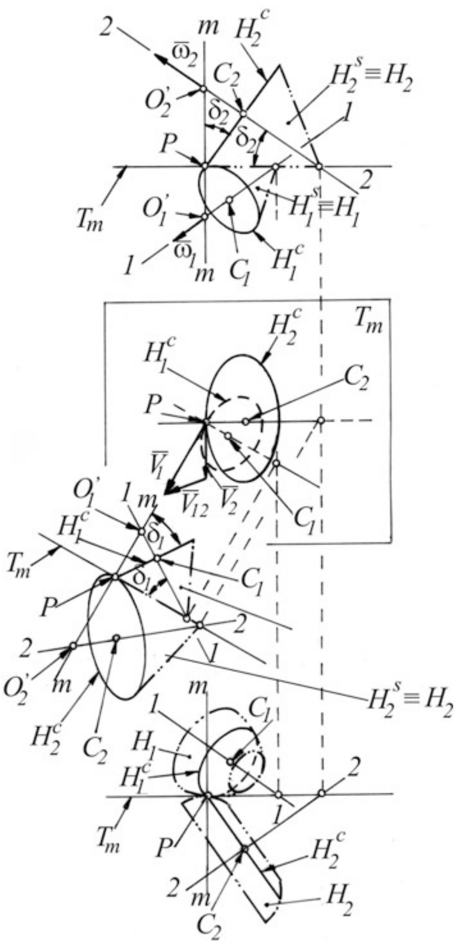

Fig. 2 Pitch configurations: $H_i^c (i = 1, 2)$ are pitch circles; $H_i^s (i = 1, 2)$ are pitch surfaces; $H_i (i = 1, 2)$ are primary surfaces with an arbitrary geometry; P is pitch contact point (pole of meshing); T_m is pitch plane; $m - m$ is pitch normal

and the relative velocity vector $\overline{V}_{12}$ at the same point, (i.e., the plane T_m where the coplanar vectors $\overline{V}_i (i = 1, 2)$ and $\overline{V}_{12}$ lie), as the normal $m - m$ to T_m at the point P are determined in a unique way as well.

- In the case considered in [13], the primary surfaces H_1 and H_2 are not defined in a unique way, since all surfaces of revolution, including the pair of circles $H_i^c (i = 1, 2)$, can be primary surfaces. The primary surfaces, discussed in [13], are simple rotation surfaces (cylinders and cones) tangent at one only point P. In practice, the algorithms for their synthesis define the diameters and the mutual position of the circles $H_i^c (i = 1, 2)$ passing through the common point P.
- It is sufficient to know the mutual position of the crossed axes of rotation $1 - 1$ and $2 - 2$, and the position of the point P (as a common point of the tooth

surfaces Σ_1 and Σ_2) in the fixed space, in order for the circles $H_i^c(i = 1, 2)$ to be completely and uniquely determined (as diameters and mutual position). The plane T_m formed by the tangents to the circles $H_i^c(i = 1, 2)$ at the point P, as the normal $m - m$ to T_m at the point P are uniquely determined as well. The mentioned upper parameters are geometric ones, since the circles $H_i^c(i = 1, 2)$ do not "put in rotation" according to the law $i_{12} = \omega_1/\omega_2$. After the law of rotations transformation begins acting, the geometric parameters defining the diameters and mutual position of $H_i^c(i = 1, 2)$, considered together with the kinematic parameters of the gear set, serve for determination of the longitudinal orientation of the conjugate tooth surfaces $\Sigma_i(i = 1, 2)$, of their pitches and of the gear module.

All said up to now is resulted in this pair of circles to be called *geometric pitch circles* [6–12]. Later, *the geometric pitch circles* will become *pitch circles* only, the plane T_m—*pitch plane*, and the normal $m - m$ to the pitch plane—*pitch normal* (see Fig. 2).

Depending on the position of the pitch circles with respect to the plane T_m, we differentiate:

- *externally contacting pitch circles* when $H_i^c(i = 1, 2)$ are situated in different half-spaces with respect to T_m;
- *internally contacting pitch circles* when $H_i^c(i = 1, 2)$ are situated in one and the same half-space with respect to T_m.

For the externally contacting configurations from Fig. 2, that written above applies to the pitch surfaces, which are alternatives to the pitch circles.

The externally tangent pitch circles have a relation to the synthesis of hyperboloid gears with external meshing (hypoid, Spiroid, helical, wormgears, etc.). As a hypothesis, the synthesis of internally tangent pitch circles can be treated as a stage of the design of new types of spatial gears with internal meshing.

An alternative to the defined pitch configurations are the so-called *kinematical pitch configurations*, consisting of *kinematical pitch surfaces* and *kinematical pitch circles,* whose synthesis is based on the synthesis of *pairs of isokinematical quasi-hyperboloids* [6]. The pitch surfaces, based on the synthesis of the hyperboloids axodes of spatial gears with crossed axes, should participate in the class of kinematic pitch configurations. Analogically to the geometric pitch surfaces, they are called pitch surfaces only [3–5]. The kinematic pitch configurations' synthesis depends on the preliminary given law of transformation of rotations $i_{12} = \omega_1/\omega_2 = \text{constant}$, and the basic geometric parameters $a_w = \text{constant}$ and $\delta = \text{constant}$ (characterizing the structure of the motions transformer) while the geometric pitch configuration synthesis does not take into account the rotations transformation law. Thus, the combination of kinematic and geometric parameters uniquely determines the pair of hyperboloids of revolution (the axodes that contacts along the instantaneous axis of the relative helical motion), the geometric axes of axodes coinciding with the axes of rotation of the hyperboloid gear shafts. In the synthesis of kinematic pitch surfaces, this axis is a locus of the kinematic pitch points chosen to be poles of meshing of the synthesized hyperboloid gears. If a point from the instantaneous helical axis is chosen as a common point of the tangent tooth surfaces Σ_1 and Σ_2, then the synthesized gear-pair has a minimal sliding velocity in a vicinity of this point (pole of meshing).

In this case, the process of optimization does not include the criterion that controls the magnitude of the sliding velocity between the tangent tooth surfaces. The kinematic pitch surfaces include the chosen conjugate parts from the axodes, as simple surfaces of revolution (cones and cylinders) that are approximations of the rotation hyperboloids at the chosen pole of meshing. If the design of the spatial gears with crossed axes is based on geometric pitch configurations, the chosen pitch contact point is defined by geometric parameters only. These parameters determine the form and dimensions of the geometric pitch surfaces, and of the blanks of the rotating links. Their synthesis does not depend on the sliding velocity at the points of contact of the conjugate tooth surfaces. In this case, the optimization process is being controlled by quality criteria, the sliding velocity here included.

3 Mathematical Model for Synthesis of Pitch Configurations with Normal Orientation of Hyperboloid Gears with External Meshing

Let two crossed axes $1-1$ and $2-2$ (being the axes of rotations of the movable links of a three-link tooth mechanism) be given in the fixed space. Their mutual position is defined by the angle $\delta = \text{constant}$ (the angle between the angular velocity vectors $\overline{\omega}_1$ and $\overline{\omega}_2$ of the moving links $(i = 1, 2)$) and the shortest distance $a_w = \text{constant}$. The concrete study is performed when $\delta \in (0, \pi)$. Each pitch circle H_i^c lies in a plane perpendicular to the axis of rotation $i - i$ of the movable link i and has a radius equal to the distance from the point P to the axis $i - i$. The pitch plane T_m is determined by the tangents at the pitch contact point P to the circles H_1^c and H_2^c. The straight-line $m - m$ is the normal to T_m at P.

The study is performed by means of the notations and the coordinate frames $S_1(O_1, x_1, y_1, z_1)$ and $S_2(O_2, x_2, y_2, z_2)$, introduced in Figs. 3 and 4.

The dimensions and the mutual position of H_1^c and H_2^c are completely determined by the cylindrical coordinates $a_i, r_i, \theta_i (i = 1, 2)$ of the contact point P in the systems $S_i (i = 1, 2)$ and by the angles $\delta_i (i = 1, 2)$ between the planes of $H_i^c (i = 1, 2)$ and the normal $m - m$. The pitch surfaces $H_i^s (i = 1, 2)$—analogs of the pitch circles are illustrated in Figs. 3 and 4.

We represent the radius-vector $\overline{\mathrm{O_1P}}$ and the unit vector $\overline{m}$ of the normal $m - m$ by $a_1,\ r_1,\ \theta_1$, and $a_2,\ r_2,\ \theta_2$, and using a_w and δ. Thus, we get the following set of equations:

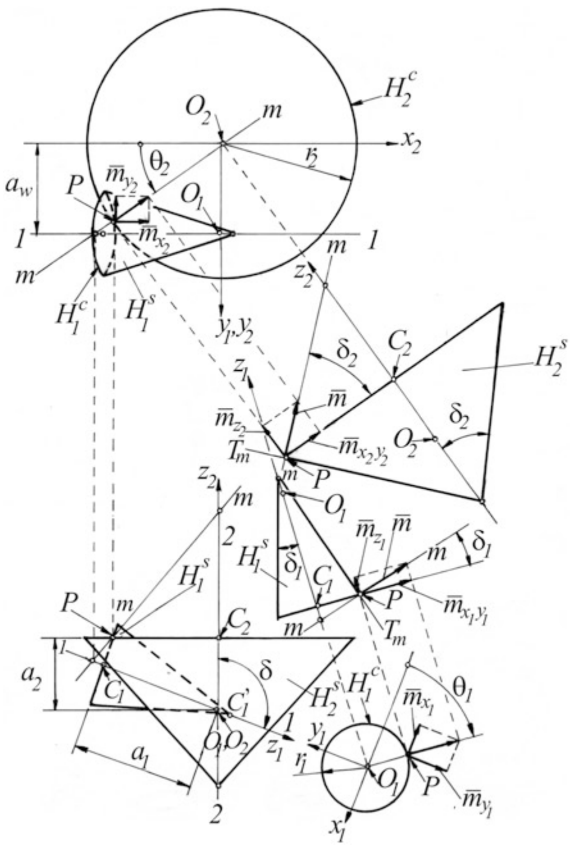

Fig. 3 Externally contacting pitch circles $H_i^c (i = 1, 2)$ and pitch surfaces $H_i^s (i = 1, 2)$, corresponding to hyperboloid gears with external meshing when $z_{2,C_2} > 0$

$$
\begin{aligned}
&r_1 \cos\theta_1 = r_2 \cos\theta_2 \cos\delta \pm a_2 \sin\delta, \\
&r_1 \sin\theta_1 = a_w - r_2 \sin\theta_2, \\
&a_1 = r_2 \cos\theta_2 \sin\delta \mp a_2 \cos\delta, \\
&\cos\delta_1 \sin\theta_1 = \cos\delta_2 \sin\theta_2, \\
&\sin\delta_1 = \sin\delta_2 \cos\delta + \cos\delta_2 \cos\theta_2 \sin\delta, \\
&\cos\delta_1 \cos\theta_1 = \sin\delta_2 \sin\delta - \cos\delta_2 \cos\theta_2 \cos\delta.
\end{aligned}
\tag{1}
$$

Here, the upper signs refer to externally contacting pitch circles $H_i^c (i = 1, 2)$ and pitch surfaces $H_i^s (i = 1, 2)$ when $z_{2,C_2} > 0$ (Fig. 3), and the lower ones—for externally contacting/tangent pitch circles $H_i^c (i = 1, 2)$ and pitch surfaces $H_i^s (i = 1, 2)$ when $z_{2,C_2} < 0$ (Fig. 4). Besides, each of the last three equations in (1) is a consequence of the other two.

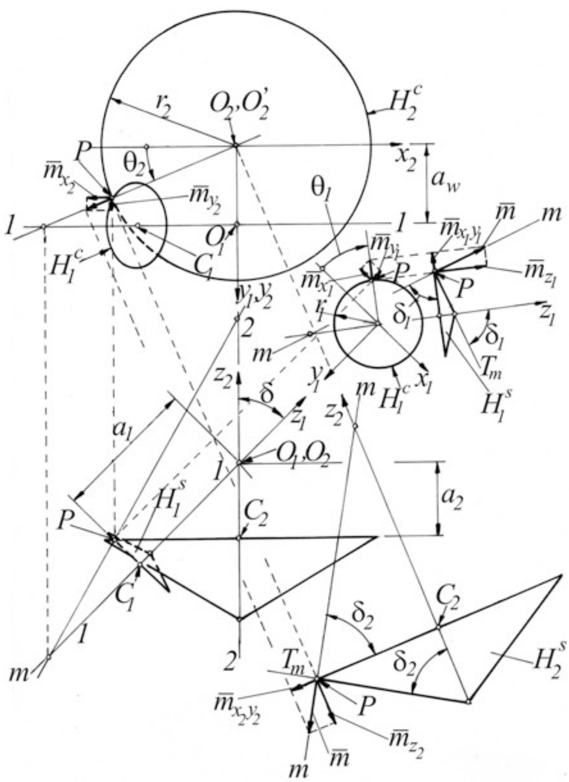

Fig. 4 Externally contacting pitch circles $H_i^c (i = 1, 2)$ and pitch surfaces $H_i^s (i = 1, 2)$, corresponding to hyperboloid gears with external meshing when $z_{2,C_2} < 0$

Later, the study of the system (1) will be performed taking into account the following geometric conditions: $\theta_1 \in \left[0, \frac{\pi}{2}\right], \theta_2 \in \left(0, \frac{\pi}{2}\right], \delta_1 \in \left[0, \frac{\pi}{2}\right), \delta_2 \in \left[0, \frac{\pi}{2}\right]$, $a_w > 0, r_i > 0 (i = 1, 2), a_i \geq 0 (i = 1, 2)$.

We will solve and study the set (1) of 5 independent equations with 10 unknowns $\delta, a_w, \delta_1, r_1, a_1, \theta_1, \delta_2, r_2, a_2, \theta_2$. For this purpose, 5 among the unknowns will be considered as free ones. Let them be $\delta, a_w, \delta_1, r_1$ and a_1. We will look for those analytical conditions that the free parameters must fulfill so that the system (1) might have a solution.

It is natural to study the following basic cases: $\delta = \frac{\pi}{2}$ (the axes of rotations $1 - 1$ and $2 - 2$ are orthogonal); $\delta \neq \frac{\pi}{2}, z_{2,C_2} < 0; \delta \neq \frac{\pi}{2}, z_{2,C_2} > 0$. The last two cases treat non-orthogonal hyperboloid gears and are illustrated in Figs. 3 and 4, respectively.

3.1 Synthesis of Orthogonally Tangent Pitch Configurations: $\delta = \frac{\pi}{2}$

Let us pay attention to the following cases that are essential for the practice:

3.1.1 $\delta_1 = 0$ and $a_1 \neq 0$

This case refers to orthogonal hyperboloid gears with external meshing when the coaxial surfaces of the link $i = 1$—reference, root and tip surfaces, are of cylindrical form.

System (1) has the following unique solution:

$$\theta_1 = 0, \tan\theta_2 = \frac{a_w}{a_1}, a_2 = r_1, r_2 = \frac{a_w}{\sin\theta_2}, \delta_2 = \frac{\pi}{2}, \tag{2}$$

in an arbitrary choice of a_w, r_1 and a_1.

The parameters in (2) define the dimensions and the position of the pitch circles corresponding to spatial high reduction gears of type Helicon® [18]. This is the borderline case that separates externally and internally tangent geometric pitch circles (Fig. 5).

3.1.2 $\delta_1 = 0$ and $a_1 = 0$

The solution of the set (1) is described with one more parameter δ_2, namely:

$$\theta_2 = \frac{\pi}{2}, \theta_1 = \frac{\pi}{2} - \delta_2, a_2 = r_1 \sin\delta_2, r_2 = a_w - r_1 \cos\delta_2. \tag{3}$$

For the existence of a solution, it is necessary for $\cos\delta_2 \leq \frac{a_w}{r_1}$.

If $a_w > r_1$, the upper condition is always fulfilled.

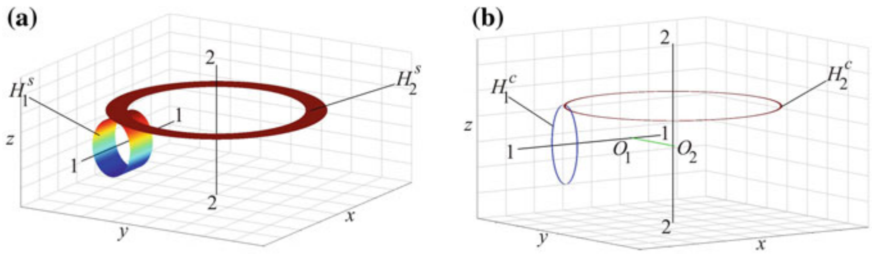

Fig. 5 Pitch configurations for the gears of type Helicon: **a** pitch surfaces $H_i^s(i = 1, 2)$; **b** pitch circles $H_i^c(i = 1, 2); a_w = 100$ mm; $\delta = 90°; \delta_1 = 0°; a_1 = 119.18$ mm; $r_1 = 31$ mm; $\delta_2 = 90°; a_2 = 31$ mm; $r_2 = 155.58$ mm

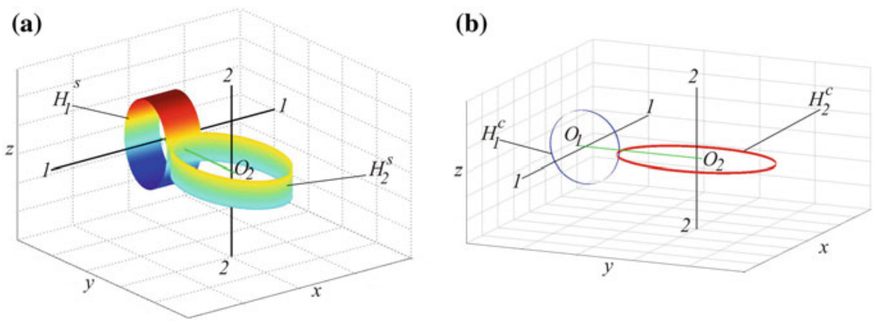

Fig. 6 Pitch configurations for orthogonal worm/helical gears: **a** pitch surfaces $H_i^s(i = 1, 2)$; **b** pitch circles $H_i^c(i = 1, 2)$; $a_w = 100$ mm; $\delta = 90°$; $a_1 = a_2 = 0$ mm; $\delta_1 = \delta_2 = 0°$; $r_1 = 30$ mm; $r_2 = 69$ mm

Pitch configurations corresponding to one particular solution,

$$\delta_2 = 0, \theta_1 = \frac{\pi}{2}, a_2 = 0, r_2 = a_w - r_1, \theta_2 = \frac{\pi}{2}, \tag{4}$$

correspond to orthogonal worm gears or helical gears (Fig. 6).

Solution (4) of (1) is the only one for which the pitch contact point P is situated on the common normal O_1O_2 of the crossed axes $1-1$ and $2-2$.

Solution (3) of set (1) of the form

$$\delta_2 = \frac{\pi}{2}, \theta_1 = 0, a_2 = r_1, r_2 = a_w, \theta_2 = \frac{\pi}{2} \tag{5}$$

defines the geometric characteristics of toroid gears whose pitch surfaces and circles are illustrated in Fig. 7.

3.1.3 $\delta_1 > 0$

In this case, the condition for the existence of geometric pitch configurations is $a_1 \geq (a_w - r_1)\tan\delta_1$, and the solution to system (1) is:

$$\begin{aligned}
\cot\theta_2 &= \frac{r_1 \tan\delta_1 + a_1}{a_w}, \sin\theta_1 = \frac{a_w}{r_1 + a_1\cot\delta_1}, \\
a_2 &= r_1\cos\theta_1, \cos\delta_2 = \frac{\sin\delta_1}{\cos\theta_2}, \\
r_2 &= \frac{a_w - r_1\sin\theta_1}{\sin\theta_2}.
\end{aligned} \tag{6}$$

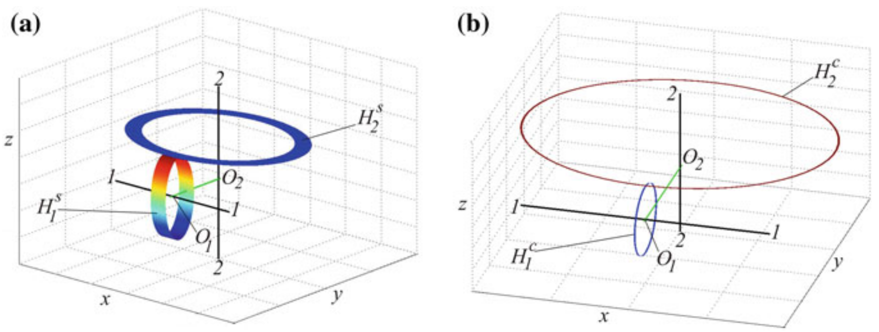

Fig. 7 Pitch configurations for toroid gears: **a** pitch surfaces $H_i^s(i = 1, 2)$; **b** pitch circles $H_i^c(i = 1, 2); a_w = 100$ mm; $\delta = 90°; \delta_1 = 0°; a_1 = 0$ mm; $r_1 = 31$ mm; $\delta_2 = 90°; a_2 = 31$ mm; $r_2 = 100$ mm

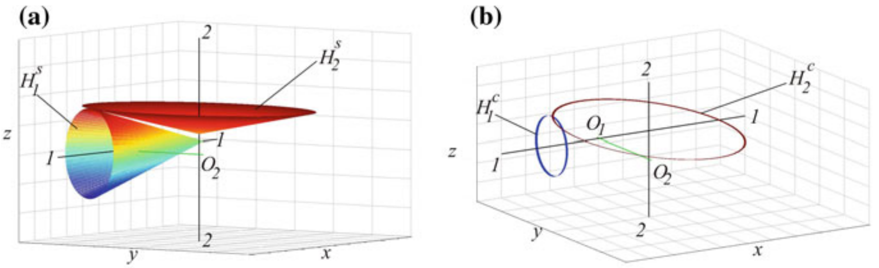

Fig. 8 Pitch configurations for orthogonal hypoid/spiroid gears: **a** pitch surfaces $H_i^s(i = 1, 2)$; **b** pitch circles $H_i^c(i = 1, 2); a_w = 100$ mm; $\delta = 90°; \delta_1 = 5°; a_1 = 100$ mm; $r_1 = 31$ mm; $\delta_2 = 83°; a_2 = 30.89$ mm; $r_2 = 139.57$ mm

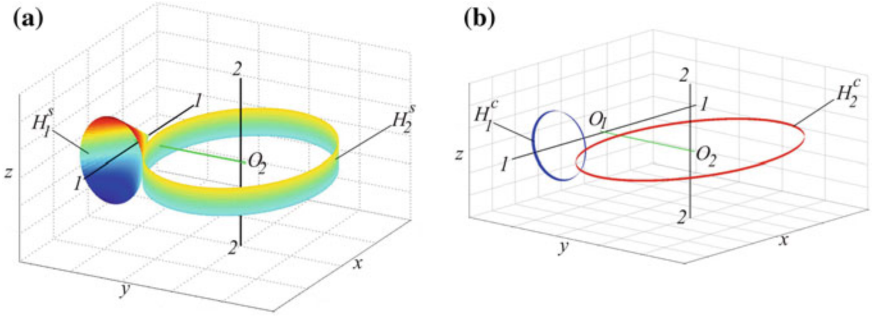

Fig. 9 Pitch configurations of a "common" wormgears: **a** pitch surfaces $H_i^s(i = 1, 2)$; **b** pitch circles $H_i^c(i = 1, 2); a_w = 100$ mm; $\delta = 90°; \delta_1 = 30°, a_1 = 39.84$ mm; $r_1 = 31$ mm; $\delta_2 = 0°; a_2 = 0$ mm; $r_2 = 79.67$ mm

The parameters of the pitch configurations of orthogonal hypoid gears and gears of type Spiroid [14] are calculated by the relations given in (6). The pitch surfaces and circles are visualized in Fig. 8.

One particular solution of (6) is the case of "common" wormgears, when

$$a_2 = 0, \theta_1 = \frac{\pi}{2}, \tan\delta_1 = \frac{a_1}{a_w - r_1}, \theta_2 = \frac{\pi}{2} - \delta_1, r_2 = \frac{a_w - r_1}{\cos\delta_1}. \tag{7}$$

The pitch surfaces and circles of this type spatial gears are shown in Fig. 9.

3.2 Synthesis of Non-orthogonal Contacting Pitch Configurations When $\delta \neq \frac{\pi}{2}$ and $z_{2,C_2} < 0$

First, it should be pointed out that system (1) has a solution if $\delta \in (0, \pi/2)$, which follows from the condition $a_2 \geq 0$.

The unknown θ_2 is calculated by the formula

$$\cot\theta_2 = \frac{(r_1 \tan\delta_1 + a_1)\sin\delta}{a_w}. \tag{8}$$

To find θ_1, we use the equation

$$\cos(\delta - \delta_1)t^2 - 2\cot\theta_2 \cdot \cos\delta_1 \cdot t - \cos(\delta + \delta_1) = 0, \tag{9}$$

where $t = \tan\dfrac{\theta_1}{2}$.

Let us consider the following cases consecutively:

3.2.1 $\delta + \delta_1 \leq \frac{\pi}{2}$

In this case, it is necessary that $a_1 > 0$ or $\delta_1 > 0$ to be fulfilled.

If, additionally, the condition

$$a_1 \leq a_w \cot\delta - r_1 \tan\delta_1 \tag{10}$$

is satisfied, which presumes $r_1 \leq a_w \cot\delta_1 \cot\delta$, then

$$\tan\frac{\theta_1}{2} = \frac{\cos\delta_1 \cot\theta_2 + \sqrt{D}}{\cos(\delta - \delta_1)}, \tag{11}$$

where $D = \frac{\cos^2\delta_1 \sin^2\delta}{a_w^2}\left[(r_1 \tan\delta_1 + a_1)^2 - a_w^2(\tan^2\delta_1 - \cot^2\delta)\right]$.

If the condition

$$a_1 \geq a_w \cot\delta - r_1 \tan\delta_1 \tag{12}$$

is fulfilled, then

$$\tan\frac{\theta_1}{2} = \frac{\cos\delta_1 \cot\theta_2 - \sqrt{D}}{\cos(\delta - \delta_1)}. \tag{13}$$

If the equality $\delta + \delta_1 = \frac{\pi}{2}$ is true, i.e., $\delta = \frac{\pi}{2} - \delta_1$, then

$$\theta_1 = 0, \delta_2 = \frac{\pi}{2}, r_2 = \frac{a_w}{\sin\theta_2}, a_2 = a_1\cos\delta - r_1\sin\delta \tag{14}$$

in condition $r_1 \leq a_1 \cot\delta$. Equalities (14) are obtained from (15), and define the pitch configurations of a non-orthogonal spatial face gear-pair (Fig. 10).

3.2.2 $\delta + \delta_1 > \frac{\pi}{2}$

In this case, if $a_1 + r_1 \tan\delta_1 \in [a_w\sqrt{\tan^2\delta_1 - \cot^2\delta}, a_w \cot\delta]$, which supposes that $\tan\delta_1 \leq \sqrt{2}\cot\delta$, then θ_1 is calculated by the equality (11).

But if $a_1 + r_1 \tan\delta_1 > \max(a_w \cot\delta,\ a_w\sqrt{\tan^2\delta_1 - \cot^2\delta})$, then θ_1 is obtained through (13).

For the calculation of the rest of the unknowns in the cases 3.2.1 and 3.2.2, we use

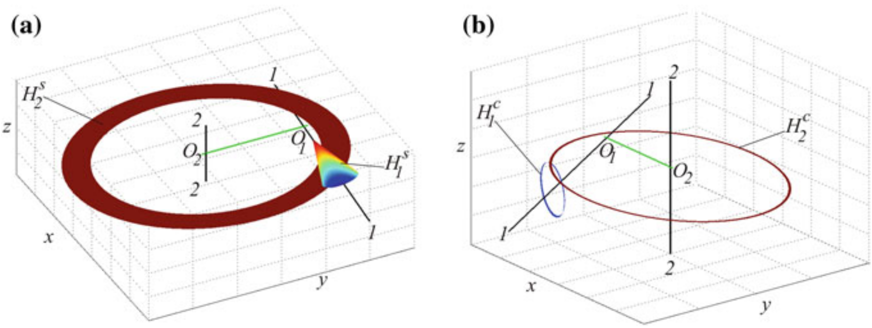

Fig. 10 Pitch configurations: **a** pitch surfaces $H_i^s(i = 1,2)$; **b** pitch circles $H_i^c(i = 1,2); a_w = 110$ mm; $\delta = 75°; \delta_1 = 15°; a_1 = 100$ mm; $r_1 = 20$ mm; $\delta_2 = 90°; a_2 = 6.57$ mm; $r_2 = 149.86$ mm

$$\begin{aligned} & a_2 = a_1 \cos\delta - r_1 \sin\delta \cos\theta_1, \\ & r_2 = \frac{a_w - r_1 \sin\theta_1}{\sin\theta_2}, \cos\delta_2 = \frac{\cos\delta_1 \sin\theta_1}{\sin\theta_2}. \end{aligned} \tag{15}$$

3.3 Synthesis of Non-orthogonal Contacting Pitch Configurations When $\delta \neq \frac{\pi}{2}$ and $z_{2,C_2} > 0$

3.3.1 $\delta_1 = 0$

Let consider the following two cases:

If $a_1 = 0$, then the solution to the set of Eq. (1) is in the form

$$\theta_1 = \theta_2 = \frac{\pi}{2}, \delta_2 = 0, a_2 = 0, r_2 = a_w - r_1 \tag{16}$$

when the condition $a_w > r_1$ is satisfied and if $\delta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, 0\right)$.

Relations (16) describe the pitch surfaces and circles of non-orthogonal worm/helical gears, as shown in Fig. 11. Only in this case of non-orthogonal hyperboloid gears, the common pitch contact point P lies on the common normal O_1O_2 of the crossed axes $1-1$ and $2-2$.

If $a_1 \neq 0$, the solution to (1) is

$$\begin{aligned} & \cot\theta_2 = \frac{a_1}{a_w}\sin\delta, \cot\theta_1 = -\frac{a_1}{a_w}\tan\delta, \\ & \tan\delta_2 = -\cos\theta_2 \tan\delta, a_2 = r_1 \cos\theta_1 \sin\delta - a_1 \cos\delta, \\ & r_2 = \frac{a_w - r_1 \sin\theta_1}{\sin\theta_2}. \end{aligned} \tag{17}$$

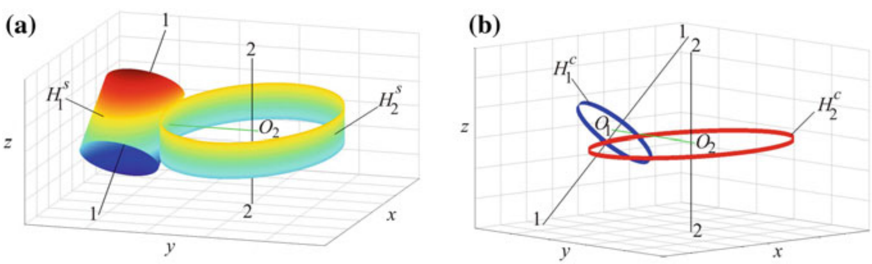

Fig. 11 Pitch configurations for non-orthogonal worm and helical gears: **a** pitch surfaces $H_i^s (i = 1, 2)$; **b** pitch circles $H_i^c (i = 1, 2); a_w = 100$ mm; $\delta = 45°; \delta_1 = \delta_2 = 0°; a_1 = a_2 = 0; r_1 = 31$ mm; $r_2 = 69$ mm

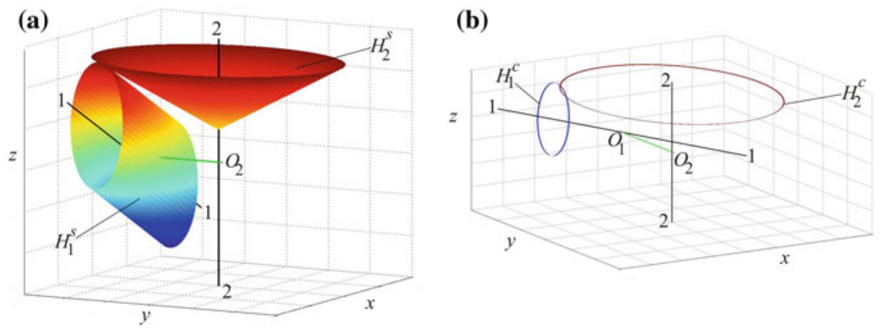

Fig. 12 Pitch configurations for non-orthogonal hyperboloid gears with cylindrical pinion and conical gear: **a** pitch surfaces $H_i^s(i=1,2)$; **b** pitch circles $H_i^c(i=1,2); a_w=100$ mm; $\delta=100°; a_1=110$ mm; $\delta_1=0°; r_1=31$ mm; $a_2=49.25$ mm; $\delta_2=76°30', r_2=140.20$ mm

The intervals for the unknowns involve the conditions $\delta>\dfrac{\pi}{2}, a_1^2>(r_1^2-a_w^2)\cot^2\delta$ (the second condition is true if $r_1<a_w$).

In Fig. 12, you can see the pitch configurations of non-orthogonal hyperboloid gears with a cylindrical pinion and a conical gear.

3.3.2 $\delta_1>0$

Now, we will pay attention to the following cases:

- Let $\delta_1=\delta-\dfrac{\pi}{2}$, which a priori involves $\delta>\dfrac{\pi}{2}$.

  Then, the solution to the set of Eq. (1) is given by

$$
\begin{aligned}
\cot\theta_2 &= \frac{r_1\sin\delta_1+a_1\cos\delta_1}{a_w}, \tan\frac{\theta_1}{2}=\frac{\sin\delta_1}{\cot\theta_2},\\
\cos\delta_2 &= \frac{\cos\delta_1\sin\theta_1}{\sin\theta_2}, a_2=r_1\cos\theta_1\sin\delta+a_1\sin\delta_1,\\
r_2 &= \frac{a_w-r_1\sin\theta_1}{\sin\theta_2}.
\end{aligned}
\tag{18}
$$

Algorithm (18) describes the geometric pitch configurations of non-orthogonal hypoid/Spiroid gearing. One concrete design is illustrated in Fig. 13.

The inequalities defined in the paper beginning

$$\theta_1\in\left[0,\frac{\pi}{2}\right], \delta_1\in\left[0,\frac{\pi}{2}\right), \delta_2\in\left[0,\frac{\pi}{2}\right], a_w>0, r_i>0 \quad (i=1,2),\ a_i\geq 0\ (i=1,2)$$

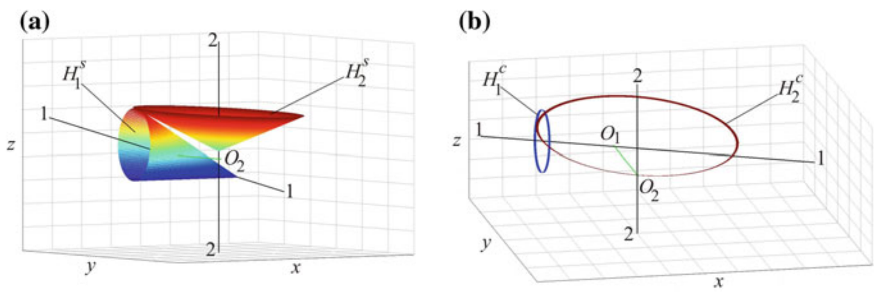

Fig. 13 Pitch configurations for non-orthogonal hyperboloid gears with conical pinion and gear when $\delta - \delta_1 = \frac{\pi}{2}$: **a** pitch surfaces $H_i^s (i = 1, 2)$; **b** pitch circles $H_i^c (i = 1, 2); a_w = 100$ mm; $\delta = 95°; a_1 = 110$ mm; $\delta_1 = 5°; r_1 = 31$ mm; $a_2 = 40.10$ mm; $\delta_2 = 76°38', r_2 = 143.17$ mm

impose the following restrictions on the free parameters:

If $a_w > r_1$, it is also necessary that $a_1 \geq (a_w - r_1) \tan \delta_1$.

If $a_w < r_1$ it is necessary that $a_1^2 \geq (r_1^2 - a_w^2) \tan^2 \delta_1$.

- Let $\delta_1 + \delta = \frac{\pi}{2}$, i.e., $\delta = \frac{\pi}{2} - \delta_1$.

From the last relation, it immediately follows that $\delta \in \left(0, \frac{\pi}{2}\right)$. Then,

$$\cot \theta_2 = \frac{r_1 \sin \delta_1 + a_1 \cos \delta_1}{a_w}. \tag{19}$$

In this case, the set of Eq. (1) has the following two solutions:

$$\theta_1 = 0, \delta_2 = \frac{\pi}{2}, r_2 = \frac{a_w}{\sin \theta_2}, a_2 = r_1 \cos \delta_1 - a_1 \sin \delta_1 \tag{20}$$

which is possible if $a_1 \leq r_1 \cot \delta_1$;

$$\tan \frac{\theta_1}{2} = \frac{\cot \theta_2}{\sin \delta_1}, \delta_2 = \frac{\cos \delta_1 \sin \theta_1}{\sin \theta_2}, r_2 = \frac{a_w - r_1 \sin \theta_1}{\sin \theta_2}, \tag{21}$$
$$a_2 = r_1 \cos \theta_1 \sin \delta - a_1 \cos \delta$$

if the condition $a_1 \leq (a_w - r_1) \tan \delta_1$ is true.

The cases below differ only in regard to the conditions imposed upon the parameters $\delta, a_w, \delta_1, r_1$ and a_1 and the formula for the calculation of θ_1.

The remaining unknowns have been calculated by:

$$\begin{aligned} \cot\theta_2 &= \frac{(a_1 + r_1 \tan\delta_1)\sin\delta}{a_w}, \\ \cos\delta_2 &= \frac{\cos\delta_1 \sin\theta_1}{\sin\theta_2}, r_2 = \frac{a_w - r_1 \sin\theta_1}{\sin\theta_2}, \\ a_2 &= r_1 \cos\theta_1 \sin\delta - a_1 \cos\delta. \end{aligned} \tag{22}$$

Let us describe the cases mentioned:

- If $\delta < \frac{\pi}{2}, \delta_1 + \delta < \frac{\pi}{2}, a_w > r_1$, and $a_1 \leq (a_w - r_1)\tan\delta_1$, then θ_1 is calculated by (11).
- If $\delta > \frac{\pi}{2}$, $\delta_1 > \delta - \frac{\pi}{2}$ and $a_1 \geq \max(0, (\mathrm{a_w} - \mathrm{r_1})\tan\delta_1)$, then θ_1 is received using (13).
- If $\delta > \frac{\pi}{2}$, $\delta_1 > \delta - \frac{\pi}{2}$ and $a_1 \geq \max(0, (a_w - r_1)\tan\delta_1)$, then θ_1 is calculated by equality (13).
- If $\delta < \frac{\pi}{2}, \delta_1 + \delta > \frac{\pi}{2}, a_w > r_1, a_1 \leq (a_w - r_1)\tan\delta_1$, and $(r_1 \tan\delta_1 + a_1)^2 > a_w^2(\tan^2\delta_1 - \cot^2\delta)$, then θ_1 is determined by means of (11).

Another variant of the pitch surfaces and circles of non-orthogonal hyperboloid gears of type hypoid or Spiroid is shown in Fig. 14.

- If $\delta > \frac{\pi}{2}, \delta_1 > \delta - \frac{\pi}{2}, a_w > r_1, a_1 \leq (a_w - r_1)\tan\delta_1$, and $(r_1 \tan\delta_1 + a_1)^2 > a_w^2(\tan^2\delta_1 - \cot^2\delta)$, then θ_1 can be calculated by both (11) and (13).

The pitch configurations illustrated above (Figs. 5, 6, 7, 8, 9, 10, 11, 12, 13 and 14) are obtained from the developed computer program. The coordinate system $S(x, y, z)$, which is shown in all figures, is a basic one. According to it, the location of the coordinate systems S_1 and S_2 (in Figs. 3 and 4) of the corresponding pitch configurations H_i^s and $H_i^c(i = 1, 2)$ is fixed. In the program created, visualization of the coordinate systems S_1 and S_2 is not intended.

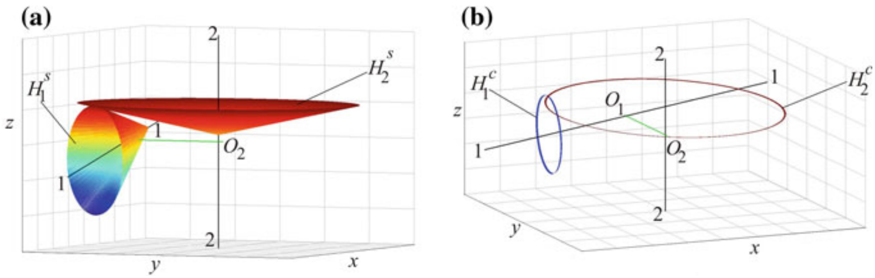

Fig. 14 Pitch configurations for non-orthogonal hyperboloid gears with conical pinion and gear when $\delta_1 + \delta > \frac{\pi}{2}$: **a** pitch surfaces $H_i^s(i = 1, 2)$; **b** pitch circles $H_i^c(i = 1, 2); a_w = 100$ mm; $\delta = 85°; a_1 = 110$ mm; $\delta_1 = 10°; r_1 = 31$ mm; $a_2 = 21.20$ mm; $\delta_2 = 83°21', r_2 = 148.77$ mm

4 Mathematical Model for Synthesis of Pitch Configurations of Hyperboloid Gears with External Meshing and Inverse Orientation of the Pitch Configurations

The study is performed by the notations and the coordinate frames $S_1(O_1, x_1, y_1, z_1)$ and $S_2(O_2, x_2, y_2, z_2)$ introduced in Fig. 15 [10]. The dimensions and mutual position of H_1^c and H_2^c are completely determined by the cylindrical coordinates $a_i, r_i, \theta_i (i = 1, 2)$ of the contact point P in the systems $S_i (i = 1, 2)$, and by the angle $\delta_i (i = 1, 2)$ between the plane of $H_i^c (i = 1, 2)$ and the normal $m - m$. The analogs of the pitch circles $H_i^c (i = 1, 2)$, which are the pitch surfaces (cones) $H_i^s (i = 1, 2)$

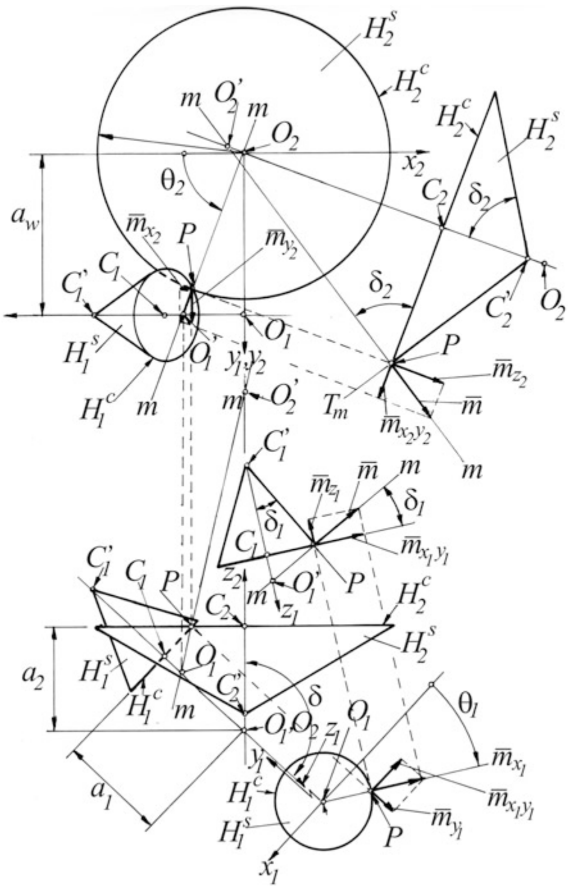

Fig. 15 Externally contacting pitch circles $H_i^c (i = 1, 2)$ and pitch surfaces $H_i^s (i = 1, 2)$ with inverse orientation corresponding to hyperboloid gears with external meshing

are illustrated in Fig. 2. The points of intersection $C_i'(i = 1, 2)$ of the axes of rotation $i - i(i = 1, 2)$ and the plane T_m are tips of the pitch cones $H_i^s(i = 1, 2)$.

Figure 15 is oriented to the synthesis of pitch configurations whose position in the fixed space ensures the design of one special group of non-orthogonal hyperboloid gears. There, the geometric pitch configurations, when they are with *inverse orientation* in the fixed space, can be seen. This type of pitch configurations is characteristic for special constructive types of non-orthogonal spatial gears, which allows for the optimal bearing of both gears from a strength point of view. The geometric characteristics: $\delta > \dfrac{\pi}{2}; z_{2,C_2} > 0; \angle\left(\overline{C_1C_1'}, \overline{O_1z_1}\right) = 180°; \angle\left(\overline{C_2C_2'}, \overline{O_2z_2}\right) = 180°$ are valid for this class of gears.

Representing the radius-vector $\overline{O_1P}$ and the unit vector $\overline{m}$ of the normal $m - m$ by a_1, r_1, θ_1 and a_2, r_2, θ_2, and using a_w and δ, we get to the following set of equations:

$$\begin{aligned}
&r_1 \cos\theta_1 = r_2 \cos\theta_2 \cos\delta + a_2 \sin\delta,\\
&r_1 \sin\theta_1 = a_w - r_2 \sin\theta_2,\\
&a_1 = r_2 \cos\theta_2 \sin\delta - a_2 \cos\delta,\\
&\cos\delta_1 \sin\theta_1 = \cos\delta_2 \sin\theta_2,\\
&\sin\delta_1 = -(\sin\delta_2 \cos\delta + \cos\delta_2 \cos\theta_2 \sin\delta),\\
&\cos\delta_1 \cos\theta_1 = \sin\delta_2 \sin\delta - \cos\delta_2 \cos\theta_2 \cos\delta.
\end{aligned} \tag{23}$$

We will examine (23), taking into account the following geometric conditions:

$$\theta_1 \in \left[0, \frac{\pi}{2}\right], \theta_2 \in \left(0, \frac{\pi}{2}\right], \delta_1 \in \left[0, \frac{\pi}{2}\right), \delta_2 \in \left[0, \frac{\pi}{2}\right], a_w > 0, r_i > 0, a_i \geq 0$$
$$(i = 1, 2).$$

Each of the last three equations in (23) is the consequence of the other two, i.e., (23) is a set of 5 independent equations with 10 unknowns: $\delta, a_w, \delta_1, r_1, a_1, \theta_1, \delta_2, r_2, a_2, \theta_2$. Therefore, each solution to (23) is a function of 5 of them (we will consider them as free ones). We suppose that the independent (free) parameters are $\delta_1, a_w, \delta_1, r_1$ and a_1. We will look for the analytical relations that have to be fulfilled so that the set (23) can have a solution.

4.1 Synthesis of Non-orthogonally Contacting Pitch Configurations with Inverse Orientation

After examining the set of Eq. (23), written in accordance with the symbols given in Fig. 15, the following three groups of solutions are established:

4.1.1 $\delta = \frac{\pi}{2} + \delta_1$

Then, the solution to (23) is of the form

$$\begin{aligned} \theta_1 = 0, \delta_2 > \frac{\pi}{2}, r_2 = \frac{a_w}{\sin\theta_2}, a_2 = r_1 \sin\delta - a_1 \cos\delta, \\ \cot\theta_2 = \frac{(a_1 - r_1 \tan\delta_1)\sin\delta}{a_w} = \frac{a_1\cos\delta_1 - r_1\sin\delta_1}{a_w}. \end{aligned} \tag{24}$$

The condition for the existence of solution (24) is $a_1 \geq r_1 \tan\delta_1$. This solution is illustrated in Fig. 16a.

4.1.2 $\delta > \frac{\pi}{2} + \delta_1$

In this case, the solution to (23) can be calculated by the formulae

$$\begin{aligned} \cot\theta_2 &= \frac{(a_1 - r_1\tan\delta_1)\sin\delta}{a_w}, \\ \tan\frac{\theta_1}{2} &= \frac{\cos\delta_1\cot\theta_2 - \sqrt{D}}{\cos(\delta + \delta_1)}, \\ a_2 &= r_1\cos\theta_1\sin\delta - a_1\cos\delta, \\ \cos\delta_2 &= \frac{\cos\delta_1\sin\theta_1}{\sin\theta_2}, r_2 = \frac{a_w - r_1\sin\theta_1}{\sin\theta_2}, \end{aligned} \tag{25}$$

where $D = \frac{\cos^2\delta_1\sin^2\delta}{a_w^2}\left[(a_1 - r_1\tan\delta_1)^2 + a_w^2\left(\cot^2\delta - \tan\delta_1\right)\right]$.

The condition for the existence of solution (25) is $a_1 \geq r_1\tan\delta_1$. This solution is illustrated in Fig. 16b.

4.1.3 $\delta > \frac{\pi}{2}, \delta_1 = 0$

Now, the solution to the set of Eq. (23) for the case of pitch configurations with inverse orientation is

$$\begin{aligned} \cot\theta_2 &= \frac{a_1\sin\delta}{a_w}, \tan\delta_2 = -\cos\theta_2\tan\delta, \\ \tan\frac{\theta_1}{2} &= \frac{\cot\theta_2 - \sqrt{D}}{\cos\delta}, a_2 = r_1\cos\theta_1\sin\delta - a_1\cos\delta, \\ r_2 &= \frac{a_w - r_1\sin\theta_1}{\sin\theta_2}, \end{aligned} \tag{26}$$

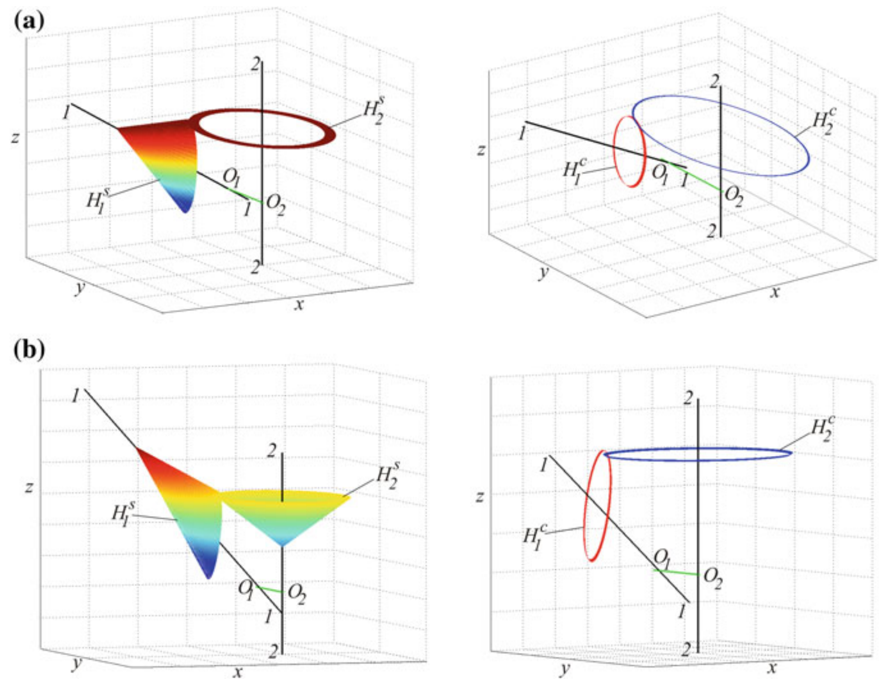

Fig. 16 Pitch configurations with inverse orientation when H_1^s is a rotation cone: **a** H_2^s is a disc, $a_w = 100\,\text{mm}, \delta = 100^\circ, a_1 = 100\,\text{mm}, \delta_1 = 10^\circ, r_1 = 30\,\text{mm}, \delta_2 = 90^\circ, r_2 = 136.75\,\text{mm}, \; a_2 = 46.91\,\text{mm}$; **b** H_2^s is a rotation cone, $a_w = 100\,\text{mm}, \delta = 110^\circ, a_1 = 100\,\text{mm}, \delta_1 = 10^\circ, r_1 = 30\,\text{mm}, \delta_2 = 75^\circ 25', \; r_2 = 126.20\,\text{mm}, a_2 = 61.87\,\text{mm}$

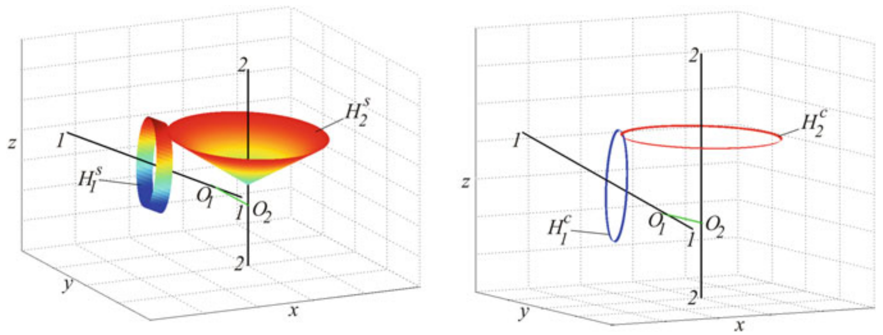

Fig. 17 Pitch configurations with inverse orientation when H_1^s is a rotation cylinder and H_2^s is a rotation cone, $a_w = 100\,\text{mm}, \delta = 100^\circ, a_1 = 100\,\text{mm}, \delta_1 = 10^\circ, r_1 = 30\,\text{mm}, \; \delta_2 = 75^\circ 54'$, $r_2 = 129.55\,\text{mm}, a_2 = 45.92\,\text{mm}$

where $D = \cot^2\theta_2 + \cos^2\delta$.

The pitch configurations, shown in Fig. 17, are visualized using algorithm (26).

5 Conclusions

Considering one contact point P, which belongs to the conjugate tooth surfaces Σ_1 and Σ_2 as a common point of the geometric pitch circles H_1^c and H_2^c, makes it possible to understand the approaches applied to the construction of mathematical models for synthesis of hyperboloid gears with both externally and internally meshing. In this context, it is essential to remind that the geometric pitch circles are used to define the longitudinal orientation of the tooth surfaces $\Sigma_i(i = 1, 2)$ contacting at P. It has to be pointed once more that these circles can be elements from the coaxial reference, root and tip surfaces. The surfaces, including the geometric pitch circles, are called geometric pitch configurations or simply pitch configurations. The pairs of geometric pitch circles and the pairs of geometric pitch surfaces form the set of geometric pitch configurations. The reference surfaces of the gears are often used as pitch surfaces in the practice of the synthesis of hyperboloid gears. It is possible that one of the pitch surfaces coincides with the tip surfaces of one of the gears. Then, the other pitch surface is an envelope of the first one, and includes the second geometric pitch circle.

In conclusion, one and the same pair of pitch configurations can stay in the base of the synthesis of different spatial gears with crossed axes of rotations, from the viewpoints of both the form of the corresponding geometric pitch surfaces and the realized law of rotations transformation.

The defined and synthesized geometric pitch configurations are basic primitives of the worked out mathematical models oriented to the synthesis and design of spatial gears with crossed axes of rotation.

The presented mathematical model for synthesis of inverse oriented geometric pitch configurations of non-orthogonal hyperboloid gears continues the part of this study that treats normally oriented pitch surfaces and circles related to the same class of mechanisms. The presented algorithms and conditions for their existence are a basic element from the process of design of non-orthogonal gear transmissions with crossed axes. The non-orthogonal character of the structure of these gears, together with the special orientation of their pitch configurations, creates conditions for synthesis of spatial gear mechanisms with innovated qualities.

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