Optimization of HCR Gearing Geometry from a Scuffing Point of View

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Abstract The paper deals with an issue of increasing the resistance of HCR external involute gearing from a scuffing point of view. It reports on a difference between involute gears with low contact ratio (LCR) and those with high contact ratio (HCR). The paper describes scuffing as the most significant damage done to the teeth flanks of HCR involute gears. In the case of warm scuffing, it is the combined action of high pressure between surfaces, high sliding speeds, and excessive contact temperature, resulting from pressure and sliding speed values, which consequently cause oil film rupture between the teeth flanks. Adopting a suitable geometry of the tooth curve profile, certain values of addendum heights for the meshing wheel will be defined according to the criteria of specific slips and corrected head shapes of the teeth of both wheels. The paper deals with the assessment and theoretical analysis of the impact of the HCR tooth profile resistance to scuffing on the basis of integral temperature criterion according to the Winter-Michaelis criterion. The basic relations for assessing the scuffing properties of an HCR involute gearing profile are derived, as well as the optimization of

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geometrical parameters of HCR gearing based on theoretical considerations on the properties of HCR gearing in terms of its resistance to warm scuffing combined with geometrical constraints against interferences. Finally, the results of the obtained optimization are compared with experimental results provided in previous research. A significant benefit in a theoretical area is the generalization of the integral temperature criterion for involute HCR gearing. By optimizing the criterion for the integral temperature of involute HCR gearing, the minimal flash temperature is obtained and the condition for the occurrence of scuffing is minimized.

Keywords HCR involute gearing · Scuffing · Flash temperature criterion · Integral temperature criterion · Optimization

1 Introduction

Nowadays, mechanical transmissions are produced in millions of special series, in the context of the automotive industry (for transferring mechanical energy from the engine to the wheels, starting an engine, moving the window, etc.), or in the area of railway engineering, industries of construction machinery, etc. Special transmissions are also used in agricultural, industrial, construction and mining equipment. Obviously, they are also used as universal mechanical transmissions in all fields of mechanical engineering, in first and foremost for connecting motors, i.e., driving units [1–3]. In this paper, mechanical transmissions for the automotive industry will be researched, i.e., the focus will be on the gears used in this type of transmission.

Mechanical transmissions, being integral parts of a driving system, when operated properly, significantly influence the reliability of the system as a whole. However, they can be found in a state of failure and thus be the cause of unplanned delays in production. Despite the fact that all conditions are provided for undisturbed functioning, failures of one or more elements can occur, causing unplanned delays, and hence leading to improper operation, or the current termination of the working of gear transmission [2].

Because of the role of transferring mechanical energy from the driving engine to the driven machine, the failure of mechanical gear transmission means the immediate termination of the entire system, the termination of its production and a delay in production, which can cause large financial losses for users of mechanical transmissions [2, 4].

Bearing in mind the great use and importance of mechanical transmissions, they need to provide high reliability in order for the delays in production or losses thus incurred to be as small as possible. Proper operation of transmissions greatly affects the quality and reliability of other machines and mechanisms into which they are incorporated. This means that during the construction of each element in a mechanical transmission, care should be taken in relation to their reliability, and at the same time, in relation to the optimal design and the requirement for achieving better technical characteristics of the transmission. To ensure a reliable and long service life, and in general, the high quality operation of the mechanical units and systems into which they are installed, it is necessary to examine in detail the characteristics of the proper operation of a mechanical transmission, the characteristics of their work at various loads, and the reasons for accidents and breakdown situations [2, 4].

Scientific and technological progress in engineering has led to increasing improvement in the use of resources, a reduction in energy intensity of production and an increase in reliability and efficiency during operation. This trend is also reflected in the field of the development of gears, which, in recent years, has seen the ability to transmit power increased, while, at the same time, the volume and weight of gears has been reduced.

Increasing the transmitted power of a gear relative to the unit of volume is, however, associated with an increase in thermal load transfer. Apart from the usual kinds of tooth damage (fracture in the heel and the formation of pitting), the increased heat load has uncovered another kind of damage, so-called scuffing. Boundary curves representing the main kinds of tooth damage are shown in Fig. 1, depending on the transmitted torque and the peripheral speed for the same lifetime [5–7].

Problems of tooth scuffing mainly occur during such transfers, which cannot be used for lubrication oils with EP additives (EP—extreme pressure). Examples of EP additives include sulphur, chlorine, phosphorus and lead. Scuffing can occur in the gear reducer for gas turbines, in which non-additive-enhanced oil is commonly used. The oil in transmissions of diesel driven ships and in the gearboxes of diesel locomotives does not provide adequate protection against scuffing.

Naturally, scuffing may arise in other cases, even assuming that the gear lubrication oils do have additives. In such cases, scuffing occurs in overloading transfers that operate at the higher peripheral speed of transfers with poorly chosen tooth geometry or that influence an excessive increase in oil temperature operation (e.g., high ambient temperature, etc.) [5].

In any case, progressive scuffing means serious damage to the shape of the tooth profile, which subsequently means removing the teeth from the service. Increased

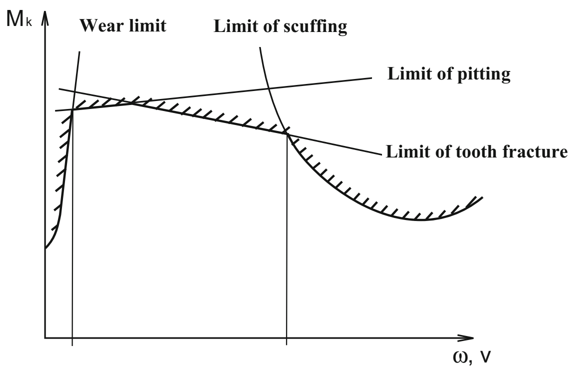

Fig. 1 Typical load carrying capacity limits of case carburized gears [7]

noise transmission over the standard border may refer, in some cases, to the reason for exclusion from the transfer operation.

The problem related to the involute gear with an extended duration of the contact period, known in the literature as HCR (High Contact Ratio), is discussed in this paper. Due to the larger path of action and the higher peripheral speed, scuffing can occur more frequently in HCR gear teeth. The objective of this paper is to analyze and discuss the possibilities for reducing the occurring of scuffing as tooth damage on external HCR gears.

In recent years, while introducing higher demands on the capacity of teeth, and while developing new materials with high resistance to pitting or high flexural strength, it has become obvious that the limit factor at the design of spur gears may, in many cases, just be scuffing [5–7].

The majority of standard methods and procedures only apply for LCR (low contact ratio) gears [5, 8, 9]. This type of damage of HCR gears has not, to date, been sufficiently researched. It has been identified that the process of scuffing can occur in HCR gears while gear processing, resulting in a direct impact on their working life.

2 Effect of Higher Contact Ratio on the Noise Level of Gearing

There is a new option involving minimizing the size of gearing, and thus the size of gearwheels as well. Reducing the dimensions of the gear leads to a greater heat load caused by the reduction of the material volume for the transfer of heat energy formed in the teeth meshing. Increased thermal stress in mechanical transmission can cause scuffing of tooth flanks. Besides the heat stress, the tendency towards scuffing damage is also dependent on the gear load, the peripheral speed, the gearing geometry, the quality of the tooth flanks and the properties of the lubricating oil. When crossing the limit criteria, the damage of tooth flanks by scuffing can lead to damaging of the gear, which is possible even after a very short time of operation [5, 7].

In the era of modern technological development, there is continuing research into new forms of tooth which could potentially meet the most demanding requirements. The intent of this article is primarily to reduce the noise of mechanical transmissions, improve them (automotive transmissions to begin with), reduce their size and increase their efficiency, all while increasing the transmitted power. In involute gear drives, the majority of geometric parameters affects the properties of this type of gearing and cannot be changed to a great extent. Therefore, it is necessary to focus on those factors which affect the properties of teeth and which still have not been sufficiently examined in detail, even though they have a significant impact on the desired characteristics of the transfer.

One such parameter is the coefficient of contact ratio of involute gears (ε_α) [5, 10]. Today, nearly all automotive transmission gears have involute gears, for which the coefficient of the contact ratio is equal to or greater than 2. Such a type of gear in the literature is generally marked as HCR (High Contact Ratio) teeth. The correct geometry for the proposed HCR gearing is complicated by the fact that there is much more to the internal primary and secondary teeth, as well as to interference, than in standard involute profiles. In terms of strength characteristics, the proposed HCR gearing is also problematic, mainly due to the lack of extensive experimental test results of such transfers. Likewise, the problem is that the current standards for strength calculation of involute gearing are contemplated with higher ε_α. Those factors hinder the optimization of geometrical parameters of HCR gearing as a virtually arbitrarily defined objective function. The aim of the article is therefore to contribute to a greater knowledge of the properties of HCR gearing, especially in terms of its resistance to scuffing and further development of the strength calculation of gearings with HCR teeth.

Contact ratio can be defined as the number of pairs of teeth in contact during the course of action. The physical significance of the contact ratio lies in the fact that it is a measure of the average number of teeth in contact during the period in which a tooth comes in and out of contact with the mating gear [3, 5, 6].

The correct working of the gear will be assured if the value of the path of contact is higher than that of the base pitch [3, 6, 11]. The ratio between the length of contact g_α, and the pitch on the base cylinder p_b is provided by the following formula:

$$\varepsilon_\alpha = \frac{\text{length of contact}}{\text{base pitch}} = \frac{g_\alpha}{p_b} = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a_w \sin \alpha_{wt}}{p_t \cos \alpha_{wt}}. \tag{1}$$

Referring to Fig. 2, the progression of tooth contact has been represented for LCR gears. At the point A, the contact commences. Each of the pairs carries a load of $F/2$. At the internal point B, the whole load is on the second pair as the first pair leaves the mesh at the point E; therefore, there is a single tooth contact. This single pair bears the load until the point D is reached. Double-pair engagement begins from here onwards, until the former pair departs from the mesh. The minimum acceptable contact ratio for smooth operation is 1.2. Gears should not generally be designed having contact ratios less than about 1.2, since inaccuracies in mounting might reduce the contact ratio even more, increasing the possibility of impact between the teeth, as well as an increase in the noise level. To ensure smooth and continuous operation, the contact ratio must be as high as possible, which the limiting factors permit. A minimum contact ratio of 1.4 is preferred, and the larger one is better. Contact ratios for conventional gearing are generally in the range from 1.4 to 1.6; hence, the number of tooth engagements is either one or two [12, 13].

When HCR gearing is used, it is not necessary to achieve a greater gear load capacity; nevertheless, there is a greater risk of interference due to a greater height of the teeth. The advantage of HCR gearing is also higher resistance (load distribution is shared between more pairs of teeth simultaneously) and lower relative noise level, which can be significantly reduced by using the integer HCR factor ε_α.

HCR profiles are more complicated than standard involute profiles; they have greater predisposition for interference and pointed tip thickness, but also an undercut of the teeth during production (primary production interference) [11, 12].

The coefficient of the contact ratio is the main indicator of HCR gearing, which differs from the commonly used standard profiles.

The geometry of involute HCR gearing is presented in Fig. 3, where the difference in geometry between HCR gearing and the standard involute gearing can be observed. Likewise, there is a difference in length of the line of action g_a (the tooth pitch on the base cylinder p_{bt} is the same as in Fig. 2).

General distribution of the applied force at the characteristic points of LCR and HCR gear tooth flanks is shown in Fig. 4. When comparing both these kinds of gearing, it is obvious that, in the case of LCR gearing, the maximum force is applied when one pair of teeth is in contact (between points BD, Fig. 4a). This can be considered to be 100% of the value of the force F. The greatest applied force in

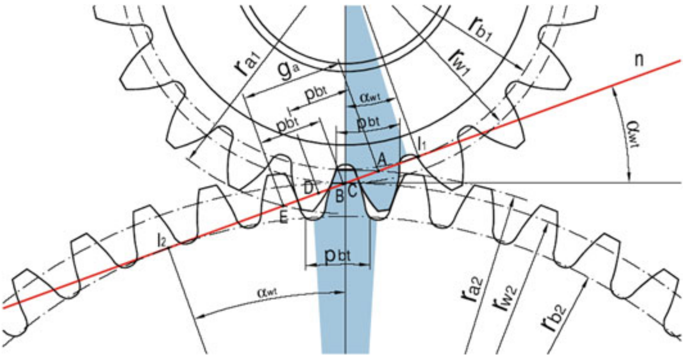

Fig. 2 Geometry of the standard involute gearing [11]

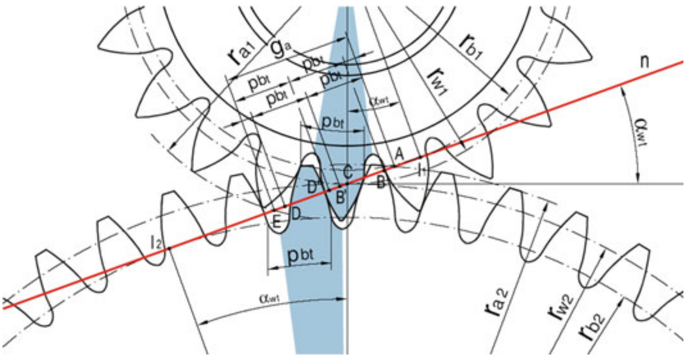

Fig. 3 Geometry of involute HCR gearing [11]

the HCR gearing (between points BB' and DD') can be considered that of about 50%, when two pairs of teeth are in contact. Consequently, the size of the applied force is decreased when three pairs of teeth are in contact. Hence, the value of involute gearing 2/3*F* is decreased to 1/3*F*, and the value 1/3*F* is decreased to 1/6*F*, meaning that the load distribution is more favorable in HCR gearing [11, 12].

Often, gears in the mechanical gearboxes of automobiles are designed with greater gear ratio, integer transverse contact ratio ($\varepsilon_{\alpha} \geq 2$) and overlap ratio ($\varepsilon_{\alpha}$ 1). These gears are characterized by their increased strength capacity [14].

While a one or two teeth pair in contact change at the mesh of the standard gear, two teeth pairs are permanently in contact at the mesh of an HCR gear with $\varepsilon_{\alpha} = 2$. Advantages of an HCR gear with $\varepsilon_{\alpha} = 2$ are based on the smoother change of stiffness at the mesh and the smooth division of the total carried force between two teeth pairs.

When $\varepsilon_{\alpha} > 2$, three pairs of teeth occur in contact along the line of action, thereby representing triple tooth contact (Fig. 5). Triple tooth contact generally has a higher load capacity and lower gearing noise. Also, the favorable property of this HCR gearing includes increasing the resistance of fatigue damage.

In high precision and heavily loaded spur gears, the effect of gear errors is negligible, so the periodic variation of tooth stiffness is the principal cause of noise and vibration. High contact ratio spur gears could be used to exclude or reduce the variation of tooth stiffness [13, 14].

Increasing the noise of a gear pair is largely conditioned by the load increase. Increasing the applied force affects the greater elastic deformation of the teeth, including greater internal dynamic forces.

The main cause of internal dynamic forces is engaging different variants of involute tooth shapes, as well as changing the number of teeth pairs in contact. Dynamic forces between the teeth depend on the speed of rotation and the number of teeth of the pinion and gear wheel.

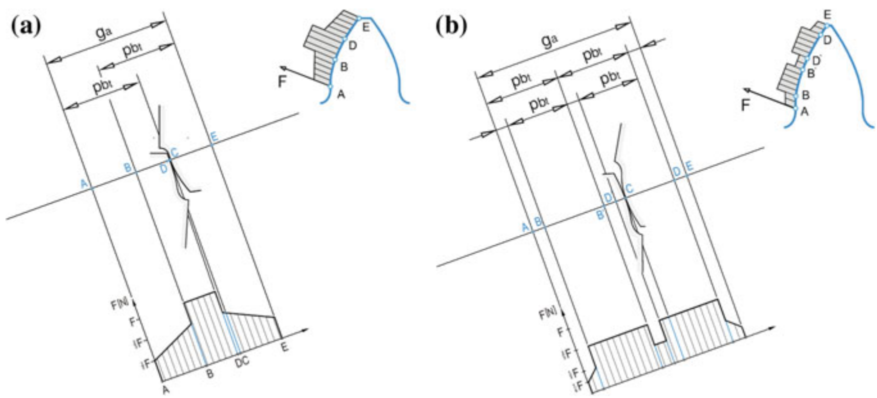

Fig. 4 Distribution of tooth load during **a** low contact ratio (LCR) and **b** high contact ratio (HCR) [11]

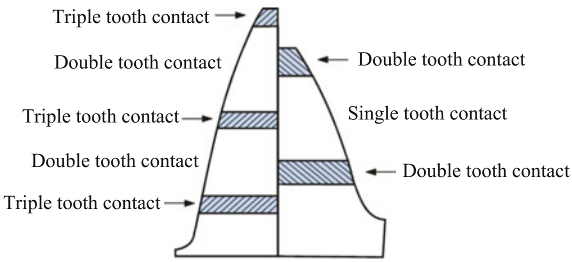

Fig. 5 Comparison of the gearing profiles

By changing some parameters, dynamic forces can be reduced. Introducing and changing the helix angle β, or changing the contact ratio ε_α, can accomplish a quite significant reduction in noise levels of a gear pair.

It is well known that increasing the average number of teeth in contact leads to excluding or reducing the vibration amplitude. First, it was established experimentally that dynamic loads decrease with an increase in the contact ratio in spur gearing [15]. Then, in order to get a further reduction of the vibration, HCR gear profiles can be optimized. Sato et al. [16] found that HCR gears were less sensitive with respect to manufacturing errors. In particular, such kinds of gear allow for larger tolerance in the tip relief length. Moreover, they found that, in the absence of a pressure angle error, the best contact ratio should be about 2; otherwise, it was better to have a contact ratio of about 1.7 or higher than 2.3. Kahraman and Blankenship [17] published an experimental work on HCR gear vibration; they found that the best behavior was obtained with an integer contact ratio, even though other specific non-integer (rational) contact ratios could minimize the amplitude of some specific harmonics of the static transmission error. It is important to note that in Ref. [17], HCR gears were obtained by modifying the outside diameter; the other macro-geometric parameters, e.g., the number of teeth, were left unchanged.

According to the results of different measurements of gear pair, the reduction of noise proved to be greatest using HCR gearing with the value of the contact ratio $\varepsilon_\alpha = 2$. A decrease in noise is caused by $\varepsilon_\alpha = 2$, because there are always two pairs of teeth in contact, meaning that when one pair of teeth comes out of contact, another pair of teeth is coming into contact; hence, the applied force is considerably smaller, since it is divided between two pairs of teeth.

3 Geometric Parameters of the Objective Function Defining the Correct Mating HCR Gearing

According to Eq. (1), it follows that $\varepsilon_\alpha = f(g_\alpha, p_{bt})$. Tooth pitch on the base circle of LCR gearing is equal to the base pitch on HCR gearing, and it is considered to be constant. This means that achieving the greatest value of the contact ratio ε_α has to be obtained by the greatest possible increase in the length of the line of action g_α. The length of the line of action g_α is calculated in following equation [3, 5, 6]:

$$g_\alpha = \sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a_w \sin \alpha_{wt}, \tag{2}$$

where the tip diameters of the pinion and gear wheel are as follows:

$$r_{a1} = r_1 + (h_a + x_1 \cdot m_n), \tag{3}$$

$$r_{a2} = r_2 + (h_a + x_2 \cdot m_n). \tag{4}$$

From Eqs. (3) to (4), it is clear that the length of the line of action g_α is directly dependent on the addendum height h_{a1}, h_{a2} and factors of addendum modification x_1, x_2. Optimization of the geometric parameters of HCR gearing can be based on the objective function to achieve the maximum value of the contact ratio ε_α for a given centre distance a_w. The main optimization parameter at this level can be the addendum heights of teeth h_{a1}, h_{a2}, and factors of addendum modification x_1 and x_2. For a given distance between the centers of the wheels, x_c can be defined as the relationship between x_1 and x_2 [11, 12].

Addendum heights h_{a1} and h_{a2} can be found from the following equations:

$$h_{a1} = h_{a1}^* \cdot m_n, \tag{5}$$

$$h_{a2} = h_{a2}^* \cdot m_n, \tag{6}$$

where h_{a1}^* and h_{a2}^* are addendum heights for the module equal to one. Furthermore, it follows that

$$x_2 = x_c - x_1. \tag{7}$$

Consequently, this implies that the contact ratio is the objective function of both addendum heights and the addendum modification factor of pinion $\varepsilon_\alpha = f\,(h_{a1}^*, h_{a2}^*, x_1)$ = max, i.e., optimization parameters h_{a1}^*, h_{a2}^*; x_1 makes a nonlinear optimization of triple constraint, with limitation requirements defined for [11, 12]:

- removal of the meshing interference,
- minimum arc thickness of the tooth tip $s_{a1,2}$,
- distribution x_c to x_1, x_2 has to be performed through balancing specific slips, strength, or a particular condition, respectively compromising their combinations [11, 12].

Therefore, the most favorable solution is obtained by increasing the addendum height. However, there are a lot of geometrical and manufacturing constraints that have to be satisfied, thus limiting the increase of the contact ratio. Description of the geometrical and manufacturing constraints and their solution are thoroughly explained and solved in [11, 12].

Due to the high load capacity of HCR gear, and since thermal power is reduced, there is a great possibility that scuffing may occur, even before pitting. The origin of fatigue damaged tooth flanks (scuffing and pitting) directly depends on the load transfer.

4 Scuffing as the Most Important Damage of HCR Tooth Flanks

Scuffing tends to occur at the head of the large slip rates with the impact of high temperatures. It appears most often in high speed and highly stressed gearings. Scuffing is not a fatigue phenomenon, as it may occur at the beginning of the operation [7, 18]. There are several analytical methods for predicting the risk of scuffing; however, the threshold for determining whether a gear set will scuff remains mostly dependent on empirical results. The method for evaluating the risk of scuffing is a function of oil viscosity and additives; the operating bulk temperature of the gear; sliding velocity; the surface roughness of the gear teeth; gear materials and heat treatments; and surface pressure. The risk of scuffing is defined by comparing the calculated tooth contact temperature with the limiting scuffing temperature; this is accomplished through gear scuffing tests for each gear lubricant [7, 18].

Scuffing is not a fatigue phenomenon, and it may occur instantaneously. Based on the severity of the damage, scuffing can be categorized as mild, moderate, or severe. Scuffing is classified as mild if it occurs only on small areas of the teeth and is confined to the peaks of the surface asperities (Fig. 6a). It is generally non-progressive. Moderate scuffing occurs in patches that cover significant portions of the teeth. If the operating conditions do not change, moderate scuffing may be progressive. Severe scuffing (Fig. 6b) occurs on significant portions of the gear tooth (for example, the entire addendum, the entire dedendum, or both). In some cases, the surface material may be plastically deformed and displaced over the tip of

(a) (b)

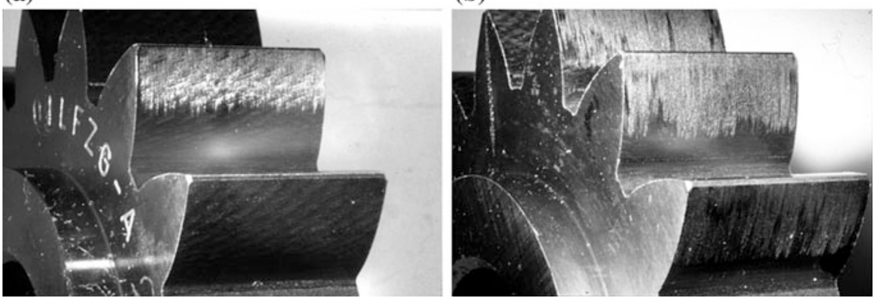

Fig. 6 Light scuffing (**a**), severe scuffing (**b**) [7]

the tooth or into the root of the tooth. Unless corrective measures are taken, severe scuffing is usually progressive [18].

Scuffing is severe adhesion that causes the transfer of metal from one tooth surface to another due to welding and tearing. The damage typically occurs in the addendum, dedendum, or both, away from the operating pitch line, in narrow or broad bands that are oriented in the direction of sliding. Scuffing may occur in localized patches if it is due to load concentrations. The scuffed area appears to have a rough or matted texture [7].

Scuffing is the process that occurs when the surfaces of two contacting bodies are joined by localized welding and then pulled apart. A material transfer occurs between the two contacting surfaces due to high metal-to-metal contact, and hence produces a weld. The high metal-to-metal contact is the result of a local failure of the gear lubricant, which has been caused by frictional heating due to high sliding speed and high surface pressure. This type of scuffing is called warm scuffing.

The scuffing traces appear in the form of streaks or scratches with rough bottoms and sides, often emerging as bands of variable depth and width oriented in the direction of the height of the tooth, affecting isolated zones or their whole width. The scuffing traces are generally more clearly marked at the tooth tip and root of the teeth in the high sliding zones.

In the case of warm scuffing, it is, in fact, the combined action of high pressure between surfaces, high sliding speeds, and excessive contact temperature resulting from pressure and sliding speed values, which causes the rupture of oil film between the tooth flanks. During the start-up or running-in of certain gears, some local scuffing of lesser importance, which is characterized by shallow traces and very fine roughness, may appear at certain points of the teeth in the zone where the contact pressure is at a maximum. In general, after a certain period of operation at reduced load, these localized traces of scuffing diminish by wear. Once this happens, the gear may operate under its nominal load. In this case, a slight increase in the lubricant viscosity will allow for better safety in service. On ground gears, localized scuffing can be observed at the tip and root of the teeth as the result of insufficient tip relief or too great a deviation in the profile. Identical phenomena can also appear near the tooth ends due to insufficient longitudinal correction or too great a helix deviation [7, 18].

When the failure occurs a long time after the start-up of the gearing, it is caused by an accidental faulty lubrication or an overload resulting from the continuous use of the machine in a manner for which it was not designed. However, if the failure occurs a short time after the start-up, one should examine whether the amount of heat generated by the gear is compatible with the choice of the gear geometry and the choice of lubricant. Objective signs of scuffing include a rapid increase of friction coefficient, wear, vibration, and a temperature increase in the total surface of the meshing wheels and lubricating oil, as well as increased noise transfer [19, 20]. Intensity of scuffing can be assessed by the change in the roughness of the tooth profile in relation to its width perpendicular to the direction of the slip. If there are deep scratches from 3 to 7 pm, they can be the cause of scuffing. When the process of scuffing gears begins, it may, depending on the conditions of friction, stop after some time (surfaces with smooth flanks) or, conversely, the amount and intensity of scuffing characters in service may increase over time. Then, it is the case of limiting, or progressive scuffing [18].

Scuffing damage can be prevented through design changes or operational/break-in changes. Design-related changes include optimizing the gear geometry/accuracy and the use of nitrided steel. Operational and break-in changes include the use of high viscosity lubricants with anti-scuff additives, reduced lubricant temperatures, and the running-in of new gearboxes at reduced loads.

5 Calculating the Tooth Scuffing Resistance According to the Criterion of Integral Temperature

One of the hypotheses about the development of the scuffing phenomenon involves a temperature criterion. It is based on the assumption that there is a limit temperature of the surface of the teeth at which scuffing occurs. These criteria can be divided into those based on local instantaneous surface temperature and those based on the mean temperature of the surface of the teeth.

Theoretically, the best criterion among these criteria is the sophisticated Blok criterion (1937). Blok was one of the earliest researchers to study scuffing and to propose a hypothesis to explain the experimental observations. In his study, he suggested that scuffing would only take place if a critical temperature is reached at the sliding interface [21]. This is based on the assumption that for each combination of oil and gears of the material, there is still a critical temperature at which the oil loses its lubricating properties and will not protect the surfaces of the teeth from intermediate metal contact. It is assumed that the lubrication is effected by alloyed mineral oil. When the local temperature in the contact area between two tooth flanks reaches the critical value, then scuffing occurs. Critical instantaneous temperature, according to this theory, for a particular combination of metal-lubricating oil is constant and does not depend on the operating conditions of the transmission. Corresponding to the temperature disorientation of oil molecules on the metal surface, it is close to the evaporation temperature of the oil, and therefore does not depend on whether the teeth are in contact or in hydrodynamic boundary friction.

Instantaneous local temperature ϑ_C at the current contact of teeth meshing together on a Blok criterion consists of two components: the local instantaneous temperature (flash temperature, Blitztemperatur, ϑ_{Bl}), which both causes friction and increases the second component, the tooth bulk temperature ϑ_0; hence, it can be written as [21]

$$\vartheta_C = \vartheta_0 + \vartheta_{Bl}. \tag{8}$$

For the practical calculation of safety against scuffing of gearing, more favorable temperature criteria are those based on the mean surface temperature. One of the reasons is the fact that, when used, it is not necessary to know the local coefficient of friction, radii of curvature, etc. These criteria, based on the mean temperature of the surface of the teeth, are the Lechner criterion (1966), the Seitzinger criterion (1971), the Winter-Michaelis criterion (1975) and calculation according to the Schauerhammer criterion (1978).

The Winter-Michaelis criterion of integral temperature (median average contact temperature) is among the most modern criteria for assessing resistance to tooth scuffing. It is based on defining the contact surface temperature of teeth in relation to the Blok theory. The calculation consists of identifying the instantaneous temperature along the mating line under the Blok and adding tooth bulk to a constant temperature. The comparative average temperature of the surface is then determined as the quotient of the integral thus calculated during the unsteady temperature along the mating line and a length image. The calculation of safety against scuffing of the teeth is a relatively simple one, and provides good agreement with measured or detected data in practice, both for pure mineral oils, but also for doped and synthetic oils. For these reasons, the integral temperature criterion appears to be the best comprehensive method published so far for calculating gearing resistance to scuffing. Therefore, in addition to the Blok criterion, it became a part of the standard DIN 3990 [8].

This paper will present the relations of integral temperature criterion for spur involute gears with high contact ratio (HCR). Since the Winter-Michaelis criterion is based on the Blok hypothesis (Eq. 8 [21]), it is first necessary to derive the formula for calculation of the instantaneous local flash temperature ϑ_{Bl}.

The instantaneous local temperature ϑ_C at the momentary contact of teeth meshing, together with the Blok criterion, consists of two components: the local instantaneous flash temperature (Blitztemperatur, ϑ_{Bl}), which causes friction, and the tooth bulk temperature ϑ_O.

5.1 *Local Instantaneous Flash Temperature* ϑ_{Bl} *for the Mated Teeth of HCR Spur Gears*

For calculation of the local instantaneous flash temperature ϑ_{Bl}, it is necessary to determine the place where the rise in temperature is the highest in the transition of the heat source.

According to Blok (flash temperature criterion), it is possible to express the flash temperature (local instantaneous flash temperature) at any point of meshing along the contact path, using the following formula:

$$\vartheta_{Bl} = 0.62\,\mu \left(\frac{F_n}{b}\right)^{0.75} \left(\frac{E_r}{\rho_r}\right)^{0.25} \frac{\left|v_{\rho 1} - v_{\rho 2}\right|}{\sqrt{\lambda_1\,\rho_1\,c_1\,v_{\rho 1}} + \sqrt{\lambda_2\,\rho_2\,c_2\,v_{\rho 2}}}. \tag{9}$$

Usually, in the literature, it is marked that λ ρ c = B_M, so it follows that

$$\vartheta_{Bl} = 0.62\,\mu w^{0.75}\left(\frac{E_r}{\rho_r}\right)^{0.25}\frac{\left|v_{\rho 1} - v_{\rho 2}\right|}{\sqrt{B_{M1}\,v_{\rho 1}} + \sqrt{B_{M2}\,v_{\rho 2}}}, \tag{10}$$

where

ϑ_{Bl}	is the flash temperature (local instantaneous flash temperature);
μ	is the mean coefficient of friction;
w	is the normal unit load ($w = F_n/b$),
F_n	is the normal force, b is the gear width;
E_r	is the reduced modulus of the elasticity of the gear wheel;
ρ_r	is the reduced radius of curvature at the point of mesh;
$v_{\rho 1,2}$	is the tangential velocity at the X profile point;
$\lambda_{1,2}$	are the coefficients of thermal conductivity of the wheel materials; and
$\rho_{1,2}$	are the specific densities of the wheel material.

6 General Principles of the Integral Temperature Criterion in HCR Involute Gearing

The integral temperature criterion is based on the middle temperatures of surface calculated on the basis of the Blok theory according to Eq. (10). The comparative medium temperature of surfaces ϑ_i (the integral temperature) consists of two temperatures: the medium value of local instantaneous flash temperature along the action line ϑ_{Blm} and the tooth bulk temperature ϑ_O [21].

Therefore, integral temperature is calculated from the following relationship:

$$\vartheta_i = \vartheta_0 + B\vartheta_{Blm}, \tag{11}$$

where B is a weighting factor which takes into consideration qualitatively different temperatures influencing the actual tooth bulk temperature ϑ_O and temperature ϑ_{Blm}, which is defined only as a comparative temperature, and does not reflect the actual size of the temperature at the contact points [21].

The medium value of the local flash temperature is determined from Eq. (10), which takes into account the medium friction coefficient μ_m taken along the meshing, so that the proportion of the integral local flash temperature ϑ_{Bl} taken along meshing and the length of the engagement line represent the temperature ϑ_{Blm}, i.e.,

$$\vartheta_{Blm} = \frac{\int_0^l \vartheta_{Bl}\,dx}{l}. \tag{12}$$

For determining the integral temperature ϑ_i from Eq. (11), it is necessary to determine the tooth bulk temperature ϑ_0. It can be assumed that the tooth bulk temperature principally consists of two parts. The first is the tooth surface temperature when it is not in contact (approximately equal to the temperature of the oil). The other part depends on the size of the difference between the integral temperature and the oil temperature, as well as from the values of the coefficients of thermal convection in the gear material and thermal transfer from the gear to the oil. If the coefficients of thermal convection and thermal transfer are included in the dimensionless coefficient A, it can be written that

$$\vartheta_0 = \vartheta_{oil} + A(\vartheta_i - \vartheta_{oil}). \tag{13}$$

The coefficients A and B in Eq. (13), resp. (11), are determined experimentally by measuring the temperature at the surface of the teeth. If Eq. (13) is substituted into Eq. (11), then it has the following form:

$$\vartheta_i = \vartheta_{oil} + \frac{B}{1-A}\,\vartheta_{Blm}, \tag{14}$$

or, simplified,

$$\vartheta_i = \vartheta_{oil} + C\vartheta_{Blm}, \tag{15}$$

where

$$C = \frac{B}{1-A}. \tag{16}$$

6.1 Calculating the Integral Temperature for HCR Involute Gearing

The size of the integral temperature is determined from Eq. (15), as

$$\vartheta_i = \vartheta_{oil} + C\frac{\int_0^l \vartheta_{Bl}\,dx}{l}. \tag{17}$$

The integral in Eq. (17) can be expressed in the form

$$\frac{1}{l}\int_0^l \vartheta_{Bl}\,dx = \vartheta_{BlE}\,X_\varepsilon, \tag{18}$$

where $\vartheta_{Bl\ E}$ is the local flash temperature at the reference point of meshing E for the contact between teeth with the contact ratio $\varepsilon_\alpha = 1$, and X_ε is a dimensionless factor taking into account the distribution of the load between mating teeth, the contact ratio of the head of the pinion and gear wheel, and the geometric characteristics of the curve tooth profile [5].

If ϑ_{Bl} and $\vartheta_{Bl\ E}$ are expressed in Eq. (18) using (10), it follows that

$$X_\varepsilon = \frac{\int_0^l \frac{F_{bt}^{0.75}}{\rho_r^{0.25}} \left|\sqrt{v_{t1}} - \sqrt{v_{t2}}\right| dx}{\frac{F_{bt\,E\,\varepsilon_\alpha=1}^{0.75}}{\rho_{r\,E}^{0.25}} \left|\sqrt{v_{t1}} - \sqrt{v_{t2}}\right| l}, \tag{19}$$

and then

$$\vartheta_i = \vartheta_{oil} + C\vartheta_{Bl\mathrm{E}} X_\varepsilon, \tag{20}$$

where

ϑ_i is the integral temperature;
ϑ_{oil} is the oil temperature;
$\vartheta_{Bl\ E}$ is the local flash temperature at the reference point E with the contact ratio $\varepsilon_\alpha = 1$; and
X_ε is the factor of load distribution for HCR involute gearing (Fig. 7).

6.2 *Determining the Factor X_ε for HCR Involute Gearing*

HCR gearing is different from the standard profiles in different load distributions along the contact line and, obviously, there are also greater tangential velocities at the beginning and end of the meshing. If Eq. (10) is analyzed more deeply, it is

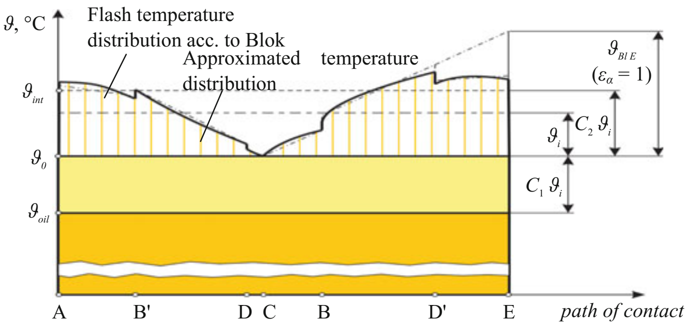

Fig. 7 Temperature distribution along the path of contact for HCR involute gearing

evident that, from the geometric parameter point of view, the main influence on the resistance to scuffing of the teeth can be attributed to the values of the tangential velocities and the value of the reduced radius of curvature.

It is obvious that, for determination of the factor of load distribution (X_ε), it is necessary to identify load distribution, the size of the radii of curvature and the tangential speed at any contact point. Basically, the main difference between the calculation of the integral temperature intensity for standard gearing and that for HCR gearing is determination of the factor of load distribution X_ε. Relations for X_ε are determined as a part of the area under the course curves of the local flash temperatures along the meshing (Fig. 7) and local flash temperatures at the point $E\varepsilon_\alpha = 1$ multiplied by the length of the meshing line (equivalent areas). The considered load distribution along the contact line is shown in Fig. 8 [12].

Equation (19) for involute gear can be simplified if it is considered that

$$\begin{aligned} v_{\rho 1} &= \rho_1\, \omega_1 \\ v_{\rho 2} &= \rho_2\, \omega_2\,. \\ u &= \tfrac{\omega_1}{\omega_2} \end{aligned} \tag{21}$$

Therefore, if considering the load distribution along the contact line (Fig. 8), and if the endpoint of contact, i.e., the point on the head of the pinion (point E), is considered to be the reference point for the calculation, then

$$\begin{gathered} X_\varepsilon = \frac{\int_0^l \frac{F_{bt}^{0.75}(x)}{\rho_r^{0.25}(x)} \left|\sqrt{\rho_1(x)\,\omega_1} - \sqrt{\rho_2(x)\,\omega_2}\right| dx}{l\, \frac{F_{bt\,E\,\varepsilon_\alpha=1}^{0.75}}{\rho_{rE}^{0.25}} \left|\sqrt{\rho_{1E}\,\omega_1} - \sqrt{\rho_{2E}\,\omega_2}\right|} \quad \text{or} \\ X_\varepsilon = \frac{\int_0^l F_{bt}^{0.75} \left|\sqrt[4]{\frac{\rho_1}{\rho_2}} - \sqrt[4]{\frac{\rho_2}{u^2\rho_1}}\right| dx}{l\, F_{bt\,E\,\varepsilon_\alpha=1}^{0.75} \left|\sqrt[4]{\frac{\rho_{1E}}{\rho_{2E}}} - \sqrt[4]{\frac{\rho_{2E}}{u^2\rho_{1E}}}\right|}\,. \end{gathered} \tag{22}$$

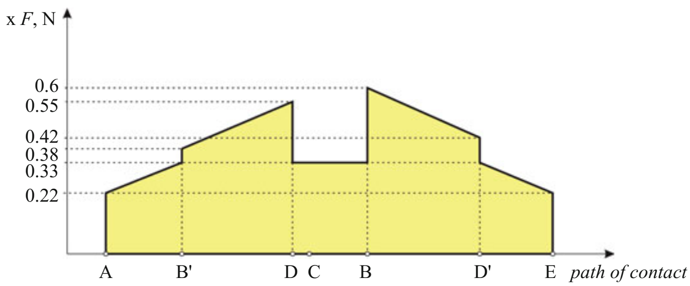

Fig. 8 Load distribution along the contact line for HCR involute gearing

However, Winter and Michaelis, in their criteria, foresaw a simplified waveform of temperatures along the contact line. Theoretical values of these temperatures apply strictly, and only at points C and E, with the change in temperature between these points being linear (Fig. 9) [12, 22]. In fact, this meshing could not occur with different radii of curvature of involute of both wheels, since the tangent at any individual point is linear, and the deviation between the actual and linear course for considering temperature is negligible.

The overall surface (P) under the flash temperature line during the entire contact of path of action can be replaced with the surface of a rectangle. The width of this rectangle is the same value as the contact line, while the height represents the integral temperature ($\vartheta_{Bl\ E}$):

$$\vartheta_{Bl} = P = \overline{AE} \cdot \vartheta_{Bl\,\mathrm{int}} = \vartheta_{Bl\,E} \cdot X_{\varepsilon} \cdot X_G \cdot X_M. \tag{23}$$

According to Fig. 9, it is possible to derive the relationship for the factor of load distribution (X_{ε}) for HCR involute gearing.

First of all, the overall surface is to be divided into six separate parts, and the relations for calculating these areas are as follows:

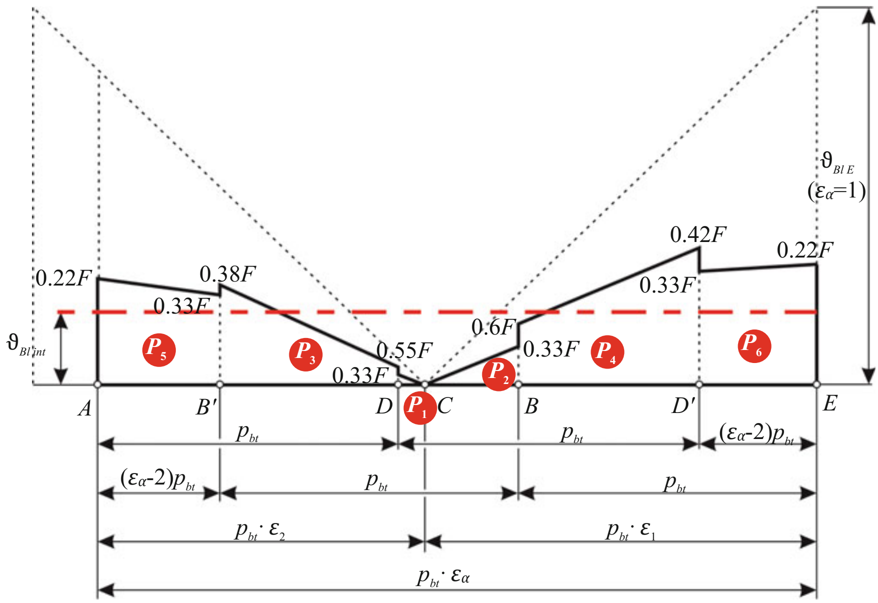

Fig. 9 Linear distribution of temperature along the path of contact for HCR involute gearing

$$
\begin{aligned}
P_1 &= \frac{\overline{CD}}{\overline{CE}}\,\vartheta_{BlE\varepsilon_\alpha=1}\cdot(0.33)^{0.75}\cdot\frac{\overline{CD}}{2},\\
P_2 &= \frac{\overline{CB}}{\overline{CE}}\,\vartheta_{BlE\varepsilon_\alpha=1}\cdot(0.33)^{0.75}\cdot\frac{\overline{CB}}{2},\\
P_3 &= \left(\frac{\overline{CB'}}{\overline{CE}}\vartheta_{BlE\varepsilon_\alpha=1}\cdot(0.38)^{0.75}+\frac{\overline{CD}}{\overline{CE}}\vartheta_{BlE\varepsilon_\alpha=1}\cdot(0.55)^{0.75}\right)\cdot\frac{\overline{B'D}}{2},\\
P_4 &= \left(\frac{\overline{CD'}}{\overline{CE}}\vartheta_{BlE\varepsilon_\alpha=1}\cdot(0.42)^{0.75}+\frac{\overline{CB}}{\overline{CE}}\vartheta_{BlE\varepsilon_\alpha=1}\cdot(0.6)^{0.75}\right)\cdot\frac{\overline{BD'}}{2},\\
P_5 &= \left(\frac{\overline{CB'}}{\overline{CE}}\vartheta_{BlE\varepsilon_\alpha=1}\cdot(0.33)^{0.75}+\frac{\overline{CA}}{\overline{CE}}\vartheta_{BlE\varepsilon_\alpha=1}\cdot(0.22)^{0.75}\right)\cdot\frac{\overline{AB'}}{2},\\
P_6 &= \left(\frac{\overline{CD'}}{\overline{CE}}\vartheta_{BlE\varepsilon_\alpha=1}\cdot(0.33)^{0.75}+\vartheta_{BlE\varepsilon_\alpha=1}\cdot(0.22)^{0.75}\right)\cdot\frac{\overline{D'E}}{2}.
\end{aligned}
$$

Secondly, the area under the polygon and line segments showing the tooth bulk temperature ϑ_O are provided by the following expression:

$$
\begin{aligned}
P &= \sum_{i=1}^{6} P_i\\
&= \frac{p_{bt}\vartheta_{BlE\varepsilon_\alpha=1}}{2\ \varepsilon_1}\ \left[-0.543\varepsilon_1\varepsilon_2+0.204\varepsilon_2^2+0.123\varepsilon_1^2+0.759\varepsilon_2+1.001\varepsilon_1-0.539\right].
\end{aligned} \tag{24}
$$

Equation (22) for the factor of load distribution can be written in the following form:

$$
X_\varepsilon = \frac{P}{l\cdot\vartheta_{BlE\varepsilon_\alpha=1}}.
$$

Considering that $l = AE = \varepsilon_\alpha\, p_{bt}$, it follows that

$$
X_\varepsilon = \frac{1}{2\ \varepsilon_1\varepsilon_\alpha}\ \left[-0.543\varepsilon_1\varepsilon_2+0.204\varepsilon_2^2+0.123\varepsilon_1^2+0.759\varepsilon_2+1.001\varepsilon_1-0.539\right]. \tag{25}
$$

For comparison, Winter and Michaelis derived that for LCR gearing the following applies [5, 22]:

$$
X_\varepsilon = \frac{1}{2\ \varepsilon_1\varepsilon_\alpha}\ \left[0.18(\varepsilon_1^2+0.7\varepsilon_2^2)+0.82\varepsilon_1-0.52\varepsilon_2-0.33\varepsilon_1\varepsilon_2\right].
$$

6.3 Calculating Local Flash Temperature $\vartheta_{Bl\ E}$ for HCR Involute Gearing

To compare the resistance of particular types of gearing against scuffing, it is necessary to identify local flash temperatures ϑ_{BlE} at the reference point. The reference point is defined by the integral temperature for HCR involute gearing as the endpoint of contact line on the head of the pinion, i.e., the point E on the contact line.

The temperature at the reference point E can be expressed using Eq. (9):

$$\vartheta_{BlE} = 0.62\,\mu_m \left(\frac{F_{nE}}{b}\right)^{0.75} \left(\frac{E_r}{\rho_r}\right)_E^{0.25} \frac{\left|v_{\rho 1E} - v_{\rho 2E}\right|}{\sqrt{B_{M1}\,v_{\rho 1E}} + \sqrt{B_{M2}\,v_{\rho 2E}}}. \tag{26}$$

If the normal force F_{nE}, acting at the point E of the straight tooth (spur gear) is expressed, the tangential force F_0 on the rolling circle can be obtained from $F_{nE} = F_0/\cos\alpha_E$. Additionally, if it is considered that the gearing of both gear wheels is made of standard steel (i.e., $B_{M1} = B_{M2} = B_M$), Eq. (26) can be expressed in the following form:

$$\vartheta_{BlE} = 0.62\,\mu_m \left(\frac{w}{\cos\alpha_E}\right)^{0.75} \left(\frac{E_r}{\rho_r}\right)^{0.25} \frac{\left|\sqrt{v_{\rho 1E}} - \sqrt{v_{\rho 2E}}\right|}{\sqrt{B_M}}, \tag{27}$$

where $w = F_0/b$.

The values E_r and B_M depend only on the material of the meshing gear wheels. Then, the coefficient of material X_M can be defined by the expression

$$X_M = \frac{E_r^{0.25}}{B_M^{0.5}}. \tag{28}$$

For standard steel, it follows that

$$\sqrt{B_M} = \sqrt{\lambda\,\rho\,c} = \sqrt{50 \cdot 7800 \cdot 10^{-9} \cdot 482 \cdot 10^3} = 13.7\ \mathrm{Nmm^{-1}\,s^{-0.5}\,K^{-1}}$$

$E_r = 2.26 \cdot 10^{11}\ \mathrm{Nm^{-2}}$.

Then, the coefficient of material is calculated as $X_M = 50\ \mathrm{K\ N^{-0.75}\ s^{0.5}\ m^{-0.5}\ mm}$.

If Eq. (28) is substituted into Eq. (27), and ρ_{rE}, $v_{\rho 1E}$, $v_{\rho 2E}$ are substituted for the relations (21), after modification and arrangement, it follows that

$$\vartheta_{BlE} = \mu_m\, w^{0.75}\, v_0^{0.5}\, a^{-0.25}\, X_M\, X_G, \tag{29}$$

where X_G is a dimensionless coefficient of the gear geometry

$$X_G = 0.62\sqrt{u+1}\,\frac{\sqrt{\sin\alpha_{wt} + \frac{l_E}{r_1}} - \sqrt{\sin\alpha_E - \frac{l_E}{u\,r_1}}}{\sqrt[4]{u\sin\alpha_{wt} - \frac{l_E^2}{r_1^2\sin\alpha_{wt}} + \frac{l_E}{r_1}(u-1)}\cdot\sqrt[4]{\cos^3\alpha_{wt}}},$$

which can be modified in the following form:

$$X_G = 1.24\sqrt[4]{\frac{\mathrm{tag}\alpha_{wt}}{\cos^2\alpha_{wt}}}\cdot\frac{\sqrt{u+1}}{2}\cdot\frac{\sqrt{\rho_{1E}} - \sqrt{\rho_{2E}}}{\sqrt[4]{\rho_{1E}\rho_{2E}}}. \tag{30}$$

The product of the first two factors in the expression (30) is, for $\alpha_{wt} = 20°$, approximately equal to one. In [8], the factor X_G is presented in a simplified form as

$$X_G = \frac{\sqrt{u+1}}{2}\cdot\frac{\sqrt{\rho_{1E}} - \sqrt{\rho_{2E}}}{\sqrt[4]{\rho_{1E}\rho_{2E}}}. \tag{31}$$

However, for the pressure angle different from 20°, there is a lower accuracy in the calculations (for example, for α_{wt} = 25°, there is a deviation of X_G of about 8%).

6.4 Calculating Medium Temperature of the Tooth Surface ϑ_{Blm} for HCR Involute Gearing

The size of the local instantaneous flash temperature ϑ_{BlE} is determined from the general Eq. (29). With respect to this relationship and Eqs. (15) and (20), there is an equation

$$\vartheta_{Blm} = \mu_m\, w^{0.75}\, v_0^{0.5}\, a^{-0.25}\, X_M\, X_G\, X_\varepsilon. \tag{32}$$

If the medium value of the coefficient of friction μ_m along the contact line is considered to be constant, then the general influence on the medium temperature value ϑ_{Blm} will be expressed with the product of the coefficients X_G and X_ε. Therefore, the occurrence of scuffing depends on the factor of load distribution (X_ε) and the factor of gear geometry (X_G).

The product of coefficients $X_G \cdot X_\varepsilon$ is crucial for assessment of the impact on the size of the temperature ϑ_{Blm}, and thus on the resistance to scuffing of HCR involute gearing. It is clear that there is strong dependence of the temperature ϑ_{Blm} on the shape of the tooth [8]. So, it is very important as to what profile is used for teeth meshing (involute, cycloidal, Novikov, etc.), since it determines the temperature in contact and the occurrence of scuffing.

If Blok's intensity of the local flash temperature of standard and HCR gearing is compared with comparable geometric characteristics, it is apparent that these temperatures are higher for HCR gearing. Figure 10 shows the plotting of Blok's local flash temperatures along the contact path for gearing loaded by M_{k1} = 1000 Nm, and with the peripheral speed v_o = 12 m/s, for LCR gearing with $\varepsilon_\alpha \approx 1.6$. Local flash temperatures for HCR gearing with $\varepsilon_\alpha \approx 2$ are presented in Fig. 11. It is obvious that the values of local flash temperatures are smaller for LCR involute gearing than for HCR involute gearing. Therefore, it should be taken into account that the occurrence of tooth damage on scuffing is more possible for HCR involute gearing [12].

7 Optimization of Geometrical Parameters of HCR Gearing Based on Theoretical Considerations on the Properties of HCR Gearing in Terms of Its Resistance to Warm Scuffing

The main goal of the paper is to extend the validity of the integral temperature criterion (according to Winter and Michaelis) on external HCR involute gearing. The value of the integral temperature of mating gear flanks can be expressed by the relation projection of the graphs. One should consider the course of loading and tangential velocity along the meshing line according to Figs. 8 and 9.

Considering Eq. (20), the value of the local flash temperature ϑ_{BIE} at the meshing point E will be determined from Eq. (26) for specific parameters of the end point E, which is in the meshing of the pinion head.

The relation for X_ε is determined to be a part of the area under the course curves of the local flash temperatures along the mating line in Eq. (25).

For calculation of the integral temperature according to Eq. (20), it is necessary to identify the proportionality coefficient C. If we consider the tooth bulk temperature ϑ_0 (the surface temperature of the tooth just before contact), the following relation can be applied (13):

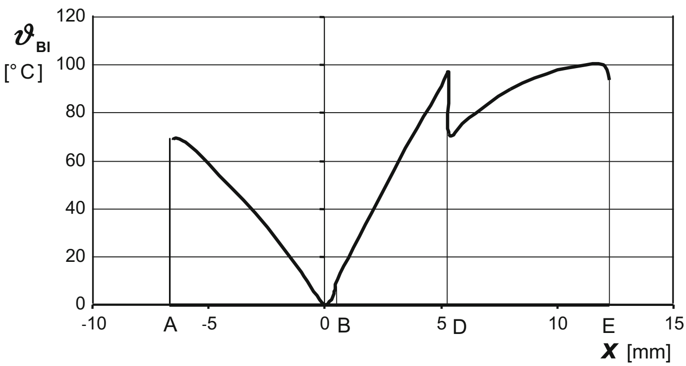

Fig. 10 Flash temperatures along the contact path of external involute gearing with an LCR profile

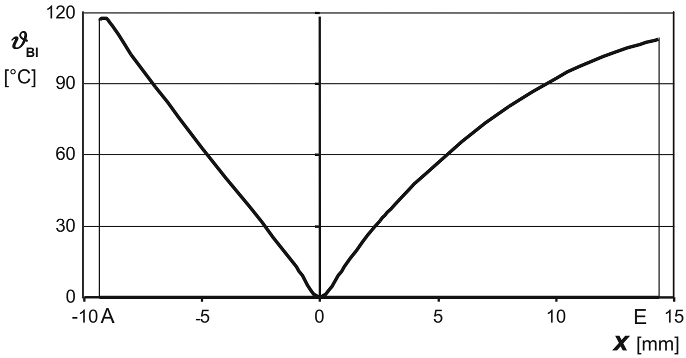

Fig. 11 Flash temperatures along the contact path of external involute gearing with the HCR profile $\varepsilon_\alpha \approx 2$

$$\vartheta_0 = \vartheta_{oil} + A(\vartheta_l - \vartheta_{oil}),$$

and, at the same time, according to Eq. (15), it is

$$\vartheta_i = \vartheta_{oil} + C\vartheta_{Blm}.$$

Comparing and taking into account the relation Eq. (20), the expression can be obtained as

$$C = \frac{\vartheta_0 - \vartheta_{oil}}{A\,\vartheta_{BIE}\,X_\varepsilon}. \tag{33}$$

The size of the coefficient C can be determined from Eq. (33), which is obtained using temperature values ϑ_0 and ϑ_{oil} determined from the measurement values ϑ_{BIE}. The coefficient A expresses the heat transfer from the oil to the gear wheel, and therefore cannot depend on the geometry of the teeth. Its size can be determined from the values of the coefficients B and C for the involute gear, which, according to [8], can be applied as B = 1.5 and C = 2.2. Then, from Eq. (16), it follows that

$$A = 1 - \frac{B}{C} = 0.31818 \quad \text{and} \quad C = \frac{\vartheta_0 - \vartheta_{oil}}{0.3\overline{18}\,\vartheta_{BIE}\,X_\varepsilon}. \tag{34}$$

The goal of the optimization is to obtain HCR involute gearing with a contact ratio factor of $\varepsilon_\alpha = 2$; yet, at the same time, the occurrence of scuffing must be avoided.

If the contact ratio has value two ($\varepsilon_\alpha = 2$), it means that two pairs of gears are always engaged. With this value of the contact ratio, the vibration and gear noise will be reduced, which is very important for the production of transmissions in the automotive industry. In order to achieve a high contact ratio, the addendum height is made higher in order to obtain a larger line of action. Geometrical and manufacturing constraints are described and their solutions are thoroughly explained in [11, 12]. There are several constraints that should be satisfied (interference during production, meshing interference, minimum thickness of the tooth head circle, slide conditions in HCR involute gearing). According to [11, 12], several constraints are provided, and they come from the limitation condition of the following:

- interference during production:

$$g_{F1} = r_{b1}\ \tan\ \alpha_t - \frac{\left(h^*_{a1} - x_1\right)\cdot m_n}{\sin \alpha_t}\ \geq\ 0, \tag{35a}$$

$$g_{F2} = r_{b2}\ \tan\ \alpha_t - \frac{\left(h^*_{a2} - x_c + x_1\right)\cdot m_n}{\sin\ \alpha_t} \geq 0; \tag{35b}$$

- meshing interference:

$$B_1 = \sqrt{r^2_{b1} + \left[a_w\ \sin\ \alpha_{wt} - g_{F2}\right]^2} \geq r_{a1}, \tag{36a}$$

$$B_2 = \sqrt{r^2_{b2} + \left[a_w\ \sin\ \alpha_{wt} - g_{F1}\right]^2} \geq r_{a2}; \tag{36b}$$

- minimum thickness of the tooth head circle:

$$s_{a1} = 2r_{a1}\left(\frac{s_{kt1}}{2r_1} + inv\alpha_t - inv\alpha_{at1}\right) \geq 0.4m_n, \tag{37a}$$

$$s_{a2} = 2r_{a2}\left(\frac{s_{kt2}}{2r_2} + inv\alpha_t - inv\alpha_{at2}\right) \geq 0.4m_n. \tag{37b}$$

Furthermore, if the contact ratio should be achieved equal to two, there is an equation for the contact ratio which represents the goal function with the aim that it should have a value of two [23].

$$\varepsilon_\alpha = \frac{z_1}{2\pi}\left[\tan\ \alpha_{at1} -\ \tan\ \alpha_{wt} - i\cdot\ \tan\ \alpha_{at2} + i\cdot\ \tan\ \alpha_{wt}\right] = 2. \tag{38}$$

Beside the geometric condition, there is another requirement for avoiding the occurrence of scuffing and consequently reducing the rapid damage of tooth flanks. Using the same variable parameters h^*_{a1}, h^*_{a2} and x_1, the flash temperature at the reference point E (Eq. 26) is considered to be the highest temperature at the mating line. Since the occurrence of scuffing depends on the flash temperature, the value $\vartheta_{Bl\ \mathrm{E}}$ should be an additional constraint requiring minimization. Since the occurrence of scuffing depends on the factor of load distribution (X_ε) and the factor of gear geometry (X_G), the product of the coefficients $X_G \cdot X_\varepsilon$, is crucial for assessment of the impact on the flash temperature and thus on the resistance to scuffing of HCR involute gearing. According to Eqs. (27), (30) and (32),

$$\vartheta_{BlE} = \mu_m \, w^{0.75} \, v_0^{0.5} \, a^{-0.25} \, X_M \, X_{GE} \, X_{\varepsilon E} = f(X_{\varepsilon E} \cdot X_{GE}) \\ \Rightarrow \quad f\left(\frac{\left| \sqrt{v_{\rho 1E}} - \sqrt{v_{\rho 2E}} \right|}{\left(\frac{1}{\rho_{rE}} \right)^{0.25}} \cdot \frac{\left| \sqrt{\rho_{1E}} - \sqrt{\rho_{2E}} \right|}{\left(\rho_{1E} \rho_{2E} \right)^{0.25}} \right) \quad \Rightarrow \quad \min \tag{39}$$

and for both gears made of standard steel (i.e., $B_{M1} = B_{M2} = B_M$), the equation for $\vartheta_{Bl\ \mathrm{E}}$ depends on variable parameters only in tangential velocities ($v_{\rho 1E}$ and $v_{\rho 2E}$), reducing the radius of curvature at the reference point E ($\rho_{r\mathrm{E}}$) and the radii of curvature of profiles at the point E (ρ_{1E}, ρ_{2E}). Therefore, additional tooth parameters should be calculated:

- tangential velocity at the reference point E

$$v_{\rho 1E} = \rho_{1E} \, \omega_1, \quad v_{\rho 2E} = \rho_{2E} \, \omega_2, \quad u = \frac{\omega_1}{\omega_2}; \tag{40}$$

- radius of curvature of profiles at the reference point E

$$\rho_{1E} = r_{b1} \cdot \tan \alpha_{at1}; \quad \rho_{2E} = a_w \cdot \sin \alpha_{wt} - r_{b1} \cdot \tan \alpha_{at1} \tag{41}$$

- reduced radius of curvature at the reference point E

$$\frac{1}{\rho_{rE}} = \frac{1}{\rho_{1E}} + \frac{1}{\rho_{2E}}. \tag{42}$$

However, in order to demonstrate this hypothesis regarding the obtainment of a minimal flash temperature at the reference point E, the conditions for achieving a maximal temperature will be found:

$$\vartheta_{BlE} = \mu_m \, w^{0.75} \, v_0^{0.5} \, a^{-0.25} \, X_M \, X_{GE} \, X_{\varepsilon E} = f(X_{\varepsilon E} \cdot X_{GE}) \\ \Rightarrow \quad f\left(\frac{\left| \sqrt{v_{\rho 1E}} - \sqrt{v_{\rho 2E}} \right|}{\left(\frac{1}{\rho_{rE}} \right)^{0.25}} \cdot \frac{\left| \sqrt{\rho_{1E}} - \sqrt{\rho_{2E}} \right|}{\left(\rho_{1E} \rho_{2E} \right)^{0.25}} \right) \quad \Rightarrow \quad \max \tag{43}$$

The factor of load distribution (X_ε) and the factor of gear geometry (X_G) depend on the variables $v_{\rho 1E}$, $v_{\rho 2E}$, ρ_{1E} and ρ_{2E}, which are also the functions of variable parameters h^*_{a1}, h^*_{a2} and x_1, for the same centre distance a_w. Therefore, it is possible simultaneously to satisfy both of these functions: the objective function which must be equal to two and additional constraints accorded by scuffing. That is the reason for the strong dependence of the temperature ϑ_{Blm} on the shape of the tooth [8].

Obtaining the parameters for HCR involute gearing for maximal temperature at the reference point E, the condition for the occurrence of rapid scuffing on teeth flanks is achieved. Experimentally, it is proven on a test rig according to FZG methodology [12]. In this way, the hypothesis is proven in regard to obtaining minimal flash temperature, and thus obtaining parameters for minimal $\vartheta_{Bl\ \mathrm{E}}$ and $\varepsilon_\alpha = 2$ as the solution to this problem.

8 Conclusions

Contact ratio is increased by increasing tooth height. Dynamic loads and noise are reduced by using high contact ratio gears. According to the results of different measurements of gear pairs, the reduction in noise proved to be best using HCR gearing with the contact ratio value $\varepsilon_\alpha = 2$. A decrease in noise is caused by $\varepsilon_\alpha = 2$, because there are always two pairs of teeth in contact, meaning that when one pair of teeth come out of contact, another pair of teeth comes into contact, while the applied force is considerably smaller, since it is divided between two pairs of teeth. Therefore, gearing in the automotive industry should be performed with $\varepsilon_\alpha = 2$ in order to reduce the noise and dynamic forces.

Due to the increased addendum height, there is a larger possibility of some interference or a pointed tooth tip to occur. Consequently, these errors should be prevented by verifying whether all equations and constraints are satisfied. Conditions for the teeth on the pinion and gear wheel are related to the following: conditions for non-occurrence of interference during the production—Eqs. (35a) and (35b), conditions for non-occurrence of meshing interference—Eqs. (36a) and (36b), conditions for minimal thickness of the tooth head circle of both gears—Eqs. (37a) and (37b), and an additional condition related to the balance of specific sliding at the beginning and end of meshing in order to reduce losses during meshing [11, 12].

Also, there is a possibility of the occurrence of scuffing due to higher tangential velocities and high pressure between teeth. Using the criterion of integral temperature, in order to avoid scuffing, the relation (39) must be minimized.

In previous chapters, two optimizations have been processed: geometric and thermal optimization. Combining these two optimization criteria, the problem of HCR gearing is solved, resulting in $\varepsilon_\alpha = 2$ with minimal possibilities of scuffing.

This geometric optimization problem of finding the optimal parameters for gears in order to equalize the contact ratio with two is very real, especially in the automotive industry. However, the analyses presented in this paper cover only external gear pairs. In the future, it would be very interesting to define the method and conditions for an internal gear pair, which is a common kind of gearing in this type of industry.

After researching the external and internal gear pairs and defining the kinematic conditions for obtaining the contact ratio of two, there is still a question as to whether the gears can achieve the expected load-carrying capacity for the defined interval of time. In other words, will the flanks of teeth sustain loading for a sufficient period of time, without the occurrence of scuffing or pitting, or some other premature damage of the teeth? This requires introduction of new constraints and conditions related to the load carrying ability of and the influence of temperature on the teeth, which are all related to the condition of the high contact ratio of gears. This complicates the existing method for solving tasks and implies defining multiple tasks for achieving the contact ratio equalized by two and loading the torques with no premature damage on tooth flanks.

Therefore, a detailed theoretical analysis of the HCR geometry had to be performed, together with one on the impact of the shape of the involute HCR teeth on its scuffing resistance using the criterion of integral temperature. Besides the theoretical analysis, certain parameters of involute HCR gearing are provided in order to obtain experimental verification of the derived results according to relations of scuffing tests. After evaluating the impact of gearing geometry on its scuffing resistance from the point of integral temperature, it can be concluded that it is possible to achieve a smaller flash temperature, and thus increase the resistance to scuffing. Certainly, besides this condition, the type of transmission oil has a very important influence on the occurrence of scuffing. In other words, scuffing occurrence can be reduced using oils for extra pressure. Experimental verification should confirm if using this type of oil is necessary or not.

This article should contribute to the generalization of the integral temperature criterion for involute HCR gearing. It has been shown that relations for integral temperature criterion for involute HCR gearing need to include a calculation of the factor of load distribution for the case $\varepsilon_\alpha \geq 2$. In the continuation of this research, the criterion for the integral temperature of involute HCR gearing can be optimized and the minimal flash temperature can be obtained. For that case, the factor of load distribution (X_ε) and the factor of gear geometry (X_G) were derived for the case of involute HCR gearing.

This expanded criterion can be used in practice to determine the characteristics of the gears, first and foremost in construction within the automotive industry. In addition, for LCR gears, pitting is the most significant damage for teeth flanks; however, in applying HCR gears, the primary tooth damage is scuffing. Therefore, the calculation of HCR gears must first be performed for scuffing, and after that, for pitting. This procedure can be directly used in designing HCR gears, or as a basis for further theoretical and experimental research of scuffing on gear flanks.

Achieving these results, involute HCR teeth on gear wheels of automotive transmissions will have equal load-carrying capacity during their entire time of operation. Since the load is always distributed on two pairs of teeth, they can resist higher loading with smaller deformation and vibration.

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