Practice of Design and Production of Worm Gears with Localized Contact

E. Trubachev, T. Savelyeva and T. Pushkareva

Abstract The paper considers the mathematical model, the main idea and examples of the practical application of a new approach to the design and production of worm gears with localized contact. Features of design for special and series worm gears under various limitations are distinguished; recommendations on the choice of parameters are given.

Keywords Worm gear · Hob · Fly-cutter · Gear design · Contact localization

1 Introduction

One of the most important results of applying our developed method for localized contact synthesis in worm-type gears [9] turned out to be an abrupt decrease in the range of expensive hobs used in diversified production of worm and spiroid gears [1, 6]. This presented an opportunity for a rather simple and effective solution to practical problems for production of both series and non-series (special) worm gears. A great number of such solutions appeared over the course of the last decade at the Institute of Mechanics of Kalashnikov Izhevsk State Technical University and from within the innovative production enterprise "Mechanik" Ltd. This deserves, in our opinion, an overview of the accumulated experience, since similar works are also vital for other enterprises that produce worm gears [3, 5].

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2 Idea and Algorithm of the Design Method

The method for analysis of machine-tool meshing parameters for generation of worm-type gearwheel teeth by means of a helicoid generating surface has been proposed in [9]. Later, it became the foundation for development of the algorithm [8] for spiroid and worm gear design on the basis of applying the assigned gear cutting tool (in the important particular case, the standard one). The scheme shown in Fig. 1 essentially corresponds to this algorithm; however, we further assume that it can also be applied to the case in which tool parameters are not assigned or are not completely determined.

The main idea of the method implies that the geometric parameters of the generating worm and its arrangement with respect to the gearwheel blank are selected in accordance with the conditions of:

– meshing of three surfaces [4], that is, the conjugated surfaces of the worm and the gearwheel and the desired generating surface at arbitrarily assigned design points:

$$\begin{aligned} \mathbf{n} \times \mathbf{v}_{12} = \mathbf{n} \times (\mathbf{v}_1 - \mathbf{v}_2) = 0, \\ \Rightarrow \mathbf{n} \times (\mathbf{v}_0 - \mathbf{v}_1) = \mathbf{n} \times \mathbf{v}_{01} = 0, \\ \mathbf{n} \times \mathbf{v}_{02} = \mathbf{n} \times (\mathbf{v}_0 - \mathbf{v}_2) = 0, \end{aligned} \tag{1}$$

where $\mathbf{n}$ is the common normal to three pointed surfaces, $\mathbf{v}_{0(1,\ 2)}$ are velocities of their motion in meshing, and $\mathbf{v}_{12}$, $\mathbf{v}_{02}$ and $\mathbf{v}_{01}$ are relative velocities of surfaces.

Here and further on, indices $_0$, $_1$ and $_2$ are related to the generating worm, the operating worm and the gearwheel, respectively; and indices $_R$ and $_L$ relate to parameters of the right and left flanks of teeth and threads and their meshing.

– constancy of the pitch and profile of surfaces of the operating and generating worms-helicoids, for which the following relations (invariant with respect to the choice of the coordinate system) are valid [10]:

$$\tan\ \gamma = -\frac{\mathbf{n} \cdot \mathbf{e}_t}{\mathbf{n} \cdot \mathbf{k}}, \tag{2}$$

$$\tan\ \alpha_x = -\frac{\mathbf{n} \cdot \mathbf{e}_r}{\mathbf{n} \cdot \mathbf{k}}, \tag{3}$$

where α_x is the axial angle of the helicoid profile, γ is the lead angle for its pitch line, and $\mathbf{e}_t$, $\mathbf{e}_r$ and $\mathbf{k}$ are the unit vectors of the tangential, radial and axial directions with respect to the axis of the helicoid.

The system composed of Eqs. (1)–(3) has a rather simple analytic solution [9] in which the preliminary assigned parameters are the machine-tool cross angle Σ_0,

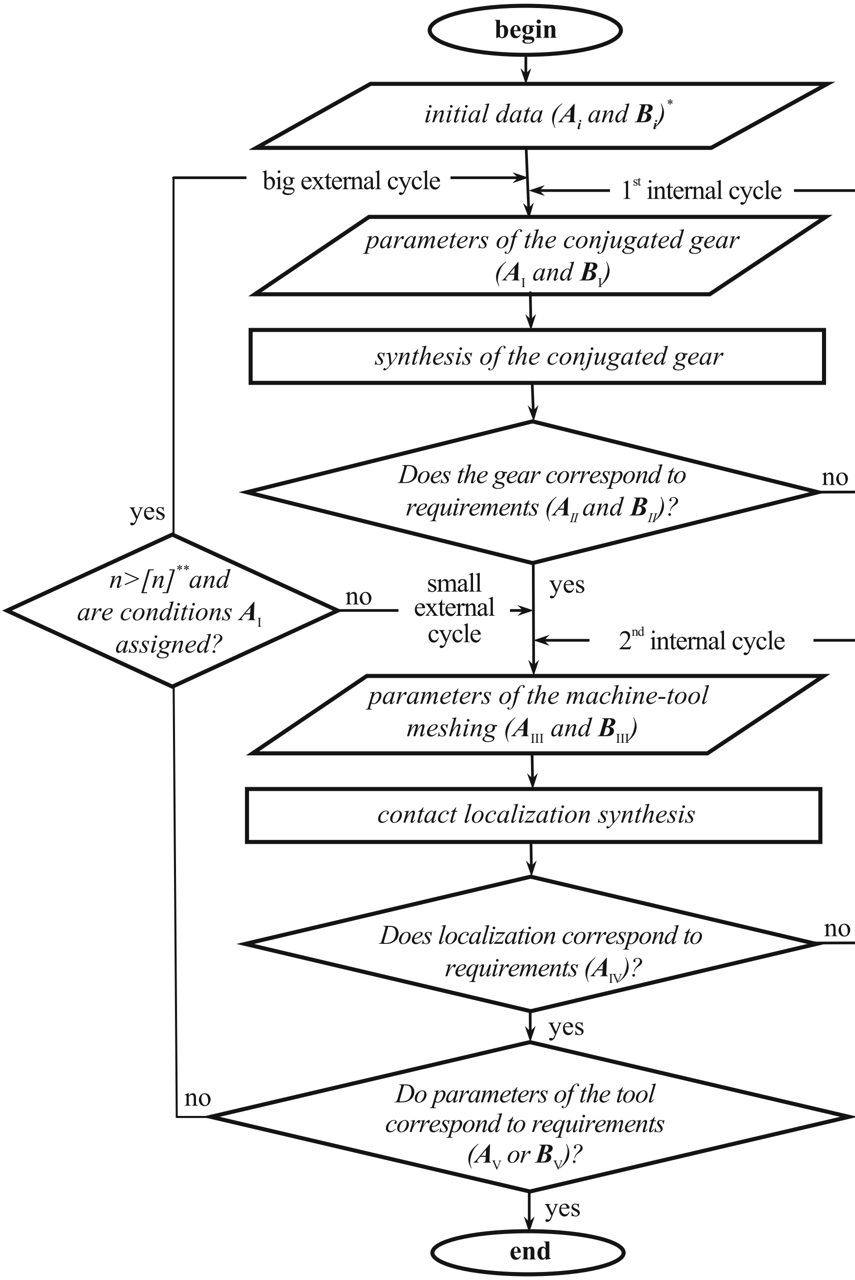


Fig. 1 Algorithm of the worm gear design when applying the assigned gear-cutting tool. *$\boldsymbol{A_i}$ are limitations-inequalities, $\boldsymbol{B_i}$ are limitations-equalities, **n is the number of iterations for a small external cycle

the number of threads z_0 and the axial module m_{x0} of the generating worm, and the parameters obtained in accordance with conditions (1)–(3) are the center distance in the machine-tool meshing a_{w0}, pressure angles $\alpha^1_{x0R,L}$ and the pitch diameter d_0 of the generating worm.

Figure 1 presents the limitations for optimization: they are the groups of parameters $\boldsymbol{A_i}$ and $\boldsymbol{B_i}$, which are related to the following *i*th steps of the design:

1. Input (assignment) of parameters of the conjugated gear.
2. Evaluation of conformity for the conjugated gear to the assigned requirements.
3. Input of initial (assigned) parameters for the machine-tool meshing.
4. Evaluation of conformity to the assigned requirements for localized contact.
5. Evaluation of conformity to the available (required) one for the generating worm.

3 Limitations and Components of the Target Function

Let us consider the essence and several examples of limitations $\boldsymbol{A_i}$ and $\boldsymbol{B_i}$ assigned at statement of the design problem and considered at different steps. In the first step, limitations $\boldsymbol{A}_{\mathrm{I}}$ and $\boldsymbol{B}_{\mathrm{I}}$ affect the following parameters:

– the gear ratio u_{12} (allowable limits of its variation or the strictly assigned value);
– the outer diameter of the worm d_{a1} and the maximum diameter of the gearwheel d_{ae2}, the center distance a_w and the cross angle Σ (limitations of the casing space, overall dimensions of blanks, possibility of assembly of the worm unit, and worm rigidity);
– the axial module m_{x1} of the worm (can be limited by rational values of the addendum modification factor of the worm $x = f\,(m_{x1})$ and the standard row of modules).

In the case when it is necessary to cut only the gearwheel that is conjugate with the existing worm, the limitations imposed on all worm parameters are related to the type $\boldsymbol{B}_{\mathrm{I}}$, that is, they have to be equal to those assigned.

In the second step, we need the correspondence of the analyzed conjugated gear to the assigned requirements (limitations $\boldsymbol{A}_{\mathrm{II}}$): level of the efficiency and load-carrying capacity (torques $[T_2]$ allowable in accordance with various criteria, fitting the limitation of the absence of undercutting and pointing at machining of gearwheel teeth and undercutting at the grinding of worm threads. The version of maximization of η and $[T_2]$ is also possible, that is, they are included in the target function of optimization. Iterations at the 1st internal cycle (Fig. 1) are subjected to meeting these requirements.

In the third step, limitations $\boldsymbol{A}_{\mathrm{III}}$ and $\boldsymbol{B}_{\mathrm{III}}$ set the allowable values of the machine-tool center distance a_{w0}, the machine-tool cross angle Σ_0 and the number of threads z_0 of the generating worm.

In the forth step, local and non-local specifications are the basis for evaluation of the geometry of the modified surface of the gearwheel teeth and correctness of the design parameters of the machine-tool setting. The following limitations $\boldsymbol{A}_{\mathrm{IV}}$ are considered here:

- allowable maximum values of the longitudinal and profile modification of teeth (corresponding to the allowable concentrations of the load and the tooth-to-tooth error at transfer of meshing which appear due to contact localization);
- allowable minimum values of the longitudinal and profile modification of teeth (corresponding to the allowable sensitivity of the meshing to the action of manufacture and assembly errors);
- completeness of surface profiling at generation (this is practically always related to the ratio of diameters and lengths of the operating and generating worms and the heights of their thread profiles);
- desirable distribution of the modification field along the tooth (contact localization on a definite area of the tooth);
- absence of pointing or undercutting of the tooth of the worm gearwheel or undercutting of the thread of the generating worm;
- absence of pointing of the tool that produces the generating worm.

Parameters of contact localization can also be components of the target function of optimization. In order to fulfill the conditions $\boldsymbol{A}_{\mathrm{IV}}$ or at optimization, iterations at the 2nd internal cycle (Fig. 1) are made.

The fifth step determines the application of the existing gear-cutting tool or, regarding the algorithm, the evaluation of the correspondence of the generating worm parameters to those chosen from a certain set of discretely assigned values, that is, limitations $\boldsymbol{A}_{\mathrm{V}}$ or $\boldsymbol{B}_{\mathrm{V}}$. These parameters are: the number of threads z_0, the axial module m_{x0}, the length of the cutting part b_0, the pitch diameter d_0, pressure angles $\alpha_{x0R,L}$, curvature radii $\rho_{x0R,L}$ and factors of the height h_0^* and thickness s_0^* of thread profile.

The set and type of limitation ($\boldsymbol{A}_{\mathrm{V}}$ or $\boldsymbol{B}_{\mathrm{V}}$) depends significantly on the choice of method for implementation of the generating worm, that is, the application of a fly-cutter or a hob for gear-cutting. In the first case, limits on the possible correction of parameters are determined by the tool rigidity; and they are limited, for example, by a wish to apply the existing mandrel with diameter $d_{mandrel}$ for the fly-cutter. When the existing hob is applied, parameters are assigned unambiguously, and their required values are obtained at the iterative mode of the design process. As is seen in Fig. 1, the corresponding external cycles can imply a return to steps I (big external cycle) or III (small external cycle) of assigning the parameters of the operating or machine-tool meshing. As a rule, during the design process, the version of the return to step III is first implemented. Then, as the possibilities of achieving the required or optimal solution have been exhausted, we return to step I, accordingly.

Let us further consider the examples of implementation of the analysis for practical cases, differing by the purpose of gears, the degree of responsibility and, therefore, the assignment of the target function of optimization and the set of limitations $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$.

4 Simple Case: Gear Parameters Are Strictly Assigned

The simplest design situation is that for the case in which it is necessary to meet the limitations $\boldsymbol{B}_{\mathrm{I}}$ of the first step focusing on producing a special and relatively simple tool—a fly-cutter. This situation appears when there is no available hob with somewhat similar parameters and gear parameters cannot be changed significantly. The typical case is the production of responsible repair pairs for complex manufacturing equipment (there is even a simpler case in which the repair problem is solved by substituting the whole gearbox, but it will not be considered here). Note that a delusion can appear here: if the situation is simple, one should always tend to reduce the design process to it. However, it is simple only from the point of view of design; and in production, it can lead to difficulties that are sometimes irresistible. These difficulties are related to the relatively low accuracy and productivity of the method for gear-cutting by means of the fly-cutter.

In this case, the design process is generally traditional. First, a conjugate meshing is designed in accordance with the assigned, and usually strict, limitations. Sometimes, the design is reduced to the checking process when the gear to be designed, in fact, repeats the prototype being replaced (often after interpretation of the specimen without a drawing). Furthermore, the machine-tool meshing with optimization of localized contact follows. Perhaps the main feature here is the accounting for conditions for application of the already available range of mandrels for fly-cutters.

Let us consider a number of gear designs for this typical case. Table 1 presents the main parameters of gears cut by fly-cutters reproducing single-thread worms at gear generation.

In all cases, parameters of the conjugated gear have been restricted by limitations of type $\boldsymbol{B}_{\mathrm{I}}$ (their equality to the assigned ones). While maintaining the initial parameters at the first stage of design and, therefore, meeting the limitations,

Table 1 Examples of gears for the case of a strictly assigned operating meshing

№	a_w, mm	u_{12}	m_{x1}/m_{x0}, mm	Σ_0	d_1/d_0, mm	$d_{mandrel}$, mm
1.	33	24:2	1.5/1.498	87.00	30/33.47	25.0
2.	64	30:2	3.15/3.105	85.00	33.7/35.40	22.8
3.	70	30:1	3.5/3.495	89.88	35/35.76	22.8
4.	164	45:1	6.0/6.0	89.85	58/60.14	44.0
5.	180	62:2	5.0/4.895	85.80	50/52.12	28.0
6.	180	40:1	6.3/6.3	89.90	100.8/110.29	95.0

the $\boldsymbol{B}_{\mathrm{II}}$ type of limitations (keeping the operation performances) eliminates additional iterations at the big external and the 1st internal cycles (Fig. 1).

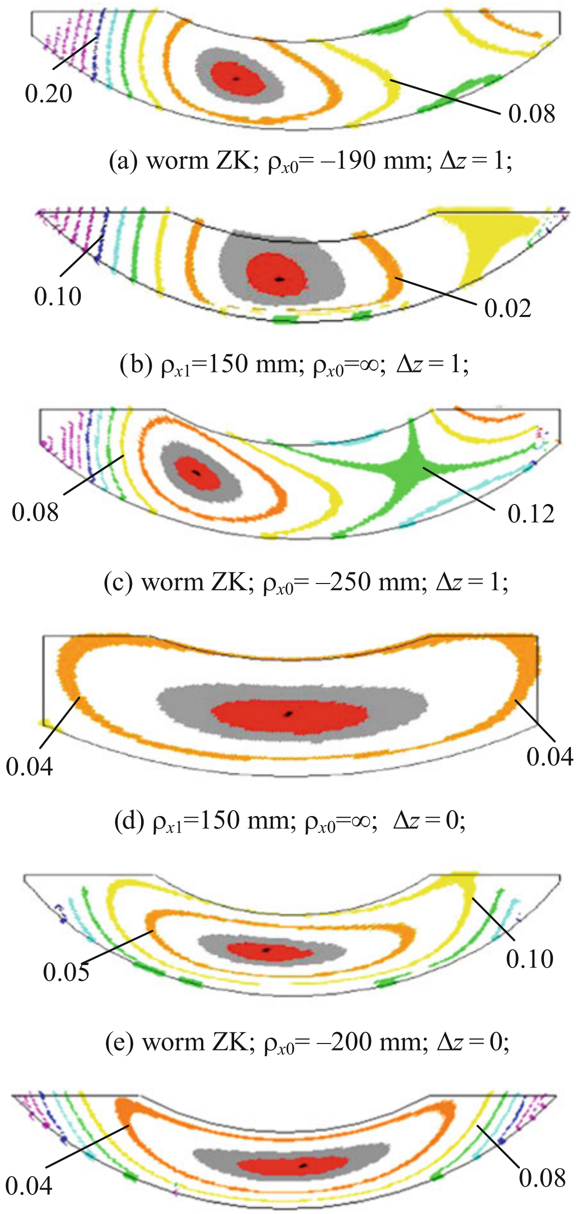
In this case, localized contact should be optimized. The profile localization is provided by the corresponding selection of the profile curvature radii for the worm ρ_{x1} (Fig. 2b, d) or the tool ρ_{x0} (Fig. 2a, c, e, f). When applying the fly-cutter, it is possible to implement profile modification by any of these versions by choosing the most reasonable one from the manufacturing and structural points of view with regard to this gear. When applying the hob, the only version is, as a rule, the selection of the curvature for the worm profile at keeping the initial profile of the tool. The value of the profile curvature is taken with account of limitations $\boldsymbol{A}_{\mathrm{II}}$ and $\boldsymbol{A}_{\mathrm{IV}}$.

Longitudinal localization is provided by variation of the cross angle at machining Σ_0 and/or the axial module of the generating worm m_{x0}. Since these variables affect the design parameter d_0, one should account for limitations-inequalities $\boldsymbol{A}_{\mathrm{V}}$ for its choice related to the available mandrel (or the desirable one, for instance, in accordance with layout considerations) for the cutter by applying the small external cycle of iterations. Thus, for samples, shown in Table 1, one of the design conditions is the application of available mandrels, including, for example, one and the same mandrel for gears № 2 and № 3.

Note that the equal degree of modification can be obtained at various combinations of m_{x0} и Σ_0 (correspondingly, for significantly different d_0). Let us give some recommendations for the choice of these parameters.

Intervals of acceptable combinations of parameters m_{x0} and Σ_0 are narrow, and the relation between parameters of the machine-tool setting and parameters of the modification fields is not obvious in the general case. Therefore, the first approximation at the iteration mode often determines the convergence to the optimal solution and, consequently, the time needed to find it. In the simplest case, the machine-tool meshing which is identical to the operating one is chosen at the first approximation. Usually, it makes sense when the numbers of threads of the operating and generating worms are equal to each other, and the difference between the diameter of worm roots and the diameter of the available mandrel is relatively small. The required longitudinal modification can be provided here by variation of one of the pair of the above-mentioned parameters, which is convenient for the setting of the gear-hobbing machine (the module taken from the range of standard values or the orthogonal machine-tool meshing). The example can be gears № 4 and № 6 in Table 1: when designing their machine-tool settings, the required localization and limitations of the mandrel diameter are provided by variation of the cross angle Σ_0 only.

Limits of the reasonable variation of the value Σ_0 mainly depend on the difference between the numbers of threads of the operating and generating worms $\Delta z = z_1 - z_0$. For $\Delta z = 0$, such a range is within the limits from 87.5° up to 91°; and for $\Delta z = 1$, it is from 84° up to 87.5°. As a rule, a higher level of tooth modification corresponds to the smaller values of the machine-tool cross angle and the module of the generating worm.



(a) worm ZK; ρ_{x0}= –190 mm; $\Delta z = 1$;

(b) ρ_{x1}=150 mm; $\rho_{x0}=\infty$; $\Delta z = 1$;

(c) worm ZK; ρ_{x0}= –250 mm; $\Delta z = 1$;

(d) ρ_{x1}=150 mm; $\rho_{x0}=\infty$; $\Delta z = 0$;

(e) worm ZK; ρ_{x0}= –200 mm; $\Delta z = 0$;

(f) worm ZK; ρ_{x0}= –200 mm; $\Delta z = 0$;

Fig. 2 Modification fields for gears: **a** № 2; **b** № 1; **c** № 5; **d** № 3; **e** № 4; **f** № 6; « 0.04 » are modification levels, mm

Distribution of modifications along the gearwheel tooth length is often unacceptably asymmetrical relative to the design point. Asymmetry essentially depends on the difference between the numbers of threads for the operating and generating worms (for instance, versions a, b, c in Fig. 2 for $\Delta z = 1$ and versions d, e, f in Fig. 2 for $\Delta z = 0$). An extremely large modification at the tooth face end from the worm thread entering the mesh can be corrected by shifting the design point to this face end: at first approximation, it is 3–5% of the total length of the tooth.

As for the considered simple case, iterations are concentrated at the second internal and small external cycles (Fig. 1), without addressing the first one and organizing the big external cycle of iterations, which is essentially simplifying the search for the solution.

5 More Complex Case: Application of the Existing Hob

In case of applying the hob from the range of existing ones, it is necessary to provide the conformity for its design parameters to the assigned ones. This is achieved through introduction of iterations at the big external cycle (Fig. 1) with variation of parameters of the first step, thus leading to additional iterations at the 1st internal cycle, since it is required that we provide the values of operation parameters within the range allowable in accordance with limitations $\boldsymbol{A}_{\mathrm{II}}$.

Let us consider certain features of the design procedure for such a case by example of the gears having the main parameters given in Table 2. Note that gearwheels for all gears have been cut with single-thread hobs. Moreover, hobs specified by the Russian standard GOST 9324–80 and intended for machining involute spur and helical gearwheels have been used for cutting gears № 8, № 9 and № 10.

In the fifth step of the design procedure, after searching for the first approximation with regard to the above-mentioned recommendation and optimizing at the 2nd internal cycle (Fig. 1), the parameters of the generating worm can take values different from those assigned in accordance with limitations $\boldsymbol{B}_{\mathrm{V}}$. Differences between the obtained design and the required parameters of the tool involve the values of discrepancies which should be accounted for at the following cycle of

Table 2 Examples of gears designed in accordance with the assigned hobs

№	a_w, mm	$u^*_{12\ initial} \rightarrow u_{12\ accepted}$	$m_{x1\ initial}/m_{x0} \rightarrow m_{x1\ accepted}$, mm	$d_{1\ initial}/\ d_0 \rightarrow d_{1\ accepted}$, mm	$[\alpha_{x0L/R}] \rightarrow \alpha_{x1L/R}$, °	Σ_0, °
7.	52.5	28:1 → 31:1	2.5/2.5 → 2.46	35/35 → 28	20.18 → 17.5	89.3
8.	60.0	24:2 → 24:2	2.5/2.5 → 2.5	60/74 → 60	20.44 → 20.0	87.2
9.	96.4	22:1 → 44:2	6.3/3.0026 → 2.963	50.4/72.5 → 58	20.00 → 17.0	86.8
10.	103.5	33:1 → 33:1	4/4 → 4.005	75/79.1 → 75	20.01 → 19.8	89.8

*The subscript $_{initial}$ designates the initial values of parameters (parameters of a reference gear), the subscript $_{accepted}$ designates the parameters accepted after the design procedure iterations (big external cycle—Fig. 1). For this purpose, the corresponding corrections are introduced to the values of the homonymic parameters of the operating worm [8] assigned at step I with account for limitations $\boldsymbol{A}_{\mathrm{I}}$:

$$\Delta m_{x0j} = m_{x0j} - [m_{x0}],\ m_{x1j+1} = m_{x1j} + \Delta m_{x0j}, \tag{4}$$

$$\Delta\alpha_{x0R,Lj} = \alpha_{x0R,Lj} - \left[\alpha_{x0R,L}\right], \quad \alpha_{x1R,Lj+1} = \alpha_{x1R,Lj} + \Delta\alpha_{x0R,Lj}; \tag{5}$$

$$\Delta s^*_{0j} = s^*_{0j} - \left[s^*_{0j}\right], \quad s^*_{1j+1} = s^*_{1j} + \Delta s^*_{0j}. \tag{6}$$

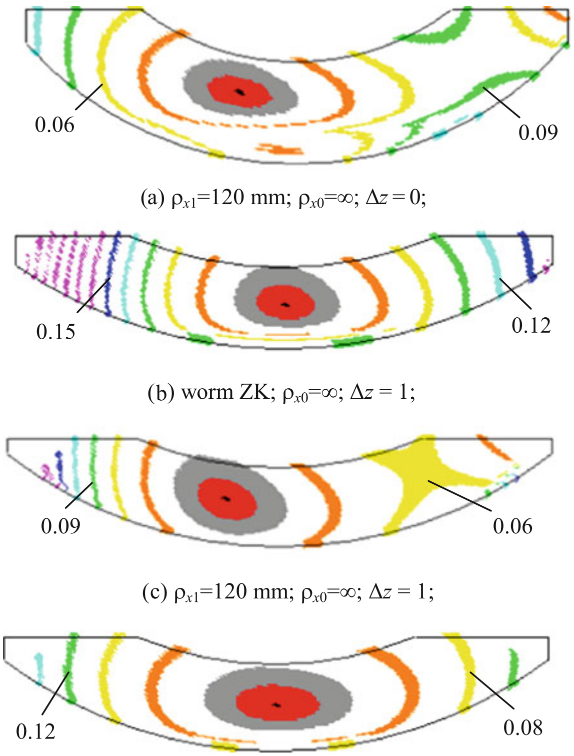
Here, j is the number of iteration of the big external cycle, $\Delta m_{x0}, \Delta\alpha_{x0R,Lj}, \Delta s^*_{0j}$ are the values of discrepancies, respectively, for the module, pressure angle and thread thickness factor for the generating worm; and the values in square brackets are assigned in accordance with limitations $\boldsymbol{B}_{\mathrm{V}}$. When varying the parameters of the first step, provision of the required strength, rigidity and durability (providing limitations $\boldsymbol{A_2}$) is also evaluated, thus increasing the number of iterations as compared to simple cases.

Similar to simple cases, for a more complex case of applying the existing hob, it is possible to obtain the required tooth modification by correction of only one of two parameters, that is, the module of the operating worm m_{x1} while maintaining the orthogonality of the operating and machine-tool meshing or only machine-tool cross angle Σ_0 (gear № 8 in Table 2) while maintaining the same modules of the worm and hob (for example, the standard ones).

Dr. Eng. S. A. Lagutin [4] was the first to pay attention to the possibility of applying the same modules for the operating and generating worms at localized contact synthesis in the worm gear. He found a simple relation between the parameters of the worm and hob for this important and convenient case. However, in the general case of gear design, it is necessary to search for the combination of Σ_0, m_{x1}, d_1 and the position of the design point presenting an optimal picture of tooth modification. It is evident from the information given in Table 2 and Fig. 3 that the above-mentioned recommendations on the choice of the design point position, machine-tool cross angle Σ_0 and relations between modules of the operating and generating worms are also generally valid for more complex cases.

Providing the level of tooth modification within the interval (0.02–0.05)m_{x1} can require a correction of parameters d_1, m_{x1}, α_{x1}, and s^* and, in certain cases, of the gear ratio u_{12}. Such a correction inevitably leads to a certain change in the operation parameters of the gears, as compared to the initial ones available prior to optimization synthesis. Both positive and negative effects are possible here, and this circumstance should be given due attention. However, in most cases, the correction is acceptable. Thus, in the worst cases for gears shown in Table 2, the decrease in efficiency did not exceed 1 to 2%, and that of torques, the result of contact failures of teeth, was under 10%, which turned to be acceptable; application of the existing tooth cutting tool gave an essential economy.

Fig. 3 Modification fields for gears: **a** № 7; **b** № 8; **c** № 9; **d** № 10



(a) ρ_{x1}=120 mm; $\rho_{x0}=\infty$; $\Delta z = 0$;

(b) worm ZK; $\rho_{x0}=\infty$; $\Delta z = 1$;

(c) ρ_{x1}=120 mm; $\rho_{x0}=\infty$; $\Delta z = 1$;

(d) worm ZK; ; $\rho_{x0}=\infty$; $\Delta z = 0$;

6 Features of Design for Highly Responsible Gears

A number of highly responsible gears which were designed and introduced into series production required a very thorough and versatile analysis. Let us consider two specific cases.

A heavy-loaded low-speed gear. We paid great attention in our works on this theme (for instance, [2]) to a special selection of parameters of the non-orthogonal worm gear, when the worm gearing becomes similar to the spiroid one in accordance with its properties: contact ratio (Fig. 4), arrangement and path velocity of contact lines, sensitivity to the action of errors. Consequently, it becomes more suitable for application in low-speed (rotational frequency of the output shaft does not exceed 200 rpm) and heavy-loaded (contact stresses are 1000–2000 MPa) drives for pipeline valves.

We have designed a range of such non-orthogonal worm gears for the gearbox of the torque of 64,000 Nm (Σ = 75°, a_w = 260 mm) [7]. In all gears, the contact has been localized in both length and height; and gearwheels have been cut by standard involute gear hobs. The main parameters of gears and machine-tool settings and the main operation performances of gears are presented in Table 3. The obtained theoretical and operational modification fields and bearing contacts are shown in Figs. 5 and 6. Note that the efficiency of such gears turned to be a little less (by 2.5% on average) compared to their orthogonal analogs. But in this case, bending loads on teeth decreased abruptly, gear sensitivity to the action of errors was reduced and the possibility arose of applying heat-hardened steel for gearwheels; in total, it led to an abrupt increase in the gear strength.

The important feature of the design and production of these gears has been the choice of the level of the longitudinal and profile modification. As opposed to orthogonal worm gears, and similar to spiroid ones, the longitudinal modification of teeth had an impact mainly on the contact ratio actually implemented (and additionally, on the tooth-to-tooth accuracy, however, this feature is not crucial for the considered gears); and profile modification had an impact on load distribution along contact lines (areas). The actual bearing contacts obtained under load reveal the correctness of the accepted degree of contact localization (Fig. 6).

Note that in order to localize the contact in non-orthogonal gears, it is reasonable to follow recommendations on the change of the machine-tool cross angle, as compared to the cross angle of the gear, which has been described above for the orthogonal version.

A loaded gear with the increased requirements for tooth-to-tooth accuracy and the backlash. It was required that we master series production of worm pairs with the centre distance of 100 mm in which the motion smoothness and the bearing contact were rated by the 6th degree of accuracy in accordance with the Russian

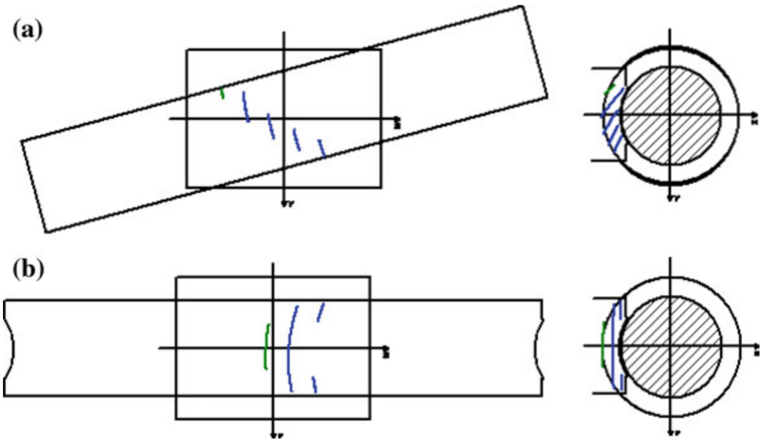


Fig. 4 Modification fields: **a** gear № 13; **b** orthogonal analog

Table 3 Non-orthogonal low-speed heavy-loaded gears

№	u_{12}	m_{x1}/m_{x0}, *mm*	d_1/d_0, *mm*	Σ_0,°	η,%	$\tilde{\sigma}_F$[b], *MPa*	
						thread	tooth
11.	81:2	4.967/5.008	102.2/87.9	70.7	45.9/(46.7)[a]	114/(285)	261/(492)
12.	68:2	6.070/6.011	106.0/97.6	72.3	49.4/(50.7)	96/(231)	111/(210)
13.	57:1	7.207/7.017	103.7/101.1	77.4	37.1/(37.9)	153/(474)	150/(399)

[a]Values in brackets correspond to orthogonal analogs
[b]Conditional values of bending stresses applied for comparative analysis

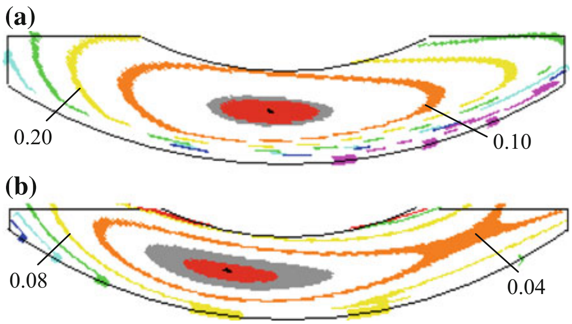


Fig. 5 Modification fields for gears **a** № 12; **b** № 11

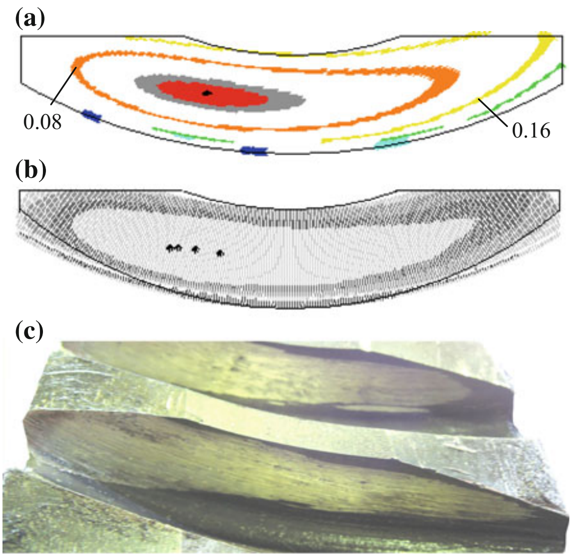


Fig. 6 Gear № 13: **a** modification field; **b** designed bearing contact; **c** actual bearing contact under load

Table 4 Versions of a highly responsible gear

№	u_{12}	m_{x1}, *mm*	d_1, *mm*	η,%	σ_H, %	σ_F, %	
						thread	Tooth
14.	48:1	3.250	44.0	68.9	100.0	100.0	
15.	50:1	3.172	45.2	65.4	96.4	109.3	106.8
16.		3.189		65.5	96.5	109.1	112.9
17.		3.201		65.6	96.1	103.5	103.9
18.	51:1	3.182	41.0	66.6	97.4	110.8	100.0
19.	50:1	3.175	44.0	66.1	96.7	127.0	111.0

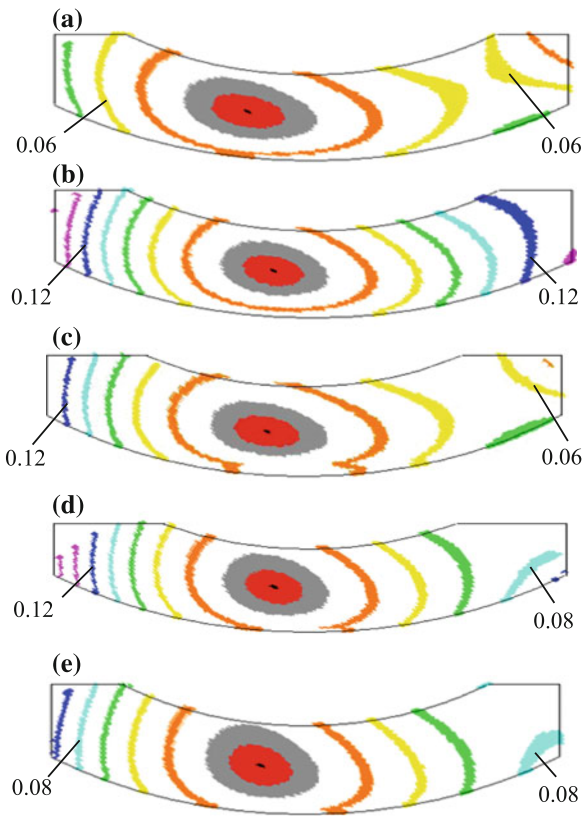


Fig. 7 Modification fields for gears: **a** № 15—0.06 mm*; **b** № 17—0.12 mm; **c** № 18; **d** № 19; **e** № 16—0.08 mm; *for gear №s 15, 16 and 17, localization within a definite range of values is provided

State Standard GOST 3675-81 (this corresponds to the standard DIN 3974-2); and the backlash had to be within the 0.03–0.06 mm range. Of course, the problem of producing a gear of the 6th degree of accuracy was mainly the problem of ensuring the appropriate level of manufacturing (equipment, tool, mounting and measuring means). However, within this range of manufacturing problems, no less important a task is the design of a gear that corresponds to the following requirements:

(a) High load-carrying capacity, both for the contact and bending strength;
(b) Application of the available precision tool, that is, involute gear hobs of a high degree of accuracy;
(c) Relatively low sensitivity of the bearing contact and backlash to the errors of the axial position of the gearwheel (assembly without adjustment).

Table 4 presents the main parameters of the versions of the gears considered. The first of the enumerated versions (gear № 14) was called the basic one: it was given by the customer as the result of designing a drive comprising a worm gear. Note here that the number of considered gears was indeed considerably greater; Table 4 presents only the "final gears" which possess better loading characteristics (contact σ_H and bending σ_F stresses on teeth), as compared to the basic version.

Three versions (gear №s 15-17) have been chosen from gears presented in Table 4 for practical implementation as those closest in accordance with the set of comparisons with the reference gear. The different degree of longitudinal localization of the contact (Fig. 7a, b, e) is provided in these gears, this degree having been chosen after practical development – directly for experimental specimens.

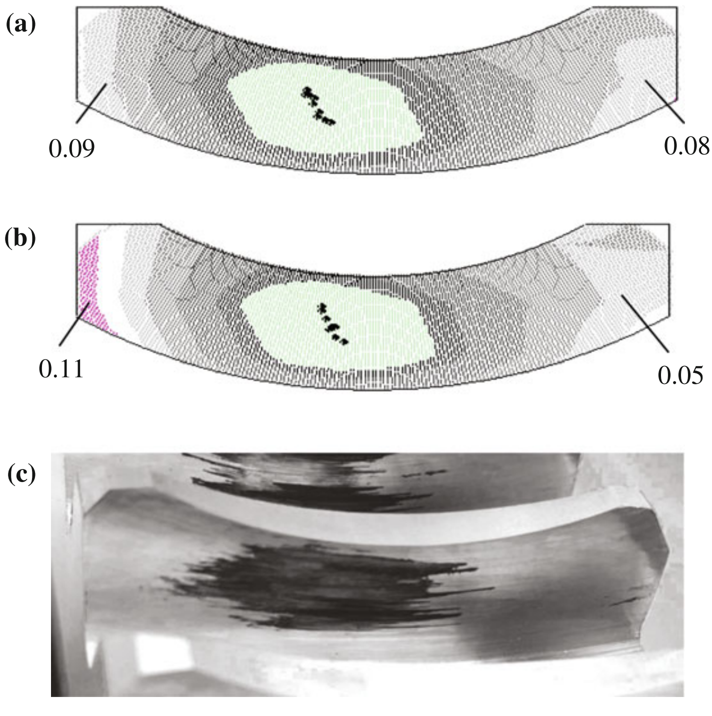


Fig. 8 Gear № 16: **a** design bearing contact at the nominal axial arrangement of the gearwheel (f_x = 0); **b** design bearing contacts at the axial shift of the gearwheel f_x = 200 µm; **c** actual bearing contact after running-in

Our developed technique here allowed us to take into account another additional limitation (which should be referred to the set $\boldsymbol{B}_{\mathrm{III}}$): the coincidence of machine-tool settings for cutting the gearwheels for the pointed versions ($\Sigma_0 = 88.8°$, $a_{w0} = 107.7$ mm). In essence, we have obtained a new, previously unknown result: one and the same gearwheel (a number of interchangeable gearwheels) can be used to mesh with somehow differing worms and to obtain the various degrees of contact localization and acceptable load characteristics. It allowed for an abrupt reduction in the time and other costs for development of the design solution.

During the working out of design decisions, gear № 16 was chosen (Fig. 7e). Its initial bearing contact turned out to have acceptable dimensions and, as expected, a low sensitivity to the action of errors (Fig. 8).

7 Conclusions

Examples of application of our developed method and software for analysis of worm gears with localized contact which have been considered in this paper illustrate possibilities for the quick and efficient solution of a number of practical problems. In our opinion, these possibilities have not been completely uncovered. For example, we deliberately did not include all existing examples of worm gears which have been produced in practice in the number of gears considered. This was done to emphasize the basic trends of the choice of parameters and assessments of gears. However, it does not mean that the method for analysis and the results presented are not applicable for those cases which would require solutions outside common tendencies. For those cases, the pointed trends also assist significantly in the search for a non-standard solution. No doubt, such solutions are also of great interest, especially for more complex cases of multi-thread gears.

The paper, for practical reasons, did not address the issue of generation and a specific type of target function. This issue is urgent from the point of view of complete formalization of the considered optimization problem and it deserves special consideration. Note here that for an experienced design engineer, the absence of this function is explicitly not the irresistible obstacle. Often, it is enough to make a subjective ranking and practical checkup of different assessments, which have been pointed out above.

In our opinion, further development should involve supplementation of the method of analysis with more specified ranges of recommended parameters, empiric relations plotted in accordance with these or those criteria and, perhaps, unification of parameters of gears and tools.

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