Development of Geometric Descriptors for Gears and Gear Tools

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Abstract Important parameters of *a gear tool* are values of deflection of the machined surface at its different points, appearing due to errors of tool set-up, its re-sharpening and so on. An important parameter of *a gear* is its sensitivity to variation in the mutual arrangement of links appearing at gear assembly and due to deformations by loads and temperature influence. It is almost impossible for a production engineer, designer, product assembler or repairman to find this information; it can be obtained through complicated computer-aided analysis, individually tailored to each specific tool and gear. The place for keeping this information may be geometric descriptors of tools and gears proposed in this paper. The geometric descriptor will allow the manufacturers to solve multiple complex tasks quickly and reliably: (a) to obtain the proper location of the bearing contact in a gear; (b) to estimate the behavior of the bearing contact and the value of cyclic variations of the gear ratio when a gear is operated; (c) to assign deviations in tool-setting parameters in order to compensate for organic errors in the re-sharpening of tool front surfaces; (d) to determine the re-sharpening parameters in order to decrease organic errors in re-sharpening or obtain the required modification of tooth surfaces and other tasks. Theoretical basics for creating geometric descriptors are kinematic methods of the classical theory of gearing, developed later in the theory of real gearing. Choosing the most valuable references for development of geometric descriptors, we have to list works [11–20, 22–24]. The previous theoretical works written by the author are essentially useful for computer-aided design of generating processes, which precedes the development of geometric descriptors [2–4, 6, 7]. In these works, investigations of generating processes are carried out through applying: (i) the concepts of fans, wedges and bunches of normal lines [2–4, 7] (one can determine surfaces generated by jogs on generating solids, including those of secondary cutting); (ii) multi-parametric enveloping [2] (surfaces of shear are determined within tool supply and withdrawal); (iii) interrelated systems of curvilinear coordinates: integral, natural, unified, regulated [3] (one can even describe the geometry of all cutting edges for any edge-type tool as a

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continuous, smooth surface differentiable at all points with two unified regulated curvilinear coordinates on it [7]). The paper also presents: (a) analysis of features for gear machining cutting by edge-type tools and requirements for the geometry of operating flanks of teeth; (b) specification of types of geometric descriptor (paper, computer-aided and combined) and of tasks solved by their means; (c) statement of theoretical basics of development of geometric descriptors; (d) approximate contents of works on development of a system of geometrical descriptors for tools and gears; (e) theoretical investigations and specification of developed computer-aided programs aimed at development of geometrical descriptors; (f) results of computer-aided simulation through these programs for generation of helical surfaces by solids of revolution, i.e., fundamentals of geometric descriptors for disk-type cutters and disks for profile grinding; (g) structure of paper geometric descriptors and basic components of one sheet of such a descriptor. The present paper does not provide examples of geometric descriptors for specific tools and gears.

Keywords Theory of generation · Machine-tool meshing · Gear · Geometry of meshing · Geometric descriptor

1 Introduction

Many industrial and household products have descriptors with their technical parameters and consumer properties.

Important parameters of gear tools are, in particular, values of deflection of the machined surface at its different points, appearing due to tool re-sharpening, errors in tool set-up or other reasons. This information is necessary for a production engineer to be able to: (a) estimate the influence of tool set-up errors on deflections of the machined surface; (b) determine the deliberate deviations of tool setting parameters to compensate for organic errors in re-sharpening; (c) determine parameters of sharpening for front surfaces of tools in order to obtain the required modification of tooth surfaces, for example, within localization of tooth contact in a gear; and to solve other similar tasks.

An important parameter of a gear is its sensitivity to variation in mutual arrangement of gear links appearing at gear assembly and due to deformations during its operation because of both loads and temperature influence. Proper presentation of these parameters could, in particular, essentially simplify derivation of an optimal bearing contact in a gear at its assembly, repair and maintenance.

It is almost impossible for a production engineer, a designer, a product assembler or a repairman to find this information; it can be obtained through complicated computer-aided analysis, individually tailored to each specific tool and gear. That is why they need a source of such information that is convenient in use and allows for solving the mentioned tasks. It is proposed that a system be developed of geometric descriptors for tools and gears to be used as the required source.

Apparently, the task of development of *geometric* descriptors has not been brought up either in Russia or abroad. The "Description of hobs at their production and operation" described in [16] solves other tasks and pursues other aims: following the re-sharpening number and amount of metal cut at front surfaces during hob operation, determination of the hob's lifetime, etc. There is a reason to believe that the *geometric component* added to such a *description* would be very useful.

There have only been two publications by the author *on geometric descriptors*: [5, 21] in 2011 and 2012, respectively. They were published in Russian and they considered geometric descriptors of gear cutting tools. The publications remained unmarked. In recent years, this idea was discussed in private conversations with gear experts. Their typical opinion is that industry needs such descriptors very much, but their development would require labor of great intensity, both in theoretical investigations and especially in the programming implementation of a system of computer-aided generation and application of descriptors. This work can be carried out only by a development team with sufficient financing. But it is already reasonable and timely to state the necessity of such a task and to work towards solving its theoretical aspects. Tolerances for the manufacture of tooth elements of advanced gears often do not exceed several micrometers, and in their production, the urgent problem arises in regard to both the geometric features of the tool and the details of the appearance of organic errors in generation by enveloping methods. Geometric descriptors would facilitate the transition to a higher level of production of gears and tools.

Geometric descriptors are also of great interest for enterprises with worn-out batches of gear cutting machine-tools which have lost the accuracy required for production. In order to use them to produce gears of proper quality, the gears need to be designed with an essentially greater level of contact localization than is required for more precise gears. The greater value of contact localization can be also obtained through special sharpening of the edge-type gear cutting tool. The geometric descriptor, in particular, will allow a production engineer to determine the shape of the tool front surface at its sharpening.

Geometric descriptors of tools can also stimulate practical implementation of new tools and gear machining techniques. For instance: (a) cutting the gearwheels for worm gears by unified hobs [11, 23] (a single-thread hob can be used here to machine gearwheels meshed with a multi-thread worm and to obtain, in this case, the localized contact); (b) a new production method for gear machining of spiroid gearwheels by cutting heads at continuous generation [22]. Geometric descriptors would be helpful in this case for estimation of suitability of available tools for cutting specific worm (spiroid) gearwheels; revealing the quality parameters of obtained gears (first of all, the assessment of the degree of contact localization and multi-pair character of meshing).

This work is the first English publication on geometric descriptors. The text is presented in as close to layman's terms as possible to make the information comprehensible for those who are unfamiliar with all the geometrical details of gears and gear machining tools. That is why the paper states the requirements for tooth flanks of advanced gears and features of edge-type tools at the very beginning.

Then, it considers the concept of a geometric descriptor, the tasks to be solved with its help and the principle of its development. The paper also gives an example of investigations to be made prior to development of a geometric descriptor, especially investigation of the generation of helicoids (helical surfaces with constant pitch) by a disk-type tool, specifically by solids of revolution—disk grinding wheels.

2 Basics of Development of Geometric Descriptors

2.1 Requirements for the Geometry of Tooth Flanks

Representation of gear geometry underwent fundamental changes in the 20th century. At the beginning of the century, it was thought that, theoretically, conjugated surfaces should participate in meshing, these conjugated surfaces giving the uniform rotation of the driven gearwheel when contacting each other at uniform rotation of the driving wheel. It was supposed in this context that manufacture and assembly errors of gear elements were absent and that no deformations occurred due to both load and temperature. By the middle of the century, the profile and longitudinal modification of teeth had become common for responsible gears (first of all, spur and helical, bevel and hypoid). That is, the tooth flank for one of the gearwheels was deliberately deviated from that of the theoretically conjugated one. The tooth thickness at the outside surface was decreased at profile modification, thus reducing the impact power of the teeth at the instant of their re-conjugation. The tooth thickness at tooth flanks was decreased at longitudinal modification, thus eliminating the appearance of edge contact of the teeth at misalignment of the axes due to gear manufacturing and assembly errors, and deformations under load.

In the 1980s, a new approach towards optimal tooth modification appeared. Contact, bending and other deformations were taken into account for all gear elements: teeth, gearwheels, bearings, shafts, joints and casings. Modification of teeth was assigned after complicated computer-aided analysis of the mode of deformation for gear elements, accounting for re-conjugation and the multi-pair character of meshing.

The largest contribution to the development theory of localized contact in bevel and hypoid gears was made by F.L. Litvin, along with his followers (Leningrad). M.G. Segal (Saratov) developed and implemented the method for designing and cutting bevel gears, assigning values for deviations at eight boundary points of tooth flanks. Methods for gear synthesis with computer-aided simulation of mode of deformation of gear elements were successfully developed by: G.I. Sheveleva (Moscow) for bevel gears; M.L. Erikhov and his followers (Kurgan) for spur, helical and bevel gears; V.I. Goldfarb (Izhevsk) and his followers for worm-type gears; "Salyut" Inc. (Moscow) for spur gears, and other scientific schools.

At the present time, the following gears are considered to be the best: (a) those with point contact at the nominal point (at the middle of a tooth); and (b) those with small deviations in the solid of a part of the gearwheel flanks at the boundaries.

2.2 Features of Complex Edge-type Tools

The teeth of all edge-type tools are identical (Fig. 1). They have front and back surfaces, forming the cutting edges (main elements of the tool—Fig. 2) at the points of intersection. Very high demands are made on the shape of the cutting edges, namely, they generate the flanks of teeth on gearwheels. One problem for complex

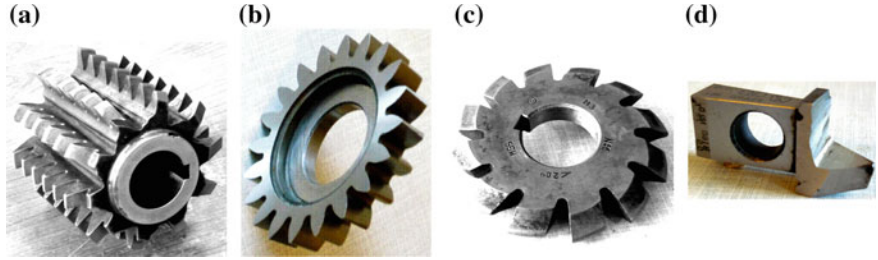

Fig. 1 Edge-type tools for cutting gearwheels: **a** involute hob; **b** shaping cutter; **c** disk-type hob; **d** cutter of a cutting head for spiral bevel gear machining

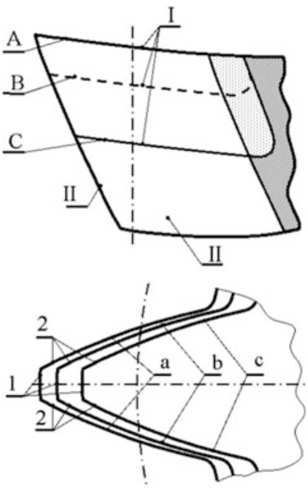

Fig. 2 Tooth and cutting edges of edge-type tools: *I* front surface. *II* back flanks. *1*, *2* top and side cutting edges: *A*, *a* for a new tool, *B*, *b* for a tool re-sharpened by 40%, *C*, *c* for a tool depleted by re-sharpening

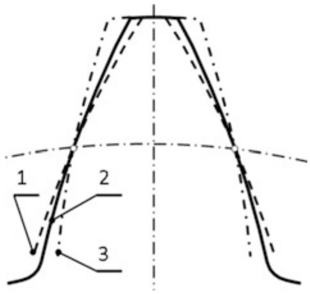

Fig. 3 Organic errors of edge-type tools: *1* new, *2* re-sharpened by 40%, *3* depleted by re-sharpening

edge-type tools is that, after their re-sharpening, the cutting edge leaves out a certain nominal surface, and it becomes all the more vital the farther the front surface strays from its nominal position. When the cutting edge leaves the nominal surface, the surface of the product will deviate from that required. This deviation results in an organic error of generation.

It is common to design an edge-type tool so that the organic error will be absent at the re-sharpening of the tool by 40% of its allowable value. As a rule, a tool that is new and re-sharpened to its maximum will cause organic errors with opposite signs (see Fig. 3). Maximum organic errors appear at the machining of wheels for worm gears: single- and double-enveloping [16, 17] and spiroid [23]. This is related to the significant sensitivity of the shape of tooth flanks for such gearwheels to variation in the hob diameter due to its re-sharpening.

There are many works on the generation of helical surfaces (which are very common in tools and gearwheels) and on investigations into the geometry for shaping cutters, involute hobs and other gear-cutting tools. In particular, they cover the following issues: (a) influence of re-sharpening on the shape of the product surface; (b) influence of errors of mutual arrangement of the tool and blank on the shape of the machined surface; (c) influence of mutual arrangement of gearwheels in a gear drive on the character of tooth contact.

There are to ways to correct organic errors in gear machining by edge-type tools: (a) changing the shape of the front re-sharpened surface; (b) changing the parameters of the tool setting with respect to the blank. One of the tasks to be solved by means of geometric descriptors of tools is to give a production engineer the possibility of reducing the organic errors of tooth flank generation to a minimum.

2.3 Geometric Descriptor of a Tool and Tasks Established with Its Application

A geometric descriptor is a document containing several pages/appendices with nomograms that allows for:

(a) determination of deviations at assigned points of the machined surface which appear due to the re-sharpening of the tool and errors in its setting and motion at gear machining—*the direct task*;
(b) determination of the parameters of the tool setting, motion and refurbishment so as to obtain the assigned deviations of the shape of the machined surface from the nominal enveloping surface—*the reverse task.*

Let us explain the above through several examples of tasks and manners of their establishment through application of geometric descriptors.

Example 1 The geometric descriptor of the grinding wheel for machining a helical surface with constant pitch of: worms, gearwheels (including the spur variety) and other parts. The main element of the geometric descriptor is the nomogram connecting the deviation of the product profile from the nominal one at any point depending on the assigned:

(a) diameter of the grinding wheel (at its invariable profile);
(b) errors of the grinding wheel position with respect to the product (for all parameters of its setting);
(c) errors of the grinding wheel profile—displacement of the whole profile and deviations at any assigned point.

The following points can be assigned here at the profile of a product or grinding wheel: (a) the point at the outside cylinder; (b) the lowest active point; (c) the point at the reference (nominal) cylinder; (d) any other point.

Nomograms in the descriptor allow for establishment of both the *direct* and *reverse* tasks. They are necessary for a designer to be able to assign the rules of the contact in a gear, and for a production engineer to be able to correct the parameters of gear machining and obtain the required bearing contact.

Example 2 *The geometric descriptor of the disk-type hob for machining a helical surface with constant pitch.* According to the contents of the descriptor, the established tasks and the methodology of application, it is close to the descriptor of the grinding wheel. The difference is that, instead of variation of the wheel diameter at the invariable axial profile, the tool re-sharpening for its front surface is accounted. The shape of the back flank inherent to the tool and the shape of the front surface to be re-sharpened are considered here. Such a supplement to the descriptor is very significant and rather complex in its computer-aided implementation.

Example 3 *The geometric descriptor of the involute hob—the main edge-type gear machining tool.* According to its functional possibilities, this descriptor is similar to the descriptor of the disk-type hob, but much more complicated. First, the content of the descriptor and a part of the established tasks are different for tools which machine worm and spur or helical gearwheels. Second, the involute hob is intended for cutting gearwheels with different numbers of teeth and factors of addendum modification. And it is desirable to have one descriptor per hob, rather than a series of descriptors for one hob per each of the products.

Who creates nomograms for geometric descriptors and how. A descriptor itself, with appendices—nomograms and instructions for application, are generated by means of special software.

Let us consider in more detail the manufacturing tasks which are established by means of geometric descriptors of tools.

Task 1 (direct). It is necessary to determine the influence of the tool position on the shape of the machined surfaces of a gearwheel:

- to determine the value of deviations from the assigned position at specified points of the surface generated by the tool on the tooth of a gearwheel. The task is established for the design position and motion of the tool and blank.
- to determine the variation of these deviations for the assigned errors of position and motion of the tool with respect to the blank. Establishment of this task will allow for estimating the applicability of this tool (for cutting a specific gearwheel) with the front surface present upon it.

Task 2 (reverse). It is necessary to determine for a specific tool:

- the position and parameters of tool motion with respect to the blank, so that the deviation of the generated surface from the design at the assigned points of the tooth will be a minimum.
- the allowable limits of errors for tool setting, so that the deviation of the generated surface from the design will be within the tolerance range.

Establishment of this task will allow for determining such optimal position and motion of this tool in machining a specific gearwheel that will provide minimum organic error of generation and provide the opportunity to specify technically-based tolerances for position and motion of the tool within gear machining.

Task 3 (reverse). It is necessary to determine the shape of the front surface of the tool and the profile of the grinding disk for re-sharpening of this surface, providing the modification of the machined surface of the tooth that minimally deviates from the required design surface. Establishment of this task diminishes the urgency of the problem of variation of organic errors of generation due to the re-sharpening of gear cutting tools.

Task 4 (reverse). Correction is necessary for position, motion or parameters of front surfaces of the tool based on measurements (by an involute profile measuring device or control-measuring machines) of gearwheels previously cut by this tool. Establishment of this task allows for at least partial compensation for the influence of a large group of random factors (not involved in geometric descriptors) affecting the generating process: deformations and dynamic processes in the system "machine—equipment—tool—part", errors of kinematic chains, and others.

2.4 Types of Geometric Descriptor

The three following types of descriptor are proposed:

Type 1. Paper descriptor, with diagrams and nomograms as its basis, created by a computer program for a specific tool according to one of two techniques:

(1) Based on its measurement at the control-measuring machine. This technique is preferable, but expensive equipment is required in this case.

(2) Based on computer-aided simulation of generating processes according to which the tool is made at the tool-making plant and, perhaps, accounting is made for measurement of several tools within one dimension type at the control-measuring machine (to obtain the statistics for a group of tools and to check the validity of accounted parameters of generation within tool manufacture). This technique is less reliable than the first one and it is appropriate only for mass production (with use of the control-measuring machine).

Drawbacks of the first type of descriptor are as follows: it is hard to provide the establishment of all tasks considered in Sect. 2.3 (especially for hobs); and it is time-consuming to solve optimization tasks here.

Type 2. Computerized descriptor, with a package of software programs as its basis, intended for establishment of all the tasks described in the previous section. Such a descriptor consists of the program package and documentation of its application.

Type 3. Combined descriptor, including:

1. Extractions from the paper descriptor—nomograms for establishing the simple tasks, with instructions and examples of their application.
2. Package of software programs for establishing complex optimization tasks, with documentation of its application.

We suppose that the main type of geometrical descriptor should be the third type —the combined type.

2.5 Tools and Gears for the Priority Certification

The most common gears in machines are the spur and the helical. That is why the system of geometric descriptors should first of all be developed for tools that machine spur and helical gearwheels: worm and disk-type hobs, shaping cutters (see Fig. 1), grinding wheels, and others. It is reasonable to choose the following sequence of the development process:

The first descriptor is for wheels for profile grinding. They are used to machine, in particular, precise involute gearwheels of high-speed and heavy-loaded gear drives, and that is why it is necessary to consider even the very smallest organic errors of generation. And since it is also the easiest geometry tool, its descriptor will be simpler than the others, including the software part. It will allow for gaining experience in the development and implementation of geometric descriptors and the creation of a computer-aided system of certification.

The second descriptor is for disk-type module and shape cutters. They will help in mastering the technique of establishing the tasks that are common to all edge-type tools: geometry of cutting edges, variation after re-sharpening, and others.

The third descriptor is for hobs—the basic tool for cutting gearwheels. As was mentioned above, they are the geometric descriptors with the most complicated development process. And several types are necessary: for machining both the gearwheels of worm-type gears and spur and helical gearwheels, involute and non-involute ones.

The fourth descriptor is for shaping cutters and generating cutters.

The consequent descriptors for other tools: module and shape end-milling cutters, milling cutters for spline grooves, honing tools, etc.

2.6 Contents of Works on Development of a System of Geometric Descriptors for Tools and Gears

Let us describe the list and sequence of steps necessary to be taken when developing such a system:

1. Systematization of types and a structure of geometric descriptors and tasks established by their means.
2. Development of the list, structure and type of diagrams and nomograms necessary to be included in geometric descriptors of tools. These graphic images are individualized for each specific tool. Their list and structure are different for various types of tool.
3. Development of mathematical models and algorithms:

   - To analyze the geometry of front and back surfaces of edge-type tools (including random and deliberate deviations in position, motion and shape of grinding wheels for relieving and sharpening).
   - To analyze the geometry of tooth surfaces generated by cutting edges which are lines of intersection of complex front and back surfaces (including intended or undeliberate deviations in position and motion of the tool with respect to the blank).
   - To process data of measurement of gearwheels and tools at control-measuring machines (for individual and batches of gearwheels or tools).
   - To develop software programs for creation of nomograms and texts of descriptors.
   - To develop software programs which are the components of the descriptor.

4. Development of software packages for solving the tasks described above.
5. Investigation of geometry of tools and gears with the help of developed software in order to determine the properties and main relations specific to these types of tool and gear.
6. Sample certification of tools and gears (based on the two previous points).
7. Refinement of a system of certification and production of gearwheels with the assigned modification of tooth flanks obtained through a special sharpening of available tools and preliminary deviations in position and motion of the tool with respect to the blank.
8. Refinement of a system of certification of gears in order to control the position and dimensions of the contact pattern when producing and repairing gears.

2.7 Theoretical Studies and Computer-Aided Design Allowing for Creation of Geometric Descriptors

The theoretical foundation of the development of geometric descriptors is the classic theory of gearing that had been created by the beginning of 1970s, and its further development as the theory of real gearing. The most valuable publications by scientists of this period that can be helpful in the development of geometric descriptors are:

Shishkov [20]—the concept of "penetration velocity"; Litvin [14, 15]—kinematic and matrix methods for analysis of meshing; Zalgaller [24]—analysis of the classical theory of envelopes; Sheveleva [19]—the concept of "wrapping" and techniques of contact stress analysis; Segal [18]—the effective method of contact localization; Korostelev [12]—kinematic parameters of load-carrying capacity of spatial gearing; Lagutin and Sandler [13, 16, 17]—the concept of "meshing space" and generation for the cases of grinding the tools surfaces and production of worm gears; Goldfarb, Trubachev et al. [11, 22, 23]—numerous investigations of worm-type gears and new types of tools for cutting of gearwheels and worms.

The main results of theoretical investigations by the author related to this theme have been stated in PhD and DSc theses [3, 6] and published in works [1, 2, 4, 5, 7–10, 21]. These studies are focused on development of kinematic methods of the theory of gearing and theoretical basics of *computer-aided simulation* of generating the surfaces of gears and tools. In [6], the methodology of the numerical analysis of gearing is developed, which does not require analytic investigation of geometry and kinematics of specific gears and machine meshing. In [8], the computer-aided implementation of this methodology is described. One of the elements of this ideology is application of generalized systems of coordinates and motions to which any active and machine meshing are reduced. In [3, 4], fundamentals of an alternative theory of generation are developed and stated based on new geometric and kinematic images and concepts: a fan, a wedge and a bunch of normal lines in jogs of profiles and surfaces [2]; acceleration of implementation [1].

An integral component of the alternative theory of generation is the application of consequent and multiple-parameter envelopes [2] and discrete and continuous curvilinear coordinates, including natural [3] and regulated with controllable placement of points along profiles [3, 21]. A distinctive feature of the developed theory of generation is that not only is the separate most important segment of a tooth considered, but so is the whole profile or surface of the tooth (or root or even the whole gear rim) [3, 7]. The profound technique for the description of any edge-type tool (shaping and various milling cutters) is developed, enabling the finding of both the generated surface and the dynamic angles of cutting. The initial tool surface will have continuous smooth curvilinear coordinates in this case [7].

Software programs developed on the basis of theoretical investigations and used when creating the geometric descriptors. Let us consider the most acclaimed programs developed by Taisin and Babichev within the project [21]:

1. *Subprogram* ***Profil*** **is** intended to work with plane lines and profiles. It allows for composing complex profiles (including those containing jogs) out of segments of typical lines in an interactive mode. Visualization of the created profile is made during this composition (see Fig. 4).

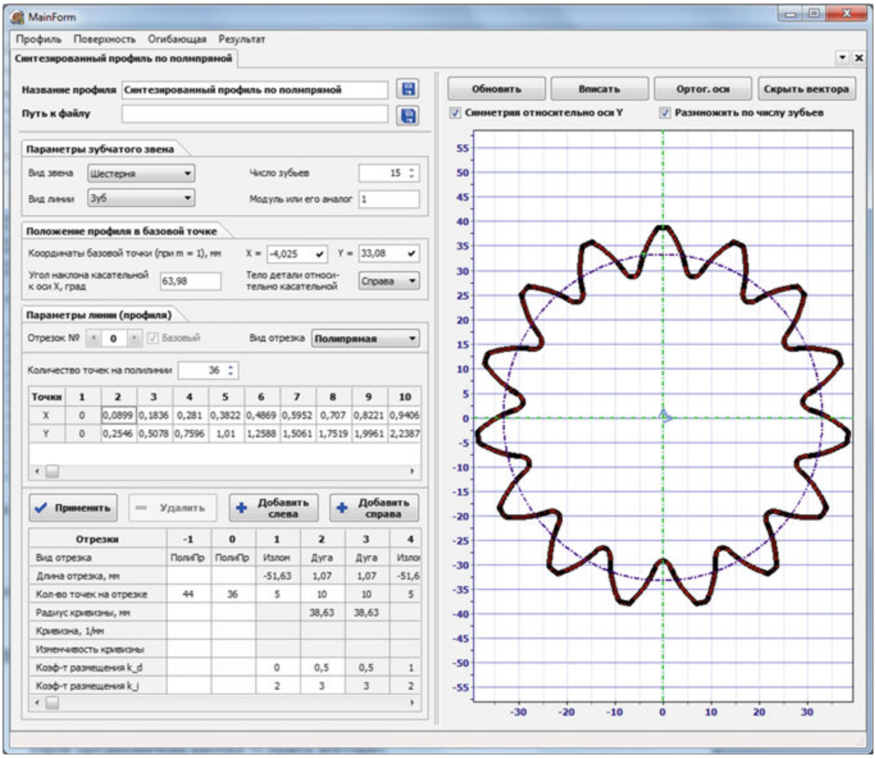

Fig. 4 The main window of the subprogram *Profil*

The profile can consist of a number of various segments (straight line, arc, involute, jog, poly-straight line, poly-arc). The subprogram ***Profil*** contains procedures and functions for determining:

- Cartesian coordinates of the point assigned by a curvilinear coordinate unique to the whole profile;
- Projections of the unit normal vector and unit vector of the tangent to the line at this point, including *the jogs of the profile* (!) where fans of normal and tangent lines are located;
- Curvature and radius of curvature at the given point.

The most attractive feature of the program is that it allows a user to control the location of points on profiles (their number and non-uniformity of location for each typical segment, including jogs).

2. *Subprogram* ***EasySurf*** assigns the areas of surfaces generated by motion of plane and space segments of lines. In the general case, the motion of the generating segment can be helical with constant or variable pitch (its special cases are linear and rotary types of motion). The generating segment of the line itself does not change its dimensions and shapes in motion.
3. *The main program* ***GrindTool*** determines the profile of the grinding wheel used to form the final designed spur or helical gearwheel, cylindrical worm, helical groove of the drill or other helical surfaces with a constant pitch. Such a surface is called a helicoid in mathematics. It is often encountered in machine parts and tools. The surface of the grinding wheel determined here can also be the initial tool surface (ITS) of form milling cutters and end-mill gear cutters for machining the pointed parts. Cutting edges of milling cutters should be located on this ITS.
4. *The main program* ***GrindZub*** analyzes the geometry within gear grinding. The main established task is to determine the surface generated by a grinding wheel. The question arises here—what is the purpose of determining the surface for which the profile of the grinding wheel has already been designed? It is evident that this exact known surface of the part (the synthesized gearwheel) will be obtained according to the designed tool (grinding wheel). This statement is valid only in the case when the grinding wheel is mounted (and will be transmitting) with respect to the part in a strictly determined way. Due to errors of mounting and motion and due to wheel dressing, deviations in the surface generated by grinding from the required designed surface occur. And though these deviations are small, they should be considered when manufacturing precise gears.

In order to develop a geometric descriptor for a specific tool or a gear according to the steps and programs described above, it is necessary to perform a thorough preliminary investigation of the appropriate type of machine-tool or operating gearing. An example of part of such an investigation prior to development of descriptors for tools with the initial tool surface as a surface of rotation is presented below in Sect. 3.

Prior to proceeding to that section, let us explain several features of the generation of surfaces by bodies of rotation for purposes of comprehension. Figure 5a shows the convolute worm. It is generated at the thread-turning lathe by a cutter with rectilinear cutting edges: two lateral and one vertex. These three edges belong to one plane (on the front surface, see Fig. 2) and generate two jogs (two singular points on the continuous profile of the cutter) in areas where they meet. The front surface (a plane) passes through the straight line intersecting the worm axis at the right angle. There is the "model" of the cutter (three rectilinear segments with two jogs between them) at the root of the worm profile in the normal cross-section. The surface of space is generated by a helical motion of the cutter profile with respect to the worm solid (see line ***I*** in Fig. 5a).

If the same convolute worm is generated not by the cutter but by the tool with the initial tool surface as the body of rotation (for instance, to grind all the space surfaces of such a worm), then it turns out that:

(1) contact of the grinding wheel surface with flanks of the worm space will be along spatial curves (see line ***II*** (ABC) in Fig. 5a);
(2) the surface of the grinding wheel will not be conical (as a consequence of the previous statement);
(3) jogs as screw lines on the worm surface can be obtained by the grinding wheel only for a specially selected cross angle during grinding (at other values of the cross angle, the transient surface will appear on the worm (see line 6–6 generated by the point of the jog 6 in Fig. 5b).

An important (and upsetting) circumstance is that the shape of the contact line ***II*** in Fig. 5a and the profile of the grinding wheel depend on the diameter of the grinding wheel and cross angle during grinding.

Another upsetting feature of the generation process is demonstrated in Fig. 5b. It shows that the continuous profile (or the surface) generates, as a rule, a continuous

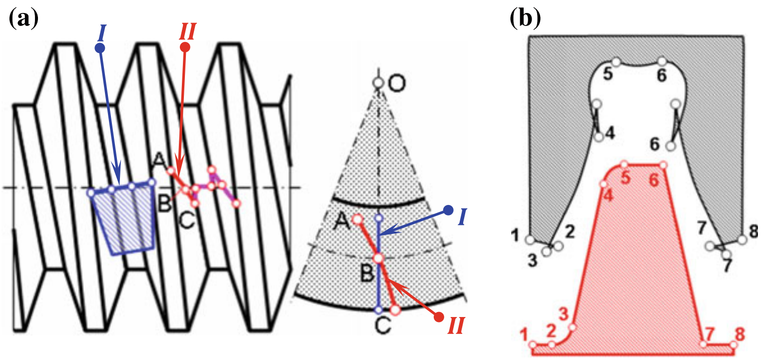

Fig. 5 Problems of the generating process: **a** *I* generation by a cutter: plane line of contact of the tool and a part; *II* generation by a grinding wheel: complex spatial lines of contact of the initial tool surface and the part; **b** undercut and looping of the conjugated profile

conjugated profile (or surface) of the part root (see this in more detail in [4]). But this continuous conjugated profile can be self-intersecting, that is, looping. It is the reason begind undercut (sloping “fish tails”; see lines 4–5 and 6–6 in Fig. 5b) and the impossibility of gear machining for a section of the assigned profile (abrupt “fish tails”; see lines 2–3 and 7–7 in Fig. 5b).

All upsetting features of generation processes should be revealed and rated quantitatively during investigation prior to the development of descriptors and should be considered during said development.

3 Investigations Preceding Development of Geometric Descriptors of Tools (by Example of the Descriptor of a Tool for Profile Gear Grinding)

3.1 Choice of a Gear Pair for Profile Gear Grinding

The object of profile grinding became a pinion of one of uniform-strength gears synthesized by us [9]. Uniform strength means the planar gearing (including spur gears) where Hertz contact stresses along the line of action are constant, that is, σ_H = const. Synthesis of such gears is reduced to establishment of a variation task [9]: it is necessary to determine a function $f(x,y)$ (line of action) and conjugated profiles $\mathbf{r}_1 = \mathbf{r}_1(u_1)$ and $\mathbf{r}_2 = \mathbf{r}_2(u_2)$ for which the chosen quality parameter (the contact stress σ_H) is equal to the assigned value at all points of the given line of action.

Figure 6 shows one such synthesized gear. Its parameters are: power at the pinion shaft P_1 = 100 kW; pinion speed n_1 = 1500 rpm; gear ratio u = 2; center distance a_w = 150 mm; face width b_2 = 36 mm; pinion tooth number z_1 = 15;

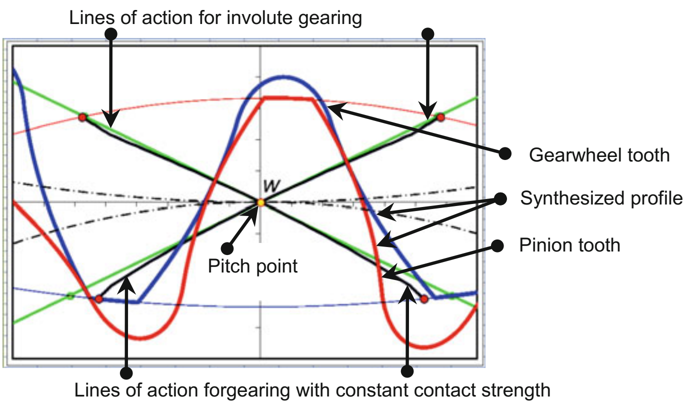

Fig. 6 Uniform-strength gear (contact stresses σ_H = const)

pressure angle at the pitch point $\alpha_w = 25.562°$; contact stresses on the main part of the line of action $\sigma_H = 1000$ MPa, and at the beginning and at the end of the line of action $\sigma_H = 700$ MPa (assuming that one tooth pair is in the contact); addendum factors for gearwheels: $h^*_{a1} = 1$ and $h^*_{a2} = 1$; module at pitch circles $m_w = 6.667$ mm; tooth thicknesses at addendum circles: $S_{a1} = S_{a2} = 3.106$ mm; contact ratio $\varepsilon = 1.327$.

For comparison, Fig. 6 also shows lines of action for an involute gear with the same pressure angle at the pitch point W $\alpha_w = 25.562°$ and the same contact stresses $\sigma_H = 1000$ MPa. According to Fig. 6, the lines of action for a synthesized gear slightly differ from the straight lines that are the lines of action in the involute analog. That is why pressure angles are close to their corresponding values for an involute gear. The tooth shape is also close to the involute one—according to analysis, deviation of profiles from involutes for a synthesized gear is less than 0.1 mm at addendums and up to 0.3 mm at dedendums as compared to the involute analog.

Figure 7 shows diagrams of quality parameters of the synthesized uniform-strength gear as compared to parameters of the involute analog. The most important quality parameter is Hertz contact stress σ_H. The values of σ_H are the same at the pitch points of both gears. Beyond the pitch point, the value σ_{Hmax} for the involute analog is greater than for the uniform-strength gear in all meshing phases.

At the beginning of tooth contact, the value σ_{Hmax} is 2.1 times greater; at the zone of a single-pair meshing, it will be greater by 10%; at the instant of the tooth coming out of the meshing, the value σ_H is greater by 30% (see Fig. 7e). The reason lays in the character of variation of profile curvature radii R along the line of action (Fig. 7 a–c): at the pinion root, the value of R is from 23 to 27 mm; at the gearwheel tooth root, there is even a small concave segment.

The value of the factor υ_{1max} of the specific sliding at the pinion tooth root in the uniform strength gear is 2.3–3.0 times less than in the involute analog (υ characterizes the intensity of an abrasive tooth wear).

The pinion of the equal strength gear shown in Fig. 6 is also the object of profile grinding, because it is better to carry out refinement of techniques, algorithms and programs for the development of geometric descriptors as applied to complex profiles consisting of diverse curves and determined by means of program software.

Results of the pinion synthesis shown in Fig. 6 are taken as the initial data for the subprogram **Profil**. After processing, they are transmitted to the main programs ***GrindTool*** and ***GrindZub***. They are the arrays: of coordinates of points of the synthesized profile; of projections of the tangent and normal lines and radii of curvature to the profile at all determined points.

3.2 Investigation of the Machine-Tool Gearing with the Initial Tool Surface as the Surface of Rotation

This section describes the results of investigating the generation process for the pinion dedendum with parameters determined in the Sect. 3.1. At the investigation,

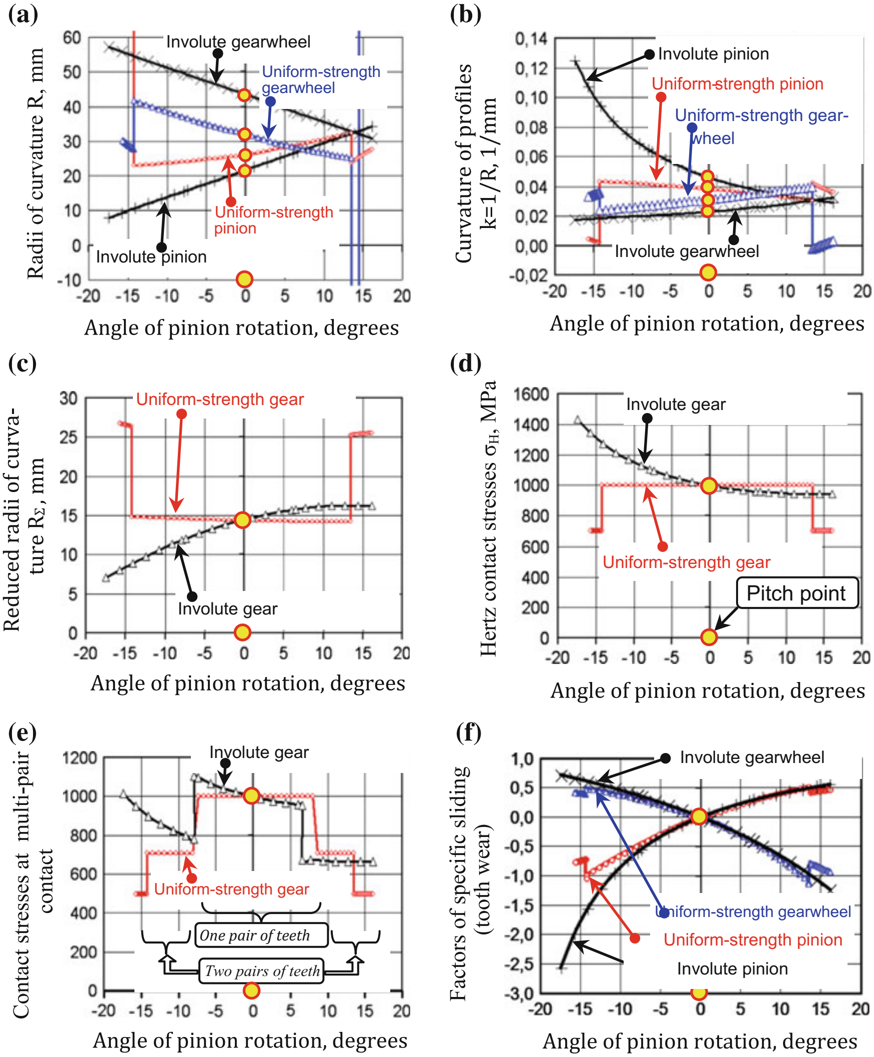

Fig. 7 Comparison of quality parameters along the whole meshing line for involute and uniform-strength gears: **a** curvature radii of tooth profiles for a pinion and gearwheel at contact points; **b** curvatures of tooth profiles; **c** reduced radii of curvature at points of tooth contact; **d** Hertz contact stresses for a single-pair tooth contact; **e** Hertz contact stresses for a single- and double-pair tooth contact; **f** specific sliding of teeth

a helix angle on pitch diameter was equal $\beta_w = 20°$ and cross angle in machine meshing $\gamma = 110°$. This investigation was carried out with the ***GrindTool*** and ***GrindZub*** programs. Figures 8, 9, 10, 11 and 12 present a selection of diagrams—results of operation of the program ***GrindTool***. Figure 8 shows the difference between normal sections of dedendums of the helicoid for the same face

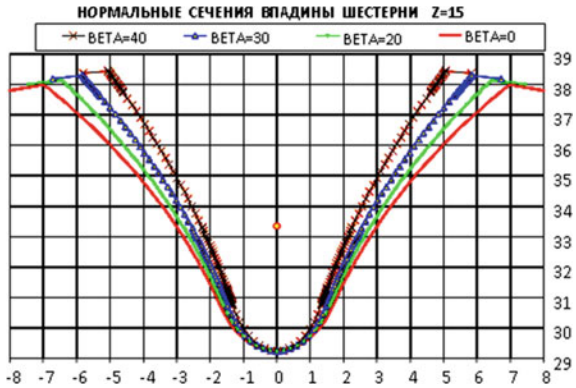

Fig. 8 Normal cross-sections of the helicoid at different helix angles of the tooth line β_w

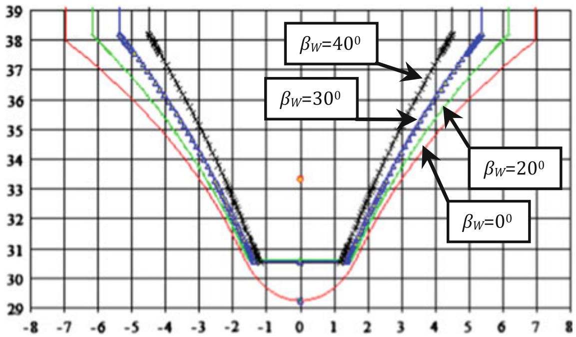

Fig. 9 Profiles of grinding wheels at different helix angles of the tooth line β_w

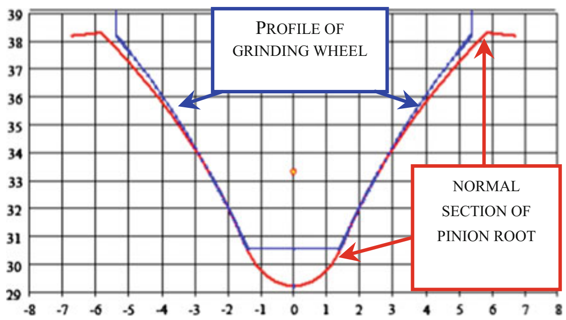

Fig. 10 Profile of normal section of the pinion with $\beta_w = 30°$

cross-section at different tooth helix angles. Figure 9 shows the difference in profiles of grinding wheels intended to machine flanks of helicoids with the same face section but with different helix angles. Note that the profile of the grinding wheel is identical to the root profile only for the angle $\beta_w = 0$. Figure 10 shows the

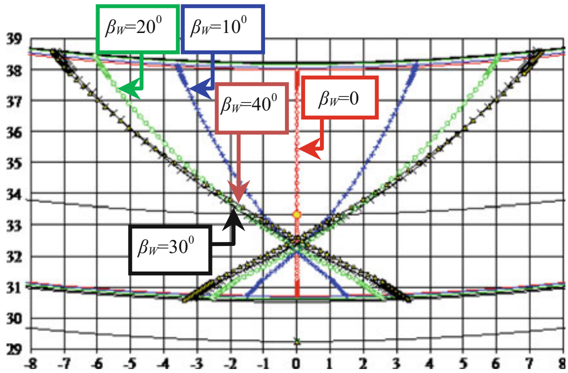

Fig. 11 Location of contact lines on the grinding wheel at different β_w

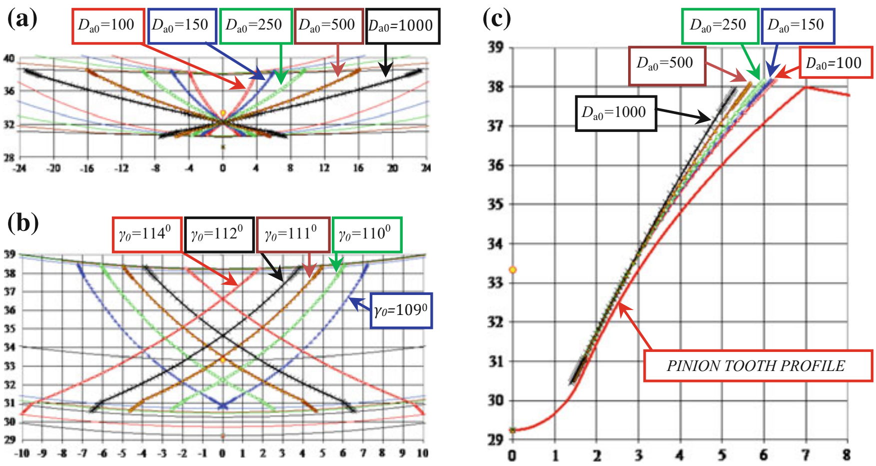

Fig. 12 Influence of the grinding wheel outer diameter D_{a0} and the cross angle γ_0 in gear machining on location of contact lines and wheel profile: **a** influence of D_{a0} on contact lines; **b** influence of γ_0 on contact lines; **c** influence of D_{a0} on the profile of the grinding wheel

difference between the *profile of the grinding wheel and profile of the normal cross-section* $\beta_w = 30°$. The greater the value β_w, the greater this difference is. Figure 11 shows that for the same face cross-sections of helicoids, the lines of contact of surfaces of grinding wheels with surfaces of helicoids are different at various inclination angles of the tooth line. Note that only for the angle $\beta_w = 0$ (spur gear) is the line of contact of tool surface and the machined part the plane line—the axial cross-section of the grinding wheel surface (see the red line for $\beta_{\mathrm{w}} = 0$). In all other cases, the lines of contact (they are also called the characteristic lines) are spatial curves. And the greater the helix angle β_w, the farther this line goes from the center distance.

Figure 12a shows the influence of the grinding wheel outer diameter D_{a0} on the line of contact (characteristic). It turned out that this influence is very significant. Thus, for a small diameter D_{a0} = 100 mm, approximately 8 mm of the tooth line are in machining: 6 mm from the right and 6 mm from the left side of the tooth. And for a large diameter D_{a0} = 1000 mm, approximately 46 mm of the tooth line are in machining: 30 mm for each side of the tooth. Therefore, the greater the diameter of the wheel, the larger the minimum tool overrun required when machining a helical gearwheel: for D_{a0} = 100 mm, it is equal to ΔS_Z = 6 mm, for D_{a0} = 1000 mm, it is ΔS_Z = 30 mm. This means that movement of the tool along the axis of the machined gearwheel should be greater than $(b + 2 \cdot \Delta S_Z)$, where b is the face width.

If D_{a0} is changed, the tool (in this case, the grinding wheel) profile should also be changed: Fig. 12c shows the dependence of the grinding wheel profile on its diameter D_{a0}. At the right, the large-scale face profile of the machined helical pinion is shown by the red solid line. At the left, there are five profiles of grinding wheels for five different outer diameters D_{a0}. It is clear that segments of a tool machining the dedendum differ insignificantly. However, segments machining the addendum are significantly different: when D_{a0} is changed two times, variation of the profile reaches 0.2–0.3 mm at the segment machining the points at the addendum. That is, organic errors in profiling (not considering the diameter of the grinding wheel) can become 100 times greater than the required accuracy of tooth profile manufacturing.

Note that all diagrams in Figs. 8, 9, 10, 11, and 12a, c are plotted for machining at the cross angle γ equal to $\gamma = (90° + \beta_w)$. That is, it was accepted that the tool is planned according to its design position with respect to the blank for which the tool inclination angle $(90° - \gamma)$ is equal to the helix angle of the tooth (β_w) *on the pitch diameter* (d_w). But according to the theory of tool design, it is known that the other angle of tool inclination, with respect to the blank, can be used; let us designate it γ_0. Figure 12b shows the influence of this cross angle γ_0 on the location of the contact lines during grinding of pinion teeth.

The general conclusion from Figs. 8, 9, 10, 11 and 12: for the considered profile of a gear dedendum, there is a rather wide range of variation in parameters for the tool (D_{a0}), the gearwheel (β_w) and their mutual arrangement (γ_0). When machining typical cylindrical gears and worms by profile grinding, there are no difficulties associated with undercut, looping or self-intersecting of the required tool profile. That is why there is a rather wide range of variation in tool parameters and tool position for achieving the improvement of quality parameters of gear machining and their optimization.

Examples of operation of the program ***GrindZub*** are not given here, since they differ slightly from the examples given in Figs. 8, 9, 10, 11 and 12.

3.3 What a Paper Descriptor Should Represent and How It Should Be Developed on the Basis of Performed Study for Specific Types of Machine-Tool and Active Meshing

We suppose that a paper geometrical descriptor for a specific meshing (machine-tool or active) will include several sheets. The structure of all sheets is similar (Fig. 13): there are two groups of scales along the edges (input and output for the established tasks) and a group of nomograms (the basis of geometrical descriptors) is located in the middle. There are also two types of scale on each nomogram: input and output. Scales of variable parameters and quality parameters can be multilevel, for example: dimensions are pointed in millimeters and inches (linear-dependent scales); or backlashes in a gear can be measured in normal and circular directions (scales with non-linear dependence). Scales of consequently applied nomograms indicate the same parameter (in Fig. 13, this relation is symbolized by dashed lines). It is desirable to make colored lines of relations and fills and to relate these colors logically with tasks to be established and types of variable parameter and quality parameter.

Each of the sheets of any descriptor is intended to establish several similar tasks (both direct and reverse) by means of available nomograms. When establishing the direct task, the input data are assigned at left scales (Fig. 13) and the result is obtained at right scales. When establishing the reverse task, the method is opposite: the input data is on the right scales and the results are on the lef. In order to simplify the work with descriptors, "trajectories of motion" are plotted on nomograms between the right and left scales for each task to be established. A part of sheets for certain tools can be similar. For example, one-two sheets of the geometrical descriptor of a disk-type cutter can be identical to one-two sheets of the descriptor of a grinding wheel for profile grinding. They are, in particular, the sheets with nomograms relating the deviations in the generated surface with parameters of

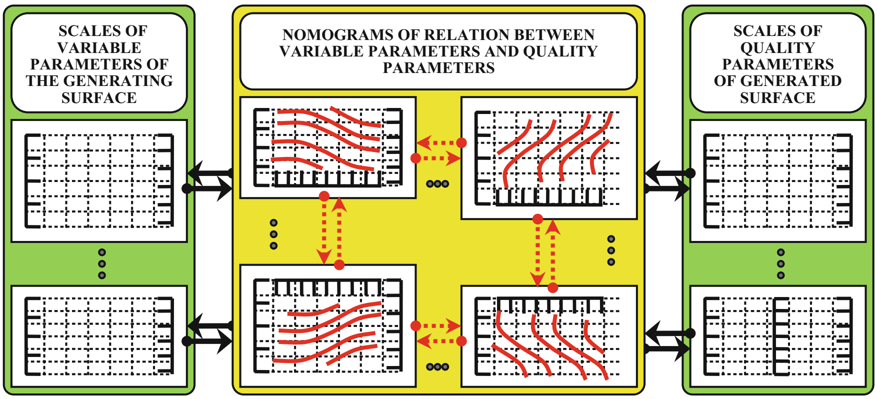

Fig. 13 Basic components of one sheet of the paper geometrical descriptor

mutual arrangement of axes of the tool and machined gearwheel. It is valid, for example, for establishing the task of determining the tool displacement after its re-sharpening (dressing), providing the required backlash in the gear.

The foundations of the development of nomograms (i.e., the main components of geometrical descriptors) are various arrays of results of thorough investigation of a specific gearing. It is reasonable to carry out the development of nomograms by means of software programs created for this purpose. It is similar to the method of development of *dynamic* blocking contours [10].

4 Conclusions

1. In order to manufacture a quality gear, one should take into account both the geometrical features of tools and the details of appearance of organic errors of surface formation by generation methods. This is impossible to do without computer-aided analysis. One easy-to-use representation of this analysis can be geometric descriptors of gears and gear-machining tools.
2. The paper described the idea of development of these descriptors, procedure and contents of the project to create them. The geometrical descriptor, in particular, will assist a designer in assigning tolerances for a gear, including rules of tooth contact, and a production engineer in determining parameters of tool shape and position providing the required accuracy of the product and obtaining the assigned bearing contact.
3. Three types of geometrical descriptors have been proposed: paper, computerized and combined; tasks established by their means have been considered. Combined descriptors are recommended as basic ones for complex edge-type tools.
4. The structure of paper geometrical descriptors and methodology of their creation are developed:

    - the basis of a geometrical descriptor is a nomogram, which demonstrates the relations and numerical dependencies between input and output parameters of the task to be established;
    - such a descriptor consists of several pages with nomograms for establishing one or two simple tasks;
    - complex tasks are established by specially developed computer-aided programs for combined descriptors;
    - paper geometrical descriptors for specific tools are generated by a special program software—the main (and the most expensive) element of the proposed system of certification;
    - the theoretical basic for creating a special program software is a number of kinematic methods of the theory of gearing focused on computer-aided methods of analysis of generation processes.

5. The paper presented: the performed theoretical studies and developed computer-aided programs intended for creation of geometrical descriptors; and the results of computer-aided simulation carried out by means of these programs for generation of helical surfaces by solids of revolution, the simulation being the basis for the creation of geometric descriptors of disk-type cutters and wheels for profile grinding.
6. Information presented in this paper is the result of the first (initial) stage of works on this project. The next planned step will involve the following tasks:
   - to finalize the list, structure and types of diagrams, schemes and nomograms to be included for geometric descriptors;
   - to finalize the software for the generation of diagrams and nomograms for the descriptors of wheels for profile gear grinding;
   - to create some sheets of the first version of the descriptor of grinding wheels.

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