Veto in Yes-no and Yes-no-Abstain Voting Systems

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Abstract The paper presents a transformation from simple "yes-no" cooperative games to simple cooperative games where players have more than two actions available to them by introducing abstentions into a yes-no voting system. The results obtained up to now are rather pessimistic (Felsenthal and Machover, Power, voting, and voting power: 30 years after, Part II (2013) [6], even call them "the curious case of the absent abstention"). We discuss in this paper the relation between the right of veto, weights of the players and quotas. Our results clarify some general properties and enable an a priori analysis to gain a better understanding of the decision-making mechanism of such decisive bodies. Examples of the United Nations Security Council and Polish president-parliament cohabitation are used to illustrate our discussion.

1 Introduction

The legislative process in many countries provides for the possibility of the so-called veto, whose task is to strengthen the quality of legislation through the elimination of possible mistakes or imperfections, and on the other hand, in situations of considerable controversy, through strengthening the conditions of its acceptance (rejection of the veto, if permitted, usually requires a larger threshold of votes "for"). For example, the need to break (if permissible within the legal system) the veto results in the law not being passed by a simple majority, but by a qualified majority of legislators. This is the case in Poland, where the passing of a law by the Sejm usually requires a majority of over 50%, while overcoming the veto of the president in the Sejm requires 3/5 of votes "for". In the US, unlike in Poland, overcoming of the veto requires 2/3 of the votes "for" of both chambers.

There appears to be a side effect (so to speak) of granting the right of veto to one instance of the legislative process: its strengthening by increasing the influence of

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this instance on the decision-making process, even when the veto is not actually used. We are considering here the decision-making process in general and not the law-making process in the parliament, albeit veto is universally associated with the so-called presidential veto. However, there are also some analogies in other decision-making processes. Examples include joint stock companies, where the holder of the so-called "golden share" can also stop the whole decision-making process, thus applying the veto in practice. In this case, the veto can not be rejected. It can be thus concluded that there are two types of veto: such that can not be rejected (i.e. veto of the first type) and such that can be rejected (i.e. veto of the second type).

One purpose of this article is to attempt to estimate the "measure" of veto for increasing the impact of the decision maker on the decision-making process. The measurement of the so-called power of veto will allow the estimation of the part of it that is not associated with correcting of legislation (decisions being taken), but is perhaps its unintended "side effect". The so-called power index is a tool to measure the power, including the power of veto. Classic power indices (Shapley-Shubik, Banzhaf and Johnston, to mention the most popular indices) are often modified, in the opinion of the authors, to better capture the feature. For example, one may observe the introduction of many modifications of the classical Shapley-Shubik power index (see, for example, O'Neil [18], Napel and Widgren [17] or Kuziemko and Werker [10]). In our paper, we do not assess which of these modifications are the best. It seems that the study of the effect of veto on the decision-making process needs no such modification.

From a theoretical point of view introduction of veto is a transformation from simple "yes-no" cooperative games to simple cooperative games where players have more than two actions available to them. Felsenthal and Machower [5] introduced the term "tertiary games", introducing abstentions into a yes-no voting system, which for example suits the case of the Security Council. Hence, they generalised the problem, which led to various attempts to better evaluate the role and power of members which are empowered with veto and doesn't empowered with veto in the same decisive body. For example, Tchantchoa et al. [22] analyse satisfaction, Freixas and Zwicker [7] multiple levels of approval and Grabisch and Lange [8] multichoice games; all for yes-no voting with abstention. The results obtained up to now are rather pessimistic: Felsenthal and Machover [6] even call them "the curious case of the absent abstention". Following this stream, we discuss in this paper the relation between the right of veto, weights of the players and quotas. Our results clarify some general properties and enable an a priori analysis to gain a better understanding of the decision-making mechanism of such decisive bodies.

The article is set up as follows. After introduction, the next section outlines the way in which decisions are modelled. This section presents preliminaries connected with the game-theoretical language of modelling and ways of calculating power indices for a simple voting game and how to take vetoes into account in these calculations. The next section presents the calculation of different power indices for games in which voters have equal voting rights, but some have an unconditional veto (voting yes/no). This section describes a procedure to define an equivalent voting game without vetoes in which players have different weights. After that, the next section presents the calculation of the Shapley-Shubik power index in games where voters have equal voting rights and there is yes-no-abstain voting. The last but one section presents the derivation of equivalent games based on a transformation of weights, quotas and the type of voting. Some general evaluations of the power of voting and non-voting players are presented together with examples of the United Nations Security Council and president-parliament system. Finally, there are some conclusions and suggestions for future research.

2 Preliminaries

Let N be a finite set of committee members, q be a quota and w_j be the voting weight of member j, where $j \in N$.

In this paper, we consider a special class of cooperative games called weighted majority games. A weighted majority game G is defined by a quota q and a sequence of nonnegative numbers w_i, $i \in$ n, where we may think of w_i as the number of votes, or weight, of player i and q as the threshold, or quota, needed for a coalition to win. We assume that q and w_j are nonnegative integers. A subset of the players is called a coalition.

A game on N is given by a map v: $2^N \rightarrow$ R with $v(\varnothing) = 0$. The space of all games on N is denoted by G. A coalition T $\in 2^N$ is called a carrier of v if $v(S) = v(S \cap T)$ for any $S \in 2^N$. The domain $SG \subset G$ of simple games on N consists of all $v \in G$ such that

(i) $v(S) \in \{0, 1\}$ for all $S \in 2^N$;
(ii) $v(N) = 1$;
(iii) of v is monotonic, i.e. if $S \subset T$ *then* $v(S) \leq v(T)$.

A coalition S is said to be winning in $v \in SG$ if $v(S) = 1$ and losing otherwise. Therefore, passing a bill, for example, is equivalent to forming a winning coalition consisting of voters. A simple game (N, v) is said to be proper, if and only if the following is satisfied: for all T⊂N, if $v(T) = 1$ *then* $v(N \backslash T) = 0$.

We only analyse simple and proper games where players may vote either yes-no or yes-no-abstain, respectively.

If a given committee member can transform any winning coalition into a non-winning one by using a veto, then that veto is said to be of first degree.

If the veto of a given committee member turns some, but not all, winning coalitions not including that member into non-winning coalitions, then that veto is defined to be of second degree.

3 Measurement of the Power of Decision-Maker

It is generally accepted that measurement of the power of the decision maker, understood as his effect on the final result, is accomplished by means of the so-called power indices. They originate from simple game theory, where they were originally used to determine the division of the so-called payoff, or the value of the game.

We consider the weighted decision-making body of the size n = card{N} in which decisions are taken by vote, with a quorum γ, the sum of weights τ and allocation of weights $\omega = (\omega_1, \ldots, \omega_n)$. We assume that each i-th voter only votes "yes" or "no". Any non-empty subset of players $S \subseteq N$ we call voter setup. For a given allocation ω and quorum γ, we say that $S \subseteq N$ is a winning configuration of voters if $\sum_{i \in S} \omega_i \geq \gamma$ or a losing one if $\sum_{i \in S} \omega_i < \gamma$. let

$$T = \left[(\gamma, \boldsymbol{\omega}) \in \mathrm{R}_{\mathrm{n}+1} : \sum_{\mathrm{i}=1}^{\mathrm{n}} \omega_{\mathrm{i}} = \tau, \ \omega_{\mathrm{i}} \geq 0, \ 0 \leq \gamma \leq \tau \right],$$

denote a set of all the coalitions size n with the sum of weights τ and quorum γ.

Most measures of power (referred to as the power indices) are used to measure the so-called a priori power of players constituting a set that is structured only in terms of voting rule. The a priori power index is therefore a vector function $\Pi: \mathrm{T} \rightarrow R_n^+$ mapping the T set of all the n size coalitions in a non-negative set of real numbers R^n. The power index, therefore, describes the expectations of the player associated with decisiveness, in the sense that his voice will affect the final outcome of the vote. There are two possible positions of such a player: pivotal position and the "swing" position.

Let $(i_1, i_2, \ldots, i_n)$ denote a permutation of players from a coalition of n players, and let the player k take the position r in this permutation, i.e. $k = i_r$. We say that if k is a player in a pivotal position with respect to the permutation $(i_1, i_2, \ldots, i_n)$, if

$$\sum_{j-1}^{r} \omega_{i_j} \geq \gamma \quad and \quad \sum_{j-1}^{r} \omega_{i_j} - \omega_{i_r} < \gamma.$$

The most common a priori index, i.e. the Shapley-Shubik index [20, 21], denoted as SS-index, is defined as the following expression:

$$SS(\gamma, \boldsymbol{\omega}) = \frac{p_i}{n!},$$

where:

pi is the count of cases in which the player i is decisive and,

n! is the number of possible different powerful orders for the permutation.

For the “swing” position the most commonly used a priori power index is the Penrose- Banzhaf index [2, 19]. Let S be a winning configuration in the coalition $[\gamma, \omega]$, and ϵ S. Being in the swing position in the configuration S means that the following inequalities are met:

$$\sum_{k\varepsilon S} \omega_k \geq \gamma \quad oraz \quad \sum_{k\varepsilon S\setminus\{i\}} \omega_k < \gamma.$$

Let s_i denote the number of changes (swings) of the member in total, and in the committee $[\gamma, \omega]$. The Penrose-Banzhaf index for a member of the coalition is defined as follows:

$$PB_i(\gamma, \boldsymbol{\omega}) = \frac{s_i}{\sum_{k \in N} s_k}.$$

Both indices can be used to measure the power of veto, although the Penrose-Banzhaf index, it seems, is better suited for this purpose as, assuming that the coalition is formed and one of its members betrays it (swings), such behaviour is, in a way, a use of the veto. Thus we think that the “swing” should be used in the measurement of the a priori power of veto. The problem is that the Penrose-Banzhaf index is not the only one based on such concept of the position of the decision maker. In addition, we know that the decision-making process in this case is sequential in nature, and only the end result is (or rather may be) vetoed. The use of other indices should also be considered: Coleman index [4] of the coalition maintenance; Coleman index of the power to initiate action; Coleman index of the power of the collectivity to act, Rae index; Zipke index; Brahms-Lake’s index; Deegan-Packel index; Holler index; or Johnston index. The latter index is a “swing” type index and according to Brahms [3] or Lorenzo-Freire et al. [11] it best describes the decision-making process, in which a veto can occur.[1]

The concept of Johnston power index [9] is based on the concept of the so-called vulnerable coalition.

The coalition is vulnerable if it includes at least one member in the “swing” position, and whose defection converts the coalition from the winning of losing. For example, in the Polish Sejm, the President, together with 232 members of parliament creates a vulnerable coalition, in which only he has all the power in the sense of Johnson: a defection by the president (the use of the veto) converts the winning coalition into a losing one (and none of the deputies alone is able to do—there is always the remaining majority of 231 deputies). Note that the president and 231 members of parliament also create a vulnerable coalition, however, the president has to share the power with each of the members: each of them is therefore equally powerful.

[1]Axiomatic characteristics of strength indices in decision-making bodies of the veto can be found in Mercik [13], while dynamic characteristics can be found in Mercik and Ramsey [16].

Table 1 The Johnston power indices for the game [1–4]

Vulnerable coalitions	Number of vulnerable coalitions	Critical defections			Fractional critical defections		
		3 votes player	2 votes player	1 vote player	3 votes player	2 votes player	1 vote player
(3, 2)	1	1	1	0	½	½	0
(3, 1)	1	1	0	1	½	0	½
(3, 2, 1)	1	1	0	0	1	0	0
Total	3	3	1	1	2	½	½
J(i)					4/6	1/6	1/6

Source Mercik [12]

Let us Consider (Table 1) the following example: the game [4; 3, 2, 1], i.e. voting where there are three voters with 3, 2 and 1 votes each. The majority needed for a decision is 4. The following are vulnerable coalitions in this game: (3, 2), (3, 1) and (3, 2, 1) (vulnerable coalitions must be winning coalitions).

Example 1 The value of the Johnston index for the Polish president-parliament system.

Table 2 shows the results of calculations for the Polish Sejm and the president (the Senate is not involved in the veto process—each majority in the Sejm rejects the veto of the Senate).[2]

The actual presidential veto in Poland is the veto of the second type, i.e. it can be rejected by a majority of 3/5 votes of the Sejm. In theoretical considerations, however, we can also assume the existence of a veto of the first kind, i.e. allow the possibility that the veto of the president can not be rejected.[3] Values for the a priori Johnston index for the situations when the president respectively does not have a right to veto, is equipped with a veto of the first type, and is equipped with a veto of the second type are shown in Table 3.

The applied a priori power index is a standardized index and therefore the results of the calculations show that in the Polish parliamentary system the position of the president can count only in the situation when he is equipped in veto of the first kind, i.e. the kind of veto that can not be rejected. Because such situations are difficult to imagine (and would mean virtual dictatorship of the president) it should suffice to compare the a priori power of the president without the right of veto and with the veto of the second kind. Using the results from Table 3 it can be calculated that the veto itself increases the strength of the president by 1,675 times, which is a significant result, although we observe a considerable imbalance of power between the president and Sejm in favour of the latter (Table 2).

[2]Details of the calculations can be found in Mercik [12]. General issues related to the calculation can be found, among other works, in Alonso-Meijide et al. [1].

[3]From the point of view of the winning coalition, this means that the president must be its member.

Table 2 The value of the Johnston power index for the Polish president-parliament system

	Johnston Power index	
	Without the party structure in the Sejm	With the party structure in the Sejm
President	0.9234	0.0067
Sejm	0.0766	0.9933
Senate	0	0

Source Mercik [12]

Table 3 The value of Johnston index for the president equipped with a different kinds of veto

President without the right of veto	President with the veto of the first type	President with the veto of the second type
0.0040	0.1190	0.0067

Source own calculations

Similar calculations can be carried out using other indices of power, although the results seem to be similar. Thus, the right of veto is an important attribute (also with regard to the president in the Polish parliamentary system) and it is possible to "value" it as a priori power of the decision maker.

Example 2 (Mercik and Ramsey [15] A priori power of the UN Security Council and its members (yes-no voting system).

Let us recall that the United Nations Security Council has 5 permanent members with power of veto, 10 non-permanent members without power of veto; in total 15 members (N = 15).[4] The quota for a decision to be passed is q = 9. All members have equal weight ($w_i = 1$, for $i = 1, \ldots, 15$). However, the yes-no voting system is not exactly the type of voting in use in the UN Security Council but yes-no voting is a good starting point for a more general analysis.

One may compare the values of a priori indices of veto and non-veto members of the UN Security Council (yes-no version). One may find the results in Table 4.

As we may see from Table 4, the ratio between the power indices of non-permanent and permanent members of the UN Security Council varies from 1 to 10.10 for the Penrose-Banzhaf power index to 1 to 105.24 for the Shapley-Shubik power index. Actually, in the literature on power indices the absolute Penrose-Banzhaf power index is the most commonly accepted one, so we may conclude that a permanent member of the council is approximately 10 times stronger than a non-permanent one if no member uses his or her right to abstain.

Let us now analyse situation when veto is introduced. In fact, abstention by a non-veto player is identical to voting against an issue. This is not the case for veto-players. In a priori analysis, on one hand all veto-players must be included in any winning coalition to prevent a veto being used. On the other hand, when a veto

[4]UN Security Council voting System [23] http://www.un.org/en/sc/meetings/voting.shtml (taken on 15.03.2017).

Table 4 A comparison of power indices for permanent and non-permanent members for the yes-no voting version of the game based on the UN Security Council from the perspective of a priori power indices. *Source* own calculations

	Relative Penrose-Banzhaf Power Index	Absolute Penrose-Banzhaf Power Index	Shapley-Shubik Power Index	Johnston Power Index
Permanent member (veto member)	0.166929	0.051758	0.196270	0.177987
Non-permanent (non-veto member)	0.016535	0.005127	0.001865	0.011006
Ratio of non-permanent/permanent members	1:10.10	1:10.10	1:105.24	1:16.17

player abstains, then a given coalition must be enlarged by another non-veto player to substitute the veto player who abstains. Let k_v denote the number of veto-players choosing to abstain, $k_v = 0, 1, 2, \ldots, v$ (for example: each permanent member of the UNSC may abstain. In this case $v = 4$). This means that in the a priori analysis the game (N, q, w) should be replaced by the sequence of games $(\mathrm{N}, \mathrm{q} + k_v, \mathrm{w})$ for $k_v = 0, \ldots, v$. It is easy to notice that for $k_v = 0$ this game is in fact the game with yes-no voting only.[5]

Example 3 (Mercik and Ramsey [14, 15] A priori power of the UN Security Council and its members (yes-no-abstain voting system).

First, we calculate the Shapley-Shubik power index directly for a given k_v. A non-permanent member is pivotal only if he or she is the ninth player in the coalition, preceded by all non-abstaining $(5 - k_v)$ permanent members and $(3 + k_v)$ non-permanent members. Note that there are $\binom{9}{3 + k_v}$ ways of choosing the players who appear before player i and 8! ways of ordering these players. The number of players that come after player i equals $(6 - k_v)$, i.e. the non-veto players who did not appear before player i. There are $(6 - k_v)!$ orderings of these players. So, for non-veto player i (i > 5) this happens in $\binom{9}{3 + k_v} 8!(6 - k_v)!$ ways. Ignoring the abstaining permanent members, there are $(15 - k_v)!$ sequential coalitions, so the Shapley-Shubik power index for a non-veto player in this game with given k_v equals $\pi_i^{SS}(N, q, w)(k_v) = \frac{1}{(15 - k_v)!} \binom{9}{3 + k_v} 8!(6 - k_v)!$.

Dividing the rest of the power equally among the $5 - k_v$ non-abstaining permanent members, we obtain a Shapley-Shubik power index for them of

[5]Note that in the UNSC example we may assume that the abstaining permanent members do not take part in the game, thus we do not need to consider the case $k_v = 5$.

Table 5 Shapley-Shubik power index for veto and non-veto members of the UN Security Council as a function of the number of abstentions among permanent members. *Source* own calculations

Number of veto players abstaining, k_v	Shapley-Shubik power index for non-permanent members	Shapley-Shubik power index for non-abstaining permanent members	Ratio of power between non-permanent and permanent members (in %)
0	0.001864802	0.196270396	0.95
1	0.006993007	0.232517483	3.01
2	0.019580420	0.268065268	7.30
3	0.042424242	0.287878788	14.74
4	0.072727273	0.272727273	26.67

$$\pi_i^{SS}(15,9,1)(k_v) = \frac{1 - \frac{10}{(15-k_v)!}\binom{9}{3+k_v}8!(6-k_v)!}{5-k_v} \qquad i \leq 5.$$

The exact values for different values of k_v are presented in Table 5.

The results shown in Table 5 support our intuition connected with the relaxation of the power of veto, i.e. introduction of abstentions. The relative power of a non-permanent member of the UN Security Council increases from 0.95 to 26.67% of the power of a non-abstaining permanent member of the council as the number of permanent members of the council abstaining increases. One result which seems initially counter-intuitive is that the power index of a lone non-abstaining veto player is actually lower than the power index of two non-abstaining veto players. This is probably due to the fact that a single veto player would need all but two of the non-permanent members to pass a motion. Hence, the non-permanent members are becoming close to veto players.

4 Formal Equivalence of Quota, Weights and Veto

Mercik and Ramsey [14, 15] showed a general formal equivalence in a priori analysis between voting games with first degree vetoes and standard weighted voting games. Consider the *n* player game where *k* players have an unconditional veto and the quota is *q*. We now define a game without veto power, but with weighted votes, that is equivalent. Let w_i be the weight of player *i* and $\bar{w}$ be the sum of these weights. Assume that the minimum weight of the votes against a motion *m* necessary to stop it being passed is the same in both games, i.e. $m = n - q + 1$, and non-veto players are given a weight of 1. Hence, we define the quota to be $\bar{w} - \mathrm{n} + \mathrm{q}$. Note that by giving each of the veto players a weight of *m* in the new game, then each of them essentially remains a veto player, since the quota cannot be attained if any veto player votes against the motion. Also, from the definition of the

Table 6 The values of quota and weight of veto player for different number of abstains. *Source* own calculations

Number of abstentions	New quota of modified Security Council	Weight of veto player
0	39	7
1	29	6
2	21	5
3	15	4
4	11	3

weights, it is simple to see that if at least $q - k$ non-veto players vote for a motion, in addition to the veto players, then the motion will be passed. It follows that the two games are equivalent, i.e. have the same set of winning coalitions. It should be noted that any higher weights given to the veto players would satisfy this equivalence relationship (as long as the quota is changed appropriately—in fact the veto players could all be given different weights $\geq m$ and the resulting game would still be equivalent).

Example 4 Equivalent simple game for United Nations Security Council.

For the United Nations Security Council where there are 5 permanent members, 10 non-permanent members (in total 15 members), we have n = 15, k = 5 and q = 9. Now, we construct an equivalent simple game without veto players: $m = n - q + 1 = 15 - 9 + 1 = 7$. Hence, the initial representation of the voting game played by the Security Council of the UN is [9; 1*, 1*, 1*, 1*, 1*, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1], where veto players are marked by stars. This is equivalent to the simple game [39; 7, 7, 7, 7, 7, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1].

Given that k_v permanent members abstain, arguing as above we can treat this as a game where $n = 15 - k_v$, $k = 5 - k_v$ and q = 9. The number of votes needed to block a bill is $7 - k_v$. Hence, we ascribe a weight of $7 - k_v$ to each non-abstaining permanent member. It follows that the sum of weights is given by $(7 - k_v)(5 - k_v) + 10$ and the quota is $(7 - k_v)(5 - k_v) + 4 + k_v$. Table 6 presents the appropriate weighted games for different k_v.

5 Summary

Analysis of the impact of the veto attribute on the position of the decision-makers allows to conclude that there are no paradoxical situations and the veto actually increases the decisive power of a certain decision maker. How strong this impact is remains an open question. Measurement of power using power index is an a priori measurement, and therefore relating to situations occurring only at certain frequency. E.g. examining the situation in the UN Security Council, we find that as the most frequent numbers of abstentions equals 0 or 1 (at least in the year 2014). One may say that in the sense of the Shapley-Shubik power index the United Nations

Security Council may be represented by the simple game $(N, q, w) = (15, 39, 7, 7, 7, 7, 7, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$ or $(N, q, w) = (14, 29, 6, 6, 6, 6, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$ respectively. These weights can be used in a certain sense as a ratio of measures of the power of permanent members compared non-permanent members and the ratio of the force non-permanent members of the Security Council to the permanent ones is at least equal to 1:6, or considering the value of the power index ranges from 1:3.5 to 1:105.25, depending how many permanent members of the council choose to abstain. It is believed that similar proportions will also occur for other decision-making bodies which introduce the veto. That means, without doubt, that the introduction of veto to the decision-making system results in empowering the decision-makers possessing it and should be preceded by an analysis of the balance of power between decision-makers, because the introduction of veto distorts the balance in itself.

Analysing the differences between the yes-no and yes-no-abstain voting systems we find that the introduction of veto in each situation (yes-no and yes-no-abstain) increases the power of the decision-makers possessing it.

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