"Cut-Glue" Approximation Based on Particle Swarm Sub-optimization for Strongly Nonlinear Parametric Dependencies of Mathematical Models

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Abstract The difficulties in the experimental and computer modeling of the static and dynamic transport media are associated with a significant non-linearity of the present model dependencies. This is due to the mechanical friction effects, backlash, aerodynamic effects and other physical phenomena. The construction of the mathematical models (MM) for such objects is most often associated with the mathematical treatment of the experimental data. The point-like dependencies of the output variables linked to the input are essentially non-linear, piecewise and sometimes discontinuous. These dependencies, when approximated with polynomial or spline functions, are difficult to control, and thus associated with large errors. The radically new solution to this problem is proposed in paper [1]. This method is called the «cut-glue» approximation (CGA) method. It is based on partitioning the simulated dependence within specified areas; approximating each fragment with the polynomial dependencies; multiplicative "cutting" for each dependence is the fragments along border area; the additive "gluing" of these fragments is grouped into a single function, as the approximated dependence model. The analytical characteristics of this approximating "gluing" function is the main feature and advantage, properties of functions allow to analyze analytically the model and its application for simulating the dynamic models of the transport media. For a broad CGA method introduction in the experimental modeling research

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practice, it is necessary to develop, justify and explore the local methodologies implementation of these stages. One of the CGA stage is the "glue" process, when the additive fragments are "glued" into a single function. The auxiliary multiplicative allocation function (MAF) is used for that. The MAF structure includes the edge steepness parameter—ε. In this paper, the algorithm is developed based on the modification of the particle swarm optimization (PSO) method, which is used in this research, together with the sub-optimization ε parameter. The hybrid PSO method smoothly includes the basic prototype of the fundamental equations defining the mechanic laws, gravitation and the stochastic "blurring" movement parameters. The developed special software tool, as a research test bed, for conducting the numerical experiments is illustrated in this paper.

Keywords Optimization · Approximation · Mathematical model · Experimental data · Heuristic · Particle swarm optimization

1 Introduction

The simulation of complex dynamic objects (aircraft, vehicles, processes and systems) leads to a differential equations system, which is based on the physical laws. Thus, this system of differential equations defines the object individual elements and their relationship, and in addition, it links the object reaction to the set of external influences. Often, this is due to the significantly different description of such nonlinear phenomena: the mechanical effects of dry friction and aerodynamic "stall" backlash effects [2–4], the complex electromagnetic and thermodynamic processes and to many other factors. Usually, these objects are not define with appropriate theoretical methods, due to the incomplete laws knowledge of these processes. In this paper, the physical tests of real objects or their scale models have been performed. The examples are, the wind tunnel simulations of the airplane or motorcar models. The experiments based on the computer modeling for the universal systems [2–8] are widely used. The models parameters are specified on the basis of these experiments. However, there are difficulties associated with a significant non-linearity present in these models [5–8].

For the technical devices, processes and systems, which are largely based on the heuristic designers ideas, when developed for the first time, the analytical modeling is generally unavailable or results in a very unsatisfactory accuracy, because it is determined by numerous hard accounted factors. In this case, it is advisable to pursue the models construction taking directly their experimental characteristics. Often, these characteristics are essentially nonlinear. In many cases, when approximating the static characteristics, it is sufficient to define a piecewise approximation. However, in dynamic models, such decision creates considerable difficulty in defining the models solution and their mathematical transformations.

The background of the approximation problem for the effective solution is published in several references [1–3, 5, 6]. The proposed CGA method is based on the multiplicative "cutting" of the modeled dependences, which are well-approximated sections, and the additive "gluing" combines them into a single analytic function. At the same time, for the used "cut" analytic function makes the final expression characterized with the analytic properties. The latter, is the main feature and advantage of the method, because it is not only numerical, but in addition, analyzes the resulting conversion model.

Today, the effectiveness of the proposed approximation solution is theoretically proved and practically confirmed only for 1- and 2-dimensional dependencies [1, 9]. In [10], the possibility and prospects of the proposed approach application for approximating the arbitrary dimension dependencies are demonstrated. This significantly broadens the method scope and its importance in the field of science and practice.

"Cut" is a process in CGA that "cut" a fragment, which approximates some experimental dependence, which is plotted on the selected fragment boundaries [1]. Mathematically, this corresponds to the experimental data single matrix cut into several fragments, while preserving the common borders. The boundaries are defined by the description accuracy condition of the section, which is approximated by the analytic function. The MAF structure includes the edge steepness parameter—ε. In a previous work [1], the author citied a condition for the most effective option, when using this parameter. This effectiveness is due to programming expertise, and, as the research shows, it is only effective for a sufficiently accurate reproduction of the approximation function for the fragment boundaries.

"Glue" is the process in CGA that provides a single analytic function, describing the researching region of the object characteristics [1]. The combined fragments, i.e., gluing them together after the "cut" operation, are obtained by performing the algebraic summation. After the series of pilot experiments, the authors have found that the variation in the ε values for the various coordinates in a multidimensional space can improve the approximation result for the "glue" procedure. It is decided to investigate the possibility of optimizing the ε parameter for each selected fragment of the specific 2-dimensional CGA problem.

2 Problem Formulation

The objective is to research MAF, its approximation characteristics and its general mathematical special properties, by providing the reception of the interval isolated fragments (IIF) from the approximated fragment function (AFF), which is locally approximating the fragments of the experimental dependences. The develop methodology for the MAF settings ensures the highest approximation accuracy.

3 Research Techniques

According to the CGA theoretical foundations the ε parameter is a real number, and to optimize it is desirable to use the algorithm from the continuity optimization methods group. In addition, the analytical complexity of the n-dimensional MAF and the resulting CGA function make use of the search optimization algorithm. One of the proven methods, to meet the challenges of finding the optimal research objects parameter, are heuristics algorithms, which are based on the groups behavior of animals, insects, etc. The obvious representative of this trend is PSO. The algorithm is similar to the behavioral characteristics of the insects swarm [11–15].

Previously, the authors carried out a number of works [16–19] on PSO research and its performance, as well as the modifications to improve the accuracy, resource finding optima efficiency, in order to expand its application functionality. In particular, its adaptation to the multiextremal tasks solution. However, the pilot investigations revealed that the 2-dimensional case, with ε_x and $\varepsilon_{y,}$ has only one value, which varies depending on the selected criterion, when considering a range of values in the matrix of the absolute errors. In this paper, we seek to research the ε parameter in 2-dimensional search space covering x and y of each selection. We need to compare the obtained and pilot research results. This implies further research, with the structural modifications and parametric optimization of the PSO mechanical model parameters.

4 PSO MM Modification

The swarming is physically connected with the movement of the real inertial particle, with the equations defining the fundamental mechanical laws:

$$X_{ti} = X_{(t-\Delta t)i} + \vec{V}_{(t-\Delta t)i}\Delta t, \tag{1}$$

$$\vec{V}_{ti} = \vec{V}_{(t-\Delta t)i} + \vec{A}_{(t-\Delta t)i}\Delta t, \tag{2}$$

$$\vec{A}_i = \vec{A}_{pi} + \vec{A}_{tri}, \tag{3}$$

$$\vec{A}_{pi} = \vec{A}_{pi}^{G} + \vec{A}_{pi}^{L},\ \vec{A}_{pi}^{Q} = \vec{G}_Q m_i m_r/(r^2+\varepsilon) \quad Q \in \{G, L\}, \tag{4}$$

$$\vec{A}_{tri} = -\mu\vec{V}_{(t-\Delta t)i}, \tag{5}$$

where: $X_{(t-\Delta t)i}$ and X_{ti}—is the previous and current position of the ith particle; Δt—is the integration interval by time; $\vec{V}_{(t-\Delta t)i}$ and $\vec{V}_{ti}$—is the speed of the ith particle in

the previous and current times; $\vec{A}_{(t-\Delta t)i}$—is the particle acceleration at previous moment; $\vec{A}^G_{pi}$, $\vec{A}^L_{pi}$—is the "biological attraction" acceleration of the particles to a global extremum and to the best position for the particle at all time; $\vec{A}_{tri}$—is the slowing by friction; $\vec{G}_Q$—is the "biological gravity" analogue; $m_i = m_r$—is the attracted particles masses biosimilars; r—is the distance between the current particle position and the current extremum; ε—is the small correction constant (ε = 1e-15, eliminating the problems at r = 0), μ—is the friction coefficient.

To simulate some features, which are inherent to the living beings, an incomplete behavior predictability model is used that form the stochastic "blur" [16] of its basic parameters (geometric, kinematic and dynamic), as defined in the Eqs. 1 and 2. For this purpose, in the model is introduced the random function, with the symmetrical distribution in respect to the nominal value and adjustable range, specifying the "blurring" part of each parameter:

$$\lambda^{\pi}(\xi) = \lambda \cdot (1 + 2\xi \cdot (rnd(1) - 0.5)), \lambda^{\pi}(\xi) \in \{\mu, \vec{G}_Q, C_Q, An, D\}, \tag{6}$$

where: ξ—is the relative departure of the distorted parameter from the nominal value; $rnd(1)$—is a random real number in the [0, 1] range; C_Q—is the apparent particles gravity center; An—is the angle of particles reflection from the consideration boundary; D—is the dissipation factor (loss of energy in an inelastic particle collision of the border).

The composition of the total acceleration is caused by the attractive forces (biological analogues) of the particles including the conditional "global" and "local" attraction.

5 MAF Parameters Influence on the Approximation Quality in IIF "Gluing"

The final step in the general CGA procedure is the cut of IIF from AAF, and their subsequent "gluing" into a single MM. According to [10], the transformation is using the following formulas:

$$f_{(i_1,\ldots i_n)}(\vec{x}) = \varphi_{(i_1,\ldots i_n)}(\vec{x}) \cdot \lambda_{(i_1,\ldots i_n)}\left(\vec{x}, \vec{x}_{(i_1-1,\ldots i_n-1)}, \vec{x}_{(i_1,\ldots i_n)}, \varepsilon_j\right), \tag{7}$$

$$\lambda_{(i_1,\ldots i_n)}\left(\vec{x}, \vec{x}_{(i_1-1,\ldots i_n-1)}, \vec{x}_{(i_1,\ldots i_n)}, \varepsilon_j\right) = \prod_{j=1}^{n} \lambda_{j_i}\left(x_j, x_{j_{i-1}}, x_{j_i}, \varepsilon_j\right), \tag{8}$$

$$\lambda_{j_i}\left(x_j, x_{j_{i-1}}, x_{j_i}, \varepsilon_j\right) = 0.25 \cdot \sigma_l\left(x_j, x_{j_{i-1}}, \varepsilon_j\right) \cdot \sigma_r\left(x_j, x_{j_i}, \varepsilon_j\right) / \delta\left(x_j, x_{j_{i-1}}, x_{j_i}, \varepsilon_j\right), \tag{9}$$

where:

$$\sigma_l\left(x_j, x_{j_{i-1}}, \varepsilon_j\right) = x_j - x_{j_{i-1}} + \sqrt{\left(x_j - x_{j_{i-1}}\right)^2 + \varepsilon_j^2};$$
$$\sigma_r\left(x_j, x_{j_i}, \varepsilon_j\right) = x_{j_i} - x_j + \sqrt{\left(x_{j_i} - x_j\right)^2 + \varepsilon_j^2}.$$

$$\delta\left(x_j, x_{j_{i-1}}, x_{j_i}, \varepsilon_j\right) = \sqrt{\left[\left(x_j - x_{j_{i-1}}\right)^2 + \varepsilon_j^2\right] \cdot \left[\left(x_{j_i} - x_j\right)^2 + \varepsilon_j^2\right]}, \tag{10}$$

$$f(\vec{x}) = \sum_{j=1}^{n} \sum_{i_j=1}^{n_j} f_{\left(i_1, \ldots i_j, \ldots i_n\right)}(\vec{x}). \tag{11}$$

The initial data for the preliminary problem solution have been generated by the authors. As a result, for the selected conditional experiment parameters: the equation in the form of a 3rd degree polynomial function with 2 variables and corresponding coefficients b is defined as follows:

$$\begin{aligned} \mathrm{F(x,y)} = {} & b_0 + b_1 x + b_2 y + b_{11} x^2 + b_{12} xy + b_{22} y^2 \\ & + b_{111} x^3 + b_{112} x^2 y + b_{122} xy^2 + b_{222} y^3, \end{aligned} \tag{12}$$

where: $b_0 = 200$, $b_1 = 8$, $b_2 = -12$, $b_{11} = 1.95$, $b_{12} = -0.18$, $b_{22} = 1.72$, $b_{111} = 0.08$, $b_{112} = 0.023$, $b_{122} = -0.08$, $b_{222} = 0.1$; 10-dimension vector x, 8-dimension vector y, defining the simulated experiment grid.

The resulting matrix is divided into 4 contiguous areas with common borders. Figure 1 displays the original experimental data. The rows in the figure correspond to the x values, the columns correspond to the y values. Figure 2 displays the data graphics.

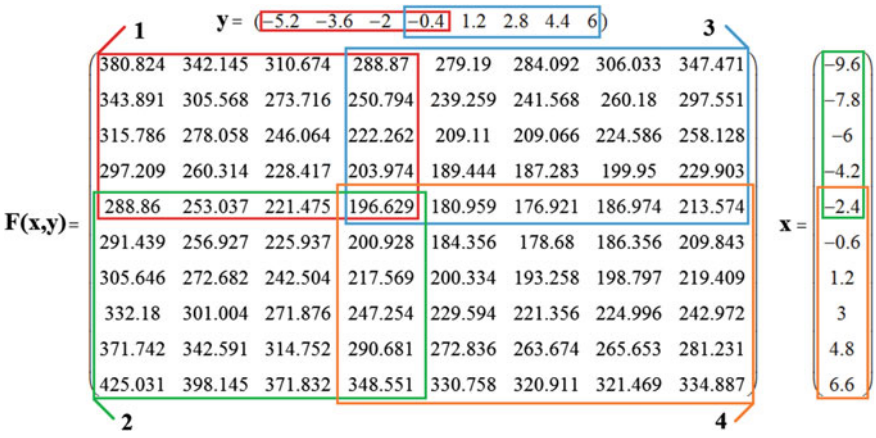

Fig. 1 Experimental data matrix

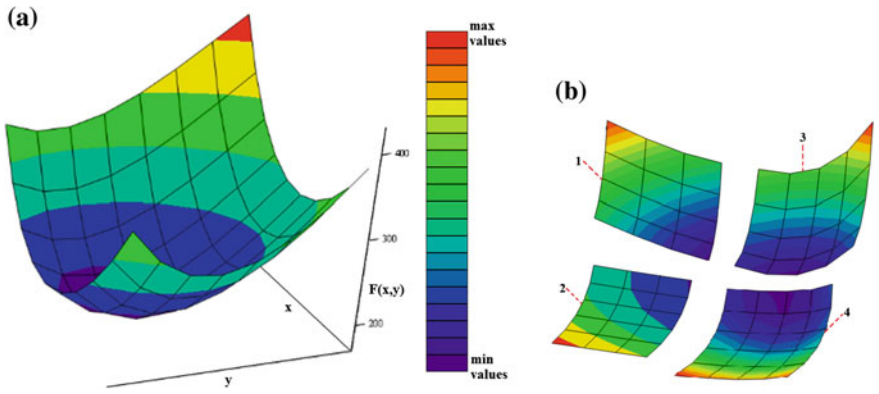

Fig. 2 Experimental data graphs: **a** entire, **b** by fragment

Table 1 Obtained b coefficients from 4-regression equation

	b^1	b^2	b^3	b^4
b_0	−8e-3	−0.904	0.3646	−0.5314
b_1	1.36	3.040	0.712	2.392
b_2	−11.82	−16.84,856	−14.65,208	−12.42,488
b_{11}	196.6	199.966,688	198.317,312	198.47,456
b_{12}	5.6587	7.05,072	8.70,432	8.00,976
b_{22}	1.5256	1.6544	2.0116	2.1404

For each fragment, the 2nd regression equation with the corresponding coefficients b (see Table 1) is defined as:

$$\varphi^i(x,y) = b_0^i + b_1^i x + b_2^i y + b_{11}^i x^2 + b_{12}^i xy + b_{22}^i y^2,\ i \in \{1,2,3,4\}, \tag{13}$$

In accordance with the problem statement, for the experiments the following data are selected: $(\varepsilon_{xi}, \varepsilon_{yi}) \in (0, 1]$, $i \in \{1, 2, 3, 4\}$, the iterations number—300, the particles number—100. In addition, other PSO mechanical model parameters were set for the problem under consideration. The assessment criterion and the minimization of the error for the entire dependence approximation and fragments in the pilot research are the mean-square deviation (MSD) and the max error in the absolute errors matrix. In addition, the experience has shown that it is appropriate to identify the matrix key areas, which also apply to the selected assessment criterion. Therefore, the objects approximation accuracy ranking can be result in several mathematical constructs derived from CGA. First, these include the entire matrix of the experimental data. Second, the following can be considered: the matrix without border values (according to (9), corner and edge function value, except the conjugating values, little dependence on ε, and, to a greater extent, depending on the order approximated). Third, the "gluing" fragments common border, not including the border values of the entire matrix (these elements are mostly depending on ε).

Fig. 3 Outputs in "Glue_MSP" software

Fourth and fifth, viewed separately the "gluing" fragments common edges (vertical and horizontal). All these CGA accuracy ranking objects are used in the current work.

In this research, the PSO agents search is done in the 8-dimensional space and their movement is displayed without releasing the projections in the subspace (3 degree maximum). For clarity, Figs. 3 and 4 illustrate the data, displayed in the window parts of the developed software, resulting from the search for an extremum in 2-dimensional space. Figure 3 shows the output data. Figure 4 shows the visualization of the swarm movement at the extremum localization time. The two experiments results are performed with the selected criteria and considering the areas defined in the absolute errors matrix, as shown in Table 2.

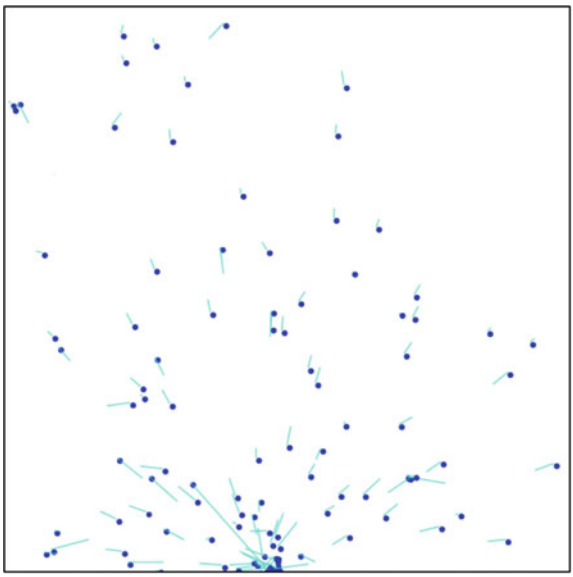

Fig. 4 Visualization the swarm movement in "Glue_MSP" software

Table 2 Experimental result

Considered values in absolute errors matrix	Criteria	MSD %	Max %	ε_{x1}	ε_{y1}	ε_{x2}	ε_{y2}	ε_{x3}	ε_{y3}	ε_{x4}	ε_{y4}
Entire matrix	MSD	9.14	26.93	1e-6	0.99	0.99	1e-6	1e-6	1e-6	0.99	0.99
	Max	9.39	26.75	1e-6	0.99	0.99	0.99	1e-6	1e-6	0.99	1e-6
Matrix without boundaries	MSD	5.62	26.87	1e-6	0.99	0.7	0.44	1e-6	8e−6	0.99	0.74
	Max	5.77	26.75	1e-6	0.99	0.99	0.99	1e-6	1e-6	0.99	1e-6
«Glue» submatrix	MSD	2.98	28.51	1e-6	0.81	0.99	0.99	1e-6	1e-6	0.47	1e-6
	Max	3.1	26.75	1e-6	0.99	0.99	0.99	1e-6	1e-6	0.99	1e-6
Horizontal of «glue» submatrix	MSD	3.65	29.16	0.99	0.99	0.99	0.99	1e-6	1e-6	1e-6	1e-6
	Max	3.71	26.77	1e-6	0.99	1e-6	0.99	1e-6	1e-6	0.99	1e-6
Vertical of «glue» submatrix	MSD	0.39	35.49	1e-6	0.99	1e-6	1e-6	0.25	0.99	1e-6	1e-6
	Max	1.06	26.75	1e-6	0.99	0.99	0.99	1e-6	1e-6	0.99	1e-6

Table 3 Test results comparison

			Research			
			Pilot	Current	Pilot	Current
		Criteria	Max %		MSD %	
Considered values in absolute errors matrix	Entire matrix	MSD	11.08	9.14	32.35	26.93
		Max	11.57	9.39	29.7	26.75
	Matrix without boundaries	MSD	7.65	5.62	31.42	26.87
		Max	7.88	5.77	29.71	26.75
	«Glue» submatrix	MSD	3.74	2.98	30.07	28.51
		Max	3.81	3.1	29.7	26.75
	Horizontal of «glue» submatrix	MSD	4.32	3.65	30.07	29.16
		Max	4.36	3.71	29.7	26.77
	Vertical of «glue» submatrix	MSD	0.54	0.39	33.49	35.49
		Max	0.9	1.06	29.7	26.75

The total operation time spent to carry out all the experiments with 300 iterations is $\sim$1.1 s. The PSO operation time spent on finding the extrema, is described in Table 2, with $\sim$160 iteration— $\sim$0.7 s. As can be seen from Table 2, the obtained values of MSD, Max, ε_x and ε_y are highly dependent on the selected criteria range values.

Table 3 contains the results obtained from the ε parameter optimization, performed for each fragment and the ε parameter optimization for all fragments within the pilot research.

The data comparison have shown the following: the total operation time spent on carrying out all experiments with 300 iterations, has reduced by $\sim$50%, MSD and max errors have decreased by $\sim$17 and $\sim$8.3%, respectively.

6 Conclusion

The following conclusions can be drawn from the current work results:

- ε parameter has not, in general, a universal value for all the fragments;
- in ε parameter optimization for each fragment, the values of these parameters depend only on the problem properties, and the visible relationships were found between them;
- ε parameter in 2 dimensions for each fragment has only one value, which varies according to the chosen criterion, and within the considered range of values in the absolute errors matrix;

- when optimizing ε parameter for each fragment, MSD and max absolute error decreased by ∼17 and ∼8.3% respectively, which is relative to the common ε parameter optimization results for all the fragments, obtained in the pilot research;
- the authors developed the PSO modification, which has proved to be an effective tool for the ε parameter sub-optimization problems solution in 2-dimensional CGA MAF.

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