Considering Diversity in Spatial Decision Support Systems

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Introduction

Many decision problems contain certain elements that are related to space [32, 35]. For example, to place a set of facilities, factors such as locations, distance, and connectivity among potential locations must be considered. Political redistricting is another example where space plays a significant role in determining the final plan that must satisfy restrictions such as spatial contiguity and compactness. We broadly refer to these as spatial decision problems.

Spatial decision problems are often difficult to solve due to many factors. Researchers have long recognized that spatial decision problems are often computationally intensive to solve [1]. This is because most spatial decision problems rely on a search algorithm to find feasible and optimal solutions from a huge set of potential solutions to the problem. The computational intensity of spatial decision problems often makes it impractical to find the optimal solution to the problem as the time used to search for the solution may become excessive. For many real world problems, even if the global optimal can be found, the solution is only optimal in the context of how the problem is simplified by removing factors that are otherwise difficult to be considered in the optimization model.

In additional to the computational burden, spatial decision problems often have multiple stakeholders who decide how the final decision should be made [38]. These stakeholders often have different goals to achieve regarding a specific problem and some of these goals are typically translated as the multiple criteria or objectives of the problems. For many problems that have multiple objectives, there may not exist a single solution that is deemed to be optimal by all stakeholders. To address

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such decision problems, a variety of solution approaches have been developed [8]. The literature, however, seems to be less concerned with how to incorporate these approaches such that a better solution can be ultimately reached. A more interesting question is, if existing solution approaches can be collectively used to provide high-quality solutions, is it useful to develop new ones? Moreover, how can we successfully incorporate different perspectives of decision makers and stakeholders to generate more robust and reliable solutions that are satisfactory to a wider group of people?

The above questions are related to an interesting topic in social science: diversity, referring to a state of *difference* exhibited in a system and its components. Recent developments have demonstrated that effectively incorporating diversity may provide better solutions to highly complex problems in social and economic domains such as long-term prediction [19]. The purpose of this paper is to explore how the concept of diversity manifests in spatial decision making and how spatial decision making can benefit by incorporating diversity in the solution process. Although this paper is focused on decision problems from an optimization perspective, many concepts developed here can also be applied to other types of decision making problems. In the remainder of this paper, I first identify the kinds of diversity in spatial decision making, and then discuss a number of approaches to incorporating diversity into geographical problem solving.

Kinds of Diversity

Let $\mathbf{x}$ be a vector of decision variables. For a spatial decision problem, at least a subset of these decision variables have spatial references, often encoded as location indices. For example, we can have $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ as indices to n locations and assign x_i to 1 if the ith location is selected for a design purpose (e.g., facility location) and 0 otherwise. We then assume $\mathbf{x}$ must be drawn from a domain denoted as $\mathbf{S}$ that defines all feasible solutions. The goal of solving a spatial decision problem is then to find an $\mathbf{x}$ such that a set of m objective functions, $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x}))^T$, can be optimized. Formally we write the problem in a generic form as

$$\begin{aligned} &\min \mathbf{f} \\ &\text{subject to } \mathbf{x} \in \mathbf{S}\,. \end{aligned} \tag{1}$$

Simon [24] suggested three steps that are commonly adopted in problem solving for a broad range of applications where decisions must be made. Starting at the intelligence step, the problem must be formulated so that alternative solutions can be found in the second step called design. In the third step called choice, a final decision must be made based on the alternatives identified. To solve a spatial decision problem, diversity is ubiquitous in all steps. For example, diversity occurs when the problem is interpreted and formulated by different stakeholders from different perspectives, solved using different methods, and presented to decision makers who have different preferences. Specifically in this paper, I examine diversity in spatial decision making from three perspective: (1) how solutions differ with respect to their decision variables and objective functions, (2) how the optimality of solutions differs and how their differences can be measured, and (3) how approaches to solving these problems differ.

Diverse Solutions

Solutions to a decision problem are typically described using two spaces: solution space and objective space. A solution space is formed by all the feasible solutions to the problem. Formally, a solution space is an n-dimensional attribute space where each dimension is one of the n decision variables, and we can denote it as a set of $\{\mathbf{x}|\mathbf{x} \in \mathbf{S}\}$. An objective space, however, is an m-dimensional space where each dimension is one of the m objective functions, denoted as $\{\mathbf{f}(\mathbf{x})|\mathbf{x} \in \mathbf{S}\}$. For spatial decision problems, a third space can also be identified: the geographic space of the solutions because each solution can be mapped and the spatial pattern shown on the map conveys meaningful messages that will be critical in the decision process [3]. Here we use a general notation of $g(\mathbf{x})$ to indicate the measure of solution $\mathbf{x}$ in the geographic space and therefore the geographic space can be denoted as a set $\{g(\mathbf{x})|\mathbf{x} \in \mathbf{S}\}$. Figure 1 illustrates the relationship between these three spaces.

The difference between solutions in the solution space can be captured using a distance measure such as the Euclidean distance

$$d_{ij} = \sqrt{\sum_{k}^{n} (x_k^i - x_k^j)^2}\,, \tag{2}$$

where x_k^i and x_k^j are the kth decision variable in solutions i and j, respectively. Using the measure in Eq. (2), the distances between the solutions in Fig. 1 are $d_{AB} = d_{BA} = 2$, $d_{BC} = d_{CB} = \sqrt{2}$, and $d_{AC} = d_{CA} = \sqrt{2}$.

The difference between two solutions can also be calculated in the objective space, again using a Euclidean distance:

$$d_{ij}^{\text{obj}} = \sqrt{\sum_{k}^{m} (f_k^i - f_k^j)^2}\,, \tag{3}$$

where f_k^i is the kth objective function value for solution i. In the hypothetical objective space in Fig. 1, it can be noted that $d_{AC}^{\text{obj}} < d_{AB}^{\text{obj}} < d_{BC}^{\text{obj}}$.

While the above two measures provide the numerical distances between solutions, one may argue that because the selected nodes in solutions A and B are adjacent in each case, they are more clustered than in solution C where the

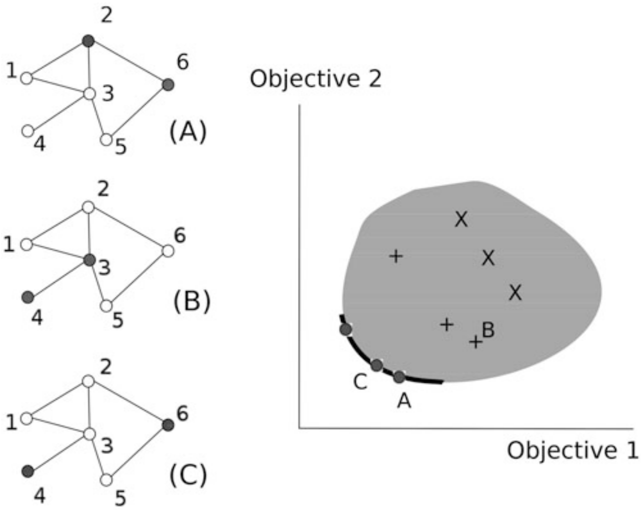

Fig. 1 A hypothetical spatial decision problem in which two nodes must be selected in a network of six nodes in order to minimize two objectives. Three possible solutions are illustrated as (A), (B), and (C). The number associated with each node is the index of the decision variable corresponding to that node. For each solution, its realization in the solution space is represented by whether a node is selected (*gray circle*) or not (*open circle*), or a set of values for the decision variables. For example, solution *A* is (0, 1, 0, 0, 0, 1). The geographic space realization is the network map shown in this figure, and each *dot* in the plot represents one of the hypothetical solutions in the objective space

selected nodes have no direct connections. Many measures can be used to reflect the geographic space of these solutions. Here we use a simple measure of the shortest distance or smallest number of edges on the path between selected nodes to illustrate the concept, and we have $g_A = 1$, $g_B = 1$, and $g_C = 3$. Accordingly, the distance in geographic space between these solutions can be simply calculated using the absolute difference between these measures:

$$d_{ij}^{\text{geog}} = |g_i - g_j|\,, \tag{4}$$

where g_i and g_j are the geographic measures of solutions i and j, respectively. In the three solutions in Fig. 1, we have $d_{AB}^{\text{geog}} = d_{BA}^{\text{geog}} = 0$, $d_{BC}^{\text{geog}} = d_{CB}^{\text{geog}} = 2$, and $d_{AC}^{\text{geog}} = d_{CA}^{\text{geog}} = 2$.

Diverse Optimality

The diversity in the objective space has two aspects. First, each solution can be identified using its objective function values as shown in Fig. 1 where the three dots marked as A, B, and C in the plot refer to the hypothetical values of the objective function values. The distance between these solutions in the objective space can therefore be simply calculated using the Euclidean distance between them.

Second, it is important to note that the multiple objectives for a problem reflect different and often conflicting goals. A consequence of such difference is the trade-off among alternative solutions, meaning there is no single solution that can be considered to be satisfactory with respect to all the goals. The trade-off among solutions can be formally understood using the concept of a domination. Here, we say a solution to a decision problem $\mathbf{x}_1$ dominates (or is better than) another solution $\mathbf{x}_2$ if and only if

$$\forall i f_i(\mathbf{x}_1) \leq f_i(\mathbf{x}_2) \ \wedge \ \exists i f_i(\mathbf{x}_1) < f_i(\mathbf{x}_2) .$$

In other words, solution $\mathbf{x}_1$ dominates solution $\mathbf{x}_2$ if $\mathbf{x}_1$ has at least one objective function value that is smaller (better) than that of $\mathbf{x}_2$, while all other objective values of $\mathbf{x}_1$ are not greater (worse) than that of $\mathbf{x}_2$. For a single objective optimization problem, there is typically only one solution that dominates all other feasible solutions. For a multiobjective problem, however, there often exists a set of solutions that are called non-dominated solutions, meaning they dominate all other solutions outside the set and each of these solutions does not dominate other members in the set. Solutions in this set are optimal and the set is often referred to as the Pareto front. In Fig. 1, the shaded area in the plot represents the objective space of the solutions, the thick curve represents the Pareto front and solutions on the curve are optimal (and therefore non-dominated) solutions.

A fundamental problem of (spatial) decision making is that the decision problem may be ill-structured because many social, economic, and environmental factors are difficult to be included in problem formulation [4, 25]. This feature suggests that the optimal solutions obtained based on the original problem formulation may become sub-optimal when new factors are considered as they often may be in real world applications. It is therefore important to understand the structure of the entire solution space instead of just the optimal ones, even if they can be found. We use the ranks of the solutions in the objective space to reveal this structure. Using the definition of dominance, we first give all the non-dominated solutions a rank of 1 (circles in the plot of Fig. 1). Then we increase the rank value to 2 and assign it to the non-dominated solutions in the remaining un-ranked solutions (pluses in Fig. 1). This process continues until all solutions are ranked.

After the ranking process is completed, we can measure the diversity of the objective space at different levels. First, we measure the between rank diversity of solutions using the inverse Simpson index [26]:

$$\frac{1}{\sum_{k=1}^{K} p_k^2}, \tag{5}$$

where p_k be the proportion of solutions that fall in rank k, and K is the total number of ranks in the solutions. The denominator is the probability that two random individual solutions have the same rank. If each solution has its own rank, we have $p_k = 1/K$ $(1 \leq k \leq K)$ and the between rank diversity is K. On the other hand, if all solutions are non-dominated (there is only 1 rank), we have a minimal between rank diversity of 1. For the 9 solutions in Fig. 1, the between rank diversity is $1/(\frac{1}{3^2} + \frac{1}{3^2} + \frac{1}{3^2}) = 3$.

Second, we can measure the diversity of solutions within each rank as the ratio between the number of solutions in the rank and a hypervolume of the solution space:

$$d_k = | \cup_{\mathbf{x} \in R_k} \mathbf{f}(\mathbf{x})| / \Pi_i^m (\mathbf{f}_i^u - \mathbf{f}_i^l) \,, \tag{6}$$

where the denominator is the hypervolume computed using the upper and lower bounds of each objective function values, $\mathbf{f}_i^u$ and $\mathbf{f}_i^l$, respectively, R_k is the set of solutions in rank k, and the numerator gives the number of unique individual solutions in rank k in the solution space.

Finally, while the above measures are aimed to provide a view for the solutions in the entire set or the ranked ones, diversity of solutions can also be measured at the level of each solution by examining the crowdedness of the neighborhood of that solution. Here we can borrow the concept of niche count from the evolutionary algorithm literature [8, 11] to measure the crowdedness around a solution:

$$n_i = \sum_{j=1}^{N} \mathrm{sh}(d_{ij}), \tag{7}$$

where n_i is the niche count of solution i, d_{ij} is the distance between individual i and j, which can be any of the distance measures discussed above (Eqs. (2), (3), and (4)) depending on what type of diversity is to be measured, and function $\mathrm{sh}(d)$ is defined as:

$$\mathrm{sh}(d) = \begin{cases} 1 - (d/\sigma_{\mathrm{share}})^{\alpha} & \text{if } d < \sigma_{\mathrm{share}} \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

where σ_{share} is a constant distance threshold that dictates the size of the neighborhood to be used for a solution, and α is a constant coefficient that reflects the weight given to a distance. In general, a high niche count suggests a high number of solutions exist around the given solution. We typically focus on solutions that have a low diversity of solutions in their neighborhood as suggested by low niche counts. Without knowing how solutions are exactly distributed in the solution space or objective space, it is often desired that each solution have a similar niche count (or local diversity).

At this point, one might reflect on why the above diversity discussion and diversity measures matter. The answer relies on how spatial decision problems are or can be solved. If there exists a magic tool that can return the exact solutions to a spatial decision problem, then none of the above discussion would matter much because the problem can be solved exactly, meaning we can find the optimal solution to the problem and therefore can make the decision consequently. For many real world decision problems, however, it is often impractical to solve the problem exactly, and it is important to find as many solutions as possible to enable an informed decision making process. More critically, the solutions found need to be diverse so that decision makers are not biased toward a certain subset of the solutions.

Diverse Toolboxes

Given the formulation of a spatial decision problem, the optimal solution can be obtained using an exact method. However, as discussed above, such an exact approach may become impractical when the size of the problem increases and additional factors must be considered in the decision process. It is critical, therefore, to explore a diverse set of solutions to the problem to enable a comprehensive examination of the solution space during the decision making process so that the decision makers can make their final choice. A second type of solution approach, called heuristics, can be used for this purpose. Heuristics are often more efficient compared with their exact counterparts, though they do not guarantee the global solutions to be found. The literature has generally suggested the effectiveness of heuristics in finding high quality solutions that are optimal or near optimal [5]. However, it has not been the focus of existing research to discuss how heuristic methods can be used to generate a diverse set of solutions to facilitate the decision process. In this section, I give a brief overview on the diversity of the solution methods. I will then discuss in the next section on how to utilize the diversity of these methods.

A large number of heuristic methods for spatial optimization problems have been developed in the past few decades. A traditional approach to developing such a method is problem-specific and lacks the flexibility of applying to other problems. The effectiveness of this type of heuristics is evident in the literature [5, 7]. For example, the vertex exchange method developed to solve the p-median problem in location-allocation models [31], though highly effective [22], cannot be directly used for other location allocation problems such as the center problems without significant modifications. Another example is the heuristic method that is specifically designed to solve political redistricting problems [33].

In general, traditional heuristic methods can be considered in different categories. A simple approach is to develop a greedy algorithm that construct a complete, feasible solution by assigning the values to the decision variables step by step. During each step, a decision variable is assigned so that the solution appears to be the best at that step, which can be simply achieved by minimizing the increase of the objective function value caused by assigning the new decision variable (assuming minimization is the goal). A greedy algorithm can often be strikingly simple to develop but the performance may not be satisfactory for many problems, especially when the problems contain many local optimal solutions.

Different from greedy algorithms, a local search algorithm starts from a complete solution to the problem that is called the current solution. The search algorithm can be used to create a neighboring solution by manipulating the current solution. The neighboring solution will be used to replace the current one if the former exhibit a better objective function value. Otherwise, the algorithm keeps searching for other neighbors. The algorithm stops when no better solution can be found. The vertex exchange algorithm for the *p*-median problem [31] is an example of local search where a neighbor solution is generated by swapping a selected vertex with other candidates.

In contrast to traditional approaches that are typically tailored to specific problems, a new set of heuristics is aimed to solve a wide range of optimization problems. These new methods, called metaheuristics collectively, include evolutionary algorithms, tabu search, simulated annealing, and ant colony optimization algorithms. A common feature of these algorithms is their root in natural processes. Evolutionary algorithms (EAs), for example, are derived from the natural selection theory [11, 12]. For an EA to find an optimal or near optimal solution to a problem, a set of solutions called a population is maintained at the same time. Each solution in an EA population is evaluated and consequently rated using a fitness function related to the objective functions of the problem. Solutions that exhibit high fitness function values often have a high chance to be used to create new solutions for the next iteration. In addition to their nature-inspired search mechanisms, metaheuristic methods also try to represent various optimization problems in a general and adaptive fashion. In EAs, for example, binary, integer, or real number strings have been used to represent solutions to numerical optimization problems in general [23], and geographic optimization problems in particular [35].

Embracing Diversity

Diversity can be incorporated into a spatial decision making process in a variety of ways. Before we start the discussion of specific incorporation strategies, let us stipulate the importance and therefore the benefits of recognizing and incorporating diversity in spatial decision analysis. First, the decision makers may wish to examine a diverse set of solutions such that important solutions, though may not be optimal according to the original mathematical formula, can be discussed and may be further modified. Second, a diverse set of solutions in the solution process can be used to maintain useful components of optimal solutions that otherwise may not exist in the “good” solutions chosen by the search algorithm. Here, I identify a number of technologies that can be used to promote or utilize diversity discussed above for the purpose of spatial decision making.

Encouraging and Maintaining Diverse Solutions

Several methods have been suggested in the evolutionary algorithm literature to maintain a diverse set of solutions. These methods try to balance two kinds of power in a search algorithm. First, a search algorithm is exploratory if it focuses on finding new solutions, especially solutions with new components that have not been found or included in those found so far during the search process. In EAs, a process called mutation is specifically designed to increase the exploratory power of search by randomly changing a portion of an existing solution with a hope of introducing new values which can then be combined with other solutions in order to construct better solutions. On the other hand, a search is exploitative if it tries to exhaustively use values in solutions found so far. In EAs, a crossover operation tries to combine two existing solutions to create new ones and therefore "exploits" current information that is already included in the two solutions. An exploitative operation tends to decrease the diversity of solutions while an exploratory one often increases the diversity.

Carefully balancing these two types of operations in a solution approach is critical for a successful search [6, 9, 37]. Some more recent work has also tested an adaptive fashion of using exploratory and exploitative search operations. For example, Tarokh [29, 30] suggest exploratory operations to be used more frequently if the lack of diversity is deemed in the current solutions. In EAs, the sharing method [11, 191] has been commonly used to reduce the chance of a solution to be selected if it is in a crowded neighborhood (measured in Eq. (7)). This concept is also used in EAs for multiobjective optimization problems where the fitness values of solutions in a crowded area in the objective space will be reduced so the solutions in less crowded areas have more chance to explore their neighborhood [8, 14].

Hybrid Solution Toolboxes

Solution approaches developed in the literature can be used in different ways. Though the common way of using these methods independently is useful, the overall performance can be improved if these methods are used collectively. One way of utilizing the diverse tools is to design a new process based on the components from existing methods. A method designed in this way can be called a hybrid method. For example, the concepts of vertex change and greedy algorithms are used to develop new and more effective hybrid methods to solve the p-median problem [21, 34]. While this type of hybridization is common in the literature [10, 16, 18, 20], a successful algorithm design may be ad hoc as many design aspects cannot be replicated in other problems.

Cooperative Methods

The recent literature has suggested another approach to incorporating different tools for problem solving. Hong and Page [13], for example, developed a general framework that includes a large number of problem-solving agents, each of which is a specific heuristic method that can be used to find a local-optimal solution to a problem. Each problem solving agent is evaluated using the average of the best solutions found. A subset of these agents is then selected to solve another set of random problems where each problem is solved sequentially, meaning one agent starts to solve it and then pass the final result to the next agent until all the agents are used. Their computational experiments on three different problem configurations suggested that a set of randomly selected agents outperformed the best agents on all cases.

In many real world problem situations, it has been observed that humans cooperate throughout the solution process and there have been different strategies in cooperating. In English, for example, it is often agreed that "two heads are better than one" [27, 28]. In this spirit, we can develop a new framework where problem solving agents work with each other through different cooperative (and sometimes non-cooperative) mechanisms, where some agents may prefer working alone while other may tend to solve a problem together with the others. There can be many cooperation strategies too. To illustrate various cooperation strategies, we discuss a recent development [36] in solving the p-median problem using two different approaches: a method called TB developed by Teitz and Bart [31], and a method called SA that is based on simulated annealing [15]. TB maintains a current solution and continuously replaces it with a better neighboring solution. TB stops when no better neighboring solution can be found. SA, however, uses a probability to accept a neighboring solution for replacement. while the acceptance probability for a better solution is always 1, SA also accepts solutions that are worse than the current one. The probability of accepting worse solutions decreases as the search progresses. SA terminates when no solution is accepted.

In this example [36], a total of seven modes of cooperation were implemented. First, TB and SA were two "work alone" modes where each ran separately and reported its own result. In addition to running these two methods independently, five cooperative strategies were also used. In a *relay* strategy, TB ran first and then the solution found by TB was used in SA; the process terminated after SA stops. A *sequential consensus* strategy was similar to relay, but the solution found by SA were passed on to TB again and the process repeated until no improvement can be made. To use a *compete* strategy, both TB and SA started independently and then, during each iteration of both methods, the current solutions were compared and the winner was used by both method for the next iteration. A *full cooperation* strategy depended on an exchange mechanism such that the two methods always exchanged their current solutions during each iteration. Finally, a *parallel consensus* strategy was developed so that both methods ran independently until they stopped and then they exchanged their best solutions found with each other; each agent then restarted

Table 1 Experiments on cooperation strategies for the *p*-median problem

Cooperation strategy	Number of optima found			Average deviation from optima		
	Best	Average	Worst	Best	Average	Worst
TB	25	5	5	0.07	0.32	0.78
SA	23	6	6	0.25	0.91	1.92
Relay	28	11	11	0.04	0.21	0.49
Sequential consensus	30	9	9	0.03	0.18	0.50
Compete	29	9	9	0.04	0.20	0.49
Full cooperation	30	10	10	0.05	0.17	0.39
Parallel consensus	34	15	15	0.01	0.11	0.24

using the solution from the other agent and continued the search process. Both kept exchanging solutions until not improvement can be made by any agents. A parallel computing environment was used to implement these methods.

Forty benchmark *p*-median problems [2] were used to test the above strategies. Each strategy was run 100 times for each problem. The best, average, and worst solutions generated in these 100 runs were used to report two summaries: the number of times these solutions were optimal for the 40 problems, and the average deviation from the known optima (Table 1). For example, the parallel consensus strategy found the optimal solutions to 34 of the 40 problems in the best case amid the 100 runs. The results clearly suggest that all the five cooperative strategies outperformed the two work-alone mode. Some strategies (e.g., parallel consensus) consistently outperformed the all other strategies, while some strategies (e.g., compete) may not necessarily outperform the other cooperative methods.

Extending the above experiment, we can consider each method as an agent that is equipped with a particular skill of solving some problems. An agent-based modeling framework, therefore, can be regarded as a platform to utilize the diversity of toolboxes in spatial decision making. In addition to such a toolbox perspective, agent-based models can also incorporate multiple players (decision makers) that have different belief systems and reflect different preferences to the decision problem. Simulation results of these models can be used by decision makers to learn interesting system behaviors.

Conclusions

The role of diversity has been recognized in many disciplines such as biology and sociology. In this paper, I attribute the importance of diversity in spatial decision making to the fundamentals of spatial decision making: multiple stakeholders with often conflicting goals, the ill-structured nature of the decision problem that leads to the need of exploring not only the optimal solutions but suboptimal solutions, and computational intensity of the solution approach. These characteristics entail the consideration of diversity for spatial decision making. This paper examines diversity in spatial decision making from three perspectives: solutions, optimality, and methods. The diversity of solutions can be identified and measured in the solution space, objective space, and their geographic space.

Considering diversity in spatial decision support systems is consistent with a postmodernist view [see, for example, 17] that adds to a computationally sophisticated environment of geocomputation. From a social or political point of view, promoting diversity in the decision process reflects a step toward a more appealing democratic process. It will be an informative debate to see if such an effort will provide us "better" decisions, the meaning of which may be beyond its methodological domain and of course is another aspect of diversity.

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