

# Multirate Systems in Cognitive Radio

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## 1 Introduction

Spectrum sensing is an important function of cognitive radios (CRs), in finding spectrum access opportunities and obtaining noninterfered spectrum for reliable communication. The purpose of cognitive radio techniques is to allow secondary users (unlicensed) to utilize the spectrum which is not occupied by the primary users (licensed) [1]. Multirate systems can perform key signal processing applications for CR systems. In wideband spectrum sensing, the wideband channel is divided into multiple nonoverlapping narrowband channels and is sensed for opportunities which are referred as multiband sensing in literature [2, 3]. The multirate signal processing techniques are useful in wideband spectrum sensing for multiband spectrum detection by the use of filter bank techniques. In cases where wideband spectrum sensing requires high sampling rates and high power consumption, multirate filter banks become a better solution [4].

In general for wideband spectrum sensing, the radio frequency (RF) front end requires wideband architecture, and the spectrum is estimated by using power spectral density (PSD). The basic method used for PSD is the periodogram spectrum estimator (PSE). The PSE is limited due to the trade-off between spectrum resolution and dynamic range because of the sidelobes of the PSE window. To overcome these limitations, multi-taper (MT) method and filter bank-based methods were utilized. The advantage of filter bank method is that it enables efficient implementation of band-pass filters using polyphase decomposition of prototype filters. A comparison between filter bank method and MT has shown that the filter bank methods are more promising compared with MT method in terms of lower computational complexities.

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In wideband multichannel spectrum sensing, FFT or filter bank-based spectrum analyzer has been considered by averaging the output samples of each subbands for detecting multiple spectral gaps. FFT and filter bank techniques have been used for sensing Wireless Local Area Networks (WLAN) such as OFDM based on IEEE 802.11 system and Wireless Personal Area Network (WPAN) with bluetooth designated to operate on 2.4 GHz ISM band [5]. Multi-resolution filter banks based on fast filter bank design with varying spectral bands have been used for sensing military radio receivers [6]. Tree-structured DFT filter bank was applied for estimating the center frequencies and spectral edges of primary user signals [7]. - Filter banks are also useful for the detection of wireless microphones in IEEE 802.22 Wireless Regional Area Network (WRAN) and estimation of center frequency to fractionally utilize the available bandwidth [8, 9]. Progressive decimation filter bank techniques (PDFB) are applicable for variable sensing resolutions to detect different bandwidths in wideband spectrum. The theoretical framework for the analysis and design of filter bank-based detectors for spectrum sensing applications in cognitive radios is also discussed in literature [10]. In this chapter we discuss the spectrum sensing techniques in CR and the application of multirate filter banks in CR applications.

## 2 Cognitive Radio

Cognitive radio system has been proposed as a promising solution to improve the spectrum utilization. The concept of cognitive radio was proposed by Joseph Mitola [11]. CR systems have intelligent mechanism for monitoring the radio spectrum to detect spectral holes and, thereby, allocate the same to secondary users without causing any harmful interference to the primary users in wideband spectrum. Particularly, CR is considered for obtaining spectrum usage characteristics across multiple dimensions such as time, space, frequency, and code. CR comprises of determining the type of signals in addition to parameters such as modulation, waveform, bandwidth, and carrier frequency occupying the spectrum [12].

FCC defines CR as *A radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation, such as maximize throughput, mitigate interference, facilitate inter-operability, access secondary markets.*

CRs are considered to be the most promising future wireless communication technology that may potentially mitigate the problem of spectrum scarcity using dynamic spectrum access techniques [13]. The underutilization of spectrum is due to the extremely low spectrum utilization in some localized temporal and geographical spectrum bands. Spectral opportunities have to be detected without any assistance from primary users. The primary users do not have any constraints to share or change the operating parameters for sharing spectrum with cognitive radio networks [14].

A CR system consists of the following entities:

*Primary User:* The users who have higher priority or legacy rights on the usage of a specific part of the spectrum are defined as primary users.

*Secondary User:* The unlicensed user, who transmits and receives signals over the licensed spectra or portions of it when primary users are inactive, is called secondary users [12].

*Spectral hole:* A band of frequencies assigned to a primary user, which is unused at a particular time and at a specific geographic location is called a spectrum hole [1].

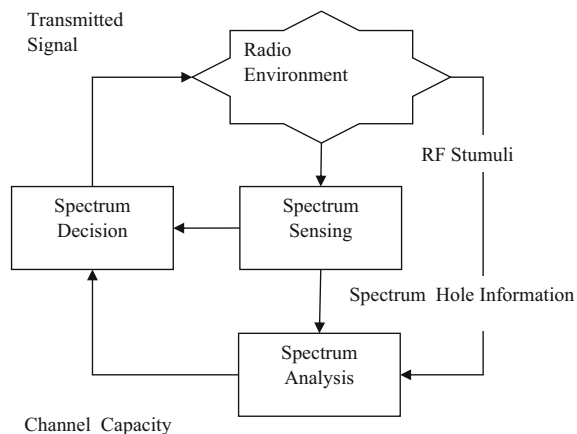
The spectral holes are classified into two types: (1) temporal and (2) spatial spectral holes. A temporal spectral hole appears when a primary user is not transmitting for a certain period of time. When the primary user transmission is confined within an area, a spatial spectral hole appears, and the secondary users can use the spectrum outside that area [15].

Cognitive radios have two main features which distinguish them from the conventional radio devices; they are *cognitive capability* and *reconfigurability* [16].

The cognitive ability allows a CR system to sense and capture the information from the surrounding radio environment. This feature allows a cognitive user to be aware of different parameters such as transmitted waveform, radio frequency spectrum, and geographical information. The gathered information are analyzed to identify any unused spectrum at a specific time and location [17]. The interaction between CR and radio environment is known as cognitive cycle. The cognitive ability of CR explained through cognitive cycle is shown in Fig. 1. A cognitive cycle consists of the following three components namely: spectrum sensing, spectrum analysis, and spectrum decision [1].

**Spectrum Sensing** In spectrum sensing, a cognitive radio observes the frequency band and gathers necessary information regarding its surrounding radio environment. Based on the information captured, the cognitive radio is able to detect spectrum holes.

**Fig. 1** Cognitive cycle



**Spectrum Analysis.** Once the spectrum holes are detected using spectrum sensing, each of the spectrum band is characterized based on the local observation of the cognitive radio as well as the statistical information of primary user network. Moreover, characteristics of spectrum holes are also analyzed and estimated.

**Spectrum Decision.** Depending on the spectrum analysis, the cognitive radio determines the operating parameters such as the data rate, the transmission mode, and the bandwidth available for transmission. The most appropriate spectrum band is selected based on the spectrum band characterization and the user requirements.

As mentioned earlier, the second key feature of a cognitive radio that distinguishes it from a traditional radio is reconfigurability. The ability of a cognitive radio to intelligently adapt to the radio environment by adjusting its operating parameters, according to the sensed environmental variations, in order to achieve the optimal performance, is referred to as reconfigurability. Cognitive wireless networks are capable of reconfiguring their infrastructure in order to adapt to the continuously changing environment. The reconfiguration actions take place in the PHY/MAC layers for the selection of appropriate technology and spectrum for operation. Different transmission access technologies can be supported by its hardware design such that transmission and reception are possible in a variety of frequencies [16, 18].

### 3 Spectrum Sensing Methods

Spectrum sensing is an inevitable part of cognitive radio systems that allows us to use the available spectrum efficiently. The different spectrum sensing methods provide the key to monitor and reuse the spectrum without interference. One of the major tasks of CR is to obtain underutilized and noninterfered spectrum for allocation of secondary users. The channel conditions keep changing due to the noise uncertainty, multipath fading, and shadowing effects in wireless channels. Therefore, there exists a need for monitoring and cooperation among secondary users for efficient spectrum utilization. The usefulness of the spectrum sensing techniques is based on the sensing performance and complexity in implementation.

Spectrum sensing techniques can be classified as noncooperative and cooperative methods. The cognitive radio acts on its own in noncooperative spectrum sensing, while in cooperative spectrum sensing, multiple CRs work together, which results in an increase of accuracy in spectrum detection and spectrum awareness. Cooperative spectrum sensing is further classified into three categories depending on how cooperating CR users share the sensing data: (1) centralized, (2) distributed, and (3) relay assisted [19, 20]. In multipath fading and shadowing environment, cooperative spectrum sensing is considered to be an effective approach. The common spectrum sensing techniques are energy detection (ED), matched filter (MF), and cyclostationary feature detection (CFD), which are discussed in subsequent sections. In filter banks, subband-based energy detection

is applied for detection of spectral holes. Apart from common spectrum sensing methods, other techniques existing in literature include eigenvalue-based methods, covariance matrix method, and wavelet-based methods [21, 22]. After spectrum sensing, the secondary users are allowed to access the spectrum holes. In order to access the spectrum holes effectively, spectrum sharing and spectrum allocation techniques are important [23]. The common spectrum sensing methods are briefly explained in the following sections.

### 3.1 Energy Detection Method

Energy detection method is further classified as traditional energy detection and subband-based energy detection.

#### 3.1.1 Traditional Energy Detection

The most widely used method of spectrum sensing is the traditional energy detection due to its low computational complexity [24]. The receiver does not require any prior knowledge of the primary user signal as energy detection is a noncoherent method of detection. The primary user is detected by measuring the energy and comparing it with a predetermined threshold. The threshold  $\lambda$  is computed using the assumed noise variance  $\sigma_w^2$  and probability of false alarm  $P_{fa}$ , which generally depends on the channel characteristics. The problem of detecting the presence and absence of signal in spectrum sensing is typically formulated by the following binary hypothesis test [25],

$$\begin{aligned} H_0 : y[n] &= w[n] \\ H_1 : y[n] &= x[n] + w[n] \end{aligned}$$

where  $y[n]$  represents the received signal,  $x[n]$  is the transmitted wireless signal, and  $w[n]$  is the zero mean complex circularly symmetric additive white Gaussian noise (AWGN). Further,  $x[n] = s[n] \otimes h[n]$  where  $s[n]$  denotes the primary user signal and  $h[n]$  the channel impulse response [26]. Hypothesis  $H_0$  represents the absence of a primary user signal and consists only the noise  $w[n]$ . On the other hand, hypothesis  $H_1$  represents the presence of primary user signal  $x[n]$  along with noise  $w[n]$ . The test statistic is computed as the energy of the received signal as given in Eq. (1),

$$T(y) = \frac{1}{N_s} \sum_{n=0}^{N_s-1} |y[n]|^2, \quad (1)$$

where  $N_s$  is the total number of samples sensed at the receiver. The test statistic follows a chi-square distribution. However, in practical cases, the test statistic can be approximated to a Gaussian distribution for large number of samples according to the central limit theorem (CLT) [27]. According to CLT any independent and identically distributed (IID) random variable with finite mean and variances approaches a normal distribution when the number of samples  $N_s$  is large enough. Therefore, the distribution of test statistics can be accurately approximated with a normal distribution for sufficiently large number of samples. The above hypothesis can be written as in,

$$T(y) \sim N\left(\sigma_w^2, \frac{1}{N_s}\sigma_v^2\right); \text{ for hypothesis } H_0$$

$$T(y) \sim N\left(\sigma_x^2 + \sigma_w^2, \frac{1}{N_s}(\sigma_x^2 + \sigma_w^2)^2\right); \text{ for hypothesis } H_1$$

where  $\sigma_x^2$  is the signal variance and  $\sigma_w^2$  is the noise variance. The presence of an active signal is determined by comparing the energy (test statistics) with a predetermined threshold. The threshold  $\lambda$  is calculated using the knowledge of probability of false alarm  $P_{fa}$  and the assumed noise variance  $\sigma_w^2$  of the received signal. The probability of false alarm  $P_{fa}$  is given as

$$p_{fa} = Q\left(\frac{\lambda - \sigma_w^2}{\sqrt{1/N_s}\sigma_w^2}\right) \quad (2)$$

and the probability of detection can be expressed as

$$p_d = Q\left(\frac{\lambda - (\sigma_w^2 + \sigma_s^2)}{\sqrt{1/N_s}(\sigma_w^2 + \sigma_s^2)}\right) \quad (3)$$

The threshold  $\lambda$  is determined from Eq. (2) as

$$\lambda = \left(Q^{-1}(P_{fa})\sqrt{1/N_s} + 1\right)\sigma_w^2 \quad (4)$$

The minimum number of samples required for spectrum sensing is obtained using Eqs. (2) and (3) [28],

$$N_{\min} = 2\left[Q^{-1}(p_{fa}) - Q^{-1}(p_d)(1 + \text{SNR})\right]^2 \text{SNR}^{-2} \quad (5)$$

### 3.1.2 Subband-Based Energy Detection

When the available wideband is split into nonoverlapping subbands, the subband-based energy detection is performed at the output of the individual subbands. The energy is computed as the test statistic at the output of each subband and compared

with a predetermined threshold. Filter bank-based methods are robust and efficient for multiband spectrum sensing where energy detection is performed at the subband level at the output of the FFT or analysis filter bank (AFB). The wideband signal is split into narrow signal bands using FFT or AFB. Similar to traditional energy detection, the subband signal can be expressed as follows:

$$\begin{aligned} H_0 : y_k[m] &= w_k[m] \\ H_1 : y_k[m] &= x_k[m] + w_k[m] \end{aligned}$$

where  $y_k[m]$  is the received signal at the  $k^{\text{th}}$  subband ( $k = 1, 2, \dots, M$ ),  $M$  is the total number of subbands with  $x_k[m] = H_k s_k[m]$ ,  $H_k$  represents the complex gain of the subbands,  $s_k[m]$  is the input signal, and  $w_k[m]$  is the noise samples of the subbands. Similar to traditional energy detection, noise follows the distribution  $w_k[m] \sim N(0, \sigma_{w,k}^2)$  and signal  $x_k[m] \sim N(0, \sigma_{x,k}^2)$  with  $\sigma_{w,k}^2$  being the noise variance and  $\sigma_{x,k}^2$  the signal variance [26]. If  $\sigma_{w,k}^2$  is the noise variance of the wideband channel, the subband noise variance is  $\frac{\sigma_w^2}{M}$ . The energy at the output of individual subbands is considered as the test statistic,

$$Y_k = \frac{1}{L} \sum_{m=0}^{L-1} y_k[m]^2 \quad (6)$$

where  $L = \left(\frac{N_s}{M}\right)$  is the number of samples in each subband with  $M$  number of subbands for sensing and  $N_s$  total number of samples received. The presence and absence of a primary user signal is written in terms of the following two hypotheses:

$$\begin{aligned} y_k(m) &\sim N\left(\sigma_{w,k}^2, \frac{1}{L}\sigma_{w,k}^4\right); \text{ for hypothesis } H_0 \\ y_k(m) &\sim N\left(\sigma_{x,k}^2 + \sigma_{w,k}^2, \frac{1}{L}(\sigma_{x,k}^2 + \sigma_{w,k}^2)^2\right); \text{ for hypothesis } H_1 \end{aligned}$$

The number of samples for each stage needs to be large enough to perform energy detection even in low SNR. The minimum number of samples required in each stage can be calculated using the relation in Eq. (5).

### 3.2 Matched Filter

Matched filter (MF) is a non-blind spectrum sensing technique with coherent detection. Prior knowledge of the primary user signal is required in MF. The known primary user information is correlated with the received signal to detect the presence of primary user signal and maximize the signal-to-noise ratio (SNR). The matched filter requires short sensing time and achieves good detection performance with low probability of missed detection and false alarm. The drawback of

this method is that it requires knowledge about primary user signal such as operating frequency, bandwidth, modulation type, and packet format. Therefore, the technique is not applicable when the information regarding the primary users are unknown [23].

### 3.3 Cyclostationary Feature Detection

Cyclostationary feature detection (CFD) technique exploits the cyclostationary features of the signal for spectrum sensing. A signal is considered to be cyclostationary if its statistical properties vary cyclically with time. When the modulated signals are combined with sinusoidal signals and pulse trains, they exhibit periodicity. The cyclostationary features are exploited from the periodicity using signal statistics such as mean and autocorrelation. The cyclic autocorrelation function (CAF) of the received signal  $x(t)$  can be expressed as

$$R_x^{(\alpha)} = E[x(t)x^*(t - \tau)\exp(-j2\pi\alpha\tau)], \quad (7)$$

where  $\alpha$  is the cyclic frequency,  $E[.]$  is the expectation operation, and  $*$  denotes complex conjugation. Using Fourier series expansion, CAF can be expressed as cyclic spectral density (CSD) [23].

When the cyclic frequency  $\alpha$  and fundamental frequencies become equal, CSD exhibits peaks. Therefore, under hypothesis  $H_0$ , the noise alone is present, and the CSD function does not exhibit peaks as the noise is nonstationary. On the other hand, in hypothesis  $H_1$ , peaks occur due to the signal and presence of noise. Therefore, CFD distinguishes the noise from the PU signal and can also be used for the detection of weak signal in case of very low SNR. CFD does not require prior knowledge of primary user waveform. The performance of CFD can be improved at a given SNR by increasing the number of samples, however at the cost of sensing time. The limitation of cyclostationary feature detection is that it requires longer processing time compared to the energy detection and matched filter detection techniques.

## 4 Wideband Spectrum Sensing

An important challenge in CR is sensing of multiple narrowband channels over a wideband spectrum. Most of the existing spectrum sensing algorithms discussed above are suitable for narrowband spectrum sensing, which exploits the spectral opportunities over narrow frequency range. To achieve higher throughput, CR needs to exploit spectral opportunities over a wide range of frequencies, from hundreds of megahertz to several gigahertz [14]. In cases where spectral opportunities are to be identified in ultra-high frequency (UHF) TV band



(between 300 MHz and 3 GHz), wideband spectrum sensing techniques are to be employed. Narrowband spectrum sensing techniques cannot be applied in this scenario as they can make only binary decision on the whole spectrum and the spectral opportunities within the wideband cannot be identified. The benefits of multichannel/wideband spectrum sensing for CR networks are the secondary user throughput capacity can be maximized and aggregate interference of primary user networks can be reduced [29]. Multiband joint detection techniques have also been utilized to maximize the secondary user throughput capacity and reduce interference of primary users [3]. The multiband spectrum sensing has a few challenges due to the following reasons as discussed in [2].

- The available wideband for spectrum sensing may not be contiguous.
- A small portion of bandwidth may be occupied by a wireless device, and the entire bandwidth may be considered unavailable. (For example, in IEEE 802.22, wireless microphone occupies only 200 kHz of a 6 MHz TV channel, and the entire TV channel would be considered occupied.)
- If a portion of signal is in deep fade, the subbands may consider that portion as a spectral hole. Therefore, if a secondary user is allocated to that portion of the spectrum, interference would occur with the existing primary user.

The multiband spectrum sensing is categorized into serial-based detectors, parallel-based detectors, and wideband-based detectors. Serial sensing is simple to implement; however, the technique is slow and undesirable when the subbands are more. Parallel sensing provides faster detection at the expense of RF components and complex signal processing. The common multiband sensing techniques use reconfigurable band-pass filters, tunable oscillators, filter banks, wavelets, and blind sensing. A comparison between the different multiband spectrum sensing methods is provided in [2]. A detailed review and comparison between the different spectrum sensing methods along with advantages, disadvantages, and challenges are also provided in [12].

Further, wideband spectrum sensing techniques are broadly classified into two types:

- Nyquist wideband SS
- Sub-Nyquist wideband SS

In Nyquist wideband spectrum sensing, digital signals are sampled at or above the Nyquist rate, and in sub-Nyquist technique the signals are sampled below the Nyquist rate. Standard analog-to-digital converters (ADC) and digital signal processing techniques are used in Nyquist wideband spectrum sensing. After the received signals are sampled, serial to parallel conversions are required for further processing of the signals. A widely used Nyquist wideband spectrum sensing technique is the filter bank-based spectrum sensing. In filter bank-based techniques, fast Fourier transform (FFT) is used to convert the signal to a series of narrowband spectra. The spectral opportunities were identified by applying the binary hypothesis test to the individual subbands. In most of the filter bank techniques, energy detection was chosen as the test statistic. The threshold for detection was jointly chosen using optimization techniques.

The sub-Nyquist approach overcomes the limitations of Nyquist approach resulting from high sampling rate and computation complexity. In sub-Nyquist sensing, the wideband signal is acquired by using sampling rates lower than Nyquist rate. The compressed sensing techniques are applied for sub-Nyquist sampling.

## 5 Filter Bank Techniques for Spectrum Sensing

The concept of multirate filter banks was proposed for spectrum sensing initially by Farhang [30]. Filter banks are implemented by shifting a low-pass prototype filter. The first subband is estimated using the prototype filter, and other subbands are obtained by modulating the prototype filter. The total bandwidth is split into narrow nonoverlapping subbands using multiple band-pass filters. Multicarrier techniques were also suggested for spectrum sensing, where OFDM was the first multicarrier technique proposed for CR [31]. OFDM was considered as a suitable candidate for CR as FFT can be used for spectral analysis and demodulator for OFDM signal. However, the limitation of using the OFDM for CR application is the presence of large sidelobes in the response of the subband filters due to 13 dB attenuation of FFT, which may lead to interference between different users because of the spectral leakage. Moreover, OFDM techniques lack high spectral dynamic range and are not suitable for detection of low-power primary users. To overcome this issue, the rectangular pulse shape in OFDM was replaced with a smooth edge pulse shape filters called filtered OFDM. Filter bank multicarrier (FBMC) and filtered OFDM become alternate solutions to overcome the above limitations. FBMC reduces the spectrum leakage compared to cyclic prefixed OFDM systems and is capable of identifying multiple users with different center frequencies and spectral gaps between users efficiently with flexibility [10]. Different FBMC schemes reported in literature include staggered modulated multitone (SMT), filtered multitone (FMT), and cosine-modulated multitone (CMT). A comparison of filter bank multicarrier methods in cognitive radio systems is presented in [32].

The spectrum efficiency can be increased by designing prototype filters with acceptable subband attenuation. Therefore, filter banks are considered to be an alternate solution for wideband spectrum sensing. To achieve high spectral dynamic range in filter banks, the length of the prototype filter also needs to be adjusted. Multi-taper method (MT) is shown as a near optimal sensing method, even though MT has high computational complexity [1, 33]. However, similar performance can be achieved with filter banks using prolate filters with lower computational complexity [30]. Discrete Fourier transform (DFT) and modified DFT filter bank with root-Nyquist filter have also been exploited for spectrum sensing in wideband cognitive radios.

Different types of filter banks have been used in CR system for varied applications. Multistage polyphase filter bank techniques were used for the detection of center frequency of primary users with low computational complexity and higher precision. FFT-based filter bank techniques have been used for sensing Wireless

Local Area Networks (WLAN) such as OFDM based on IEEE 802.11 system and Wireless Personal Area Network (WPAN) with bluetooth designated to operate on 2.4 GHz ISM band. Multi-resolution filter banks based on fast filter bank design with varying spectral resolution were applied for spectrum sensing in military radio receivers. Tree-structured DFT based filter banks have also been used for estimating the center frequencies and spectral edges of primary user signals.

### 5.1 Sensing Architecture Based on Filter Banks

Filter banks consist of an analysis filter bank (AFB) and synthesis filter bank (SFB). Synthesis filter banks are sufficient to extract the signal components of each subband from the wideband RF signals. The basic filter bank spectrum sensing architecture is illustrated in Fig. 2. The RF module is followed by wideband ADC to sample the RF signal. Different filter bank structures like cosine-modulated filter bank (CMFB), DFT, and polyphase DFT can be considered. In case of complex modulated filter banks, the complete filter bank structure can be realized using complex modulation of a single prototype filter.

In general, multiband sensing utilizes energy detection techniques due to the reduced computational complexity. Different methods such as periodogram method, multi-taper method (MTM), and filter bank methods have investigated energy detection for spectrum sensing in literature. Farhang has shown in [30] that DFT filter banks based on energy detection are more promising in terms of accuracy if noise variance is known. Energy detection is the most common method as it has low computational and implementation complexities. Energy (power) is computed at the output of individual subband and considered as the test statistic. The presence and absence of the signal is detected by comparing the energy with a predefined

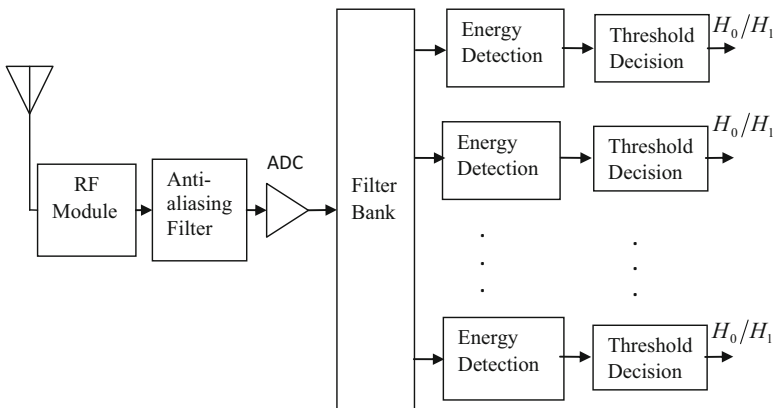


Fig. 2 Sensing Architecture Based on Filter Banks

threshold as explained in Sect. 3. The threshold is a function of probability of false alarm and noise variance of the channel.

Different filter bank structures have been used in CR for spectrum sensing like cosine-modulated filter banks and DFT-FFT-based filter banks.

## 6 Multirate Filter Banks

The multichannel filter banks can be implemented using cosine modulation, FFT, DFT, or modified DFT filter banks. The filter banks can be implemented using complex modulation of a single prototype filter [34]. Multirate filter banks are designed with basic multirate signal processing techniques such as decimation and interpolation [4]. The polyphase representation in multirate is useful for computationally efficient implementation of polyphase filter banks. In general the analysis and synthesis subbands of the filter bank are simultaneously generated by applying an appropriate modulation scheme to the linear phase finite impulse response (FIR) prototype filter. The significance of prototype filter design in the implementation of filter banks to improve the overall performance is well proven. Therefore, the design of prototype filter is vital in the implementation of filter bank structures.

In general, filter bank designs can be categorized into two types:

- Perfect reconstruction (PR)
- Near-perfect reconstruction (NPR) or quadrature mirror filters (QMF)

Perfect reconstruction filters are alias-free filters, where the output is a delayed version of input. However, the implementation of PR filters is computationally complex, and for practical applications, NPR filters are adequate. The filter bank implementation focuses on NPR as they provide improved alias suppression in the subbands by relaxing PR constraint. As the same prototype filter is employed in the analysis and synthesis banks, the NPR filters have polyphase matrices which are paraunitary and, hence, have favorable numerical properties.

Perfect reconstruction (PR) filter banks satisfy the condition that the reconstructed signal

$\hat{x}(n)$  need to be a scaled and delayed version of the input  $x(n)$ .

$$\hat{x}(n) = cx(n - n_0) \quad (8)$$

The reconstructed signal can be represented using z-transform as

$$\hat{X}(z) = T(z)X(z) \quad (9)$$

The PR condition indicates that aliasing is canceled, and distortion function  $T(z)$  is forced to be a delay. The optimization of the prototype coefficients for perfect reconstruction is highly nonlinear. In case of NPR or approximate reconstruction, the analysis and synthesis filters  $H_k(z)$  and  $F_k(z)$ , respectively, are chosen in such a

way that the adjacent subband aliasing gets cancelled. The distortion function  $T(z)$  is approximately a delay. The approximate systems mentioned are called pseudo-QMF banks and are acceptable for practical applications.

Extensive research has been carried out to find an optimal prototype filter for complex modulated filter banks. The optimization techniques for prototype filter design can be categorized into three types:

- Frequency sampling techniques
- Window-based techniques
- Direct optimization of filter coefficients

In order to overcome the limitation of having  $M$  different transfer functions, which provide perfect reconstruction, the complex modulated filter banks can be realized from a single low-pass prototype filter. The complex modulated filter banks generally use the basic pseudo-quadrature mirror filter principle. The filter banks are realized by equidistant frequency shifts of a prototype filter. For near perfect reconstruction in the filter banks, the low-pass prototype filters have to satisfy the following conditions [35]:

1. Prototype filter has to be band limited.

$$|H(e^{j\omega})| \approx 0 \quad |\omega| > \frac{\pi}{M} \quad (10)$$

2. Frequency response of prototype filter has to be pairwise power complementary.

$$|H(e^{j\omega})|^2 + \left| H\left(e^{j\left(\frac{\pi}{M}-\omega\right)}\right) \right|^2 \approx 1, \quad 0 \leq \omega \leq \frac{\pi}{M} \quad (11)$$

The advantages of such filters are twofold:

- The cost of implementing  $M$  analysis filter bank includes the cost of one prototype filter and modulation overhead. The cost of  $M$  synthesis filter is similar to an analysis filter.
- Optimization of prototype filter alone is required for the implementation of filter bank structure.

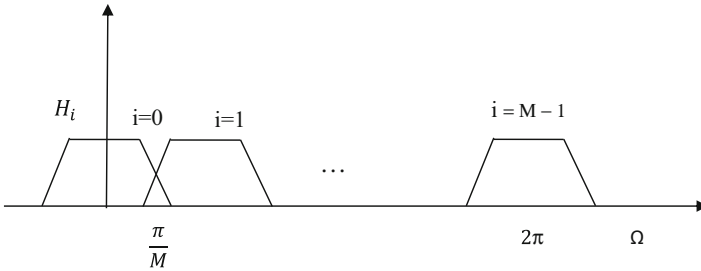
The prototype filters should have sufficient stopband attenuation to suppress the aliasing components. The realization of DFT, cosine modulation, and DFT-based polyphase filter banks are discussed in the following subsections.

## 6.1 DFT Filter Banks

In DFT filter bank, the  $M$  analysis filter banks,  $H_k(z)$ ,  $k=0, 1, \dots, M-1$ , are realized by frequency shifting the transfer function  $H(z)$  of the prototype filter  $h(n)$ . The impulse response of the prototype filter is multiplied by a factor  $e^{jn\Omega_0}$ .



**Fig. 3** Frequency response of prototype filter



**Fig. 4** Frequency shifted version of the prototype filter for analysis filter banks

Further, the frequency response of the prototype filter  $H(e^{j\Omega})$  is shifted right as  $H(e^{j(\Omega-\Omega_0)})$ . The  $M$  analysis filter response can be expressed as

$$H_i(e^{j\Omega}) = H\left(e^{j\left(\Omega - \frac{2\pi i}{M}\right)}\right), i = 1, 2, \dots, M - 1 \tag{12}$$

In case,  $W_M = e^{j2\pi/M}$ , the Z-transform of the analysis filters can be written as

$$H_i(z) = H(zW_M^i) \tag{13}$$

The frequency response of the prototype filter and the shifted versions of the prototype filter for the generation of subbands filter are shown in Figs. 3 and 4, respectively.

### 6.2 Cosine-Modulated Filter Banks

The cosine-modulated filter banks (CMFB) are also pseudo-QMF and satisfy the NPR conditions. Cosine-modulated filters can easily maintain maximally decimated NPR condition. Among the NPR FIR filter banks, CMFB is considered to be simple both in terms of design and implementation complexities. Initially, the prototype filter is designed satisfying the power complementary and band-limiting

conditions specified in Eqs. (10) and (11). In an  $M$  channel CMFB, the impulse responses of the analysis and synthesis filters are  $h_k(n)$  and  $f_k(n)$ , respectively. The filter banks are cosine-modulated versions of the prototype filter  $h(n)$ . The prototype filter is linear phase FIR Type I filter. The analysis and synthesis filters are given by Eqs. (14) and (15) for  $0 \leq n \leq N-1$  and  $0 \leq k \leq M-1$  as in [4],

$$h_k(n) = 2h(n) \cos \left[ \frac{\pi}{M} \left( k + \frac{1}{2} \right) \left( n - \frac{N-1}{2} \right) + (-1)^k \frac{\pi}{4} \right] \quad (14)$$

$$f_k(n) = 2h(n) \cos \left[ \frac{\pi}{M} \left( k + \frac{1}{2} \right) \left( n - \frac{N-1}{2} \right) - (-1)^k \frac{\pi}{4} \right] \quad (15)$$

where  $k=0, 1, 2, \dots, M-1$  is the number of subbands in the filter bank. By choosing a linear phase FIR Type I filter, the phase distortion can be eliminated completely. The amplitude distortion is reduced, when the band-limiting condition stated in Eq. (10) is satisfied and if the filters are pairwise power complementary as in Eq. (11), the aliasing error can also be reduced.

### 6.3 Spectrum Sensing with Cosine-Modulated Filter Bank

Among the different filter bank-based methods such as orthogonal multiplexed quadrature amplitude modulation (OQAM), cosine-modulated multitone (CMT), and filtered multitone (FMT), CMT is more desirable as it provides higher bandwidth efficiency compared to FMT and lower sidelobes than OQAM. The performance of spectrum estimation is characterized by different parameters such as frequency resolution, spectrum leakage, and estimation time. The above three parameters can be regulated using CMFB with a proper design of prototype filters. In wideband spectrum sensing, signals are filtered using CMFB followed by power spectrum estimation [36]. CMFB can detect primary users over contiguous channel having different bandwidths. A transceiver framework based on cosine-modulated filter bank was proposed in [37] for cognitive access to TV white spaces. The spectrum sensing is performed using the system model described in Sect. 3 for subband-based energy detection.

### 6.4 Polyphase Filter Banks

Polyphase filter bank structure reduces the complexity of the filter bank implementation using the noble identities of multirate systems. Polyphase filter banks are efficiently designed using FFT when the number of subbands  $M$  is a power of two. The DFT filters can be modified to get a better stopband attenuation compared to 13 dB of DFT at the cost of one prototype filter. The polyphase decomposition of

the prototype filter enables to implement the filter bank in an efficient manner [4, 38]. The transfer function of a FIR prototype filter  $h(n)$  is given by

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n} \quad (16)$$

The transfer function in Eq. (16) can be decomposed into polyphase components as in Eq. (17)

$$\begin{aligned} H(z) = & \sum_{n=-\infty}^{\infty} h(nM)z^{-nM} + z^{-1} \sum_{n=-\infty}^{\infty} h(nM + 1)z^{-nM} + \dots \\ & + z^{-(M-1)} \sum_{n=-\infty}^{\infty} h(nM + M - 1)z^{-nM} \end{aligned} \quad (17)$$

Eq. (17) can be written in short as in Eq. (18).

$$H(z) = \sum_{l=0}^{M-1} z^{-l} E_l(z^M) \quad (18)$$

The above equation represents a Type-1 polyphase filter. Similarly,  $l^{\text{th}}$  polyphase component of the filter bank is defined as

$$E_l(z) = \sum_{n=-\infty}^{\infty} e_l(n)z^{-n} \quad (19)$$

where,  $e_l(n) = h(Mn + l)$ . The Type-2 polyphase decomposition of the Eq. 10 can be expressed as

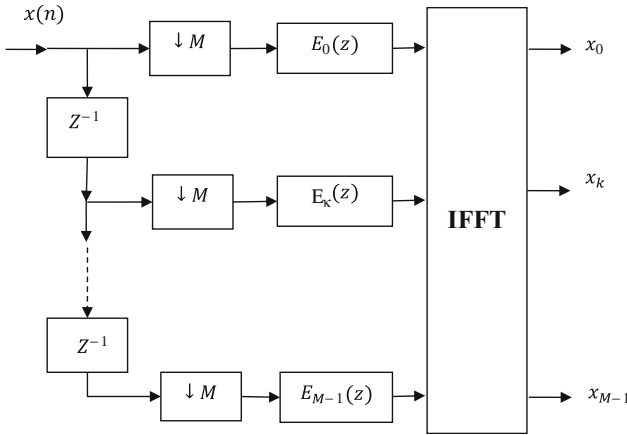
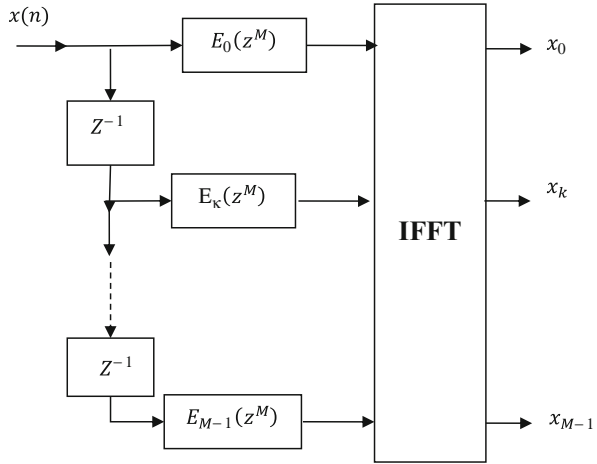
$$H(z) = \sum_{l=0}^{M-1} z^{-(M-1-l)} R_l(z^M) \quad (20)$$

Polyphase implementation simplifies the theoretical results, and computationally efficient filter banks can be realized. The filter bank implementation with uniform DFT bank using polyphase decomposition is shown in Fig. 5. Using noble identities the polyphase uniform DFT filter bank structure with decimators can be redrawn as shown in Fig. 6. Since the downsampler are shifted toward the input side, the polyphase subband filters are computed at a low sampling rate, which reduces the computational complexity by a factor of  $M$ . Due to the polyphase decomposition of prototype filter, the polyphase subband filters are shorter compared to the original filters by a factor of  $M$ .

The computational complexity of polyphase filter banks is  $N + M \log_2 M$ , where  $N$  is the length of the prototype filter. Polyphase filter bank reduces the computational complexity to a large extent compared to the complexity  $NM$  of direct



**Fig. 5** Uniform DFT bank using polyphase decomposition



**Fig. 6** Polyphase filter banks with efficient implementation using noble identities

implementation. The important advantage of multirate polyphase filter banks is that it allows efficient implementation of filter bank structure due to the polyphase decomposition. Moreover, the entire computational complexity of the polyphase filter bank is reduced to the design of a prototype filter and  $M$  point FFT.

## 7 System Model for Filter Bank Spectrum Sensing

The wideband signal for spectrum sensing is localized to various subband frequencies using filter bank structures. The available spectrum band is divided into  $M$  nonoverlapping uniform subbands, where  $M$  is the number of subbands in the

filter bank. The output of each subband,  $x_k(n)$ , is assumed to be a random process, obtained from a random process input  $s_k(n)$  passing through a linear subband filter of frequency response  $H_k$ , where  $x_k(n) = H_k s_k(n)$ . The received signal  $y_k(n)$  can be modeled as [26],

$$y_k(n) = x_k(n) + w_k(n), k = 0, 1, 2, \dots, M - 1,$$

where  $x_k(n)$  is the active signal and  $w_k(n)$  is the additive white Gaussian noise with zero mean and variance  $\sigma_w^2$ . In order to detect the presence of primary user signal, a binary hypothesis is defined as [40],

$$\begin{aligned} H_{0,k} : y_k(n) &= w_k(n) \text{ absence of signal} \\ H_{1,k} : y_k(n) &= x_k(n) + w_k(n) \text{ presence of signal} \end{aligned}$$

We consider the test statistic as the energy at the output of each subband as given by

$$y_k(n) = \frac{1}{L} \sum_{n=0}^{L-1} x_k^2(n)$$

where  $L = \left(\frac{N_s}{M}\right)$ , is the number of samples in each subband and  $k = 0, 1, 2, \dots, M - 1$ . When the number of samples is increased, the chi-square distribution approximate to a normal distribution from the central limit theorem as discussed in Sect. 3.

The presence of an active signal in a specified subband can be determined by comparing the energy in that subband with a predetermined threshold. The available wideband is divided into  $M$  nonoverlapping subbands using filter banks. Energy detection is performed on the output of each subband and compared with a predefined threshold. Depending on the threshold decision, a subband is considered to have the presence of primary user referred as '1' or absence of primary user referred as '0' (spectral hole). This is referred to as binary detection and illustrated in Fig. 7.

## 7.1 Calculation of Threshold

The threshold  $\lambda$  can be calculated using the knowledge of probability of false alarm  $P_{fa}$  and noise variance  $\sigma_w^2$  of the received signal as explained in Sect. 3. The energy detector gives the best performance with known noise variance and performance deteriorates when the noise variance is uncertain. To improve the detection performance, the noise variance can be estimated at the receiver before spectrum detection using noise variance estimation techniques.

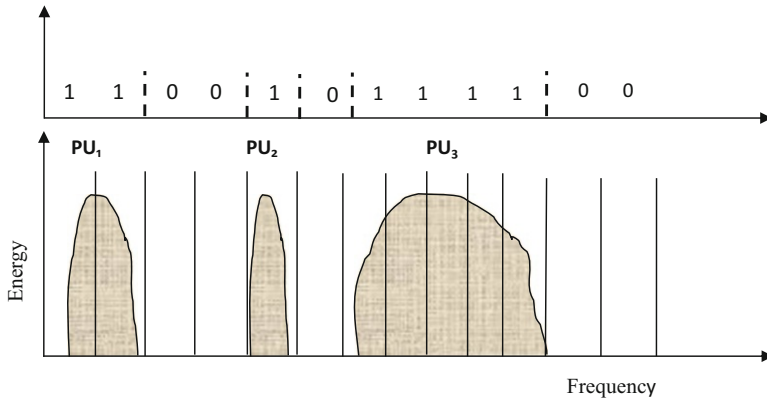


Fig. 7 Illustration of binary detection in wideband spectrum sensing

### 8 Multistage Filter Bank Spectrum Sensing

Spectrum sensing can be performed using filter bank structures for varying granularity

bands. The number of subbands  $M$  determines the granularity of sensing bandwidth. For spectrum sensing with finer granularity bands,  $M$  needs to be increased, whereas for spectrum sensing with coarser granularity bands,  $M$  needs to be decreased. The advantage of varying granularity band is that the spectrum utility and re-usability can be effectively increased. Moreover, the same structure can be used for spectrum reallocation due to the flexibility offered by the filter bank structure.

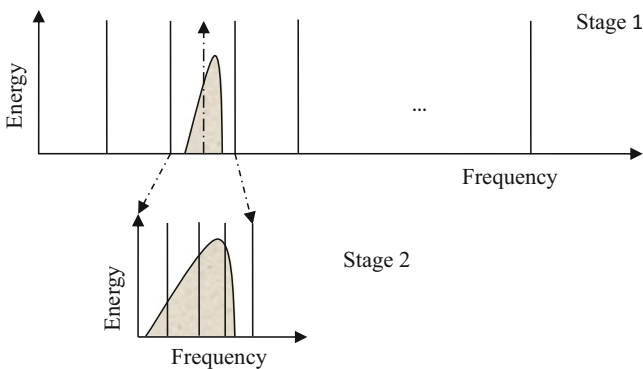
The granularity of the filter banks can be chosen specifically if the bandwidth of the primary users is known a priori. The energy is computed at the output of individual subbands as the test statistics. The threshold is calculated for specified probability of false alarm  $P_{fa}$  and known noise variance  $\sigma_w^2$ . The bandwidth efficiency could be effectively increased using finer granularity bands for spectrum sensing. When the number of subbands is increased, the spectral resolution of the filter banks gets increased, and better detection performance is achieved. The computational complexity can be reduced with efficient structures using polyphase filter bank discussed in subsequent sections. However, finer granularity bands increase the computational complexity of the filter bank structure used for spectrum sensing. In order to reduce the computational complexity, multistage filter bank structures can be considered. In the subsequent sections, we discuss multistage CMFB and multistage polyphase filter banks [9, 39].

## 8.1 Spectrum Detection with Multistage CMFB

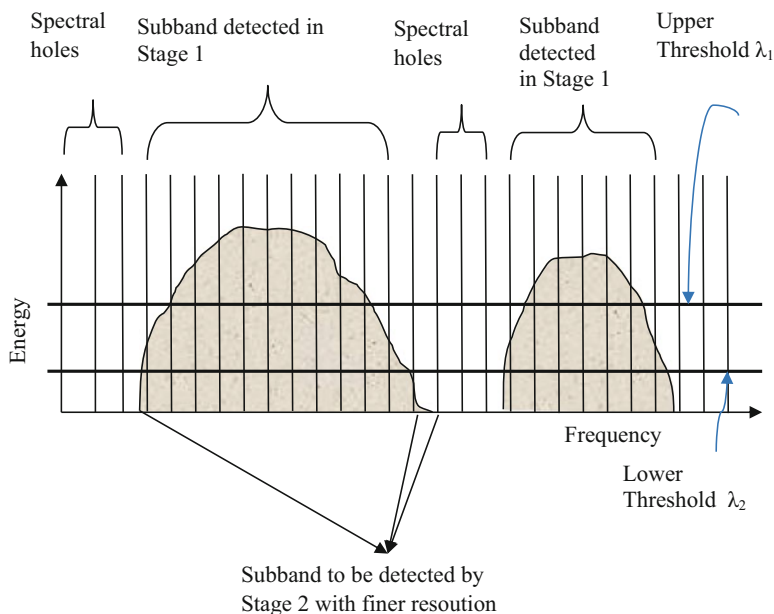
Better sensing performance can be achieved in filter banks with finer resolution. However, this would increase the computational complexity of the filter bank structure. Therefore, to overcome the computational complexity, multistage filter banks and tree-structured DFT filter banks are investigated for sensing from coarser to finer resolution [7, 8]. In multistage or multi-resolution filter banks, at the initial stage, the total bandwidth is sensed using coarser spectral resolution (smaller number of subbands). The bandwidth of interest is identified depending on the sensing decision, and only those frequency bands are further sensed with finer spectral resolution. Multi-resolution filter bank techniques include fast filter bank (FFB) based on frequency-response masking (FRM), coarser to finer spectrum sensing using wavelet transforms, and FFT-based multi-resolution spectrum sensing using multiple antennas [6].

In case of multistage filter bank spectrum sensing, the available bandwidth is initially divided into nonoverlapping subbands with coarser spectral resolution of  $M_1$  subbands as illustrated in Fig. 8. When narrowband users appear in wideband spectrum and the bandwidth of sensing is sparse as shown in Fig. 8, the subbands of interest can be detected in the first stage. The detected subbands can be sensed further with finer spectral resolution in the next stage with a spectral resolution of  $M_2$  subbands. As the narrowband users are identified in coarser resolution, only the detected subbands are sensed further with finer resolution. Therefore, the computational complexity is reduced, and better sensing performance can be achieved.

Multistage spectrum sensing can be performed by defining two thresholds based on different probability of false alarm  $P_{fa}$  depending on the channel conditions. Energy detection is performed using the predefined thresholds with different probability of false alarms, calculated as discussed in Sect. 3. Two thresholds,  $\lambda_1$  and  $\lambda_2$ , are the calculated based on different probability of false alarm ( $\lambda_2 > \lambda_1$ ). If the energy is above the threshold  $\lambda_2$ , it can be concluded as the presence of primary



**Fig. 8.** Illustration for multistage filter bank spectrum sensing from coarser to finer spectral resolution



**Fig. 9** Illustration of threshold decision with multistage spectrum sensing

user. If the energy is below  $\lambda_1$ , it is decided as the (absence of primary user) presence of a spectral hole or absence of primary user, and if the energy is between  $\lambda_1$  and  $\lambda_2$ , there is a possibility of spectral hole within the subband.

The multistage spectrum sensing with energy distribution and thresholds are explained in Fig. 9. Only the subbands having energy between  $\lambda_1$  and  $\lambda_2$  have to be sensed in the next level with finer granularity. Multiple spectral gaps can be identified in an efficient and flexible way using the multistage methods with reduced complexity since the whole band need not be sensed with finer granularity.

The significance of multistage spectrum sensing is summarized as follows:

1. The probability of detection is improved with finer granularity bands.
2. Multistage filter banks reduce the computational complexity as the whole band need not be sensed with the finer granularity.
3. Spectrum sensing can be performed from a coarser to finer spectral resolution.

## 8.2 Spectrum Sensing with Polyphase Filter Banks

Wideband spectrum sensing using filter banks has proved to be robust and efficient. Filter bank-based physical layer design for CR systems was introduced to perform simultaneous spectrum sensing and transmission. Filter bank techniques can reduce computational complexity and improve spectral analysis in cognitive radio

applications. For fractional utilization of spectrum, the center frequency and spectral edges of the primary user need to be estimated which can be done using polyphase filter banks. The complexity in filter bank implementation can be reduced to a large extent using polyphase filter bank structures. Polyphase filter banks are efficiently designed using FFT when the number of subbands  $M$  is a power of two. The problem of estimating the center frequency and spectral edges of primary users in a wideband spectrum using polyphase filter banks is discussed below.

### ***8.3 Multistage Polyphase Filter Banks***

Multistage polyphase filter bank method detects the presence of primary user and identifies the spectral holes. In addition, for fractional utilization of spectrum bands, the center frequency of the primary user can be estimated with higher precision using filter banks and subband-based energy detection along with centroid/center of mass method [8, 9]. It is well known that the detection accuracy depends on the number of subbands  $M$  in the filter bank. The computational complexity of the filter bank increases with higher values of  $M$ . However, the complexity is reduced by using multistage polyphase filter bank structure. The primary users are detected by computing the signal energy (power) at output of the individual subbands. The algorithm for the detection of unused spectrum (spectrum holes) starts from a coarser spectral resolution (smaller number of subbands) at the first stage to reduce computational complexity. Single user and multiuser scenarios are considered in wideband for spectrum sensing using multistage polyphase filter banks. The detection of single and multiple users in widebands is elaborated in the following subsections.

### ***8.4 Single User Detection in Wideband Spectrum***

Polyphase filter are useful in detection of narrowband single user in a wideband spectrum [9, 38]. For example, consider the detection of wireless microphone (WM) in the presence of a signal that follows IEEE 802.22 WRAN standard. In IEEE 802.22 WRAN standard, spectrum sensing has to be done to allow television (TV) services and wireless microphones to coexist. WMs are low-power licensed users and are allowed by Federal Communications Commission (FCC) to operate on vacant TV channels without causing interference. The detection of WM is difficult due to the low power transmission (typically 50 mW for 100 m coverage) and small bandwidth occupancy (200 kHz). In IEEE 802.22 WRAN standard, when a WM appears anywhere in the TV channel, the whole channel of 6 MHz has to be evacuated to avoid interference. However, TV channels can be utilized fractionally

when the exact position of the WM is detected [8, 41]. Hence, there are several challenges in the detection of WM signals/narrowband users.

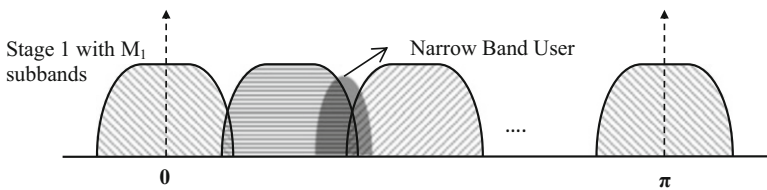
Multistage polyphase filter bank method can detect the presence of WM and estimate the center frequency of the WM with better precision by using the centroid method. The centroid-based technique can detect the presence of WM in the first stage itself, when spectrum of WM lies partly in one subband and partly in adjacent subband. When the WM is detected in the first stage, it reduces the computational complexity and latency and achieves fast sensing. However, if WM appears exclusively within a single subband, an additional stage is required to detect and estimate the center frequency of WM with finer spectral resolution. In such cases, WM can be detected in the second stage without ambiguity.

The multistage polyphase filter banks can be designed to detect the presence of WM anywhere within a TV channel (6 MHz) and to estimate the center frequency of WM taking into account the following scenarios:

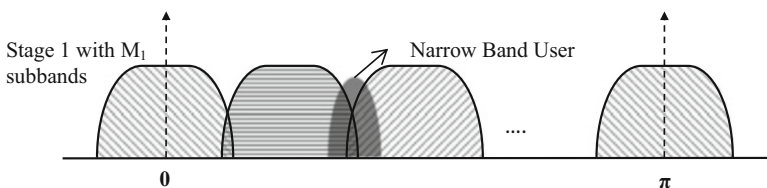
**Case 1:** If the signal spectrum of WM lies partly in one subband and partly in the adjacent subband as shown in Fig. 10, the center frequency of WM can be either in one of the subbands or between two subbands. The center frequency in such a case is estimated using the centroid method as described in the subsequent sections.

**Case 2:** If the signal spectrum of WM is in the middle of two adjacent subbands as shown in Fig. 11, the energy at the output of two subbands will be equal. That is, the center frequency of WM is at the midpoint of the two subbands. Therefore, finer level of detection is not necessary, which in turn reduces the computational complexity and latency.

**Case 3:** If the signal spectrum of WM appears exclusively within a subband as shown in Fig. 12, the output of first stage is passed to the input of the next stage filter



**Fig. 10** Case 1: Narrowband user appears anywhere between two consecutive subbands



**Fig. 11** Case 2: Narrowband user appears exactly between two subbands

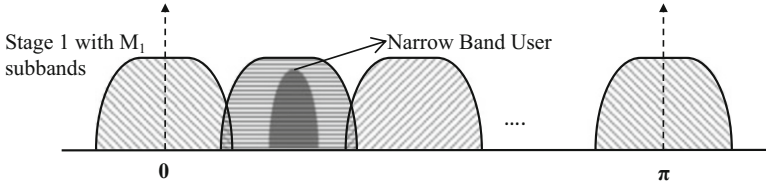


Fig. 12 Case 3: Narrowband user appears exclusively within a subband

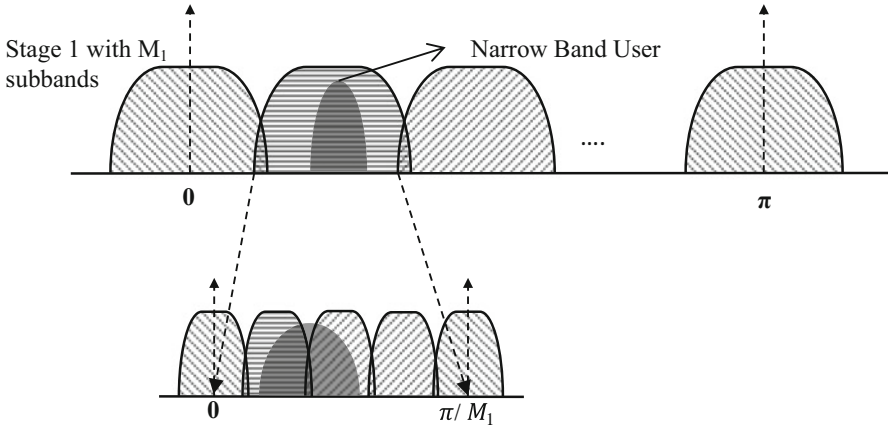


Fig. 13 Detection of narrowband users using two-stage filter bank

bank to estimate the center frequency with a finer spectral resolution. The process is illustrated in Fig. 13 where only two stages are required to detect the presence of the WM and to accurately estimate its center frequency. The center frequency is estimated in the second stage using DFT polyphase filter bank and centroid method.

The procedure followed for multistage spectrum sensing is briefed in the following two steps and illustrated in Fig. 13:

*Step 1:* The bandwidth of sensing is divided coarsely into  $M_1$  subbands and sensed through the  $M_1$  subband DFT polyphase filter bank. Energy detection is performed at the output of each subband, considering energy (power) as the test statistic to decide the presence or absence of the WM in the subbands. The detection and estimation of center frequency of WM as per Case 1 or Case 2 is done in the first stage. If the signal spectrum of WM is as per Case 3, the output of the sensed subband is further processed with finer resolution as in Step 2.

*Step 2:* The output of first stage is sensed in the next level with  $M_2$  subbands. The signal energy (power) at the output of the subband is considered as the test statistic. At this level, the spectrum is sensed with a spectral resolution of  $\pi/M_1M_2$ .



The proposed method can be summarized as:

- (i) If WM appear anywhere within consecutive subbands (Case 1 and Case 2), the center frequency of WM is estimated accurately using the centroid method in single stage.
- (ii) If WM appears anywhere exclusively within any subband (Case 3), the output of the sensed subband is further processed with finer resolution as in Step 2.

### 8.5 Center Frequency Detection Using Centroid Method

The center frequency of the narrowband frequency can be detected when the narrowband user appears between the two subbands. The center of each subband represents the energy in that subband resolution as shown in Fig. 14. The energies can be modeled as a trapezoid, and the center frequency can be calculated from the centroid of the trapezoid. The top edge of the distribution can be defined using a linear function as,

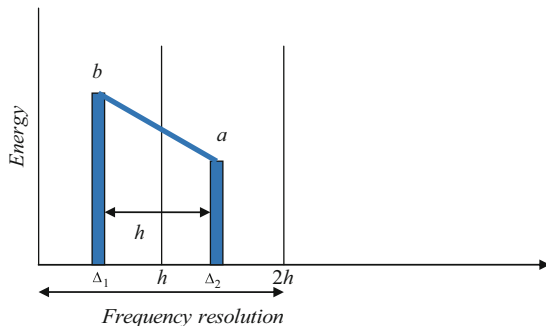
$$f(x) = b + \frac{x}{h}(a - b).$$

The area of the trapezoid is given as  $A = \frac{h}{2}(a + b)$ .

The centroid in the  $x$  direction is computed as

$$\begin{aligned} A\bar{x} &= \int_0^h xf(x)dx \\ &= \int_0^h x\left(b + \frac{x}{h}(a - b)\right)dx \\ &= \frac{h^2}{6}(2a + b) \end{aligned}$$

Fig. 14 Illustration of centroid method



where  $\bar{x} = \frac{h}{3} \left( \frac{2a+b}{a+b} \right)$ , and  $A$  is the area of the trapezoid. Here,  $\bar{x}$  represents the centroid of the trapezoid.

In case of equal energy at the output of individual subband, i.e., when  $a = b$ , the midpoint can be verified as  $\bar{x} = \frac{h}{2}$ . The center frequency of the narrowband user is related to  $h$  which represents the granularity of filter bank  $M$ ,  $a$  and  $b$  are related to the energies  $E_1$  and  $E_2$  of the adjacent subbands. The minimum of two subband energies are represented as  $a$ , i.e.,  $a = \min(E_1, E_2)$  and maximum as  $b = \max(E_1, E_2)$ . Thus, the estimated center frequency  $\hat{f}_c$  can be expressed as follows:

$$\hat{f}_c = \frac{M}{3} \left( \frac{2a + b}{E_1 + E_2} \right)$$

A generalized expression is obtained by considering the energies of subband  $E_i$  and adjacent subband  $E_{i+1}$ .

$$\hat{f}_c = \frac{M}{3} \left( \frac{2\min(E_i, E_{i+1}) + \max(E_i, E_{i+1})}{E_i + E_{i+1}} \right)$$

The centroid method provides better accuracy in center frequency estimation when the number of detected subbands is almost two. If the number of detected subbands is beyond two, the centroid method does not provide accurate estimation of center frequency. Since the energy distribution can no longer be modeled as a trapezoid, the top edge cannot be written as a linear function. In such cases, a center of mass method is used for estimation of center frequency.

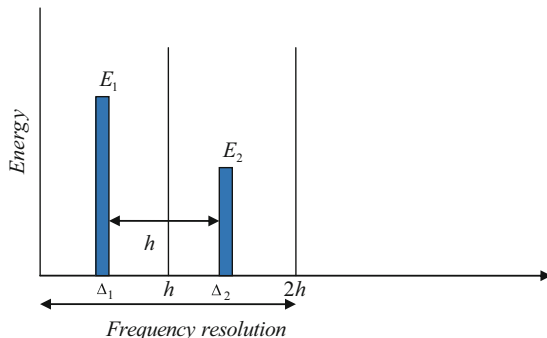
## 8.6 Center Frequency Detection Using Center of Mass

The center of mass method can also be used to estimate the center frequency. The energy of the subbands is related to the mass, and the distance is related to the frequency. Consider the energy in two subbands as  $E_1$  and  $E_2$ , let  $\Delta_1$  and  $\Delta_2$  be the center point of spectral resolution in the subband bins as shown in Fig. 15. The center frequency for the energy bins can be obtained using the law of center of mass as

$$\hat{f}_c = \frac{E_1 \Delta_1 + E_2 \Delta_2}{E_1 + E_2}$$

The expression can be extended for different subbands, and the center frequency can be calculated using the relation,

**Fig. 15** Illustration of center of mass



$$\hat{f}_c = \frac{\sum_{k=1}^n E_k \Delta_k}{\sum_{k=1}^n E_k}$$

Thus the polyphase filter banks using the multirate system can be utilized for wideband spectrum sensing in CR. The filter banks are useful for varied applications such as detection of spectral holes and estimation of center frequencies of different primary users. The polyphase filter banks reduce the computational complexity and provide efficient realization of filter bank structure for wideband spectrum sensing.

## 9 Conclusions

The multirate filter bank-based spectrum analysis is applicable for multiband spectrum sensing in cognitive radio applications. Different filter banks have been analyzed for spectrum sensing in CR such as FFT, cosine-modulated filter banks, and polyphase filter banks. From the performance analysis of the different filter banks, the polyphase filter banks exhibit more reliable and efficient detection performance. Polyphase filter banks take advantage of the low spectral leakage property of the different subbands in the filter bank structure, which enables them to enhance the performance of multiband spectrum sensing in cognitive radio networks. Moreover, the computational complexity of polyphase filter banks is reduced when compared with other filter banks. In addition, the same filter bank sensing architecture can be applied for transmission and reception of signals in vacant subbands using channel adaptation techniques.

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