Chapter 9
The Role of Culture and Ecology in Visuospatial Reasoning: The Power of Ethnomathematics

Kay Owens

Abstract Reasoning is critical in mathematics but research has shown that people reason not only with numbers and algebra but also visuospatially (also called spatial thinking and visual reasoning). This kind of reasoning occurs with representations but also in everyday living. In other words, ecology and culture impact on mathematical reasoning and learning. To investigate this premise, Papua New Guinean Indigenous cultures and their mathematical activities were appropriate as they are rich in visuospatial reasoning. There is evidence of the strength of this reasoning and that it may or may not be associated with words. The challenge is how to enlist this cultural strength in schools.

Keywords Visuospatial reasoning · Culture · Ecology and mathematics · Mathematical activities · Ethnomathematics in schools

9.1 Introduction

Mathematics education has its fair share of controversy. This chapter discusses two of these. The first is whether visualisation can play a role in mathematics and be evaluated. The second is whether mathematics is universal. In 1992, the International Group for the Psychology of Mathematics raised the former. Visual imagery and visualisation were hot topics. There were discussions around whether visualisation was internal or external (Goldin 1998).

Needless-to-say, both were regarded as feasible and important but how could you assess visualisation? What role does it play in mathematics? Imagistic processing, that is visualising, played a critical role in problem solving along with symbols, language, affect, and heuristics, all interacting (Goldin 1987). Spatial abilities (Eliot 1987; Tartre 1990) and visual imagery psychology (Shepard 1975)

K. Owens (✉)
Charles Sturt University, Bathurst, NSW, Australia
e-mail: kowens@csu.edu.au

© Springer International Publishing AG 2017

M. Rosa et al. (eds.), *Ethnomathematics and its Diverse Approaches for Mathematics Education*, ICME-13 Monographs,
DOI 10.1007/978-3-319-59220-6_9

were being drawn together, generally as a result of social considerations pertaining to education.

One early paper illustrated that proof can be established through visuospatial reasoning (Dreyfus 1991). Dreyfus argued that the validity of a proof is judged by experts who know the topic and that visual representation is an important part of deductive reasoning. He illustrated this with the problem of finding points with the sum of the distance to two intersecting lines being a constant given length. An illustration of this proof is given in Fig. 9.1.

He visually but logically explained the proof based on the diagram. This visual explanation can be dynamically illustrated either with a program like Geogebra or with a string that can slide on two perpendicular T bars that move along the given lines. Whether dynamic or static, the visual information is essentially used in a valid analytical argument. Since then the whole area of dynamic geometry software has been readily accepted as a visual means of proof. Figure 9.2 gives the example for the formula of a triangle as ½ × base × perpendicular height in which vertices at C are on the line *e* and perpendicular to the side AB; hence the perpendicular height is the same in all triangles with the common base of AB.

If a figure does not *mess up* when pulled, then properties of the shapes have been used to construct the shape. It is also a way, through measurements, to show that changes in shapes do not necessarily affect other properties.

In 1992, the importance of visualising was being described by adults and children in solving problems such as those involving:

(a) Pentominoes: how many shapes can be made with five squares joined by their sides, describe them in terms of symmetries and their possibility of being a net for an open cube,
(b) Tangrams: to make squares and explore the area of different combination of shapes in terms of area of the smallest triangle,
(c) Match stick designs and modifications: requiring embedding and disembedding shapes, and

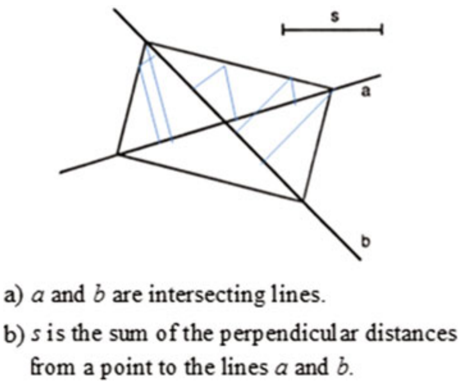


Fig. 9.1 The sum of the lengths of the perpendicular lines from each point to the lines *a* and *b* are equal. *Source* Based on Dreyfus (1991)

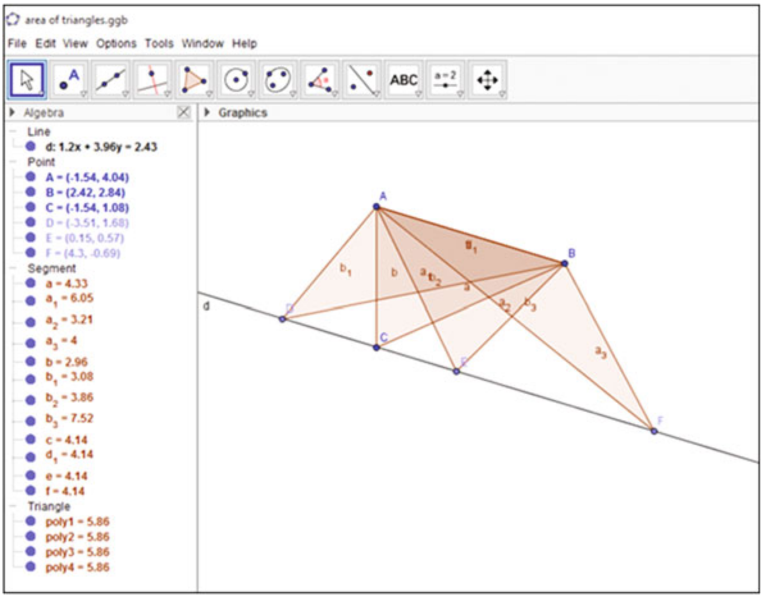


Fig. 9.2 Dynamic geometry image illustrating the area of a triangle formula (from *Geogebra*). *Source* Personal file

Fig. 9.3 Modifying pentomino *shapes* to create new ones using visuospatial reasoning. *Source* Personal file

(d) Pattern blocks: the size of their angles and those of tangrams, making enlargements.

At times they were able to capture their thinking in words but at other times there were systematic movements with the materials and both similarities and differences in the ways that people solved the problems (Owens 1992a, 1992b, 1993). It was, as Skemp (Campbell-Jones 1996) said, "out there on the table". During the pentomino problem one child said "in my mind, I pictured my hand moving the pieces around the shape" illustrating how she was mentally moving the last square around the line of four squares (see Fig. 9.3).

A close analysis of the videotapes of other children's work on the pentominoes suggested a similar tactic was being used, children were modifying one shape to form another (Owens 2015). First they often had to decide what is a shape? At first some were unsure of the question as they could not make triangles, rectangles or squares—that was the limit of their idea of a *shape* due to labels being the only emphasis in schools. Some initially considered that only familiar, symmetrical shapes for which they had a name would satisfy the word *shape*. Gradually they realised that they needed to consider other shapes and at that stage, they began modifying shapes in their head momentarily before trying something on the table.

Visuospatial reasoning was involving children in different kinds of visual imagery: holistic/pictorial, dynamic, action, pattern, and procedural visuospatial reasoning (Owens 2015, pp. 40–59). These types of imagery were first suggested by Presmeg (1986) from a high school study of children doing algebra. In brief, holistic/pictorial mental imagery are images generally of one or a few discrete representations of a drawing or object. Dynamic imagery is like the computer changing a shape or like transforming a shape such as from 2D to 3D shape or by rotating, turning, folding, or reflecting in one way or another. Action imagery involves bodily movements and may be thought about or embodied in various ways. Pattern imagery is where the pattern of the arrangement of the image parts is fairly dominant. If the imagery is well established then procedural imagery is simply following the steps or movements directing the changes to the imagery (Owens 2004; Owens et al. 2003). It is an efficient step in that the processing has been internalised.

This is similar to the idea of reification of the concept (Presmeg 2006) or the advanced structuring level of Pirie and Kieren's (1991) model. This model showed that understanding had a core of primitive knowing and image making, image having, and property noticing in order to formalize a concept with the support of imagery. Thus one important aspect of visuospatial reasoning is the diversity of types of visual imagery that help problem solving in a diversity of ways. Rivera (2011) has also emphasised the diversity and nature of visuospatial reasoning used in school learning indicating its importance in the agenda of schools.

Earlier training studies (Owens 2015, Chapter 2) suggested that visuospatial reasoning could be engaged through training, and even short training if pictures are not present in one's background to understand pictures (Bishop 1983). Recently there has been a renewed interest in visuospatial reasoning such as its malleability (Sinclair et al. 2016; Uttal et al. 2013). Thus teaching can improve this capability.

The second controversy was about the universality of mathematics. This chapter contends that people develop mathematics and, as Dreyfus (1991) suggested, mathematical visual reasoning is accepted as proof by informed people. To explore this further, everyday use of visuospatial reasoning of a differentiated population was investigated. Papua New Guinea has such a population and so an appropriate population in which to explore, with PNG colleagues, visuospatial reasoning which appeared to be important in everyday mathematical decision-making.

9.2 Research Project

The purpose of this project was to draw together many years of research to answer the question of how visuospatial reasoning is used in cultural mathematical activities in Papua New Guinea. A review of the literature of mathematical activities of PNG cultures and similar cultures in the Pacific in particular alerted me to understanding how visuospatial reasoning would be used in PNG. In particular, visuospatial reasoning was inextricably intertwined with cultural beliefs and practices (Owens 2015).

The research involved both ethnographic studies and questionnaire studies, and interrogation of 250 written reports of cultural mathematical practices by tertiary education students who were familiar with their cultural practices or undertook their own ethnographic studies. The questionnaires particularly asked about measurement practices and language using open-ended questions. These were collated and results from the same language group were compared and collated. Respondents often talked about time and location as well as area, length and volume.

Data from neighbouring language groups were also compared and focus groups were held to check the key ways in which measurement was being used. The use of visuospatial reasoning was part of these discussions although we talked about this topic as "how are people thinking to make those decisions?" and asked clarifying questions such as "What do you mean by measuring a garden area 'by leg'?".

In addition, we (Wilfred Kaleva, myself, and other interested colleagues with whom I was travelling) carried out interviews out of their places at the university or in towns. These were semi-structured interviews of people whom we knew had participated regularly in cultural activities, taking care to interview both coastal and highlands people.

However, most of the data came from ethnographic studies in villages. To get to villages in PNG, one often has to fly and/or walk, and sometimes take muddy roads in the backs of trucks or a dinghy or canoe for several hours. Villages generally have no power and no reticulated water; people have food gardens and sometimes cash crops, and use the bush and/or sea, river or swamp for food. Although we talk about all PNGians as Melanesians, with languages that are Austronesian Oceanic or Non-Austronesian (from multiple Families and Isolates), it should be noted that there are 850 languages and hence cultures in PNG.

Overall these cultural practices vary considerably although there are similarities and often cross-cultural marriages among neighbouring language groups. I was able to draw on 15 years of living in PNG and many village visits, yarning (talking, discussing) and videoing, during that time and subsequently. The particular ethnographic studies for the measurement project were carried out with educators who spoke the language or neighbouring languages and who had previously undertaken ethnomathematical studies. In particular, I want to acknowledge Charly Muke, Rex Matang, and Serongke Sondo. We also speak the ubiquitous lingua franca Tok Pisin.

All visits were negotiated with the community, generally in advance but if that was not possible due to transport difficulties, on arrival. We stayed in village huts. We observed and took videos of people going about their daily activities and often asked questions to see how they were thinking or why they were doing particular things. We particularly observed and listened to Elders, generally in a group, who might have demonstrated some of their activities if they were not in the process of building a house or making a canoe or another object. We sat and yarned for hours. We joined in activities where we had the skills. We walked along tracks with villagers, visiting gardens and other significant places.

From this rich data, I explain in this chapter what is meant by visuospatial reasoning in cultural contexts and how significant it is to mathematical activities and mathematics. It is an important mathematical thinking process that is under-utilised in schools and under-recognised in mathematics education especially in assessments that use paper-and-pencil tests.

9.3 Explaining Visuospatial Reasoning

The combined describing word *visuospatial* indicates that visual and also spatial sensing, perception and imagery are involved. The spatial aspects include the kinaesthetic and embodied sensing, perception and imagery that comes particularly with movement. Visuospatial reasoning is also about spatial relationships in and between objects, figures and positions. It involves spatial abilities such as visuospatial manipulations, alternative perspectives, and re-seeing in different ways including holistically, by completion, and in parts (Tartre 1990). Some spatial abilities such as efficient image rotation, image integration, adding detail, and image scanning but not image generation time as well as visual memory appear to assist visuospatial reasoning (Poltrock and Agnoli 1986).

Visuospatial reasoning involves generating images and generating representations, analysing and modifying both in an empirical way, predicting spatial change, and change in other fields represented spatially. It is about reasoning logically and visuospatially, making judgements, sometimes instantaneously or imperceptibly or by gazing (Mason 2008), but most importantly through applying knowledge and prior experience. Lohman et al. (1987) said "spatial ability may not consist so much in the ability to transform an image as in the ability to create the type of abstract, relation-preserving structure on which these sorts of transformations may be most easily and successfully performed" (p. 274).

Knowledge and experience are two areas that are most affected by culture and ecology whether influenced by technology, home, school, foundational cultural practices, activity, modelling, language or observation. For example, Jodie (a pseudonym) for whom English was a second language had realised that when the teacher said "bigger angle" that she was not referring to a sharper angle but to the larger angle thought of as the arms moving further apart (marked by thumb and finger) of the shapes. She later explained this as the arms being "more spread out".

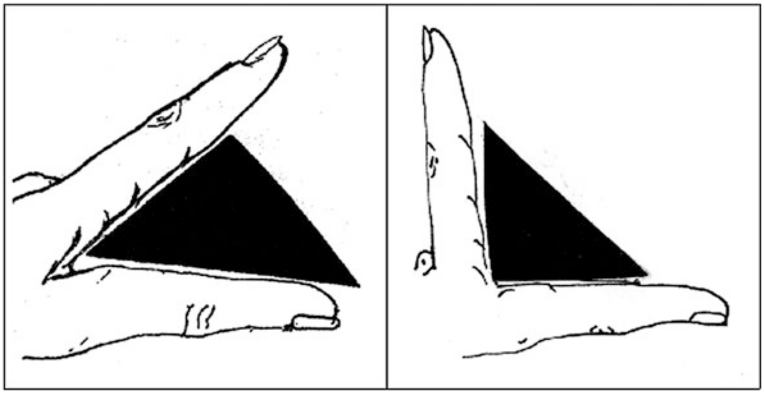


Fig. 9.4 A child's visuospatial reasoning about the size of an angle. *Source* Personal file

Her experience with the materials and background learning English at school influenced her visuospatial reasoning (Fig. 9.4).

The use of the fingers being spread apart as part of the description of angle size was engaging an embodiment of angle for visuospatial reasoning.

9.3.1 Spatial and Embodied Reasoning

Embodied knowledge is important, for example, for those who are used to change gears and turn a corner in a car, remember a phone number by touching the numbers on the phone. It is also known by those who feel the motion of water on the hull of a canoe, or the pull of a string from a flying kite or a swimming fish, or even the order of movements to create a string figure ('cat's cradle'). These pursuits are embedded in cultural contexts as illustrated by Fig. 9.5.

Sensing the feel of the swell of the sea may be learned by lying in the hull as well as by paddling and being out on the canoe feeling the wind and noting the impact on the sail also helps generate embodied visuospatial reasoning. Thus selecting the angle of a paddle, setting the position of the outrigger of a canoe, knowing the distance between places by the amount of time experienced by the body in moving between the places, assessing angles and slopes by gesturing with the hand, stretching out arms or parts of arms to assess lengths, will all be spatial decision-making times about objects in space, supported visually.

Comparisons made from one time to another are much easier when spatial, embodied reasoning is practiced (Owens 2015). In some traditional Indigenous communities people assess slope of the troughs made from the sago palm bark or the volume of water needed for sago processing (Fig. 9.6).

Fig. 9.5 Embodied cultural activities: sailing and paddling a canoe and making string figures. *Source* Personal file

Fig. 9.6 A PNG coastal activity: Sago processing; a PNG highland's approach to making a *mumu. Source* Personal file

The right amount of steam to cook karuka nuts in a dirt *mumu* (oven) is also estimated by visuospatial reasoning as they consider the size of the mumu and the amount of heat from the stones and the amount of steam escaping as they pour the water through a hole onto the hot stones (Fig. 9.6).

9.3.2 Experience and Estimation

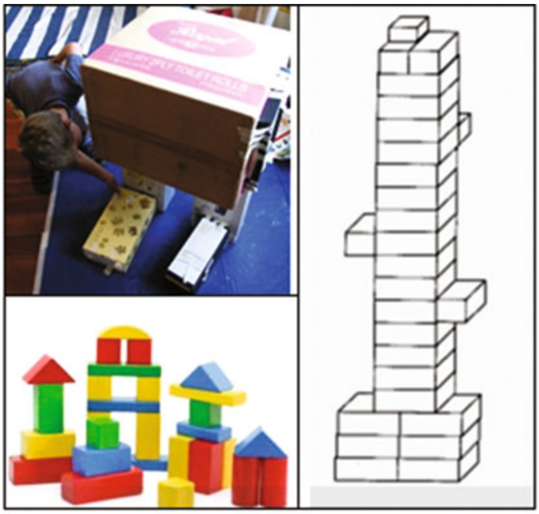
Visuospatial reasoning is not unknown in western, especially farming, cultures. Repeating activities over time leads to greater efficiency and estimation in a form of informed scientific trail-and-error approach. Farmers might consider the amount of dryness and growth of plants to determine the number of cattle or sheep or length of time for them to graze on the paddock.

In three-dimensional play, children spatially sense how their block towers are balancing—examples are given in Fig. 9.7 (the sketch is from Ness and Farenga 2007). They use visuospatial reasoning through rotating and joining objects to create their play ideas. Children learn through play how to move their arm and fingers to hit a marble. Children also move in a confined space to solve spatial problems (Lábadi et al. 2012). Children use geometric ideas such as corners and distances to solve spatial problems. Children do not necessarily require language to explore these spaces and make decisions (Learmonth et al. 2008).

Many cultural activities in Papua New Guinea entail mathematical visuospatial reasoning to estimate and balance objects (Owens 2015). For example, when a group of men are building a house from bush materials, they will make many judgments such as the slope of the roof, the area of grass for thatching with kunai, the selection of trees for 'rafters', or the number and size of limbom palms to split for the sago-roofing panels or for floors or walls.

Figure 9.8 shows men modelling the angle of the roof to ensure it is steep enough for the rain to run off but not too steep because then the kunai thatching will fall off. It also has to be appropriate for an internal wall the height of a man's hand above his head and with side walls as high as his shoulder.

Fig. 9.7 Block play. *Source* Personal file



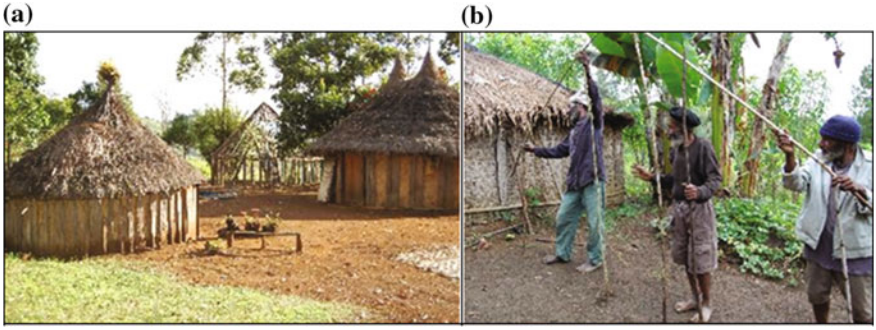


Fig. 9.8 Visuospatial reasoning for house building in Papua New Guinea. *Source* Personal file

In houses on the coast, sago-leaf roofing requires estimates of the size of sago trees to be cut down to supply the roofing along with the number of tulip trees whose inner bark is used to make rope. In addition, men will select bamboo lengths to split for floors or pitpit lengths for weaving for walls (see Fig. 9.10).

Visuospatial reasoning may begin with visualising lengths in the vertical that will become sloping or horizontal lengths in house building. In other cases there is a connection between lengths and areas or volumes. For other cases such as the use of palms, the overall size of the tree indicates whether it will be sufficient or not for the manufacture of roof panels or walls. In these cases, it seems there are ways of visualising these sizes associated with lengths. This is understandable if, for example, garden widths are of a fixed width or shapes are consistent (e.g. rectangular, square or trapezoidal as I have noted in different villages).

However, some highlands people have also added the length and width for a garden area but the decision on gardens is linked to more important matters such as quality of the soil, distance from the village, relationship of the people in the family or clan to the distribution of land, etc.

In terms of measuring the volume or mass of pigs which are important exchange and feast items, people might regard the count as more significant or they may use one or more length measures in their discussions of size (Fig. 9.9).

In most areas, two small pigs are just deemed as equal to a large one or the pigs are lined up in size so marking them off and noting the number and size for equality. Nevertheless, some cultural groups keep a track of the lengths of exchange pigs on a long rope which they add to as exchanges occur.

Some people's sweet potato (*kaukau)* mounds for exchange may be carefully piled into a cone shape and compared using a stick to measure height and rope for the circumference of the base of the cone. Again the complex relationships associated with the feast or exchange are more important than the actual size comparison. Hence various systems have been developed to provide rough estimates of equality.

Fig. 9.9 Lining up pigs for important exchanges in Papua New Guinea highlands. *Source* Google Images

Fig. 9.10 Weaving of walls in different ways in a PNG coastal village, a woven basket with lid from pandanus leaves by coastal Keapara Elder in PNG, and a basket and small purse from Timor Leste. *Source* Personal files

Figure 9.10 shows an example created by a Keapara elder. Her mental imagery of the pattern and how it will be created in terms of the width of the pandanus which is split into narrow sections to create the coloured finished pattern.

This process involves considerable estimation as the initial length depended on the size of a cereal box around which the pattern was created. It has an inner lining of wider strips similar to the base. In Timor Leste, women will also judge sizes to make a tight fitting lid on a small purse or on the basket. The hexagonal design is also carefully created for the particular basket size.

9.3.3 Estimations and Decisions Reflecting Ratio

Experience of creating with bush materials has led to some people having special skills in estimating the result of a raw product being used. Rather than thinking of the number of additional posts for a house, it is seen as half as much again—ratio thinking. This leads to how much more roofing, walls, floor, and posts; how much help will be needed; and how many gardens for a feast to recognise assistance or status. Thinking visuospatially in ratios does not require measurements and calculations.

However, not all amounts of the various items for the house are increased by half as much again. For example, the back and front walls, floor and roof might be half as much again but the side walls will not. For a nine post house, half as much again requires three more posts. If the roof slopes from the sides as well as the front and back, this will impact on the total amount of roofing required. Thus the sections of the roof are considered separately.

If the side slope was to half way along the length between posts then the overall increase can be set as the same amount for the side roofs and front and back to that point but the middle section is doubled as shown in Fig. 9.11.

In the case of a roof going up to one central point as in a curved ended rectangular house or round house as built in the highlands, the building's size will have an effect on roof and rafter lengths, reflecting trigonometry ratios. Figure 9.12 shows examples of men modelling these kinds of houses. Since the model was smaller than the example and since these houses could vary for family units or larger men's houses, certain aspects need to be considered. In practice, lengths can be shorter or longer to reflect these ratios.

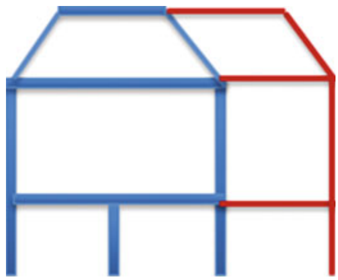


Fig. 9.11 Front view of a three × three post house extended in length by half to be a four × three post house. *Source* Personal file

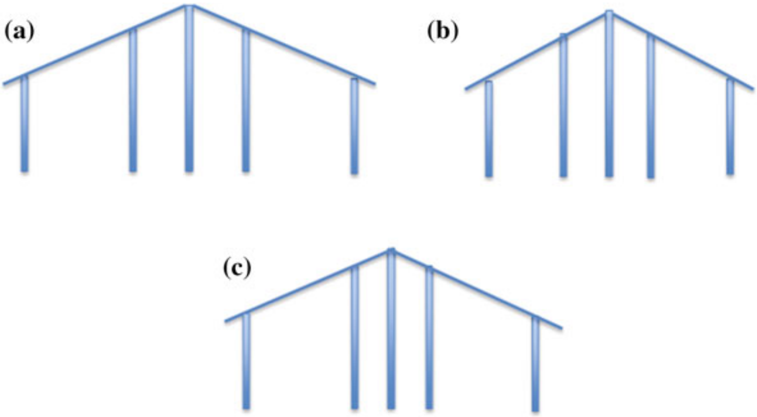


Fig. 9.12 **a** A highlands house front and **b** The model house with same lengths but larger angle on the roof or **c** The smaller middle section. *Source* Personal files

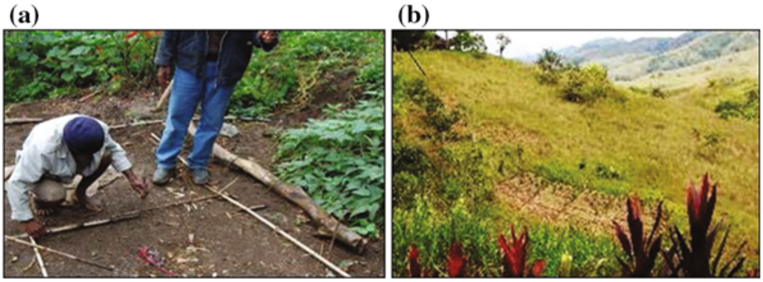


Fig. 9.13 **a** Measuring half way with an available stick, took the middle mark too far and **b** Tessellating area units are seen but not recognised as a measuring unit until pointed out. *Source* Personal files

Figure 9.12a illustrates the full house and then the model but the men kept the wall height equal to their shoulder height and the internal wall height to the raised arm height. They discussed whether the roof would now be too steep and whether they would have to compromise with the interior wall and overall height. They could see that the angle (unmeasured) would make the roof steeper.

The compromise for the same angle was to reduce the floor spacing from thirds for the outer sleeping areas and the middle talking areas so the middle section was slightly smaller as shown in the third house design. It was also possible for the width of the house to change allowing more people to sleep in the two ends. Each end space marked by the internal walls was divided into two and in Fig. 9.13a, the men are finding this halfway mark (see section on Geometry Knowledge below).

A further aspect of ratio was linked to the volume of a round house. If these houses are larger as in the background of Fig. 9.8a, then the ratio of the amount of materials needed for the walls, the rafters, and kunai for roof together with labour is associated with the size of the radius and circumference of the base circle. In fact, so strong is the aspect of ratio and labour that a person also has to consider the rate associated with the size of the food gardens needed to provide a feast for the helpers.

However, another significant aspect of the base of a house and its volume is that a large house also needs more warmth. A small house is not so cold with a little fire and one person, said one Elder. Similar ratio decisions associated with thanking labourers is associated with the size of a *mumu* pit for in ground cooking. Its shape might be partly determined by the number of pigs required but also for other foods like sweet potatoes.

9.4 Mathematical Conceptual Knowledge

While visuospatial reasoning is evident in processes such as mathematical problem solving, it is also evident in conceptual understanding. In particular, it is evident in geometry and measurement knowledge and in a sense of number. Relationships between numbers is particularly strengthened through visuospatial reasoning, a basis of Les Steffe's early work on early counting (Steffe 1991).

9.4.1 Geomtry Knowledge and Visuospatial Reasoning

In Fig. 9.13 to decide the centre of a line joining two points, a man took a long stick and marked from both points where the end of the stick fell. This made it easier to estimate the middle point even though the chosen length was more than half. The ends of the stick were recognised as equidistant from the end points and the middle point as shown in Fig. 9.14.

In house building, different groups of people know that for a rectangle, not only the opposite walls are equal but the corners must be right angles. Sometimes, people and sticks are placed in a line along the direction of the two arms of the right angle and the person at the corner has a good sense of the straightness of the line of people and whether it a right angle. They do it *by eye* was a common response to this visuospatial approach.

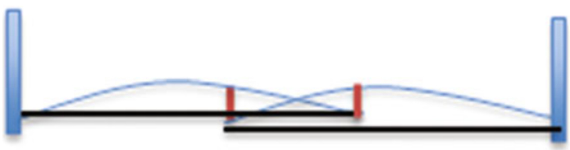


Fig. 9.14 A stick (*horizontal*) is used from both ends (*tall sticks*) to mark new points (*short sticks*) nearer the middle. *Source* Personal files

Others used the fact that not only the opposite sides have to be equal but the diagonals are also equal. For example, one man explained that, when he was a boy, he was in the way of his father measuring the diagonals with a rope. He immediately visualised that if the diagonals were different lengths, a parallelogram would be made. This lived experience with associated visuospatial reasoning was strong whereas in school, shapes are just labelled.

For a house on posts, the posts will be marked in one row equally spaced, then equal lengths will be taken to place the next row of posts. Checks are made by eye and with equal length sticks and ropes. Folding the rope for the full length of the house into three will position the centre row of posts. Equal lengths were known to form squares so each post was on the vertex of a square. When planting cash crops, two sticks are used to get equilateral triangles.

Only two sticks are needed as the first row is placed equally spaced and then the sticks are used for the other two sides of the equilateral triangle. Short sticks are put in the ground to check all the holes will be in the right position like a map. The pattern of tessellated triangles is checked out with straight oblique lines. “It is a beautiful pattern of triangles” said one villager.

Where round houses are common (Fig. 9.8a), people know a lot about circle geometry and volumes of cylinders and cones. They know about segments, equal and unequal lengths across circles, and different angles subtended by different chords of a circle. People know how much will spread out on a platform that is a segment of a circle, how much space will be available for sleeping, how much for drying nuts and storing belongings on the rafters at the top of the cylindrical part of house.

In one student’s project he illustrated and described the use of two ropes to make an isosceles right-angled triangle. The shorter rope had two knots at the ends and the other three with one knot bisecting the rope with two knots at the ends. The first rope was the length of the hypotenuse if the other rope formed the other two sides with the knot at the vertex. Without knowing Pythagoras’ theorem, this had become the practice to form right angles in his village. Another student’s project noted how walking parallel to each other from one path to another when the paths were not at right angles and the two people walked away from the path but not perpendicularly, that a trapezium was formed. Despite the sides formed by the walking not being drawn, the student could visualise the overall shape.

9.4.2 Visuospatial Reasoning with Numbers

Visuospatial displays are often associated with the size of number in PNG as this is part of reciprocal exchanges. For example, Paraide (2010) showed some of the display of “bride price” exchange at a marriage. Interestingly, these bundles and rings are Fig. 9.15.

This sense of large numbers is significant in terms of estimates and values. People immediately recognise the amount in the displays while the displays

Fig. 9.15 Paraide's (2010) study of shell money in fathoms displayed in bundles and rings of 100 and 150 fathoms. *Source* Paraide (2010)

themselves add value and respect to the exchange. In addition, generosity is evident when the fathom length is generously larger than it should or used to be. The groupings involved in each display are also significant. For example, they can be in 10s or 100s. In addition, there are displays of bunches of bananas and baskets of food like sweet potato, taro and green leaves.

Counting is often accompanied by gestures. For example, people will often use the right hand to bend down the fingers of the left hand from the little finger to the thumb as they count 1, 2, 3, 4, 5 or the same of pairs or tens. Body-part tally is another visuospatial display in which parts of the body are used to signify numbers in order from the little finger up the arm and across the head and down the other side (Owens et al. 2017; Saxe 2012). The particular parts varies from language to language (Dwyer and Minnegal 2016).

Visuospatial reasoning is evident, for example, by pointing to 10 on their body when they tally with body parts especially when decimal counting and currency arrived in the village. How far a larger number is past the ten is quickly associated with the appropriate body part and it can also be a combination of smaller numbers with practice. Saxe (2012) also noted that the complete group for the Oksapmin had changed from 26 to either 20 or in a few cases 30, still represented by a body part as part of the body-part counting system.

In other languages, whole complete groups or groups of 4, 5, 8, 10, 12, or 20 may be accompanied by an action or object grouping. For example, in Hagen, people count to four on one hand, up to eight on the other and then slap the fists together for the complete group of 8. If they want to then count to 10 they will bend down the thumbs or in Gawigel dialect wipe these on their lips for the complete group of 10 in a combination of two counting systems with different cycles, the cycles of 8 dominating at least to $8 \times 8 \times 8$. In Waghi, people point to different body parts to represent different hundreds. When counting items, men will often stride down the lines of pigs or gifts gesturing to the items as they count. In Hagen and other areas, one person will to ten while another man keeps track of the tens.

9.4.3 Visuospatial Reasoning with Measurements

While a unit of volume or area is not specified, represented, nor generally used in these many different language communities, people do see and make area units in woven objects or garden plots. For example in Fig. 9.12b, the drains in the garden show area squares. These are recognised quickly to indicate the size of the garden through visualising and comparing with other gardens made with similar drainage systems and squares at another time or place. In another community, the gardens may be long rectangular shapes but often they are also marked off with a tankard bush or banana for every group of plants (5 or 10 or 20 depending on the plants, garden or culture). The number of plants in a garden is quickly estimated by the space they take up.

For example, people will know how many plants in a nursery by just looking or in the cash crop plot. They can estimate how many coffee bean kilograms they will make from a tree, or balls of string from a particular size tulip tree. Similarly, women estimate how wide to make a *bilum* (continuous string bag—see images in Owens 2015) for the number of balls of string they have. When sailing, men know how far they are from land by the birds they see as some birds will fly further from land than others but the winds and swell will be taken into account in estimating the amount of time it might take to reach the land. The time is not measured in hours but in the change in position of the sun. Similarly, distances are imagined in terms of known distances especially when walking.

Interesting measures or comparisons are used for making traps. The fence to direct an animal inside needs to be of a specific height and length and at a slight angle. It has to be positioned on an animal track recognised again visuospatially. Similarly if a noose trap is to be made the parts and weights need to be well devised. While part of this may be through trial and error, nevertheless the actual design requires a strong visual image. The same can be said of the making of a canoe. The prow has a certain shape but no drawings and often no measurements are made. They are made and modified according to need and available trees. Even the selection of trees from the supply on one's land requires carefully remembering of the types, sizes, shapes, and positions of many trees and estimates of their future growth and use. People feel to get the thickness of the hull correct. They estimate the positioning of the outrigger visuospatially but they will test this for balance and make adjustments to the ties.

9.4.4 Position

Grid maps are not available in the bush or ocean settings of Papua New Guinea. Nevertheless, people can find their way through the jungle or across the seas. As in many island nations, the stars, swells, birds and the position of the sun during the day assist the navigators on the sea. There are multiple ways of telling position include time taken to walk, slopes of the land, particularities of plants and natural features. Reference lines such as a river or coast are used and various places on these are demarcated so that one can find a place considered in terms of distance from the river on the left or right looking downstream. Often places are denoted as a specific area, for example, quadrants of a valley or areas with different vegetation. The source and flow of the river are noted. Tracks that are less frequently used are noted by slight foot marks or cut branches where an earlier hunter has passed through and cleared the track of overhanging shrub. Songs and stories of the man whose track it is are recited en route connecting activity with specific features of the landscape. From a young age, children can find places in the bush to gather food such as nuts, fruits and mushrooms. They reason about the features of the environment as they search for them having learnt where certain things are likely to grow.

9.5 Visuospatial Reasoning and Decision Making

One might ask how decisions are made if measurements especially with area units are not counted or calculated. Although the garden plots in the highlands may be square or rectangular, there are many that are trapezium shaped while on the coast, much is planted more opportunistically such as where an old tree root might help to hold the soil in the heavy rains. Visuospatial memory of different areas with multiple features such as quality of soil and distance from the village are taken into account.

Decisions are around sharing according to customary relationships and expectations and the people who are involved in the discussions may use length measures as part of the discussions, comparisons of lengths, or visual area comparisons. Similar comparisons as mentioned above under ratios are made for a range of decision-making requirements.

9.6 School Programs to Maintain Cultural Visuospatial Reasoning

Many participants thought that there was no connection between what was done at home and what was done at school. The languages were different, explaining school mathematics in home language or the lingua franca was not easy. If schools brought these common knowledges into the classroom with models, children's geometry knowledge would be far advanced of the linear trajectories of western school mathematics, often dominated by labelling shapes. The following examples illustrate this point:

- Two ropes used to form a right-angled isosceles triangle, one rope for the sum of equal sides so the midpoint becomes the vertex with the right-angle, and the other the hypotenuse. These ropes can then be used for right-angles in other situations.
- The physical walk to make parallel lines with a stick held between the two people. Other straight lines such as paths cutting across the parallel lines will form a trapezium (non-parallel paths) or parallelogram (parallel paths) or rectangle (parallel, orthogonal paths).
- The use of equal sticks or half rope lengths to form joined squares.
- The use of equal diagonals to ensure a rectangle rather than a parallelogram.

These are new introductions to these specific shapes based on well-established visuospatial reasoning from cultural practices rather than static proto-type images that might be misleading (e.g. all triangles are equilateral triangles with a horizontal base). Furthermore, these activities also emphasise properties that might not always be noted in school. This practical approach to school mathematics requires a model of teaching that incorporates culture.

Two programs in PNG harness these cultural ways of thinking mathematically: an elective at the University of Goroka on *Mathematics, Language and Culture* for secondary teachers (Owens 2014) and an inquiry approach in elementary schools (Owens et al. 2015). The inquiry model brings these village activities into the classroom to ensure that school mathematics is built on this background knowledge and visuospatial reasoning.

9.6.1 Teacher Education

The former project involved secondary teachers in preservice or inservice courses to establish the cultural mathematics in a particular activity of a community, generally their own, and to detail the activity and the mathematics involved in the activity and to link it to the syllabus for Grades 7–10 or 9–12 indicating how the mathematical thinking involved in the cultural activity would support school mathematics learning of various topics. It was clear from the reports how proud students were of their Elders and community members who shared the details of the activities with them indicating how this activity not only engaged them in self-regulated learning but also in establishing their mathematical thinking identity from their cultural identity (Owens 2014).

One teacher told the seasons by where the sun rose over the mountain range to the east of the village (PNG is just south of the equator). A secondary teacher used the curved vines that looped between the hand-rail suspension vine and the walking platform of a suspension bridge as a metaphor for a sine wave, an adequate image of the sine wave. Another student used a diagram of the framework for a wig-headdress to consider a parabolic curve using the width of his finger as a unit of measure, although again it was hard to fit the parabola to the curve of the wig.

This beginning grasp of the school concepts came with considerable cultural and mathematical pride and was a good image for a teacher to create in preparing for what was to be a school mathematical shape.

9.6.2 An Inquiry Model of Teaching in Elementary Schools

The inquiry model for the inservice elementary teachers was developed to be used over a week of five one-hour lessons. It specifically took the cultural mathematics as a starting point for engaging beginning school students. The outline of the inquiry process is based on Murdoch (1998). The purpose is for children to think and do mathematics through activities linked to cultural practices but extended to school expectations, to go further with the idea and connect it to other mathematical ideas. Children are expected to have a sense of belonging with the new ideas in culture and school through a good transition from known to new ideas.

The teachers were encouraged to plan for a week. They were first to notice key ideas such as what the new pattern and relationship were and how that leads to solve a problem. The teacher needed to identify what the children already knew as often children knew more from cultural activities than the teacher considered relevant. They also had to take account of the child feeling comfortable with school mathematics and how they can make it more appropriate by considering a place or cultural activity to introduce and develop a school mathematical idea.

They had to consider resources more broadly than just the chalk board at the front and a book for the children to copy off the board. They had to think about places to visit and people who could assist with the cultural mathematics. They needed to plan readily available materials for exploring, comparing, measuring, recording, and modelling; game cards, spinners, paper, etc. that might also be used in the classroom especially to generate conversation and reinforcement of new mathematical knowledge. The days given in the model below were adaptable to the situation.

Day 1

Tuning In

- Motivating
  - real world experience such as going outdoors to see something
  - telling a story
  - showing a video
  - Examples:

    house building Jiwaka;
    triangle gardens;
    hand clapping game
- Planning to find out
  - What questions to ask?

– What processes and equipment can be used?
– What we know and how might we extend this knowledge?

Finding Out

- Observing, noticing, comparing, measuring, discussing mathematical patterns

Day 2
Sorting Out

- Discussing, modelling, comparing, making a table, drawing a diagram, finding same and different

Day 3
Going Further (Thinking more deeply)

- Applying to other numbers or another situation, reading and discussing the maths book, using symbols, playing a game, solving an open problem

Day 4
Making Connections

- Summarising the mathematics and linking to other mathematics, whole class discussion or story writing
- Taking Action
- Share at home, solve a real problem, apply to a game

Day 5
Sharing, discussing, reflecting, evaluating

- Children explain the mathematics, write a maths story, write their own summary, say what new mathematics they have learnt
- Teacher reviews and decides where to next

Teachers also had to consider assessment. Some had learnt to record whether students had achieved some behavioural objectives such as adding two-digit numbers. They mostly recorded from observation of oral answers in class and in older children from written answers to questions involving symbolic operations recorded on the chalkboard.

However, they were not aware of diagnostic assessment or interview or questioning assessment. In order to reinforce the details around how young children learn to count, do arithmetic and learn about space and geometry and measurement, the teachers were taught how to ask diagnostic interview questions. The questions were similar to those found in overseas diagnostic assessments such as Count Me In Too (NSW Department of Education and Training 1998). To plan learning, teachers need to assess what their children know by observing ways children try things, what they say, how they problem solve, what they write, what they ask to make clear or to extend their exploring.

The teachers planned and tried out asking open-ended questions that allowed children to give more than one answer according to their understanding. Students were encouraged to visualise to operate on groups of numbers such as adding onto the larger number. Interestingly the extensive use of gestures and counting with fingers and toes both assisted visualisation but not necessarily assisted in getting students to stop counting by ones.

Interestingly, the project encouraged teachers to ask the interview questions in Tok Ples. This meant that teachers really had to consider what the questions meant in terms of their cultural ways of thinking. This task indicated how a number of teachers had not made the link between home mathematics and school mathematics. To follow the meaning through, there were discussions around not only the meaning of specific words but also the visuospatial reasoning by which the teachers understood the words and/or the mathematics.

For example, Wahgi language speakers north of the major east-west flowing river had used the term "source" for north as their small rivers started in what the English speakers were calling north, a direction they did not need in their local language. However, other dialect speakers south of the major river had tributaries starting in the south so for Wahgi speakers there was confusion. Nevertheless, it is clear how the visuospatial reasoning occurred for the speakers.

In another area, the teachers from the western coastal area of Central Province were discussing the idea of an area unit and how they might be able to speak of that notion. Interestingly, they began to refer to a "group" of area units such as a row of squares. It was the same word that they used for a composite group of ten, their counting system being a base 10 system. When they discussed addition they used the words for "joining together", which is commonly used across PNG in different languages.

Subtraction led to further discussion with the two meanings of difference and take away. They tended to use the idea of "take away" which they favoured in their discussion to "separate into two groups" but they also were happy to consider difference in terms of "compare and find out how much more one group was".

Similarly visuospatial embodiments and actions are used in western school mathematics classrooms. However, it was the first time some of these teachers had really worked through the true meaning of "addition" and "subtraction", transliteration words from the English addition and subtraction.

9.7 Conclusion

Given the number of everyday mathematical activities in which people participate in PNG, it is unsurprising that they reason with their gestures and bodily movements in a range of mathematical conceptual zones. However, it is only by identifying that visuospatial reasoning is being used that a link can be made to school mathematics. This rich cultural mathematical way of thinking needs to be harnessed for advancement of students succeeding in school mathematics.

This chapter shows that visuospatial reasoning was evident when spatial and embodied imagery was used to reason. Many times the pursuits are repeated for learning as contexts might require modifications. For example, the winds and swells vary so that learning to navigate and sail needs many visuospatial perceptions and reasons to be considered. Activities requiring spatial and embodied imagery could be encouraged in school mathematics by having students experience concepts in a physical way such as in outdoor mathematics or modelling cultural activities, walking trajectories for functions or illustrating concepts with gestures.

Typically angles appear to lend themselves to these approaches. For example, the consequences of a modification in an activity such as a ball rolling down a slope or bouncing off a wall require visuospatial reasoning by associating the visual trials and results with the language associated with the angles. Volume and mass activities would also strengthen understanding of these concepts, especially in distinguishing them from the dominance of length. Such activities encourage comparison and informal ratio ideas. For example, when one factor such as length is increased, the resultant comparative changes in volume and mass are estimated and compared.

To build model structures that work best when symmetry assists with balance and stresses encourages strong visuospatial reasoning. Actually participating in group activities to work collaboratively ensures estimations and comparisons are communicated in the reasoning process. A number of shape properties could be recognised. For example, the equal radii of circles, the lines of symmetry denoted by centre points of lines, and equal areas of opposite faces of a prism may be visualised and discussed. These kinds of physical activities require problem solving and hence the vehicle for using visuospatial reasoning.

Similarly the use of visual displays of numbers could strength the conceptualisation of numbers and could be set within a story or dramatisation. Such opportunities may strengthen multiplicative thinking and an understanding of large numbers. At the same time the cultural mathematics is valued and cultural beliefs and systems acknowledged and intertwined.

Mapping too is best associated with outside physical activities that could be representing a larger area of interest to the children such as their community, its roads and tracks, and landmarks. Nevertheless, the links from culture and environment to school mathematics need to be set into a mathematical inquiry going further from one setting into a new setting that is emphasising the structures of school mathematics, its representative methods, and its vocabulary.

These examples illustrate how visuospatial reasoning as advanced in this chapter recognises a virtually untapped resource made evident by ethnomathematics. An emphasis on visuospatial reasoning advances our general understanding of mathematics. This chapter illustrates the meaning of visuospatial reasoning and its ubiquitous but varied role in mathematical thinking. This kind of thinking needs to be recognised worldwide as a significant and legitimate way of thinking mathematically. It has analytic value as well as conceptual and memory value for mathematics and mathematics education.

If problem solving and visuospatial reasoning take precedents over rote-learning content, then teachers are not blinkered by school mathematics and are more able to incorporate cultural mathematics into the school classroom. The advantage of linking culture to school mathematics is preservation of culture, use of cultural identity to promote mathematical identity, and better school mathematics education (Owens 2014).

References

Bishop, A. (1983). Space and geometry. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 176–204). New York, NY: Academic Press.

Campbell-Jones, S. (Writer). (1996). *Horizon*: *Twice five plus the wings of a bird*. London, UK: BBC.

Dreyfus, T. (1991). On the status of visual reasoning in mathematics and mathematics education. In F. Furinghetti (Ed.), *Proc. 15th Conference of International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 33–48). Italy: PME.

Dwyer, P., & Minnegal, M. (2016). Counting systems of the Strickland-Bosavi languages, Papua New Guinea. *Language and linguistics in Melanesia, 34*(1), 1–36.

Eliot, J. (1987). *Models of psychological space: Psychometric, developmental, and experimental approaches*. New York, NY: Springer-Verlag.

Goldin, G. (1987). (a) Levels of language in mathematical problem solving; (b) Cognitive representational systems for mathematical problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 59–65, 125–145). Hillsdale, NJ: Lawrence Eribaum.

Goldin, G. (1998). The PME working group on representation. *Journal of Mathematical Behavior, 17*(2), 283–301.

Lábadi, B., Horváth, D. Á., & Palotai, R. (2012). Do infants encode feature and geometry information in a two-dimensional space? *Infant Behavior and Development, 35*(3), 408–416.

Learmonth, A. E., Newcombe, N. S., Sheridan, N., & Jones, M. (2008). Why size counts: Children's spatial reorientation in large and small enclosures. *Developmental Science, 11*(3), 414–426.

Lohman, D. F., Pellegrino, J. W., Alderton, D. L., & Regian, J. W. (1987). Dimensions and components of individual differences in spatial abilities. In S. H. Irvine & S. E. Newstead (Eds.), *Intelligence and cognition* (pp. 253–312). Dordrecht, The Netherlands: Nijhoff Publishers.

Mason, J. (2008). Being mathematical with and in front of learners: Attention, awareness, and attitude as sources of differences between teacher educators, teachers and learners. In T. Wood (Ed.), *International handbook of mathematics teacher education* (Vol. 4, pp. 31–56). Rotterdam, The Netherlands: Sense Publishers.

Murdoch, K. (1998). *Classroom connections: Strategies for integrated learning*. Armadale, Victoria, Australia: Eleanor Curtain.

Ness, D., & Farenga, S. (2007). *Knowledge under construction: The importance of play in developing children's spatial and geometric thinking*. Lanham, MD: Rowan & Littlefield.

NSW Department of Education and Training. (1998). *Count me in too*. Retrieved from http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/countmeintoo/index.htm

Owens, K. (1992a). Spatial mathematics: A group test for primary school students. In K. Stephens & J. Izard (Eds.), *Reshaping assessment practices: Assessment in the mathematical sciences under challenge*. Melbourne, Australia: Australian Council for Education Research.

Owens, K. (1992b). Spatial thinking takes shape through primary school experiences. In W. Geeslin, & K. Graham (Eds.), *Proc. 16th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 202–209.). University of New Hampshire, Durham, NH: PME.

Owens, K. (1993). *Spatial thinking processes employed by primary school students engaged in mathematical problem solving* (PhD). Deakin University, Geelong, Victoria, Australia. Retrieved from http://dro.deakin.edu.au/eserv/DU:30023339/owens-spatialthinking-1993.pdf

Owens, K. (2004). Improving the teaching and learning of space mathematics. In B. Clarke, D. Clarke, G. Emanuelsson, B. Johansson, D. Lambdin, F. Lester, A. Wallby, & K. Wallby (Eds.), *International perspectives on learning and teaching mathematics* (pp. 569–584). Gothenburg, Sweden: Göteborg University National Center for Mathematics Education.

Owens, K. (2014). The impact of a teacher education culture-based project on identity as a mathematics learner. *Asia-Pacific Journal of Teacher Education, 42*(2), 186–207. doi:10.1080/1359866X.2014.892568

Owens, K. (2015). *Visuospatial reasoning: An ecocultural perspective for space, geometry and measurement education*. New York, NY: Springer.

Owens, K., Edmonds-Wathen, C., & Bino, V. (2015). Bringing ethnomathematics to elementary teachers in Papua New Guinea: A design-based research project. *Revista Latinoamericana de Etnomatematica, 8*(2), 32–52.

Owens, K., McPhail, D., & Reddacliff, C. (2003). Facilitating the teaching of space mathematics: An evaluation. In N. Pateman, B. Dougherty, & J. Zilliox (Eds.), *Proceedings of 27th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 339–345). Honolulu, Hawaii: International Group for the Psychology of Mathematics Education.

Owens, K., Lean, G., Paraide, P., & Muke, C. (2017). *The history of number: Perspective from Papua New Guinea and Oceania*. New York. NY: Springer.

Paraide, P. (2010). *Integrating Indigenous and Western mathematical knowledge in PNG early schooling* (PhD). Deakin University, Geelong, Victoria, Australia.

Pirie, S., & Kieren, T. (1991). Folding back: Dynamics in the growth of mathematical understanding. In F. Furinghetti (Ed.), *Proceedings of the 15th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 169–176). Assisi, Italy: Program Committee for the International Group for the Psychology of Mathematics Education.

Poltrock, S., & Agnoli, F. (1986). Are spatial visualization ability and visual imagery ability equivalent? In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (Vol. 3, pp. 255–296). Hillsdale, NJ: Lawrence Erlbaum.

Presmeg, N. (1986). Visualisation in high school mathematics. *For the Learning of Mathematics, 6*(3), 42–46.

Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education* (pp. 205–304). Rotterdam, The Netherlands: Sense Publishers.

Rivera, F. (2011). *Towards a visually-oriented school mathematics classrooms: Research, theory, practice, and issues*. New York, NY: Springer.

Saxe, G. (2012). *Cultural development of mathematical ideas: Papua New Guinea studies*. New York: Cambridge University Press

Shepard, R. (1975). Form, formation, and transformation of internal representations. In R. Solso (Ed.), *Information processing and cognition: The Loyola Symposium* (pp. 87–122). Hillsdale, NJ: Lawrence Erlbaum.

Sinclair, N., Bartolini Bussi, M. G., de Villiers, M., Jones, K., Kortenkamp, U., et al. (2016). Recent research on geometry education: An ICME-13 survey team report. *ZDM* (pp. 1–29). doi:10.1007/s11858-016-0796-6

Steffe, L. (Ed.). (1991). *Epistemological foundations of mathematical experience*. New York, NY: Springer-Verlag.

Tartre, L. (1990). Spatial skills, gender, and mathematics. In E. L. Fennema, G. (Ed.), *Mathematics and gender*. New York, NY: Teachers College Press.

Uttal, D., et al. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin, 139*(2), 352–402. doi:10.1037/a0028446