Chapter 14
The Critical-Reflective Dimension of Ethnomodelling

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The images of peace are ephemeral. The language of peace is subtle. The reasons for peace, the definitions of peace, the very idea of peace have to be invented, and invented again. Children, everybody, here's what we do during war: In a time of destruction, create something. A poem. A parade. A community. A school. A vow. A moral principle. One peaceful moment
Maxine Hong Kingston—The Fifth Book of Peace

Abstract This discussion is related to critical-reflective dimensions of ethnomodelling, which is the creation of indeed the exploration of contexts that develop creativity in students in order to enable them to solve problems they face in their own sociocultural contexts. The critical-reflective dimensions of ethnomodelling allows learners the opportunity to develop a sense of purpose and their own potential by using mathematics to examine and solve problems they themselves choose and deem important. In this chapter, the author also discusses the importance to use both *emic* and *etic* approaches in the implementation of ethnomodelling in investigations. Emic approach is essential for an intuitive and empathic understanding of mathematical ideas, procedures, and practices developed by the members of distinct cultural groups while etic approach is important for cross-cultural comparisons that demand standardized categories to facilitate communication. This context enables ethnomodelling to bring cultural elements to the modelling process because it is the study of mathematical phenomena within a culture. If mathematics is considered as a universal language, then the process of the elaboration of an ethnomodel is the act of creating a mathematical poem. Thus, working with learners in assisting them in this creation is the act of ethnomodelling.

Keywords Critical-reflective dimension · Emic approach · Ethnomodelling · Etic approach

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14.1 Introduction

Most critical-reflective dimensions of ethnomodelling occur when educators create supportive environments that allow learners to practice and develop critical mathematical abilities and to practice the use of the tools that enable them to solve increasingly complex problems faced in their own sociocultural context. This allows them to develop the competencies, abilities, and the skills that encourage them to focus on the data and problems they are motivated to explore, value and are engaged in. It is hoped that they develop the opportunity to see mathematics as worthy of deeper exploration.

In the past, the process of teaching and learning mathematics was a top-down phenomenon that has contributed to the development of traditional curricula and is related to an educational system that creates cynical, unmotivated, disengaged and passive learners. The purpose of ethnomodelling, as outlined here, is to propose a pedagogical action that engages both educators and students to become active, critical, and reflective participants in resolving problems they face in their own community or cultural context.

14.2 Conceptualizing Critical-Reflective Efficiency

This researcher was initially introduced to general curriculum and instructional issues with professors and colleagues who operated within the paradigm that was initially developed by Tyler (1949). The brilliance of Tyler's model is that it was one of the very first curriculum models and survives to this day as a simple and to me, elegant model. Tyler's theory was based on four fundamental questions that became known as the *Tyler Rationale*:

1. What educational purposes should the school seek to attain?
2. What educational experiences can be provided that are likely to attain these purposes?
3. How can these educational experiences be effectively organized?
4. How can we determine whether these purposes are being attained?

Tyler's model deeply influenced my early teaching and work. And so, when I first came across ethnomathematics, this writer naturally looked at it through the Tyler paradigm. There are certainly problems, and not a little controversy with Tyler's paradigm, there is not space to go into it here, but the four questions remain fundamental to my thinking and filter what I find interesting to my work. This model means that the community must participate in and be fully present in creating school experiences.

This writer is, after all, a former public school teacher trained in the western United States, who still wants to know how *this* might affect *my* practice, *my* students, and *my* community and the practices of my pre-service teachers as well.

Some may find it strange, but it still forms my core or my base as I work with ethnomathematics and modelling, and it provides a foundation to what later I found as a *critical* mathematics education. It also has consistently reminded me to return to and look at the data, the proof for me as a mathematics educator, that things work (or do not work) in classrooms, and in our communities.

On the other hand, one of the more important characteristics of a critical educator has become the ability to develop processes that allow for the analysis of data by both educators and learners that are related to ongoing and day-to-day phenomena. Critical perspectives in relation to the social conditions that affect the experiences of our students help to develop workable strategies needed to solve problems.

This is a transformational form of learning, and is based on the context and previous experiences of our learners. In this context, educators create conditions that help learners to challenge predominant opinions. By using their own context and their experiences while learning to apply a data-based critical reflection to these problems, students are able to develop a rational discourse in order "to create meanings necessary for the structural transformation of society" (D'Ambrosio 1990).

Rational discourse then becomes the form of dialogue in which all parties have the same rights, responsibilities and duties to engage in, claim, and test the validity of their observations in an environment free of prejudice, fear, and social and political domination. It develops sound arguments for action and allows us to develop plans that allow learners to transcend simple, often prejudicial opinions not based on data. It gives learners the opportunity to enter into dialogue, resolve conflicts, and collaborate.

In this type of discourse, intellectual honesty, the elimination of prejudices, and the use of critical analysis based on data and facts, are important aspects that allow for dialogue to happen rationally and freely (Rosa and Orey 2007). In this educational environment, the processes of discourse, conscious or mindful work, intuition, creativity, criticality, and emotion are important elements that combine to help students develop their own critical-reflectiveness as they learn to move away from emotion-laden opinions and focus on the data related to the problem.

In the context of ethnomodelling, *ethno* refers to differences in culture, which are combinations of, and are influenced by, geography, climate, language, history, religion, customs, institutions and on the subjective self-identification of people. These differences also include distinctions based on racial and religious oppression or nationality. Individuals from distinct cultural groups have diverse worldviews that are a product of centuries; they are far more fundamental than differences among political ideologies including the social, economic, and cultural backgrounds that define civilization as a cultural entity (Huntington 1993).

In addition, culture is expanded to include also the cultures of differing professional groups and age classes as well as social classes and gender (D'Ambrosio 1996). Culture is also defined as a social group's design for surviving in and adapting to its environment (Bullivant 1993). Despite these definitions, culture is considered as the ideations, symbols, behaviors, values, knowledge and beliefs that are shared by a community (Banks and Banks 1997).

The essence of our many diverse cultural groups is related to our many cultural artifacts, tools, or other tangible cultural elements as well as the ways we have come to interpret, use, and perceive them. Artifacts are used in distinct cultures in different ways as well as for diverse purposes. Thus, there is certainly a need for the development for a broader definition of culture to ensure that the mathematical knowledge developed by the members of distinct cultural groups is included in this process. Most of the most interesting problems we may encounter, are thought to be unsolvable, though they can be resolved using a different kind of heuristics. Differently from the traditional constrained view of modelling, ethnomodelling uses a combination of:

1. Organized structures and ethnomodels[1] to represent sociocultural information.
2. Diverse ways of manipulating organized information (algorithms).
3. The mechanical and linguistic realizations of these ways, ethnomodels, and structures.
4. The application of these ways, ethnomodels, and structures in different cultural groups and society.

The aim of ethnomodelling is not to question the foundations of modelling, but to understand the way that mathematical ideas, notions, procedures, and practices are elaborated, presented, and communicated among members of distinct cultural groups. It also aims at encouraging the search for cultural ideas, procedures, and mathematical practices, their examination, and adoption into the mathematics curriculum.

The implications for ethnomodelling are that modelling of cultural constructs may be considered as a symbol system organized by an internal logic of the member of distinct cultural groups. Thus, these members develop models of an ongoing set of mathematical ideas, procedures, and relations of one individual to another in order to identify or examine patterns and their interactions in relation to their own cultural contexts.

14.3 Emic and Etic Approaches to Mathematical Knowledge

When researchers investigate the mathematical practices used by members from distinct cultural groups, they often find distinctive, often curious characteristics of mathematical ideas, notions, procedures, and practices that are possible to label,

[1]Ethnomodels are cultural representations that facilitate the understanding of systems taken from the reality of the members of distinct cultural groups. They can be considered as external representations that are precise and consistent with the scientific and mathematical knowledge that is socially constructed and shared by these members (Rosa and Orey 2010).

explain or translate with ethnomodelling. However, an outsider's understanding of these *cultural traits*[2] is an understanding that emphasizes unimportant cultural features and might misinterpret essential concepts. The challenge that arises from this perspective is how to extract the culturally bound mathematical ideas; notions, procedures, and practices of others without letting the culture of the investigator interfere with the data or the culture of the members of a cultural group under study or observation. This happens when cultural group members have their own interpretation of their culture (emic) opposed by or simply misread by an outsider's interpretation (etic) of it.

There are two approaches to be considered in order to investigate and study what can often be thought of as novel, new or *different* mathematical ideas, concepts, procedures, and practices developed by the members of any given cultural group. By becoming mindful of these approaches, one can learn that there is more to diverse ways and we refrain from relegating profound mathematical patterns, ideas and ways of doing to the *folkloric* or *exotic*:

1. *Etic* approaches may be defined as the external or outsiders' view on beliefs, customs, behaviors, and scientific and mathematical knowledge of the members of a given cultural group. This approach relies upon the extrinsic concepts and categories that have meaning for scientific and researcher observers who are the sole judges of the validity of an etic account. It equates culture to "the collective programming of the mind which distinguishes the members of one group or category of people from another" (Hofstede 1997, p. 5). The focus of this approach is the comparison of one culture to another. Researchers that follow the etic approach in cross-cultural research generally look for universal or culture-free concepts and theories. They search for variables and constructs common to all cultures, which are compared in order to discover how they are different from or similar to each other. It is equated with the objective explanation of sociocultural phenomena from the external point of view. These individuals are considered as *culturally universal* (Sue and Sue 2003).
2. *Emic* approaches may be defined as the internal or the insiders' view about their own customs, beliefs, behaviors, and scientific and mathematical knowledge. Hence, it is an approach of understand behaviors, beliefs, customs as well as ideas, procedures and scientific and mathematical practices. This approach focuses on the intrinsic cultural distinctions that are meaningful to these members whether the natural world is distinguished from the supernatural realm in the worldview of that specific culture (Pike 1954). This approach is identified with the sympathetic comprehension of subjective experience from the internal

[2]The terms cultural traits are used for simple behavior patterns that are transmitted by a society and to which the society gives recognition and meaning. They are learned meaning system that consists of patterns of traditions, beliefs, values, norms, meanings and symbols that are passed on from one generation to the next and are shared to varying degrees by interacting members of distinct cultural groups (Ting-Toomey and Chung 2011).

> point of view because it focuses upon understanding issues from the viewpoint of the subjects being studied. In this context, culture may be defined as “the *lens* through which all phenomena are seen. It determines how these phenomena are apprehended and assimilated. (…) it is the *blueprint* of human activity. It determines the coordinates of social action and productive activity, specifying the behaviors and objects that issue from both” (McCracken 1988, p. 73). Consequently, the emic approach investigates how local people think, how they perceive and categorize the world, their rules for behavior, what has meaning for them, and how they imagine and explain things (Kottak 2006). Thus, the members of distinct cultures are the sole judges of the validity of the descriptions of their own cultural ideas, notions, procedures, and practices. These individuals are considered as *culturally specific* (Sue and Sue 2003).

The terms *emic* and *etic* were coined by anthropologist Pike (1943, 1954; Headland et al. 1990) who stated that there are two approaches in the study of cultural group systems, which contain the point of view of either the *insiders* or the *outsiders*. Unfortunately, misunderstandings that arise may imply that two dichotomous approaches to cultures have arisen. It is often presupposed that this methodological dichotomy corresponds to the opposition between behaviors of members of distinct cultural groups.

Ethnomodelling helps to engage researchers and educators in a dialogue between emic and etic approaches. The goal is for the emic approach is to be on par with, indeed respected equally along with that of the etic. For example, the results of research conducted by Eglash et al. (2006) show that mathematical ideas, procedures, and practices are merely analyzed from an *etic* approach such as in the application of the symmetrical classifications from crystallography to textile designs and patterns. Rosa and Orey (2010) and Eglash et al. (2006) affirmed that ethnomathematical researchers often use the term translation to describe the process of modelling by using the application of etic academic mathematical representations to translate mathematical phenomena.

In some cases, the *translation* from *emic* to *etic* mathematical knowledge is direct and simple such as studies related to counting systems and calendars. In other cases, the mathematics as *embedded* in processes such as iteration in beadwork and in Eulerian paths in sand drawings in which the act of translation could be considered as a process of mathematical modelling (Eglash et al. 2006).

However, it is necessary to be cautious in this process because it is easier to strictly use numeric systems and counting procedures rather than to look at the embedded mathematical knowledge that is used in architecture and crafts that requires the application of modelling (Eglash et al. 2006). This tells us that culture, is essential to how we think, apply and use and even develop new forms of mathematics. Ethnomodelling can help us to approach this goal.

14.4 The Emphasis of Ethnomodelling

It is our experience that modelling relates to the mathematical ideas, notions, procedures, and practices that describe *systems* taken from the reality of the members of distinct cultural groups that help them to describe their world by using small units of information named ethnomodels that compose a representation for outsiders' understanding. In this regard, they learn to develop and retain the necessary information needed to solve any given problem they face daily and share how this occurs with others.

Therefore, the emphasis of ethnomodelling is related to the processes that help the construction and development of mathematical knowledge, which includes important aspects of collectivity, creativity, and inventiveness. Thus, it is impossible to imprison mathematical concepts in registers of univocal[3] representation of reality and its universal explanation (Craig 1998). Thus, it is not possible to conceive mathematics as a universal language because its principles, concepts, and foundations are not the same everywhere around the world (Rosa and Orey 2007).

In this regard, the "choice among equivalent systems of representation can only be founded on considerations of simplicity, for no other consideration can adjudicate between equivalent systems that univocally designate reality" (Craig 1998, p. 540). This means that the processes of production of mathematical ideas, notions, procedures, and practices operate in the register of interpretative singularities regarding the possibilities for the symbolic construction of the mathematical knowledge developed by the members of distinct cultural groups.

In this context, the etic approach claims that knowledge developed by the members of any given cultural group has no necessary priority over its competing emic claims. Therefore, it is necessary to depend on acts of translation between emic and etic approaches (Eglash et al. 2006). In this regard, cultural specificity may be better understood with an attempt toward comprehending and respecting the background of contextualized methods independent of the subjectivity of the observers.

From an ethnomodelling perspective, the emic constructs are accounts and descriptions of behaviors, beliefs, customs, and mathematical ideas, procedures, and practices that are expressed and analyzed in terms of the conceptual schemes and categories that are regarded as meaningful (consciously and unconsciously) by the members of distinct cultural groups. This means that these constructs must be in accordance with the perceptions and understandings deemed appropriate by the insider's culture. The validation of emic knowledge comes with a matter of consensus of the local people who must agree that these constructs match the shared perceptions that portray the main characteristics of their culture. Emic knowledge can be obtained through either an elicitation or an observation. In this regard,

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[3]Univocal representations provide only one meaning or interpretation of reality, which leads individuals to develop only one explanation or conclusion.

external observers may infer local perceptions, while being careful and respectful of their perspective, context, and setting.

Etic constructs are the accounts and descriptions of behaviors, beliefs, customs, and mathematical ideas, procedures, and practices that are expressed and analyzed in terms of the conceptual schemes and categories that are meaningful and appropriate to the community of scientific observers. An *etic* construct must be precise, logical, comprehensive, replicable, and observer-researcher independent. The validation of *etic* knowledge thus becomes a matter of logical and empirical analysis, in particular, the logical analysis of whether the construct meets the standards of comprehensiveness and logical consistency. *Etic* knowledge may also be obtained through elicitation as well as observation because local people possess scientifically mathematical valid knowledge.

One of the primary issues raised in mathematics education is concerned with the actual position of researchers, educators, and students in relation to the *etic* (culturally universal) and *emic* (culturally specific) approaches. Most researchers and educators may operate from an *etic* position because they believe that their own mathematical ideas, concepts, procedures, and practices occur in the same way in every culture (Rosa and Orey 2013). This, of course, is not quite right since we are in danger of basing our beliefs on what is often considered *superior* Western ideas and ignoring alternative solutions, perspectives and experiences. As a consequence, it is important to argue here that educators should not use the etic approach to acquire the emic approach because of the danger of colonization of the local knowledge (emic) by the western knowledge (etic).

Once again, this writer cannot emphasize enough the importance that members of every cultural group have come to construct, develop, acquire, accumulate, and diffuse mathematical knowledge. It may be different, it may not launch a rocket to the moon, or navigate an airliner from A to B, but it exists, no matter how humble, unusual or technical. In this regard, minimal modifications in our pedagogical practices are required because they consider mathematical knowledge universal and equally applicable across cultures. Therefore, if the assumption regarding the universal origin, process, and manifestation of mathematical knowledge is similar across cultures, then general guidelines and strategies for the pedagogical work would appear to be appropriate to apply in all cultural groups.

Many researchers and educators who take on an emic approach often believe that many factors such as cultural values, morals, and lifestyle come into play when mathematical ideas, notions, procedures, and practices are developed in regards to the cultural backgrounds of the members of distinct cultural groups. Since students come from different cultures, they have developed different ways of doing mathematics in order to understand and comprehend their own cultural, social, political, economic, and natural environments.

Given this context, our students may come to operate from an emic rather than an etic approach. According to this assertion, it is paramount that educators and teachers alike acknowledge that our lifestyles, cultural values, and different worldviews influence the development of the students' mathematical knowledge because its development arises from cultural contexts. This is one of the most important educational issues currently confronting researchers and educators because it is pointing directly at how worldwide current guidelines and standards for mathematical instruction are culturally bound (Rosa 2010).

14.5 Emic and Etic Approaches in Ethnomodelling

Considering both research and educational fields, which approach must be applied? Should researchers and educators be based on the culturally universal or culturally specific? These questions allow us to argue that some professionals believe in a "cultural universality", which focuses on similarities and minimizes diverse factors that lead to alternative views, while others take on techniques and beliefs of cultural specificity, which focus on cultural differences. Then, the question is whether it is necessary to understand a cultural specificity that requires a specific theoretical basis and concepts (emic) against the background of universal and generic theories and methods (etic).

Over the last three decades, there has emerged a demand for local psychologies besides the Western or Euro-American psychology. Therefore, it is necessary to support the valorization and use of emic approaches in investigations in order to show that many theories and methods seem to be susceptible to the differences in distinct cultures and, consequently, demand cultural contextualization. Conversely, it is most certainly naive to state that the members of distinct cultural groups do not share universal mathematical characteristics. Globalization, the Internet, cultural interchange, even tourism has encouraged a *mudding of the waters*, so to speak. This is not a bad thing, as this exchange, causes new ideas to emerge.

For example, Bishop (1991) stated that many of our everyday activities as well as those of diverse members of cultural groups, involve a substantial amount of mathematical application. In this regard, there are six universal activities practiced by the members of any cultural group. These activities are: (1) Counting, (2) Measuring, (3) Designing, (4) Locating, (5) Explaining, and (6) Playing.

They provide the fundamental facets used to probe the traditional or daily living practices of all members of the human species. These universals are inseparably intertwined with other aspects of the daily life of the members of any cultural group. Through a study of these applications, it is possible to glimpse the wonder of how humans have came to resolve problems, communicate and maybe see how their early experiences incorporated mathematics. However, even though these activities may be universal, it is important to recognize that they are merely generalized perspectives in relation to those who may or may not share similar cultural-historical characteristics and perspectives. Once again, it is naive to believe that mathematical concepts do not reflect the cultural values and lifestyles of the members of any given cultural group.

In this regard, perhaps a better approach to opposing views may be to try to understand the universality of mathematical ideas, notions, procedures, and practices, which are relevant to researchers and educators and to consider them linguistically, that is as different accents or regional dialects. It is also necessary to state that these approaches may take into consideration the relationship between cultural norms, values, attitudes, and the manifestation of mathematical knowledge in different educational fields.

Researchers and educators must also take into account their own worldviews. If they become more aware of themselves and their worldviews and values, then they can become increasingly more open to apply ethnomodelling in their pedagogical practices. This may lead them to a clearer decision between the two approaches. Many scholars have come to believe mathematics activity as highly cultural (D'Ambrosio 1990; Eglash 1997; Rosa and Orey 2007). For example, mathematicians do not agree on the nature of mathematics, debating whether this subject is culturally bound (internalists) or culture-free (externalists) (Dossey 1992).

Internalists such as Bishop (1988) and D'Ambrosio (1985) have shown how mathematics is a cultural product, which is developed as a result of various activities such as counting, locating, measuring, designing, playing, and modelling. Other mathematicians such as Kline (1980) are considered externalists because they believe mathematics is a culturally free activity. Thus, they do not believe in the connection between mathematics and culture. For example, the results of the study conducted by Rosa and Orey (2013) showed that the majority of teachers possess an externalist view of mathematics, which means that they perceive mathematics as culture-free while few of these professional possess an internalist view of mathematics because they perceive mathematics as a cultural product.

Much of the research in ethnomathematics provides strong challenges to the primitivist view that local societies have developed only simplistic or folkloric technologies and associated mathematics (Ascher 2002; D'Ambrosio 1990; Eglash 1997; Gerdes 1991; Horne 2000 Rosa and Orey 2010; Zaslavsky 1999). It is crucial to show that the members of these societies developed sophisticated mathematical practices,[4] not just trivial, exotic or folkloric examples by directly challenging the epistemological stereotypes most damaging to their mathematical knowledge. These practices include diverse economic practices; geometric principles in craftwork, architecture, and the arts; numeric relations found in measuring, calculation, games, divination, navigation, and astronomy; and a wide variety of other artifacts and procedures (Eglash et al. 2006).

Ethnomodelling often uses the term *translation* to describe the process of modelling local systems with a *Western* academic mathematical representations. However, as with all translations, the success is always partial, and intentionality is one of the areas in which the process is particularly tricky or difficult to share. Often

[4]Sophisticated mathematical practices often reflect deep design themes that provide cohesive structures to many of the important local knowledge systems such as cosmological, spiritual, and medical for the members of distinct cultural groups. Examples of sophisticated mathematical practices include the pervasive use of fractal geometry in African design and the prevalence of fourfold symmetry in Native American design (Eglash et al. 2006).

local designs are merely analyzed from a Western view, such as the application of symmetry classifications from crystallography to local textile patterns (Eglash et al. 2006).

Ethnomathematics also makes use of modelling, but here it attempts to use modelling to establish and translate the relations between the local conceptual framework and the mathematics embedded in local designs, traditions, or contexts. Therefore, ethnomathematically speaking, mathematics can be seen as arising from emic rather than etic origins. In some cases, this process of *translation* to Western mathematics is direct and simple: as with counting systems and calendars. In other cases, the mathematics is *embedded* in a process such as iteration in beadwork, and in Eulerian paths in sand drawings, on in counting and measuring (Eglash et al. 2006).

According to this context, ethnomodelling can be considered as the application of "ethnomathematical techniques and the tools of mathematical modelling [that] allows us to see a different reality and give us insight into science done in a different way" (Orey 2000, p. 250). In order to solve problems, students need to understand alternative mathematical systems and they also need to be able to understand more about the role that mathematics plays in a societal context (Rosa and Orey 2007). This aspect promotes a deeper understanding between diverse mathematical systems and traditions by using mathematical modelling. Which is, to repeat the process of the translation and elaboration of problems and questions taken from systems that are part of the reality of the students or community (Rosa and Orey 2013).

As early as 1993, D'Ambrosio defined a *system* as a part of reality, which is to be integrally considered. In this regard, a system is a set of items taken from a certain student context and/or reality. The study of a system considers the study of all its components and the relationship between them. Mathematical modelling is a pedagogical strategy used to motivate students to work on the mathematics content and helps them to construct bridges between the mathematics of school and the mathematical concepts they use in their own reality.

For example, D'Ambrosio (2002) commented about an ethnomathematical example that naturally comes across as having a mathematical modelling methodology. In the 1989–1990 school year, a group of Brazilian teachers studied the cultivation of grape vines that were brought to Southern Brazil by Italian immigrants in the early twentieth century. This was investigated because of the cultivation of grapes and the production of wine is directly linked to the culture of the people in that region. Both Bassanezzi (2002) and D'Ambrosio (2010) believed that this particular case study is an excellent example of the connection between ethnomathematics and mathematical modelling by applying ethnomodelling (Rosa and Orey 2007).

Once again, educators and teachers should search for problems taken from students' reality that translate their deepened understanding of real-life situations through the application of culturally relevant activities. This process enables students to take a position such as sociocultural, political, environmental, and economic in relation to the system under study. According to Rosa (2010), the main objective of this pedagogical approach is to rehearse the established mathematical context that allows students to see the world as consisting of opportunities to employ mathematical knowledge that helps them to make sense of any given situation.

14.6 Distinctions Between Emic and Etic in Ethnomodelling

Our ongoing development of ethnomodelling serves as a vehicle to transfer meaning and value from the culturally constituted world around us to mathematical procedures. The communications of these ideas are represented in the effect of our cultural outlook, or lens, on mathematics concepts developed by distinct cultural groups—either ours, or the *other*. When considering this, it is worth noting that from an emic perspective, culture may not be seen as a construct apart from and causing the development of, mathematical practices.

Answers to the most fundamental anthropological questions, including the origins of humanity, the characteristics of human nature, and the form and function of human social systems are part of the worldview of every culture. Mathematicians and anthropologists have been acculturated to some particular worldview, and they therefore need a means of distinguishing between the answers they derive as acculturated individuals and the answers they derive as anthropological observers/researchers. Defining *emics* and *etics* in epistemological terms provides a reliable means of making that distinction.

From an applied perspective, definitions of culture from emic and etic approaches might be considered as two sides of the same coin. Emic knowledge is essential for an intuitive and empathetic understanding of a culture, and it is essential for conducting effective ethnographic fieldwork. It is important to state that emic approaches do not intend to directly compare two or more differing cultures, but to promote a complete understanding of the cultural group under study. The methods used in conducting emic research do not provide *culture-free* measures that can be directly compared. Instead, they provide *culture-rich* information. The choice of emic versus etic approaches depends on several important factors, including the nature of the research question, the investigators' resources, and the purpose of the study.

In accordance to this context, traditional mathematical models do not fully take into account the implications of the cultural aspect of human social systems. This cultural component is critical because its accounts "emphasize the unity of culture, viewing culture as a coherent whole, a bundle of practices and values" (Pollak and Watkins 1993, p. 490) that are incompatible with the rationality of mathematical models. However, in the greater context of mathematical knowledge that defines mathematics as more than that taught from a western-academic sense of mathematics, and includes a historical-cultural perspective of time and place as described above, the cultural component then encourages us to respect the wide variation of mathematics found word-wide.

Again, this comes from viewing mathematical practices as socially learned and diffused to members of distinct cultural groups and includes the mathematical practices with an internal logic. Then, it is the process by which transmission takes place from one person to another that is central to elucidating the role of culture in the development of mathematical knowledge. Culture then comes to play a far-reaching and constructive role with respect to teaching/learning mathematical practices.

If mathematical knowledge consists of socially learned, transmitted, and diffused mathematical practices, then, in accordance to Read (2004), the cognitive aspects of the human brain play a relatively minor role when constructing models of behaviors of sociocultural systems. The cognitive aspect of the brain that is needed in this framework is primarily related to the decision processes by which members of distinct cultural groups either accept or reject a conduct as part of one's own repertoire of behaviors. In regards to mathematical knowledge, it appears that there are two ways in which we recognize, represent, and make sense of mathematical phenomena.

First, there is a level of cognition that we share, to varying degrees, with the members of distinct cultural groups. For example, we all locate, we all judge distance and time, we all count and order objects, we all distinguish larger for smaller amounts or objects, and we all seem to enjoy patterns. This level would include cognitive modelling that we may do at a non-conscious level that serves to provide an internal organization of external mathematical phenomena and provides the basis upon which a mathematical practice takes place.

Second, there are many culturally constructed representations of external mathematical phenomena that provide internal organization for this phenomena, but where the form of the representation arises or differs is through formulating an abstract, conceptual structure that provides form and organization for external phenomena in a variety and diversity that need not be consistent with the form and patterning of those phenomena as external phenomena; that is, the cultural construct provides a constructed reality.

The implications for mathematical modelling of human systems are that our modelling of a cultural construct may be based on our symbol systems that are organized by our own internal logic or *grammar*. There is modelling of the ongoing set of behaviors and relationships of one individual to another, such as the networks we use to identify an actual pattern of interactions of individuals along one or more dimensions is deemed to be relevant for the organizational form of individuals making up a social unit.

According to this discussion, in the emic approach, the information and observations reflect the target population's own vocabulary, scientific and mathematical knowledge, conceptual categories, language of expression, religion, and cultural belief system. The word contrasts with etic, that refers to the information collected in terms of a conceptual system and the categories used by researchers or *other* outsiders. It is often necessary to use the local language or dialect and gather information in a very open-ended, nondirective way. The participants' perspective on the phenomenon of interest should unfold as the participants perceive it (emic) not as the investigators view it (etic approach).

For example, when we use the pile sort technique and ask informants to "group the food items in any groups they wish to or any way that they happen to think of", the resulting groups are *emic* categories. In the etic approach, observations and data are constructed in the researcher's system of categories and definitions. In other words, models and ethnography should be mutually reinforcing to each other. Ethnographers, if not blinded by prior theory and ideology, should come with an informed sense for the differences that make a distinction from the point of view of people being modeled. They should, in the end, be able to tell outsiders what matters most to insiders in a critical and reflective manner.

14.7 Theoretical Basis for a Critical-Reflective Dimension in Ethnomodelling

Critically reflective teaching places both educators and learners at the center of teaching and learning processes. These classrooms become active laboratories where educators coach students to develop intuitive, creative and a databased criticality by applying pedagogical approaches to real life situations. The act of teaching becomes a social and cultural activity that introduces students to the process of knowledge creation instead of passively receiving information (Freire 2000).

Currently, the debate between what are often conflicting teaching approaches continues mostly due to the over emphasis on ranking, evaluation and testing; where teaching is more or less centered on the memorization and the testing of content in preparation for further testing. Simply put, I reject this approach. The need to elaborate a mathematics curriculum that promotes critical analysis, active participation, and reflection on social transformation by students (Rosa and Orey 2007) is critical as humankind engages in resolving serious political, environmental and social problems that threaten all of us, everywhere.

It is important to call here for a curriculum change that prepares, indeed encourages, educators and learners to become critical, reflective, and responsible citizens, not as a by-product of a standard curriculum and instructional practices, but as a fundamental goal for all learners. This aims to find practical solutions to the many real problems faced by society that can actively engage learners to apply and value mathematics. It is impossible to teach mathematics or other curricular subjects in a way that is not neutral or insensitive to the experiences of students (Fasheh 1997).

Ethnomodelling focuses on critical-reflective efficiencies that engage students in relevant and contextualized activities allowing them to be involved in the construction of their own mathematical knowledge. In this context, Rosa and Orey (2007) argue that the theoretical basis for critical-reflective dimensions of ethnomodelling has its foundations in both Sociocultural Theory and the Critical Theory of Knowledge.

It involves the mathematical practices developed, used, practiced, and presented in diverse situations in the daily life of the members of these groups. This context is holistic and allows those engaged in this process to study mathematics as a system or an ecology taken from their own contextual reality in which there is an equal effort to create an understanding of all components of these systems as well as the interrelationship among them (Rosa and Orey 2007).

Ethnomodelling takes in consideration the essential processes found in the construction and development of mathematical knowledge, which includes often curious and unique aspects of collection, creativity, and invention. It is the study of mathematical phenomena within a culture since it is a social construction that is culturally bound, which applies cultural elements to the modelling process.

14.7.1 Sociocultural Theory

A fundamental aspect for this is through socialization, where knowledge is best constructed when students work in groups and learn to work cooperatively. Construction of knowledge is connected to other knowledge areas in an interdisciplinary manner. It is through real social interactions among students from diverse socio-cultural groups that learning is initiated and built (Vygotsky 1986).

In ethnomodelling processes, diverse socio-cultural environments greatly influence student cognition. Collaborative work among educators and learners makes learning more effective because it generates higher levels of engagement in mathematical thinking through the use of socially and culturally relevant activities, and this makes use of *dialogical constructivism* because the source of knowledge is based on social interactions between students and environments in which cognition is the result of the use of *cultural artifacts* in these interactions. These artifacts act as vehicles allowing students to understand problems they face in their own community (Rosa and Orey 2007).

14.7.2 Critical Theory of Knowledge

Studies of Habermas' *Critical Theory of Knowledge* reinforce the importance of social contexts in the teaching and learning process. Habermas demonstrates how critical consciousness in learners is increased as they analyze social forces around them. This occurs through the use of strategies such as interpersonal communication, dialogue, discourse, critical questioning, and the use of problems taken from their own reality. The effects of social structure influence distinct knowledge areas taken by individuals from their social context. As well there are three generic knowledge domains: *technical*, *practical*, and *emancipatory* (Habermas 1971).

14.7.3 Knowledge or Prediction

The ability to predict, using technical knowledge, is determined by how individuals manipulate their environmental contexts. It is gained through working with diverse empirical investigations and is governed by technical rules. In ethnomodelling processes, students learn to apply this as they collect data by coming to observe and document attributes of specific phenomena, verify if a specific outcome can be produced and reproduced, and know how to use rules to select different and efficient variables to manipulate and elaborate mathematical models.

In this process, students improve their ability to communicate by using data-based hermeneutics (written, verbal, and non-verbal communication) to verify social actions/norms modified by communication. It is here that meaning and interpretation of communicative patterns interact to construct and elaborate understandings that serve to outline agreements in social performance.

14.7.4 Knowledge or Criticism and Liberation

The process of gaining insight emancipates individuals from institutional forces that often limit and control their lives and is equally important to the development of critical-reflective dimensions. How we come to determine our own unique opinions, answers and solutions to the social condition around us forms an essential objective. Knowledge is used to liberate individuals from outdated, often oppressive modes of social domination by developing the tools needed to exercise databased decision-making. In the ethnomodelling process, these insights are gained through mathematical modelling.

Ethnomodelling encourages learners to recognize what is needed to solve problems. Knowledge is gained by reflecting on the data that they themselves observe, collect, and analyze. Learning is linked to the growing technical knowledge of learners and their experiences gained in conjunction with the social and cultural aspects around them through dialogical activity that enables understanding of data collected. In the ethnomodelling process, this approach helps students to learn to take responsibility and/or ownership of their own knowledge and processes. Knowledge and ideas are then translated in interdisciplinary and dialogical ways as modelers begin to focus on the data used as instruments for social transformation.

14.8 Critical-Reflective Dimensions of Ethnomodelling

Currently, there is little consensus for specific epistemologies in the critical-reflective dimensions of ethnomodelling. However, Rosa and Orey (2010) state that the main objectives of these dimensions are to:

1. Provide learners with mathematical tools necessary to study, act on, modify, change and transform their own reality.
2. Teach that deep learning of mathematics starts from the social and cultural contexts of students by providing them with opportunities to develop logical reasoning and creativity.
3. Facilitate the learning of mathematical concepts that help students build knowledge in mathematics so that they are able to understand their social, historical and cultural contexts.

Ethnomodelling encourages learners to describe, inquire about, and investigate problems coming from their immediate sociocultural context, where they work with real problems that they have a personal connection to, and learn to use mathematics as a language for translating, understanding, simplifying, and solving problems. Even this process seems to be similar to the development of mathematical modelling, ethnomodelling is the process in which members of distinct cultural groups come up with different mathematical tools that help them to organize, analyze, solve, and model specific problems located in their our own sociocultural contexts. These tools allow theses members to identify and describe specific mathematical ideas, techniques, and procedures by schematizing, formulating and visualizing problems in different ways, and discovering relations and regularities in their own cultural practices (Rosa and Orey 2010).

Modelers critically intervene in their own reality by obtaining a mathematical representation of the situation by means of reflective discussions related to the development and elaboration of the findings in their own models (Rosa and Orey 2007). As they come to use mathematics to describe a setting, opinion, or problem-situation, ethnomodelling is akin to writing a mathematical poem. Critical-reflective dimensions of ethnomodelling are based on:

1. An increasing comprehension and understanding of sociocultural context in which learners live through reflection, critical analysis and actions based on data.
2. Learners borrow existing systems they study in the context of symbolic, systematic, analytical, and critical aspects.
3. Starting from given problem-situations taken from their own sociocultural context, learners make hypotheses, test them, fix and improve upon them, draw inferences, generalize, analyze, conclude, and make decisions about the object under study.

From this perspective, ethnomodelling makes use of three gradually more complex mathematical modelling pedagogical phases based on the research developed by Barbosa (2001). I would like to take time now to share a couple of cases, or steps that we move students through as we engage in developing ethnomodels.

14.8.1 Case 1: Educators Choose a Problem

In this type of practice, the teacher chooses the situation and then describes it for students, more often than not by using preselected textbook or classic examples. This is in accordance to curriculum content where the teacher provides students with the appropriate mathematical tools needed in the elaboration of classic mathematical models. In our experience, this is often the first step as learners learn to integrate ethnomodelling strategies in their experience (Rosa and Orey 2007). These first experiences in many ways are related to Halpern's (1996) *critical thinking* that involves a range of thinking skills that leads toward desirable outcomes; and Dewey's (1933) *reflective thinking* that focuses on the process of making judgments about what has happened or was observed.

This approach allows students to solve classic problems, such as the study of the calculation of the circumference of the earth by Eratosthenes. By being shown how to describe a problem, organize variables and data, set up related equations to enable them to translate real situations into mathematical terms; they see how to make observation of patterns, testing of conjectures, and the eventual estimation of results. These are important processes for later stages as they become increasingly more and more autonomous and sophisticated in their ability to model. This most certainly introduces them to the process of mathematization, and allows students to construct and look at classic mathematical models.

14.8.2 Case 2: Educators Suggest and Elaborate the Initial Problem

At this stage, the teacher gives a common theme to the students, such as repaving a certain street, river pollution or transportation fares, and then modelers investigate the problem by collecting data, formulating independent hypotheses, and making necessary modifications in order to develop their mathematical models, that they share between groups. Students themselves are given more autonomy to participate in the activities proposed as they develop their own modelling awareness. One of the most important stages of this modelling process is related to the elaboration of a set of assumptions, which aims to simplify and solve the model to be developed (Rosa et al. 2012).

14.8.3 Case 3: Educators Facilitate the Modelling Process and Engage in Ethnomodelling

Educators at this stage facilitate the modelling process by allowing students to choose and justify their own theme. The *ethnomodelers* are encouraged to develop a project in which they are responsible for all stages of the process: from justification for and formulation of the problem to its validation and final presentation. At this stage, supervision by the teacher is akin to coaching, and involves encouragement in the modelling process. It also means the teacher, may teach short lessons to the groups that enable them to learn and understand new mathematical content needed to elaborate their model. This enables a vital critical-reflective engagement in proposed activities by encouraging modelers to develop and justify their own hypotheses and opinions based on their data and research.

Once again, this is very much like writing a poem or essay, at some point the learners must be freed from traditional limitations and the rote learning practices of mathematical grammar and prescribed textbook problems in order to practice developing their own prose and mathematical poems. During the development of ethnomodels, problems are chosen and suggested by the modelers themselves and are used to reflect critically on the aspects involved in the situation modeled. These are related to:

1. The diverse interdisciplinary connections they encounter;
2. Access and uses of many forms of technology; and
3. The discussion of environmental, economic, political, cultural, and social issues.

The use of mathematical content in this process is directed towards a critical databased analysis of problems faced by the members of their own community.

14.8.4 One Example of Ethnomodelling

The results from a conversation during a morning walk with students along a street in Ouro Preto encouraged exploration and development of a simple model that explored the relationship between mathematical ideas, procedures and practices that developed connections between community members and formal academic mathematics.

By observing the architecture along the wall of one of the schools in Ouro Preto, this professor and his students were able to converse and explore and eventually determine ways to relate functions of three types of curves: exponential, parabolic, and catenary to the patterns found on its wall (Rosa and Orey 2013). In this case I wonder aloud if the pattern shown in Fig. 14.1 was a series of parabolas catenaries or exponential functions.

After examining the data collected they measured various curves on the wall of the school and attempted to fit them to functions (exponential and quadratic) through mathematical models and came to the conclusion that the curves on the wall of the school closely approximated that of a catenary curve function.

Fig. 14.1 Curves on the wall of the school. *Source* Photo by the author

14.8.5 An Emancipatory Approach in the Critical-Reflective Dimension of Ethnomodelling

Because these pedagogical practices offer an open activity that allows us to apply multiple perspectives to solve a given problem, this is to my mind, related to the emancipatory aspect of mathematics. However, the *open* nature of the (ethno) modelling activity may be difficult for students to establish and for them to develop models that satisfactorily represent the problem under study (Barbosa 2001). Thus, the coaching, dialogical, and mediator roles of educators are vital during the ethnomodelling process; this is why this approach is considered an extension of critical theories of knowledge, and forms the emancipatory aspects by addressing social-political issues.

According to the Brazilian National Curriculum for Mathematics (Brasil 1998), all students must develop their own autonomous ability to gather data, solve problems, make decisions, work collaboratively, and communicate effectively. This cannot be done in environments that focus on uniform curriculum and testing. The approach as outlined here is based on developing *emancipatory powers*, where students are encouraged to become flexible, adaptive, reflective, critical, and creative citizens.

This perspective is related to sociocultural dimensions of mathematics, and is directly associated with the political dimension of the ethnomathematics program as it develops "actions that guide students in transition processes from subordination to autonomy in order to guide them towards a broader command of their rights as citizens" (Rosa and Orey 2016). It emphasizes the role of mathematics they see in their own context, by developing the ability to analyze a problem in relation to critical and reflective thinking aspects. As well, the role of modelling processes are used can be used to solve everyday challenges present in contemporary society.

Ethnomodelling may be understood as a language to study, understand, and comprehend problems faced daily by the community used to develop their own mathematical prose. For example, this approach is used to analyze, simplify, and solve daily phenomena in order to predict results or modify the characteristics of

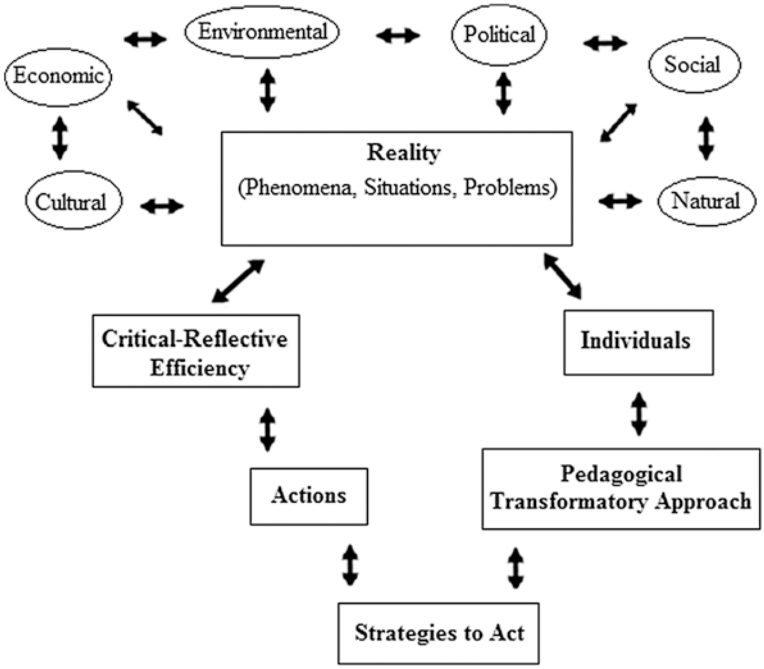

Fig. 14.2 Critical-reflective ethnomodelling cycle. *Source* Rosa and Orey (2015)

these phenomena (Bassanezzi 2002). Figure 14.2 demonstrates this *Critical-Reflective Ethnomodelling Cycle*.

Reflections on the reality modeled become a transforming action (Rosa and Orey 2007), and this system allows modelers to make their own representations by using strategies that enable them to explain, understand, manage, analyze, and reflect their own unique problem-situation.

The application of critically-reflective dimensions of modelling allows mathematics to be seen as a dynamic language used to explain phenomena and used to resolve important conflicts and problems; and is related to the reality of students as when they are passionately involved in sports or video gaming; where gamers deal with more and more abstractions, increasing in difficulty, and the development of and the creation of new tools, where the formulation of new concepts and theories allows them to solve (beat) the problem.

14.9 Final Considerations

Ethnomodelling enables students to question ideas and themes that are develop to help them to understand, explain, and make predictions related to the phenomena under study through the elaboration of ethnomodels that represent these situations

(Rosa and Orey 2007). In this context, ethnomodels are related to processes that critically check parameters selected for the solution of models gleaned from holistic interconnected contexts that the modelers themselves find valuable.

It is not possible to explain, know, understand, manage, and cope with reality outside of realistic, interconnected and holistic contexts (D'Ambrosio 1990). In the critically reflective dimensions found in ethnomodelling, it is impossible to work only with isolated theories or techniques that facilitate models to be memorized, tested upon and quickly forgotten. Ethnomodelling gives access to creativity, conceptual elaboration, and the development of logical, reflective, and critical thinking and empowers modelers to connect what they learn to what they experience outside of school.

Fundamental characteristics for critical-reflective ethnomodelling are based on opinions stemming or that may emerge from databased analysis of problems the students found meaningful. For example, Rosa and Orey (2015) discuss a typical exercise given to students in trigonometry classes: *From the top of a cliff, whose height is 100 m, a person sees a ship under a depression angle of 30°. Approximately, how far is the ship from the cliff*? They argue that this exercise does not provide the development of the critical-reflective dimension of ethnomodelling because students can apply a simple mathematical model related to the tangent function, $tg30° = 100/d$, in order to determine the distance from the base of the cliff to the ship.

In the process of modelling this problem, it is important to discuss with students the assumptions that have been previously established as a critical analysis of its solution since some generalized simplifications of reality were established such as the assumptions that the ocean is flat, the cliff is perfectly vertical to the straight line chosen to represent the distance from the base of the cliff to the ship, a straight line can reasonably approximate the distance from the base of the cliff to the ship, and the curvature of the Earth is ignored. These assumptions are not critically discussed nor reflected with the students.

The critical perspective of students in relation to social conditions that affect their own experience helps them to identify common problems and develop strategies used to solve them (D'Ambrosio 1990). Thus, the analysis of sociocultural phenomena focuses on the development of critical-reflective dimensions of ethnomodelling by using contextualized situations in the mathematics teaching and learning process (Rosa and Orey 2007). Thus, ethnomodelling is based on the ongoing development, comprehension and understanding of reality by the students themselves, and makes use of the reflections, analysis, and critical action they take. When students begin to study the symbolic, systematic, analytical and critical aspects, ethnomodelling can then explain different ways of working with reality.

Critically-reflecting becomes a transformational action that reduces complexity of the reality by allowing students to explain it, understand it, manage it, and find solutions to the problems they themselves have developed. There is a certain intuitive aspect to this application of mathematics that in this time of destruction allows both teachers and students to create something beautiful such as a mathematical poem. Why we are teaching learners to create *mathematical poems*?

Learning to triangulate their opinion by using data gleaned from an emic-etic approach reminds me of a story shared with me by Allen (1992):

The Fable of the Roasted Pig

Once upon a time, a forest where some pigs lived caught fire, and all the pigs were roasted. People, who at that time were in the habit of eating raw meat only, tasted the roasted pigs and found them delicious. From that time on, whenever men wanted roasted pork they set a forest on fire.

Due to the many bad points of *the system*, complaints grew at an increasing rate, as the system expanded to involve more and more people. It was obvious that "the system" should be drastically changed. Thus every year there were a number of conventions, and congresses, and a considerable amount of time and effort was spent on research to find a solution. But apparently no way of improving the system was ever found, for the next year and the year after and the year after that there were more and more conventions and congresses and conferences. And this went on and on and on (…).

Those who were experts on the subject put down the failure of the system to a lack of discipline on the part of the pigs, who would not stay where they should in the forests; or to the inconstant nature of fire, which was hard to control; or to the trees, which were too green to burn well; or the dampness of the earth; or the official method of setting the woods on fire or (…) or (…).

There were men who worked at setting the woods on fire (firemen). Some were specialists in setting fires by night, others by day. There were also the wind specialists, the *anemotechnicians*. There were huge compounds to keep the pigs in, before the fire broke out in the forest, and new methods were being tested on how to let the pigs out at just the right moment.

There were technicians in pig feeding, experts in building pig pens, professors in charge of training experts in pig pen construction, universities that prepared professors to be in charge of training experts in pig pen construction, research specialists who bequeathed their discoveries to the universities that prepared professors to be in charge of training experts in pig pen construction, and…

One day a fireman named John Commonsense said that the problem was really very simple and easily solved. Only four steps need to be followed: (1) the chosen pig had to be killed, (2) cleaned, (3) placed in the proper utensil, and (4) placed over the fire so that it would be cooked by the effect of the heat and not by the effect of the flames.

The director general of roasting himself came to hear of the Commonsense proposal, and sent for John Commonsense. He asked what Commonsense had to say about the problem, and after hearing the four-point idea he said:

"What you say is absolutely right–in theory, but it won't work in practice. It's wasteful. What would we do with our technicians, for instance?"

"I don't know," answered John.

"Or the specialists in seeds, in timber? And the builders of seven-story pig pens, now equipped with new cleaning machines and automatic scenters?"

"I don't know."

"Can't you see that yours is not the solution we need? Don't you know that if everything was as simple as all that, then the problem would have been solved long ago by our specialists? Tell me, where are the authorities who support your suggestion? Who are the authors who say what you say? Do you think I can tell the engineers in the fire division that it is only a question of using embers without a flame? And what shall be done with the

> forests that are ready to be burned - forests of the right kind of trees needed to produce the right kind of fire, trees that have neither fruit, nor leaves enough for shade, so that they are good only for burning? What shall be done with them? Tell me!"
>
> "I don't know."
>
> What you must bring, are realistic solutions, methods to train better wind technicians; to make pig sties eight stories high or more, instead of the seven stories we now have. We have to improve what we have; we cannot ignore history. So bring me a plan, for example, that will show me how to design the crucial experiment, which will yield a solution to the problem of roast reform. That is what we need. You are lacking in good judgment, Commonsense! Tell me, if your plan is adopted, what would I do with such experts as the president of the committee to study the integral use of the remnants of the ex-forests?"
>
> "I'm really perplexed," said John.
>
> "Well, now, since you know what the problem is, don't go around telling everybody you can fix everything. You must realize the problem is serious and complicated; it is not so simple as you had supposed it to be. An outsider says, "I can fix everything."
>
> But you have to be inside to know the problems and the difficulties. Now, just between you and me, I advise you not mention your idea to anyone, for your own good, because I understand your plan. But, you know, you may come across another boss not so capable of understanding as I am. You know what that's like, don't you, Eh?"
>
> Poor John Commonsense didn't utter a word. Without so much as saying goodbye, stupefied with fright and puzzled by the barriers put in front of him, he went away and was never seen again. It was never known where he went. That is why it is often said that when it comes to reforming the system, Commonsense is missing.
>
> Anonymous

We know that we need to give learners more time to explore, to build new ideas, to express themselves mathematically, yet many of our school systems are more concerned or worried about testing, and less about true, or deeper learning. Ethnomodelling engages our educators and students in the process of teaching and learning mathematics. Many of our students remark that it was the first time they were able to use, explore and engage in mathematics in any real, free, or honest fashion, and they were motivated to continue studying more mathematics, because they could see its power, its elegance, it worth.

It is clear that mathematics provides the foundation of the technological, industrial, military, economic, and political systems and that in turn mathematics relies on these systems for the material bases of its continuing progress. It is important to question the role of mathematics and mathematics education in arriving at the present global predicaments of humankind (D'Ambrosio 2010). So many of our learners learn mathematics, yet rarely have the opportunity to see for themselves how it is used, how it can resolve conflict, how it connects to their lives, how it allowed them to turn a conflict into one peaceful, yet empowered moment by writing their own mathematics poem.

References

Allen, D. W. (1992). *Schools for a new century: A conservative approach to radical school reform*. Santa Barbara, CA: Praeger.

Ascher, M. (2002). *Mathematics elsewhere: An exploration of ideas across cultures*. Princeton, NJ: Princeton University Press.

Banks, J. A., & Banks, C. A. M. (1997). *Multicultural education issues and prospectives*. Needham Heights, MA: Allyn and Bacon.

Barbosa, J. C. (2001). *Modelagem matemática*: concepções e experiências de futuros professores [Mathematical modelling: Conceptions and experiences of prospective teachers]. Doctorate thesis in Mathematics Education. Rio Claro, SP: UNESP.

Bassanezzi, R. C. (2002). *Ensino-aprendizagem com modelagem matemática* [Teaching and learning with mathematical modelling]. São Paulo, SP: Contexto.

Bishop, A. (1988). *Mathematical enculturation: A cultural perpective on mathematics education*. Dordtrecht: Kluwer.

Bishop, A. (1991). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht, The Netherlands: Springer.

Brasil. (1998). *Parâmetros curriculares nacionais (PCN):* matemática [National curricular parameters prospects] (pp. 29–47) Boston, MA: Allyn and Bacon.

Craig, E. (1998). *Routledge encyclopedia of philosophy: Questions to sociobiology* (Vol. 8). Now York, NY: Routledge.

Bullivant, B. (1993). Culture: Its nature and meaning for educators. In J. A. Banks & A. M. Banks (Eds), *Multicultural education issues and à prática* [Mathematics education: From theory to practice]. Campinas, SP: Papirus.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44–48.

D'Ambrosio, U. (1990). *Etnomatemática* [Ethnomathematics]. São Paulo, SP: Editora Ática.

D'Ambrosio, U. (1996). *Educação matemática: da teoria à prática* [Ethnomathematics: From theory to practice]. Campinas, SP: Papirus.

D'Ambrosio, U. (2002). *Etnomatemática: elo entre as tradições e a modernidade* [Ethnomathematics: Links between tradition and modernity]. Belo Horizonte, MG: Autêntica.

D'Ambrosio, U. (2010). Mathematics education and survival with dignity. In H. Alro, O. Ravn, & P. Valero (Eds.), *Critical mathematics education: Past, present and future* (pp. 51–63). Rotterdam, The Netherlands: Sense Publishers.

Dewey, J. (1933). *How we think*. Lexington, MA: D. C. Heath and Company.

Dossey, J. (1992). The nature of mathematics: Its role and its influence. In D. A. Grouws (Ed.), *Handbook of research in mathematics teaching and learning* (pp. 39–48). New York, NY: Macmillan.

Eglash, R. (1997). When math worlds collide: Intention and invention in ethnomathematics. *Science, Technology and Human Values, 22*(1), 79–97.

Eglash, R., Bennett, A., O'Donnell, C., Jennings, S., & Contorino, M. (2006). Culturally situated design tools: Ethnocomputing from field site to classroom. *American Anthropologist, 108*(2), 347–362.

Fasheh, M. (1997). Is math in classroom neutral or dead? A view from Palestine. *For the Learning of Mathematics, 17*(20), 24–27.

Freire, P. (2000). *Pedagogy of the oppressed*. New York, NY: Continuum.

Gerdes, P. (1991). *Etnomatemática: cultura, matemática, educação* [Ethnomathematics: Culture, mathematics, education]. Maputo, Moçambique: Lulu.

Habermas, J. (1971). *Knowledge and human interests*. Boston, MA: Beacon Press.

Halpern, D. F. (1996). *Thought and knowledge: An introduction to critical thinking*. Mahwah, NJ: L. Erlbaum Associates.

Headland, T. N., Pike, K. L., & Harris, M. (1990). *Emic and etics: The insider/outsider debate*. Newbury Park, CA: Sage.

Hofstede, G. (1997). *Culture's consequences: International differences in work-related values*. Newbury Park, CA: Sage.

Horne, C. (2000). *Geometric symmetry in patterns and tilings*. Oxford, UK: Woodhead Publishing.

Huntington, S. P. (1993). The clash of civilizations? *Foreign Affairs, 72*(3), 22–49.

Kline, M. (1980). *Mathematics: The loss of certainty*. New York, NY: Oxford University Press.

Kottak, C. (2006). *Mirror for humanity*. New York, NY: McGraw Hill.

McCracken, G. (1988). *The long interview: Qualitative research methods*. Newbury Park, CA: Sage.

Orey, D. C. (2000). The ethnomathematics of the Sioux tipi and cone. In H. Selin (Ed.), *Mathematics across culture: The history of non-western mathematics* (pp. 239–252). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Pike, K. L. (1943). *Phonetics: A critical analysis of phonetic theory and a technic for the practical description of sounds*. Ann Arbor, MI: University of Michigan.

Pike, K. L. (1954). *Emic and etic standpoints for the description of behavior*. Glendale, IL: Summer Institute of Linguistics.

Pollak, R., & Watkins, S. (1993). Cultural and economic approaches to fertility: Proper marriage or mésalliance? *Population and Development Review, 19,* 467–496.

Read, D. (2004). Mathematical modelling issues in analytical representations of human societies. *Cybernetics and Systems, 35*(2–3), 163–172.

Rosa, M. (2010). *A mixed-methods study to understand the perceptions of high school leader about English language learners (ELL): The case of mathematics*. Doctorate dissertation. College of Education. Sacramento, CA: CSUS.

Rosa, M. & Orey, D. C. (2007). A dimensão crítica da modelagem matemática: ensinando para a eficiência sociocrítica [Critical dimension of mathematical modelling: Teaching for social-critical efficiency]. *Horizontes, 25*(2), 197–206.

Rosa, M., & Orey, D. C. (2010). Ethnomodelling: A pedagogical action for uncovering ethnomathematical practices. *Journal of Mathematical Modelling and Application, 1*(3), 58–67.

Rosa, M., & Orey, D. C. (2013). The mathematics of the curves on the wall of the Colégio Arquidiocesano and its mathematical models: A case for ethnomodelling. *Journal of Mathematical Modelling and Application, 1*(8), 42–62.

Rosa, M. & Orey, D. C. (2015). Social-critical dimension of mathematical modelling. In G. A. Stillman, W. Blum, & M. S. Biembengut. (Orgs.). *Mathematical modelling in education research and practice*: *Cultural, social, and cognitive influences* (pp. 385–395). NewYork, NY: USA: Springer.

Rosa, M. & Orey, D. C. (2016). State of the art in ethnomathematics. In M. Rosa, U. D'Ambrosio, D. C. Orey, L. Shirley, W. V. Alangui, P. Palhares, & M. E. Gavarrete (Eds.) *Current and future perspectives of ethnomathematics as a program* (pp. 18–23). ICME13 Topical Surveys. London, England: SpringerOpen.

Rosa, M., Reis, F. S. & Orey, D. C. (2012). A modelagem matemática crítica nos cursos de formação de professores de matemática [Critical mathematical modelling in the mathematics teacher education program]. *Acta Scientiae, 14*(2), 159–184.

Sue, D. W., & Sue, D. (2003). *Counseling the culturally diverse: Theory and practice*. New York, NY: Wiley.

Ting-Toomey, S., & Chung, L. C. (2011). *Understanding intercultural communication*. New York, NY: Oxford University Press.

Tyler, R. (1949). *Basic principles of curriculum and instruction*. Chicago, IL: University of Chicago Press.

Vygotsky, L. (1986). *Thought and language*. Cambridge, MA: The MIT Press.

Zaslavsky, C. (1999). *Africa counts: Number and pattern in African cultures*. Chicago, IL: Lawrence Hill Books Chicago Review Press.