Chapter 12
Ethnomathematics and Its Pedagogical Action in Mathematics Education

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Abstract Ethnomathematics school practices favor respect for, solidarity and cooperation with, the others. It is associated with the pursuit of peace. The main goal of ethnomathematics is building up a civilization free from truculence, arrogance, intolerance, discrimination, inequity, bigotry, and hatred of the others. These are basic questions that define philosophical and ideological postures, which are the roots of a holistic theory of knowledge, looking into the generation, the individual and social organization, and the institutionalization, transmission and diffusion of knowledge. This concept of ethnomathematics is primeval in recognizing the emergence of perceptions of space and time and the techniques of observing, comparing, classifying, ordering, measuring, quantifying and inferring that are different styles of abstract thinking in the school curricula. This is can be achieved by the application of the trivium curriculum, which is an innovative ethnomathematical approach that needs more investigations to address its pedagogical purposes.

Keywords Conceptual dimension · Ethnomathematics · Mathematics curriculum · Pedagogical dimension · Trivium

12.1 Introduction

Many students live in a world dominated by fear, uncertainty, and arrogance, as well as there is a general feeling that the planet is moving toward some form of catastrophic, either economic, political or environmental in nature (Rosa 2000). The most visible are the population and environment crises. Another, more subtle,

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but equally lethal crisis is the humankind's relations to its extensions, institutions, ideas, as well as the relationships between the many individuals and groups that inhabit the globe (Hall 1976; Toffler 1990). It seems that faith in humanity has been forgotten as part of the rationalistic model that characterizes much of contemporary society (D'Ambrosio 2011).

Many educators know that citizens must have specific knowledge and abilities to solve problems in an increasing complex and dynamic society. Teachers realize that students become motivated when they are involved in their own learning. According to Freire (2000), this is especially true when students are encouraged to deal directly with the issues of greatest concern to themselves and their communities.

Traditional school curriculum has neglected the contributions made by minority groups and non-dominant cultures. Given these conditions, educators need to prepare students to live and succeed in this uncertain world. While this crisis of confidence may contribute to the reason for many old skills giving ways to new technologies, it is the capacity to explore and discover, the freedom to succeed without fear of failure that combine to prepare and empower students to take change of their own future. Educators have the obligation to help students to uncover, understand, and comprehend the power of mathematics in their lives.

In this context, Rosa (2000) argues that traditional school curricular programs that focus on basic *skills* are more often than not a source of discouragement, anxiety, and repetition, especially for students who have been subjected to this form of mathematics for years on end. All students deserve access to curricular activities that allow them to see the usefulness, power, and excitement that mathematics can offer.

One of the most important objectives in teaching mathematics should be the development of mathematical capacity in all students. In addition to teach students techniques and tools to solve mathematical problems, they need to learn more than basic mathematical algorithms. All students need to extend their understanding to include how mathematics connects to other disciplines, to problems in society and the environment, and how diverse people around the world use it.

All students need to be encouraged to develop and apply higher-level critical and reflective thinking skills. They must learn mathematical concepts, they need to learn to question, to take risks, verbalize ideas, listen to others' ideas, and to analyze their own and others' ideas. This form of critical thinking must be practiced regularly within mathematical contexts.

A primary goal of learning mathematics is to develop powerful and critical students to be better citizens. The investigation and exploration of ethnomathematical relations can help to develop such students' talent. The teaching of mathematics must emphasize the development of ideas, topics, and themes relevant to the current and future needs of the students (Rosa 2000). The integration of ethnomathematics into the mathematics curriculum enables students to reason both sequentially and holistically to allow them to appreciate distinct relations among different forms of mathematics.

For these reasons, teaching mathematics today is a challenging job. It is necessary that teachers are prepared to develop students' mathematical abilities, critical attitudes, and empower them with mathematical expertise and confidence by including mathematical ideas from different cultures and assist students to acknowledge the contributions that individuals from distinct cultural groups have made to the development of mathematical knowledge.

Ethnomathematics is a pedagogical tool that helps teachers and students to understand both the influence that culture has on mathematics and how this influence results in diverse ways in which mathematics is used and communicated (D'Ambrosio 2006). In this regard, ethnomathematics should become central to a complete study of mathematics.

12.2 Ethnomathematics: Theory or Practice?

People frequently ask: *Is ethnomathematics a theory or a practice?* In order to answer this question, it is necessary to reply with another question: *Is there a practice without a theory supporting it? Is it justifiable to have a theory without a practice?* One of the main objectives of this chapter is to discuss the false the dichotomy between theory and practice.

The main reasons to bring ethnomathematics to schools are to:

1. Demystify school mathematics as a final, permanent, absolute, unique form of knowledge. There is a current misperception in societies, very damaging, that those who perform well in mathematics are more intelligent, indeed *superior* in relation to others. This erroneous impression given by traditional forms of teaching is easily extrapolated to religious, ideological, political, and racial creeds.
2. Illustrate intellectual achievement of various civilizations, cultures, peoples, professions, and genders. Western mathematics is absolutely integrated with conquest and colonialism that came to dominate the entire world. The acceptance, forced or voluntary, of western mathematics and western knowledge in general leads to the acceptance of behavior and values, of ideas like *the winner is the best, the losers are to be discarded*. More than any other form of knowledge, mathematics is identified with the winners. This is true in history, in professions, in everyday life, in families, and in schools. The only possibility of building up a planetary civilization depends on restoring the dignity of the losers and both winners and losers, moving together into the new. This requires respect for each other. Otherwise, the losers will direct their efforts to become winners and the winners will do the best to protect themselves from the losers, thus generating confrontation.

School ethnomathematics practices encourage respect, solidarity and cooperation with the other. It is thus associated with the pursuit of Peace. The main goal of ethnomathematics is to support the building up of a civilization free of truculence, arrogance, intolerance, discrimination, inequity, bigotry, and hatred.

These are basic questions that define philosophical and ideological postures. These postures are found in the roots of a holistic theory of knowledge, looking into the generation, the individual and social organization, and the institutionalization, transmission and diffusion of knowledge, as studied in the Program Ethnomathematics (D'Ambrosio 2015). Repeating what is written in many of D'Ambrosio's papers, his concept of ethnomathematics is primeval. It recognizes, in every corner of the planet, the different emergence of perceptions of space and time and the techniques of observing, comparing, classifying, ordering, measuring, quantifying and inferring and, as consequently, different styles of abstract thinking.

In each corner of the planet and at every time, individuals have developed strategies to satisfy the pulsions of survival and transcendence. These strategies are synthesized in three words: the *techné* $\approx$ *tics* [i.e., the ways, modes and styles, the arts and techniques] that people developed for *mathemá* [i.e., for explaining, learning and understanding, knowing and coping with] their *ethno* [i.e., their natural facts and phenomena and the social, cultural, mythical and imaginary environment]. This etymological exercise led D'Ambrosio to construct the concept of *tics* of *mathema* in distinct *ethnos*, or *ethno* + *mathema* + *tics*, hence, by rearranging the words, ethnomathematics.

Ethnomathematics as a research paradigm is much wider than traditional concepts of mathematics and ethnicity and any current sense of multiculturalism. *Ethno* is related to the members of distinct groups identified by cultural traditions, codes, symbols, myths, and specific ways of reasoning and inferring (D'Ambrosio 1985). Hence, ethnomathematics is the way that members of various cultural groups mathematize[1] their own reality because it examines how both mathematical ideas and practices are processed and used in daily activities. It is also described as the arts and techniques developed by individuals from diverse cultural and linguistic backgrounds to explain, to understand, and to cope with their own social, cultural, environmental, political, and economic environments (Rosa 2010).

There is no contradiction with this concept of ethnomathematics and the universally accepted concept of academic or scholarly mathematics. Indeed, a very synthetic view of ancient cultural history shows that the universally accepted concept called western mathematics is an elaboration of the specific way of the peoples of the *ethno* of the Mediterranean basin organized their *tics* of *mathemá* in that region, hence their own ethnomathematics. Since there were many civilizations

[1]Mathematization is a process in which members from different cultural groups come up with different mathematical tools that can help them to organize, analyze, comprehend, understand, and solve specific problems they face in the context of their real-life situation. These tools allow them to identify and describe a specific mathematical idea or practice in a general context by schematizing, formulating, and visualizing a problem in different ways, discovering relations, discovering regularities, and transferring a real world problem to a mathematical idea through mathematization (Rosa and Orey 2010).

in this broad region, they had many contacts and cultural encounters and their ethnomathematics, as every one, went through reformulations.

It is important to recall the encounters of civilizations around the Mediterranean, mainly those of Ancient Iraq, of Egypt, of Israel, of Greece, of Persia, of Rome, and many others. They were close enough to have mutual influence, through a dynamics of cultural encounters, and eventually gave origin to Greek philosophy, which was synthesized in the *Elements*, composed by Euclid of Alexandria, *c*300 a.C., and which inaugurated what became known as the *Euclidean Style*, by creating a specific kind of narrative and criteria of truth. Although no authentic complete copy is extant, the *Elements* became the canon of what is now called western mathematics.

For religious reasons, it was rejected by Christianity in the period known as the Early Middle Age, but in this period it was preserved and advanced in Hellenistic academies in Northern Africa, especially in Alexandria, and by Arabic and Islamic scholars. After the Crusades, the *Euclidean Style* was absorbed by Europe in the periods known as the Late Middle Age and Renaissance. This gave rise to the corpus of knowledge now called western, academic or school mathematics or, simply, mathematics.

This corpus of mathematics knowledge came to be organized as a discipline in the 15th century, and was successful as the theoretical foundation of modern science and technology and of the capitalist economic system. This discipline spread through Europe and after the great navigations, conquests and subsequent colonialism, to the entire world.

The colonial process established school systems in the occupied territories, which was continued after independence and which has prevailed to the present. In the schools, mathematics was and continues to be central in schools, in all levels, and also in the universities and in research. Mathematics became necessary for the commerce, for the production and for the economy systems, for technology and for sciences.

A major cause of social exclusion is the lack of competencies in mathematics. It results in difficulties in employment and in many common daily activities. It is the same as in illiteracy and innumeracy that excludes individuals from participation in society. It is the same as in learning to read and write, in learning to deal with basic quantities (arithmetic) and forms (geometry) that are essential in every cultural environment. They are essential as communicative instruments.

Good mathematical learning occurs with social communication and cultural interaction through dialogue and negotiation of meaning of the symbolic representations between teachers and students. To include and exclude differences and mathematical traditions brought to the school community by the students is a moral decision that governments and curriculum developers must consider.

In order to understand diverse mathematical cognitive strategies, it is necessary to envisage students in the context of their own sociocultural environment, which often includes a variety of traditions, behaviors, religions, and languages. This "approach helps the organization of the pedagogical action that occurs in classrooms through the use of the local aspects of these mathematical practices" (Rosa and Orey 2015a, p. 140).

Modifications in the classroom pedagogical practices are required because mathematical activities are not universal and they are unequally applicable across cultures. Thus, if the assumption that the origin, process, and manifestation of mathematical knowledge are not similar across cultures, then universal guidelines and strategies for the pedagogical work would appear to be inappropriate to be applied to all the members of distinct cultural groups (Rosa 2010).

Mathematics is central to school systems all over the world; it dominates and, sometimes is considered the most important subject. But in every society it has not eliminated the practices of the ethnomathematics of the many culturally identified groups. The ethnomathematics of the *invisible society*, of the *non-elite population*, which produces and provides for the basic needs of the people, and of the upper classes, is present and practiced. For example, craftsmen and retailers are responsible for small scale manufacturing, for producing and selling basic goods, like food, clothing, and other utilities for daily consumption.

These professionals deal with certain branches of the economy and they provide non-material needs such as religious and popular rituals and festivals, for popular medicine, for the arts and for sports and for the many other social and cultural activities. All these, craftsmen, retailers and other professionals, rely on people's knowledge and on the traditional wisdom, passed from generation to generation, and from their peers. It is impossible to deny that *official* competencies coexist with other people's competencies to deal with daily life.

To build a civilization that rejects inequity, arrogance, and bigotry, education must give special attention to the redemption of peoples that have been, for a long time, subordinated and must give priority to the empowerment of the excluded sectors of societies.

12.3 Conceptual Dimension of Program Ethnomathematics

The program ethnomathematics contributes to restoring cultural dignity and offers the intellectual tools for the exercise of citizenship. It enhances creativity, reinforces cultural self-respect, and offers a broad view of the mankind. This program offers the possibility of harmonious relations in human behavior and between humans and nature. Intrinsic to it is the ethics of diversity:

- Respect for the other (the different).
- Solidarity with the other.
- Cooperation with the other.

It is important to return to the question in the beginning of this chapter: is Ethnomathematics research or practice? Ethnomathematics arises from research, and this is the reason for calling it a Program. But, equally important, indeed what justifies this research, are the implications for curriculum innovation and development, for teaching and teacher education, for policy making, all focusing the effort to erase arrogance, inequity, and bigotry in society.

As discussed above, the theoretical approach of a program ethnomathematics recognizes the cultural dynamics found in the encounters between cultures, which result in the coexistence of both the *official* and people's competencies. All of this links the historical and epistemological dimensions of the program ethnomathematics, which brings new light into our understanding of how mathematical ideas are generated and how they evolved through the history of humankind. It is fundamental to recognize the contributions of various cultures and the importance of the dynamics of cultural encounters.

Culture, understood in its widest definition, includes aspects of art, history, languages, literature, medicine, music, philosophy, religion, science, and technology. It is characterized by shared knowledge systems, by compatible behavior and by acceptance of an assemblage of values. Research in ethnomathematics is, necessarily, transcultural and transdisciplinarian. The encounters between cultures are examined in its widest form to permit exploration of more indirect interactions and influences, and to permit an examination of subjects on a comparative basis.

At this moment, it is important to reinforce the concept that ethnomathematics should not be confused with ethnic-mathematics, as many researchers and educators mistakenly understand it. In this regard, it is necessary to stress the importance of the use of the denomination of the *Program Ethnomathematics* because ethnomathematics is not a final form of knowledge, but it is composed of a theoretical basis and a practice that is in permanent re-elaboration. Just like the various systems of knowledge, such as mathematics, religion, culinary, dress, sports and gaming, and several other practical and abstract manifestations of the human species in different contextual realities.

Ethnomathematics is permanently subjected to revision and reformulations. Although dealing primarily with space, time, classifying, comparing, which are practices proper to the human species, ethnomathematics examines the codes and techniques that we use to express and communicate our reflections on these practices, which are, undeniably, contextual. To express these ideas that are related to the assumptions of a research program, the neologism, *ethno* + *mathema* + *tics* was coined. In this context, D'Ambrosio (2007a) argues that:

> The resistance against Ethnomathematics may be the result of a damaging confusion of ethnomathematics with ethnic-mathematics. This is caused by a strong emphasis on ethnographic studies, sometimes not supported by theoretical foundations, which may lead to a folkloristic perception of ethnomathematics (p. IX).

According to this point of view, D'Ambrosio (2007b) argues that an etymological elaboration explains the meaning of ethnomathematics, which is a construct that uses the roots *ethno* that means the natural, social, cultural and imaginary environment, *mathema* that means explaining, learning, knowing, coping with, and *tics*, which is a simplified form of *techné*, meaning modes, styles, arts and techniques. Hence, ethnomathematics is a theoretical reflection on the *tics* of *mathema* in distinct *ethnos*. This etymological root caused much criticism since it does not reflect the etymology of academic mathematics.

For example, the term *mathema* is not related to mathematics, which is a neologism based on a Greek origin that was introduced worldwide in the 15th century during the colonization process through navigation. D'Ambrosio (2006) argues that the program ethnomathematics is not *mathema* + *tics* because it tries to understand and explain various knowledge systems such as mathematics, religion, culinary, and other practical and abstract manifestations of humanity in different and diverse cultural contexts.

12.4 Pedagogical Dimension of the Program Ethnomathematics

In general, education was classified as a commodity by the World Commerce Organization, which is a special agency of the United Nations Organization. Since then, Education has become increasingly subordinate to big corporations and the result is that contents and methodologies are out of context. This affects particularly Mathematics Education. The main concern is attaining pre-decided goals of proficiency, which favor uniformity and sameness and surely leads to the promotion of docile citizens and irresponsible forms of creativity. Tests are the best instruments to support this corporate aspect of education. Tests penalize creative and critical education, which leads to the intimidation of the new and to the reproduction of the current model of society.

Education, particularly mathematics education, must focus on the immediate questions facing the world, which includes both social and environmental threats. It is necessary that mathematics educators address these questions and the pursuit of peace in all four dimensions: individual peace, social peace, environmental peace and military peace (D'Ambrosio 2000). This is the relation of the program ethnomathematics with peace, ethics and citizenship.

It is important to insist that the program ethnomathematics is not ethnic mathematics, as some commentators interpret it. Of course, one has to work with different cultural environments and try to describe mathematical ideas and practices of other cultures, acting as an ethnographer. This is a style of doing ethnomathematics, which is absolutely necessary. Cultural environments include indigenous and also urban populations. In urban populations and in the periphery of major cities we must look into the practices of culturally identifies groups, through their professional practices, as retailers and laborers, as farmers, as artisans. The scenario is of individuals facing specific situations and problems in their daily life.

Individuals develop their own ad hoc practices to deal with the specific situations and to solve the problems they face, sometimes sharing their practices with their peers. When the ad hoc responses reveal that they are efficient in dealing with similar situations and solving similar problems, they are organized as methods, with specific jargon, and they are acquired as common practices by the culturally identified group. It is natural that practitioners ask: why does this method work? This is the moment a theory comes into the scenario. Individuals and groups reflect and theorize on their methods and ideas. This is the proper ground to germinate inventions, to propose the new. This is the most important element for the program ethnomathematics.

Synthesizing the arguments of the paragraph above, research and practice in ethnomathematics are the responses to three basic questions:

1. How are ad hoc practices and solution of problems developed into methods?
2. How are methods developed into theories?
3. How are theories developed into inventions?

The answers to these questions results in the learning process of methods and practices of individuals and their peers in cultural identified groups. This is an ethnographic approach to ethnomathematics. According to this context, the program ethnomathematics is a *creative insubordination* approach in Mathematics Education because it challenges the conventional view that Science, which includes Mathematics, is a uniquely modern Western phenomenon.

Science develops wherever there is a systematic search for the understanding of daily phenomena even though there is an absence of a systematic process or achieving certain results, which is a set of an ordered sequence of fixed steps in a recognized method. Indeed, the satisfaction of survival and transcendence that exists in every cultural group and is present during the developmental process of its members is resolved by the application of ad hoc solutions to deal with problems they face daily.

The improvement of these ad hoc practices leads to the development of methods as well as the search for comprehending and explaining hose these methods lead to the evolution of theories. Thus, the intellectual adventure of the humanity is synthesized in the expanding process of knowledge, which is given by:

ad hoc solutions ⇨ *methods* ⇨ *theories* ⇨ *innovations*

The program ethnomathematics develops this process further by trying to understand how procedures and practices, ethnographically studied, develop into methods, which are related to the studies on cognition, and how methods develop into theories, which are related to the studies on epistemology, and how theories develop into inventions, which are related to the studies on creativity.

An important issue in the Philosophy of Science is the acceptability of a theory, which acquires the reliability of a community recognized as elite specialists. Yet, it is difficult to deny that ad hoc solutions and methods to solve new problems that are both generated and socialized by the members of distinct cultural groups. In this case, these elites appropriate or expropriate the methods and become responsible for building up theories.

In this context, it is important to understand that ethnomathematics arises from research, which is the main reason to name it as a program. However, it is equally important to justify that the relevance of this research program is its implications for teaching, teacher education, policy making, and for the school curricula innovation and development, as well as its efforts to erase arrogance, inequity, and bigotry in society (D'Ambrosio 2007b).

This innovation in the mathematics curriculum is related to the elaboration of activities that motivates students to recognize that mathematical knowledge is part of their everyday life. Therefore, this curriculum enhances students' ability to make meaningful mathematical connections by deepening their understandings of all forms of mathematics (Rosa and Orey 2011).

In the current context, there is a necessity for innovative ethnomathematics-based programs to identify and seek teaching-learning practices directed towards its pedagogical action. The great challenge for researchers in ethnomathematics is to conduct studies and elaborate pedagogical practices that are in accordance with the philosophical-theoretical objectives of this program (Rosa and Orey 2015a).

As an example of innovation in the development of the school practices, the "trivium curriculum for mathematics proposed by D'Ambrosio (1999) is an important innovative ethnomathematics approach that needs more investigations in order to address pedagogical purposes" (Rosa and Orey 2016, p. 18). The proposal of this trivium is an answer to the criticism of the lack of balance between the quantitative and qualitative data in the mathematics curriculum.

12.5 Trivium Curriculum for Mathematics

The *dambrosian* proposal for the reconceptualization of the mathematics curriculum is based on the trivium, which is composed by three strands: literacy, matheracy, and technoracy. This curriculum provides, in a critical and reflective way, the communicative, analytical, and technological instruments that help students do develop necessary skills and abilities to perform daily activities in the twenty-first century.

Thus, Rosa and Orey (2015b) argue that this curriculum contributes to the development of school-based activities developed on an ethnomathematics perspective. This approach allows students to use communicative, analytical and technological instruments that are essential to the exercise of their rights and duties that are necessary to practice of citizenship as well as to the critical and reflective reading of daily phenomena that occur in their daily life.

12.5.1 Literacy as Communicative Instruments

Literacy is the capability that individuals possess to process information that are impregnated in their daily lives through the use of media, internet and mobile learning by means of reading, listening, speaking, writing, calculating, designing, and representing. This ability to process information is also performed by using signs, gestures, numbers, calculators, and computers (D'Ambrosio and D'Ambrosio 2013).

Mobile learning and internet enables educational possibilities, with constantly evolving information streams. However, the vastness possibilities of the internet and mobile learning can be a hindrance to those individuals who cannot effectively sift through to analyze and interpret the material presented and found online. Mobile learning integrates the use of mobile computing such as the application of small, portable, and wireless computing and communication with e-learning that facilitates the acquisition of information and communication through technological devices (Valdes and Valdes-Corbeil 2007).

Therefore, strong literacy skills and abilities are a key tool used when students discern and interpret information, enabling them to use them to its full potential in order to "support learning experiences that are collaborative, accessible, and integrated with the world beyond the classroom (Valdes and Valdes-Corbeil 2007, p. 54). These communicative instruments facilitate the integration of schools and communities through a cultural dynamics that allows them to exchange knowledge by processing information in this social interaction. This approach helps students to reduce cultural and communication barriers between schools and communities by using synchronous[2] and asynchronous[3] communication channels (Rosa and Orey 2016).

Currently, reading includes the competency of numeracy, which is the interpretation of graphs and tables, and other ways of informing individuals. Reading even includes the understanding of the condensed language of codes. These competencies also relates to getting information. The power of literacy relates to the individuals capacity to apply these skills to effectively connect, interpret and discern the intricacies of the world in which they live.

Since the role of the teachers is important to initiate and conduct this process, they need to know the sociocultural contexts of their students. In this regard, teachers guide the choice of a theme that is more meaningful to the students in order to drive a transformational action in the school community. By using these communicative instruments, students are able to analyze, understand, comprehend, process, and respond the stimuli offered by the modern phenomena such as inflation, urban growth, consumption, production sectors, elections, health, environmental, and educational policies.

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[2]Synchronous communication literally means *at the same time* since it occurs in real time and can take place face-to-face. This kind of approach involves live communication either through sitting in a classroom, telephone conversations, instant messagings, online chattings, or teleconferencing via video/webconferences (Rosa and Orey 2016).

[3]Asynchronous communication literally means *not at the same time* since it is not immediately received or responded to by those involved. This kind of communication usually involves a set of weekly deadlines, but otherwise allows students to work at their own pace by using emails and message board forums which allow them to communicate on different schedules (Rosa and Orey 2016).

Rosa and Orey (2015b) stated that once the theme is chosen, teachers must prepare students for the field ethnographic work, which objective is to collect qualitative and quantitative data that must be analyzed and interpreted in order to allow them to formulate their questionings and conjectures to help them to solve problems they face in their school community.

12.5.2 Matheracy as Analytical Instruments

Matheracy is the capability individuals possess to infer, propose hypothesis, and drawing conclusions from data, which helps them to analyze and interpret signs and codes so they are able to propose and use models to find solutions to problems they face in their lives through the elaboration of abstract representations of the real world (Rosa and Orey 2015b).

It provides symbolic tools that help students to develop their own creativity in order to allow them to understand and solve problems and situations. It is a first step toward an intellectual posture, which is almost completely absent in our school systems. Regrettably, even conceding that problem solving, modeling, and projects can be seen in some mathematics classrooms, the main pedagogical importance is usually given to numeracy or the manipulation of numbers and operations (D'Ambrosio and D'Ambrosio 2013).

The elaboration of models helps students to read the world in order to acquire tools that allow them to propitiate in a clear and concise ways the search for solution to problems they face daily. Rosa and Orey (2015b) argue that the use of matheracy is affected by the analysis of the relation between variables that are considered essential to the understanding of the given phenomenon. In this approach, teachers need to act as mediators of this process by instrumentalizing students with the necessary tools that help them to analyze and interpret solutions to these problems in order to allow them to reflect about the issues involving society.

Thus, matheracy is the domain of skills, strategies, and abilities that empower students to be aware of the way in which they explain their beliefs, traditions, myths, symbols, and scientific and mathematical knowledge. According to D'Ambrosio (2008) states that representations given by these elements enable the expansion of reality by incorporating the sociofacts,[4] mentifacts,[5] and

[4]Sociofacts are structures and organizations of a culture that influences social behavior and the development of scientific and mathematical knowledge, which include families, governments, educational systems, sports organizations, religious groups, and any other grouping designed to develop specific sociocultural activities. Sociofacts define the social organization of the members of distinct cultural groups because they regulate how individuals' function in relation to the other members of a specific group (Rosa and Orey 2015b).

[5]Mentifacts refer to the shared ideas, values and beliefs such as religion, language, mathematics, sciences, viewpoints, law, and knowledge that are developed and diffused by the members of distinct cultural groups from generation to generation (Rosa and Orey 2015b). The main issue in

artifacts[6] regarding mathematical simulations and models. Rosa and Orey (2016) affirm that these competencies allow students to have access to a diverse set of signs, codes, symbols, and methods that are essential to the development of the decision-making process.

12.5.3 Technoracy as Technological Instruments

Technocracy is the critical and reflective capability that individuals possess to use and combine different technological instruments from the simplest to the most complex ones in order to help them to solve problems they encounter in everyday activities (Rosa and Orey 2016).

The operative aspects of the familiarity with technology, in most cases, is inaccessible to most individuals, yet basic ideas behind technological devices, their possibilities and dangers, and the morality supporting the use of technology, are essential issues to be raised among individuals. History shows that ethics and values are intimately related to the development of technological progress.

Mobile learning involves the use of mobile technology, either alone or in combination with other information and communication technology to enable learning anytime and anywhere. Learning can unfold in a variety of ways: individuals are able to use mobile devices to access educational resources, connect with others, and create content, both inside and outside classrooms. Mobile learning also encompasses efforts to support broad educational goals such as the effective administration of school systems and improved communication between schools and families.

Technoracy is an important feature of scientific and mathematical knowledge as well as its reification as technological artifacts manifest itself in technological tools that translate ways of dealing with natural, social, natural, cultural, political, and economic contexts. These environments allow the development and incorporation of technological tools used in specific sociocultural contexts (Rosa and Orey 2015b).

Technoracy uses different technological instruments that serve to mathematical purposes such as simple, graph, and scientific calculators, *softwares*, computational

(Footnote 5 continued)

the empowerment of individuals is the transition from the elaboration of mentifacts (theorizing about the events and phenomena that occur in everyday life) to the development of strategies and actions that are adequate to solve new problems and situations (D'Ambrosio 2006).

[6]Artifacts are the cultural objects, primarily material items and technologies created by members of distinct cultural groups. It is the technological subsystem composed of material objects as well as techniques of their use. Such objects are tools and other instruments that enable individuals to feed, clothe, house, defend, transport, and solve daily problems by using scientific and mathematical techniques and tools as well as informal and non-standardized mathematical knowledge found in other sociocultural contexts (Rosa and Orey 2015b).

programs, and simulators even though the manipulation of different types of equations that allow them to analyze, interpret, and evaluate models during the decision-making process in order to act and transform reality.

It is important to emphasize that a trivium mathematics curriculum intends to build a generalizable and applicable common body of mathematical knowledge by valuing locally, relevant, and specific mathematical knowledge. According to this point of view, Rosa and Orey (2015b) state that this curriculum seeks to make connections between local and academic knowledge, which can be jointly developed in a dialogical and interdisciplinary way.

It is important to highlight that the trivium curriculum addresses connections between the general aspects of academic mathematics (matheracy) and locally relevant mathematical practices developed by the members of distinct cultural groups (literacy) who use technological instruments (technoracy) to evaluate diverse ways to represent mathematical ideas, procedures, and practices as well as to assess the reasonableness of the results and their contextualizations.

12.6 Incorporating Ethnomathematics into the Mathematics Curriculum

A question that usually arises is about the objective of incorporating ethnomathematics in the curriculum. Would not be enough to teach the mathematics that will be useful to students, to pass in tests, to apply for jobs, or to improve their own professional practice? True, these are the immediate objectives expected by students, by parents and by society as a whole. It is important to do not deny these immediate expected objectives. But education cannot be resumed to practical immediate objectives. It is even more important to prepare students to face the new, to aim further than what they have now. They have to acquire high esteem of themselves.

There is a similarity with the offering of languages other than the mother language. Of course, it is important to learn and to use properly the language of the family and of the community. If we are proficient in only one language, we are less equipped to be successful in the modern World. No one denies this. But there is another factor, of cognitive nature, that applies also to mathematics. Studies in cognition show the advantages of knowing other languages.

There are evidences that the use of one language favors the utilization of another language. Bilingual speakers have to select the best from two competing options. This may be determined by the social context, but cognitive resources are activated. A very difficult question is which one to select in a determined circumstance. Hence, a possible explanation for the selection has to do with control systems recruited into linguistic processing.

According to this context, what do we expect of a good education? The answer is:

- To promote full citizenship by preparing the individual to be integrated and productive in society, through the transmission of values and the clarification of both their responsibilities and their rights in society.
- To promote the expression of the creativity of every individual, by encouraging people to achieve their potential at the highest level of their interest and ability, which leads to progress of the society.

To achieve full citizenship, no one can deny that the proficiency in mathematics is absolutely necessary. Like being illiterate, without proficiency in basic school mathematics the individual is excluded from participation in society. It will be difficult to have employment, to be a conscious consumer, to perform daily routines. But we have data that confirms the failure of school systems around the world in providing proficiency in basic school mathematics.

We have to question the meaning of basic school mathematics. There are no valid arguments that justify insisting on teaching geometry as organized 25 centuries ago, arithmetic as organized more than 10 centuries ago and algebra invented about 5 centuries ago, as it is done in most schools in the world. Teachers teach the way they were taught, ignoring progresses occurred during their professional career.

This is particularly serious in mathematics, which changed so much since the evolution of computers, calculators and informatics in various forms. The insistence in teaching geometry, arithmetic and algebra, and even calculus, as knowledge frozen for many centuries, memorizing techniques and availing the proficiency in tests, is unsustainable. The result is that students are bored and reject mathematics. As a consequence, students are not acquiring necessary competencies for full citizenship.

Of course, some basic geometry, arithmetic and algebra are important tools to solve real problems, but the access to these tools is possible, thanks to modern technological resources, without the need of memorizing techniques. The role of the teacher is to propose interesting real problems. If there is interest of the students, they will look for tools. Then, the teacher acts listening to the students and learning their approach to solve the problems, and in many cases their approach rely on ethnomathematics learned from their parents and from their communities.

The elaboration of a mathematics curriculum that is based on students' knowledge allows teachers to have more freedom and creativity to choose mathematical topics to be covered in the lessons. Through dialogue with the students, teachers can apply mathematical themes that help them to understand the content of the mathematics curriculum. Hence, teachers can engage students in the critical analysis of the dominant culture as well as of their own culture (Rosa and Orey 2011). This approach is extremely valuable and must be respected by the teacher and by fellow students.

At the same time, students may compare different approaches to deal with the problem, which reveals the existence of multiple ethnomathematics, which enriches the learning experience. Surely, the teacher may also reveal her/his own way of dealing with the problem, based on School Mathematics and the appropriate use of technological devices. The various approaches lead to possible new approaches, combining aspects of different ethnomathematics and of school mathematics. This is an important example of the dynamics of cultural encounters.

The most important aspect of the pedagogical dimension of the program ethnomathematics is the mutual exposition of different cultural approaches to face a situation or problem. This mutual exposition, with full mutual respect, is responsible to advances of knowledge throughout the evolution of the human species. This is the essence of the dynamics of cultural encounters.

Not only in mathematics, the dynamics of cultural encounters forms the great hope of new approaches concerning the assembly of environmental crises, particularly the fast exhaustion of water supplies, and to face the mounting health problems, and to confront the social and religious tensions around the world.

Regarding health care, the scientific developments since the Renaissance have led to a scientific and highly technical form of medicine, displacing traditional medicine of indigenous populations. The traditional practices survived, even if they became prohibited as criminal practices. Now, the World Health Organization has launched a project to support countries in developing proactive policies and implementing action plans that will strengthen the role Traditional Medicine keeping populations healthy (WHO 2013).

The eminent cultural historian Geoffrey Ernest Richard Lloyd refers to the encounter of Western medicine and traditional medicine saying that:

> (…) the possibilities of mismatch between what biomedicine [with a battery of tests to call on] pronounces to be the case and what individual patients feel, are unlikely ever to be completely removed. If so, alternative styles of medicine, with their more or less articulate elites to promote them, are likely to continue to bear witness to the complexities of our understanding of what it is to be truly well, and it would surely be foolhardy to suppose that biomedicine has nothing to learn from its rivals (Lloyd 2009, p. 92).

It is important to paraphrase what Geoffrey Ernest Richard Lloyd says by just replacing a few words and stating that the:

> (…) possibilities of mismatch between what Mathematics [with a battery of frozen knowledge and tests] pronounces to be an instrument to solve real life problems, are unlikely ever to be completely removed. If so, alternative styles of Mathematics, which are Ethnomathematics practiced by cultural identifies groups, are likely to continue to bear witness to the complexities of our understanding of what it is to face problems posed by real life, and it would surely be foolhardy to suppose that Academic and School Mathematics has nothing to learn from its rivals Ethnomathematics (D'Ambrosio and Rosa 2016, p. 7).

A good education cannot be limited to promote full citizenship, to succeed in employment, in being a conscious consumer and in performing well in daily routines. These are immediate, necessary goals, but not enough for a good education. A good education should also help students to attain their personal satisfaction, attaining higher objectives in life, according to their interest and abilities. This requires raising their self-esteem and creativity.

12.7 An Ethnomathematics Curriculum

Schools and classrooms cannot be isolated from the communities in which they are embedded. If schools are considered part of a community with defined cultural practices, then classrooms might be understood as environments that facilitate pedagogical practices in these learning environments.

When students come to school, they bring with them values, norms, and concepts that they have acquired in their sociocultural environment. Some of these ideas and procedural concepts are mathematical in nature. However, mathematical concepts developed in the school curriculum are presented in a way that may not be related to the students' cultural backgrounds.

It has been hypothesized that low attainment in mathematics could due to the lack of cultural consonance in the curriculum. Moreover, there is evidence from research that the inclusion of cultural aspects in the curriculum have long-term benefits for mathematics learners, which means that cultural aspects contribute to the recognition that mathematics is part of daily life of the students (Eglash et al. 2006; Rosa and Orey 2013).

The pedagogical work towards an ethnomathematics perspective allows for a broader analysis of the school context in which mathematical practices transcend the classroom environment because they embrace the sociocultural context of the students. Hence, pedagogical elements necessary to develop curricular activities are found in the school community. The field of ethnomathematics posits some possibilities for innovative educational initiatives that help to reach this goal (Rosa and Orey 2015b).

It is important to recognize that ethnomathematics is a research program that guides educational pedagogical practices. In this context, there is a need to examine the embeddedness of mathematics in culture, drawing from a body of literature that takes on the cultural nature of knowledge production into the mathematics curriculum (Rogoff 2003).

Mathematics as part of the school curriculum must reinforce and value the cultural knowledge of students rather than ignore or negate it. It is argued that this mathematics curriculum seek to change the way mathematics teachers construct their learning environments by producing teachers who are able to facilitate a mathematics learning environment grounded in real life experiences and to support students in the social construction of mathematics (Rosa and Orey 2007).

The trend towards ethnomathematical approaches to the mathematics curriculum reflects a comprehensive development in mathematics education. These approaches are intended to make school mathematics more relevant and meaningful to students and to promote the overall quality of education. It is necessary to investigate conceptions, traditions, and mathematical procedures developed by the members of a particular cultural group with the intention of incorporating these practices into the mathematics curriculum. In this direction, Rosa and Orey (2011) plead for a more culturally sensitive view of mathematics to be incorporated into the school curriculum.

This mathematical approach is presented as a cultural response to students' needs by making connections between their cultural background and mathematics, which supports the view that mathematics is conceived as a cultural product which has developed as a result of various activities. A learning environment using this type of curriculum would be full of examples that draw on the students' own experiences that are common in their own cultural contexts (Rosa and Orey 2008). Ethnomathematics uses these cultural experiences as vehicles to make mathematics learning more meaningful and to provide students with insights of mathematical knowledge embedded in their sociocultural environment. Students understand the nature of mathematics as they become aware of the presence of mathematical knowledge in their own culture.

In an ethnomathematical curriculum, it is necessary that teachers develop a different approach to mathematics instruction that empowers students to understand mathematical power more critically by considering the effects of culture on mathematical knowledge and work with their students to uncover the distorted and hidden history of the mathematical knowledge. According to Rosa (2010), this methodology is essential in developing the curricular practice of ethnomathematics and culturally relevant education: through the investigation of the cultural aspects of mathematics and an elaboration upon mathematics curricula that considers the contributions of people from other cultures, students' knowledge of mathematics becomes enabled and enriched.

12.8 Final Considerations

Modern Civilization is failing in supplying the humanity with the basic strategies for survival and, at the same time, it is denying cultural identities and social justice. Words like democracy, freedom and equity are political slogans suggesting affirmative action.

However, they have been used by social movements and political factions to draw applause and to recruit and align cadres, which are soon voided by internal contradictions and conflicts. There are no real benefits for the people. This opens a space for reactionary counter-action, which are well coordinated and silent, and gain space in maintaining the *status quo*. People see international corporations displacing public schools, all over the world co-opting educators for new programs and new methods of instruction, with the fallacious objective of getting better results in tests.

The proposals favor the economic interest of corporations, but fail to develop the dignity of the individual and the imperatives of justice, compassion and peaceful modes of resolving conflicts, aiming at a new future, in which the humanity can live together in grace and peace. It is necessary to recall the appeal of Bertrand Russell and Albert Einstein in the Pugwash Manifesto (1955):

> There lies before us, if we choose continual progress in happiness, knowledge, and wisdom. Shall we, instead, choose death, because we cannot forget our quarrels? We appeal as human beings to human beings: Remember your humanity, and forget the rest. If you can do so, the way lies open to a new Paradise; if you cannot, there lies before you the risk of universal death (Russel and Einstein 1955).

The program ethnomathematics is an answer to this appeal. It is a theoretical framework that establishes the foundation for organizing practices and systems of explanations developed by the species throughout its evolution in order to survive and to transcend.

According to this discussion, a comprehensive view of mathematics curriculum is implicit in an ethnomathematical perspective. All students possess potential for understanding and communication through a variety of mathematical systems within distinct cultural contexts. This approach allows students to gain new perspectives on human potential and on the organization of the mathematics curriculum. Therefore, mathematics can only be truly learned and taught if it includes culture, natural language, and visual representations that are culturally relevant to learners and teachers alike.

An ethnomathematical perspective in the mathematics curriculum helps all participants to come to understand and appreciate alternative viewpoints, cultural diversity, natural language, mathematics, and visual representations which form a unique system for meaning-making. In this context, reorienting teaching and learning to include ethnomathematics can engage and excite students about learning and encourages them to see themselves as being able to do mathematics by validating their own cultural experiences, which serves as an essential component of understanding and celebrating the differences between diverse cultural groups.

Ethnomathematics is a system of knowledge that offers the possibility of a harmonious relation among humans and between humans and nature. It contributes to restoring cultural dignity and offers the intellectual tools for the exercise of citizenship. It enhances creativity, reinforces cultural self-respect and mutual respect, and offers a broad view of humanity.

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