Chapter 5
Measuring the Dispersion of the Players

Abstract The purpose of this chapter is to introduce the concepts of dispersion in the aim of soccer analysis. A set of different measures have been proposed to identify the level of dispersion between teammates and between opponents. Based on that, a summary of the dispersion measures, definitions, interpretation and graphical visualization will be presented on this chapter. The measures of Stretch Index, Surface Area, Team Length and Team Width and lpwratio will be introduced throughout the chapter. The case studies presented involve two five-player teams in an SSG considering only the space of half pitch (68 m goal-to-goal and 52 m side-to-side) and another eleven-player team in a match considering the space of the entire field (106.744 m goal-to-goal and 66.611 m side-to-side) even though only playing in half pitch.

Keywords Position data · Georeferencing · uPATO · Soccer · Collective behavior · Team's dispersion

5.1 Stretch Index

5.1.1 Basic Concepts

The Stretch Index of a team is calculated as the average distance of a team's players and the Geometrical Center, giving a notion of the compactness of the team.

Definition 5.1 [1] The Stretch Index, considering both axes, at a given instant t can be calculated by:

$$SI(t) = \frac{\sum_k^N \sqrt{(p_{xk}(t) - C_x(t))^2 + p_{yk}(t) - C_y(t))^2}}{N}, \tag{5.1}$$

where $C(t)$ represents the Geometrical Center of the team, N the number of players in the team, $P_{xk}(t)$ the position along the longitudinal axis for player k at instant t, and $P_{yk}(t)$ the position along the vertical axis for player k in instant t.

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Definition 5.2 [1] The Stretch Index on a single axis, for a given instant t, is given by the following expression:

$$SI_x(t) = \sum_{k}^{N} |(p_{xk}(t) - C_x(t))^2)|, \tag{5.2}$$

where $C_x(t)$ represents the x coordinate for the Geometrical Center at instant t and $P_{xk}(t)$ the position along the longitudinal axis for player k at instant t.

Remark 5.1 The same formula is applicable for the calculation along the vertical axis, only replacing $P_{xk}(t)$ for $P_{yk}(t)$ and $C_x(t)$ for $C_y(t)$.

Definition 5.3 [1] The average values of the Stretch Index both for the coordinates, along the two axes and for each axis separately, is given by the following equation:

$$\overline{SI} = \frac{\sum_{k}^{N} SI(t)}{N_t}, \tag{5.3}$$

where N_t represents the total number of time instants measured.

Remark 5.2 This same formula is applicable for $\overline{SI_x}$ and $\overline{SI_y}$, by replacing $SI(t)$ for $SI_x(t)$ or $SI_y(t)$, respectively.

5.1.2 Real Life Examples

The results obtained by a player from Team A in the SSG are presented in Table 5.1 with intervals of 30 s and for the entire 3 min in Table 5.2.

The results obtained by a player from Team B in the SSG are presented in Table 5.3 with intervals of 30 s and for the entire 3 min in Table 5.4.

A screenshot of a representation of the Stretch Index captured from the uPATO software is displayed in Fig. 5.1.

Table 5.1 Values obtained for the Stretch Index of Team A in an SSG, for periods of 30 s

Period of time (s)	x axis (m)	y axis (m)	Both axes (m)
[0; 30[	22.8927	5.6728	24.3221
[30; 60[	43.8870	18.9101	49.7405
[60; 90[	35.6433	10.6641	39.1172
[90; 120[	30.2275	12.5269	34.8085
[120; 150[	36.0131	14.0653	41.0395
[150; 180]	20.1069	5.3325	21.6990

Table 5.2 Values obtained for the Stretch Index of Team A in an SSG, in the entire period of time of 3 min

Period of time (s)	x axis (m)	y axis (m)	Both axes (m)
[0; 180]	31.4557	11.1946	35.1151

Table 5.3 Values obtained for the Stretch Index of Team B in an SSG, for periods of 30 s

Period of time (s)	x axis (m)	y axis (m)	Both axes (m)
[0; 30[	23.5324	15.6531	29.5884
[30; 60[	16.7984	13.6148	24.0102
[60; 90[	22.6443	16.5140	29.8536
[90; 120[	14.9087	12.3292	21.4474
[120; 150[	23.3738	19.5423	32.5294
[150; 180]	20.0127	16.9829	27.7828

Table 5.4 Values obtained for the Stretch Index of Team B in an SSG, in the entire period of time of 3 min

Period of time (s)	x axis (m)	y axis (m)	Both axes (m)
[0; 180]	20.2070	15.7707	27.5298

Fig. 5.1 Screenshot of the uPATO showing an example game animation with the representation and values of the Stretch Index for both teams

5.1.3 General Interpretation

Stretch Index was introduced in basketball to measure the expansion and contraction of space, in both axes (longitudinal and lateral), demonstrated by a team during a match [2]. This measure represents the mean deviation of each teammate to the geometrical center [2, 3]. Stretch Index is also known as radius [4].

Dispersion of the players depends on contextual variables and mostly on the moment of the game (with or without possession of the ball) [5]. Dispersion is greater in attacking moments (with possession of the ball) and smaller in defensive pressure (without possession of the ball), in the case of soccer [6]. This follows the main idea that in attacking moments it is necessary to spread the players to attract the opponents towards the outside of the middle and in defensive moments it is necessary to keep the teammates closer to guarantee fewer spaces for opponent's penetration [7].

This measure works in longitudinal, lateral and/or global, thus different information can be used. In the case of the Stretch Index for the longitudinal axis it can be computed to measure specific situations of counter attacks in which a greater dispersion in goal-to-goal direction can be observed. In the other hand, greater dispersions are found in side-to-side during positional attack (ball circulation). Considering the defensive moments, both axes will drastically decrease in comparison to attacking moments. However, longitudinal dispersion can be used to classify the defensive pressure against positional attacks in which large values of dispersion may suggest that forward teammates are too far away from the defensive colleagues.

5.2 Surface Area

5.2.1 Basic Concepts

The Surface Area of a team is calculated as the area of the polygon defined as the convex polygon with the least number of vertices that can encompass all of the teams's players, and where the potential vertices are defined as the positions of the players.

Definition 5.4 [8, 9] Given a set of points, the following algorithm is applied to define the Convex Hull:

Algorithm 1: Convex Hull algorithm.

```
Create a simplex of d+1 points
for each facet F do
    for each unassigned point p do
        if p is above F then
            assign p to F's outside set;
        end
    end
end
for each facet F with a non-empty outside set do
    select the furthest point p of F's outside set
    initialize the visible set V to F
    for all unvisited neighbours N of facets in V do
        if p is above N then
            add N to V
        end
    end
    the set of horizon ridges H is the boundary of V
    for each ridge R in H do
        create a new facet from R and p
        link the new facet to its neighbours
    end
    for each new facet F' do
        for each unassigned point q in an outside set of a facet in V do
            if q is above F' then
                assign q to F"s outside set
            end
        end
    end
    delete the facets in V.
end
```

Definition 5.5 [9, 10] Given the coordinates of the n vertices that compose the Convex Hull of the team, the Surface Area is given by the following equation:

$$SA = \frac{|(x_1y_2 - y_1x_2) + (x_2y_3 - x_3y_2) + \cdots + (x_ny_1 - x_1y_n)|}{2}, \tag{5.4}$$

where (x_i, y_i) are the coordinates of the *ith* vertex of the Convex Hull.

5.2.2 Real Life Examples

The results obtained by both teams in the SSG and another team in the match are presented in Table 5.5 with intervals of 30 s and for the entire 3 min in Table 5.6.

A screenshot of a representation of the Surface Area captured from the uPATO software is displayed in Fig. 5.2.

Table 5.5 Values obtained for the Surface Area of both teams in an SSG and obtained from a team in a match, for periods of 30 s

Period of time (s)	Surface Area (m^2)		
	SSG		Match
	Team A	Team B	Team A
[0; 30[	83.0395	247.7091	485.1521
[30; 60[	550.1110	154.9699	282.3502
[60; 90[	263.9492	277.3812	352.4851
[90; 120[	320.9839	131.7475	135.3420
[120; 150[	397.8896	303.0914	420.1875
[150; 180]	72.6705	245.4415	228.5462

Table 5.6 Values obtained for the Surface Area of both teams in an SSG and obtained from a team in a match, in the entire period of time of 3 min

Period of time (s)	Surface Area (m^2)		
	SSG		Match
	Team A	Team B	Team A
[0; 180]	281.3740	226.6473	317.4336

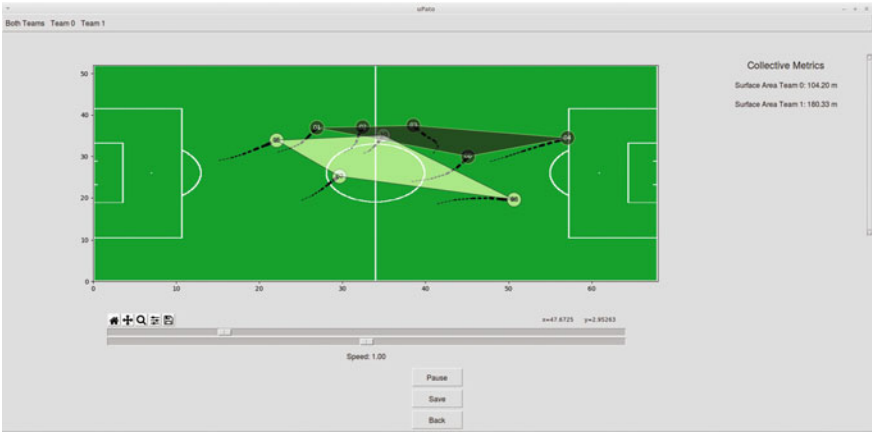

Fig. 5.2 Screenshot of the uPATO showing an example game animation with the representation and values of the surface area for both teams

5.2.3 General Interpretation

Surface Area represents the area of a polygon constituted by all teammates (dots) [11]. The measure can be defined as the total space covered by a team considering the area within the convex hull [9]. The Surface Area measures the contraction and expansion of teams across the soccer match, as does the Stretch Index [3]. However, this measure represents all the area covered by the team, while the Stretch Index only measures the mean deviation to the Geometrical Center of the team.

Typically, the area of the teams is significantly bigger in possession of the ball than without possession [3, 5]. Moreover, the variability of Surface Area decreases across the game [4, 12], thus suggesting a stabilization of the team in attacking and defensive moments.

In a study conducted in elite Spanish soccer teams values between 800 and 2800 m^2 in attacking and [1000; 2000] m^2 in defensive moments were found [13]. Values in attacking varied between 1638 and 1831 m^2 in possession of the ball and [1277; 1369] m^2 without possession in an elite Portuguese team [14].

The visualization of Surface Area can help coaches identify the space between sectors (defensive, middle and forward) and the dispersion of the team in specific moments. The generated triangulations can provide an immediate analysis of the collective behavior, particularly in moments of positional attack and defensive 'block'.

5.3 Team Length and Team Width

5.3.1 Basic Concepts

The Team Length and Width is defined by its most advanced and rear players for the length, and its rightmost and leftmost players for the length.

Definition 5.6 [4] Given a set of points for team player positioning along a time-series of length N, P, and where P_x and P_y represent the set of longitudinal and lateral coordinates for every player of the team on every measured time instant, the Team Length and Team Width on a given time instant i can be calculated as follows:

$$t_l(i) = max(P_x(i)) - min(P_x(i)) \tag{5.5}$$

$$t_w(i) = max(P_y(i)) - min(P_y(i)), \tag{5.6}$$

where t_l represents the Team Length, t_w represents the Team Width and $(P_x(i), P_y(i))$ represents the set of coordinates of the team's players in instant i.

5.3.2 Real Life Examples

The results obtained by both teams in the SSG are presented in Table 5.7 with intervals of 30 s and for the entire 3 min in Table 5.8.

The results obtained by the team in the match are presented in Table 5.9 with intervals of 30 s and for the entire 3 min in Table 5.10.

Table 5.7 Values obtained for the team width and team length of both teams in an SSG, for periods of 30 s

Period of time (s)	Team A		Team B	
	Team width (m)	Team length (m)	Team width (m)	Team length (m)
[0; 30[	25.2440	7.2677	30.7833	15.2246
[30; 60[	40.2148	18.7316	20.2804	15.5792
[60; 90[	36.7722	11.7441	28.5761	15.9253
[90; 120[	31.0945	15.2547	20.1710	14.5367
[120; 150[	37.0654	14.6651	27.0824	18.7516
[150; 180]	21.1475	6.6030	24.8196	19.4085

Table 5.8 Values obtained for the team width and team length of both teams in an SSG, in the entire period of time of 3 min

Period of time (s)	Team A		Team B	
	Team width (m)	Team length (m)	Team width (m)	Team length (m)
[0; 180]	31.9227	12.3758	25.2784	16.5720

Table 5.9 Values obtained for the team width and team length of a team in a match, for periods of 30 s

Period of time (s)	Team width (m)	Team length (m)
[0; 30[	23.2272	37.0941
[30; 60[	24.6797	21.2807
[60; 90[	26.5411	23.4404
[90; 120[	10.3152	28.7250
[120; 150[	26.3454	27.8867
[150; 180]	18.1004	23.2040

Table 5.10 Values obtained for the team width and team length of a team in a match, in the entire period of time of 3 min

Period of time (s)	Team width (m)	Team length (m)
[0; 180]	21.5368	26.9402

5.3.3 General Interpretation

The Team Length represents the maximum length of a team considering the minimum and maximum position of a player in the longitudinal (goal-to-goal) direction [4]. The same application is applied in the case of Team Width (side-to-side) [4]. Therefore, the length and width measures provide information about how stretched are the two farthest players of a team in longitudinal and lateral directions.

Coaches can use this information to understand the optimal distances to readjust some tactical tasks in training sessions based on usual length and width found in official matches. An interesting approach used the width and length to suggest specific sizes to work positional attack and counter-attack in small-sided games [15]. Moreover, coaches can compare the length and width in different moments of the match and identify in which moments the extreme size can be associated with critical moments (e.g., shots, goals).

5.4 Length per Width Ratio

5.4.1 Basic Concepts

The ratio between a team's length and width is the Length per Width Ratio (lpwratio) of that team.

Definition 5.7 [16] Given t_l and t_w as the Team Length and Team Width of a team on a given instant, the lpwratio of a team is given by the following equation:

$$lpwratio = \frac{t_l}{t_w}. \tag{5.7}$$

5.4.2 Real Life Examples

The results obtained by both teams in the SSG and the other team in the match are presented in Table 5.11 with intervals of 30 s and for the entire 3 min in Table 5.12.

A screenshot of a representation of the lpwratio captured from the uPATO software is displayed in Fig. 5.3.

5.4.3 General Interpretation

The lpwratio quantifies the relationship between the length (maximum distance between the two farthest players in longitudinal direction) and width (maximum

Table 5.11 Values obtained for the lpwratio of both teams in an SSG and a team in a match, for periods of 30 s

Period of time (s)	lpwratio		
	SSG		Match
	Team A	Team B	Team A
[0; 30[	0.2970	0.4971	1.6110
[30; 60[	0.4570	0.8310	0.8886
[60; 90[	0.3188	0.5621	0.8915
[90; 120[	0.4507	0.9115	2.9751
[120; 150[	0.3614	0.6977	1.1507
[150; 180]	0.3218	0.8415	1.3280

Table 5.12 Values obtained for the lpwratio of both teams in an SSG and a team in a match, in the entire period of time of 3 min

	lpwratio		
	SSG		Match
[0; 180]	0.3677	0.7238	1.4741

Fig. 5.3 Screenshot of the uPATO showing an example game animation with the representation and values of the width, length and lpwratio for both teams

distance between the two farthest players in lateral direction) during the match [16]. The authors of this measure argue that small variations of this measure may suggest a team's higher adherence to width and concentration on "principles of play" [16]. In the other hand, large variations of this ratio may suggest a more individual and less collectively coordinated approach to the soccer game [16].

Coaches can use this measure to classify the teams in different moments of the match. Some teams will tend to play in counter-attack, thus increasing the length and reducing the width. In the other hand, teams that opt to attack with circulation

of the ball will have increased width and reduced the length. This relationship will help to understand some patterns of play. The same case can be applied for defensive moments in which teams that opt to defend in 'block' closer to the goal will present decreased length and teams that opt to extend the defensive 'block' for middle or forward zones of the pitch will have increased length.

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