

Chapter 4

Metrics to Measure the Center of the Team

Abstract This chapter discusses a set of metrics involving the Geometrical Center (or centroid) of one or both teams. Each section describes a different metric, including the associated formulae and definitions, representations if valid, and an interpretation on what the metric can convey the user. The following measures will be presented: Geometrical Center; Longitudinal and Lateral Inter-team Distances; Time Delay between teams' movements and Coupling Strength. The case studies presented involve two five-player teams in an SSG considering only the space of half pitch (68 m goal-to-goal and 52 m side-to-side) and another eleven-player team in a match considering the space of the entire field (106.744 m goal-to-goal and 66.611 m side-to-side) even though only playing in half pitch.

Keywords Position data · Georeferencing · uPATO · Soccer · Collective behavior · Centroid

4.1 Geometrical Center

4.1.1 Basic Concepts

The average of the positions of all players of a team results in the Geometrical Center of the team, as the position that is center to the polygon formed by all of the team's players.

Definition 4.1 [9] The Geometrical Center of a team, $C(i)$, for any given instant i , is given by the following equation:

$$C(i) = \left(\frac{\sum_k^N p_{xk}(i)}{N}, \frac{\sum_k^N p_{yk}(i)}{N} \right) \tag{4.1}$$

where N represents the number of players in the team, $p_{xk}(i)$ the position along the longitudinal axis for player k in instant i , and $p_{yk}(i)$ the position along the vertical axis for player k in instant i .

4.1.2 Real Life Examples

The results obtained by both teams in the SSG and by another team in the match are presented in Table 4.1 with intervals of 30 s and for the entire 3 min in Table 4.2.

A screenshot of a representation of the Geometrical Center captured from the uPATO software is displayed in Fig. 4.1.

Table 4.1 Values obtained for the Geometrical Center of both teams in an SSG and one team in a match, for periods of 30 s

Period of time (s)	SSG		Match
	Team A (m)	Team B (m)	Team A (m)
[0; 30[(33.1874, 27.8405)	(27.8414, 20.2657)	(78.3172, 36.8925)
[30; 60[(30.5711, 32.4622)	(22.5556, 25.7821)	(69.1305, 36.7852)
[60; 90[(36.2524, 28.3947)	(33.6091, 22.0747)	(73.8001, 37.4438)
[90; 120[(34.4705, 34.2036)	(29.9851, 29.4189)	(92.7840, 32.9777)
[120; 150[(33.3639, 28.6499)	(28.7977, 20.0259)	(71.7747, 36.8492)
[150; 180]	(32.6036, 34.1118)	(27.4019, 27.4541)	(86.3340, 36.1387)

Table 4.2 Values obtained for the Geometrical Center of both teams in an SSG and one team in a match, in the entire period of time of 3 min

Period of time (s)	SSG		Match
	Team A (m)	Team B (m)	Team A (m)
[0; 180]	(33.4010, 30.9450)	(28.3535, 24.1718)	(78.6921, 36.1801)



Fig. 4.1 Screenshot of the uPATO game animation with the representation and values of the Geometrical Center visible for both teams in the example SSG

4.1.3 General Interpretation

The Geometrical Center represents the middle point of a team [3]. Centroid, wcentroid or team's center have been also used as synonymous of Geometrical Center [5, 7, 9]. The Geometrical Center based on the Euclidian distance of the dots (players) provides useful information about the oscillation of the middle point of the team during the match and in specific circumstances [1]. In some cases the Geometrical Center has been used to identify the intra- and inter-team coordination tendencies in a temporal series [6], namely to monitor the in- and anti-phase relationships between Geometrical Centers from both teams [1, 3, 6]. In such analysis, some studies suggested that non-synchronization of geometrical centers is associated with critical moments [1, 5].

The Geometrical Center can be used by coaches to identify the specific position of the middle point of the team during defensive and attacking moments and associate such point with the Geometrical Center of the opponent team. Moreover, it can be used to control the distances of the farthest players to the center of the team. The association of the Geometrical Center with the oscillation of the ball can be also useful to check the capacity of the team to move collectively based on the position of the ball.

4.2 Longitudinal and Lateral Inter-team Distances

4.2.1 Basic Concepts

The different inter-team distances can be calculated through the difference between the positions of the geometrical centers of the two teams.

Definition 4.2 [6] The instantaneous Longitudinal Inter-team Distance can be calculated by the following equation:

$$iLoID = |x_{t1} - x_{t2}|, \quad (4.2)$$

where x_{tk} , $k = 1, 2$, represents the x coordinate of the Geometrical Center of team k on the calculated instant.

Definition 4.3 [6] The instantaneous Lateral Inter-team Distance is given by the following equation:

$$iLaID = |y_{t1} - y_{t2}|, \quad (4.3)$$

where y_{tk} , $k = 1, 2$, represents the y coordinate of the Geometrical Center of team k on the calculated instant.

Definition 4.4 [6] The instantaneous total Inter-team Distance is given by the following equation:

$$iID = \sqrt{(x_{t1} - x_{t2})^2 + (y_{t1} - y_{t2})^2}, \quad (4.4)$$

where (x_{tk}, y_{tk}) , $k = 1, 2$, represents the coordinates of the Geometrical Center of team k on the calculated instant.

Definition 4.5 [6] The average Inter-team Distance can be calculated through the following equation:

$$\overline{iD} = \frac{\sum_{t=1}^N iID(t)}{N}, \quad (4.5)$$

where $iID(t)$ is the Inter-team Distance on a given instant t , and N the total of measured time instants. The same equation can be adapted for the average Longitudinal and Lateral Inter-team Distances, by replacing $iID(t)$ with $iLoID(t)$ or $iLaID(t)$, respectively.

4.2.2 Real Life Examples

The results obtained by a player in the SSG are presented in Table 4.3 with intervals of 30 s and for the entire 3 min in Table 4.4.

A screenshot of a representation of the total Inter-team Distance captured from the uPATO software is displayed in Fig. 4.2.

No results are presented for the match because this metric required the existence of data from both teams, which is not available in the evaluated match.

Table 4.3 Values obtained for the Longitudinal and Lateral Inter-team Distances in an SSG, for periods of 30 s

Period of time (s)	x axis (m)	y axis (m)	Both axes (m)
[0; 30[5.3484	7.5770	9.2713
[30; 60[8.0405	6.6773	10.4342
[60; 90[3.4501	6.3190	6.8505
[90; 120[4.4987	4.7876	6.5583
[120; 150[4.5607	8.6208	9.7583
[150; 180]	5.2162	6.6662	8.4489

Table 4.4 Values obtained for the Longitudinal and Lateral Inter-team Distances in an SSG, in the entire period of time of 3 min

Period of time (s)	x axis (m)	y axis (m)	Both axes (m)
[0; 180]	5.1857	6.7746	8.4471



Fig. 4.2 Screenshot of the uPATO showing an example game animation with the representation and values of the Inter-team Distance

4.2.3 General Interpretation

The association between geometrical centers of both teams can be used to classify the synchronization during positional attacking, counter attack or defensive pressure. Longitudinal and Lateral Inter-team Distances represent the distance between Geometrical Centers of both teams in the two axes [6]. In the study conducted by the authors of this measure [6] a 3-sec window was implemented to monitor the capacity of this measure to anticipate critical match events.

The variability of the distance between both Geometrical Centers depends on contextual factors. However, this can be useful to assess which game situations approach or distance teams. Moreover, coaches can use such measure to classify the influence of specific small-sided and conditioned games in the spatio-temporal relationship. The classification of specific defensive (defensive 'block' or defensive transition) or attacking moments (positional attack or counter-attack) can be also made using this measure of distance in longitudinal and lateral axes.

4.3 Time Delay Between Teams' Movements

4.3.1 Basic Concepts

The Time Delay between two teams is the time difference a team takes to adjust to a change of positioning of the other team. This is approached by measuring the time it takes for the Geometrical Center of a team to approach the other team's center after its movement.

Definition 4.6 [2, 8] Given a time-series with N measurements, where gm_{i1} and gm_{i2} sets of points representing the Geometrical Center of each team across the time-series, a window size w , a time lag τ on the integer interval $-\tau_{max} \leq \tau \leq \tau_{max}$ and a time index i , a pair of time windows, W_x and W_y can be selected from the time-series, for each coordinate of the Geometrical Center. The time windows are given by:

$$W_x = \begin{cases} \{x_i, x_{i+1}, x_{i+2}, \dots, x_{i+w_{max}}\} & \text{if } \tau \leq 0 \\ \{x_{i-\tau}, x_{i+1-\tau}, x_{i+2-\tau}, \dots, x_{i+w_{max}-\tau}\} & \text{if } \tau > 0 \end{cases} \quad (4.6)$$

$$W_y = \begin{cases} \{y_{i+\tau}, y_{i+1+\tau}, y_{i+2+\tau}, \dots, y_{i+w_{max}+\tau}\} & \text{if } \tau < 0 \\ \{y_i, y_{i+1}, y_{i+2}, \dots, y_{i+w_{max}}\} & \text{if } \tau > 0 \end{cases} \quad (4.7)$$

Definition 4.7 [2, 8] The cross-correlation between the time windows can be defined as:

$$r(W_x, W_y) = \frac{1}{w_{max}} \sum_{i=1}^{w_{max}} \frac{(W_{x_i} - \mu(W_x))(W_{y_i} - \mu(W_y))}{sd(W_x)sd(W_y)}, \quad (4.8)$$

where $\mu(W_x)$ and $\mu(W_y)$ are the mean of W_x and W_y , and $sd(W_x)$ and $sd(W_y)$ are the standard deviation of W_x and W_y .

Definition 4.8 [8] Given the cross-correlation values for the different time windows of τ lag, the Time Delay for each instant is given by the following equation:

$$TD(i) = \max(r(W_x, W_y)(i)), \quad (4.9)$$

where i represents the time instant of the time-series of N length.

To better represent the tendency of the time delays over a period of time, the graphs exemplified in Fig.4.3 are created through the use of Fourier Series and linear regressions.

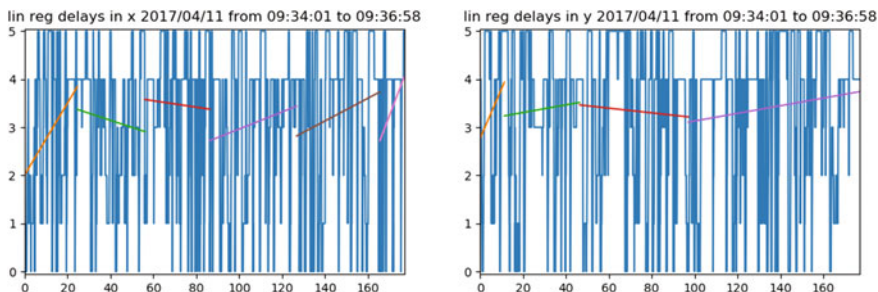


Fig. 4.3 Example of the graph illustrating the time delays between two teams and their tendency during a period of a game. The values presented in both axes are in seconds

Definition 4.9 [4] Given a set of time delays of length N , the graphical representation for the trend estimate of the time delay between the teams is calculated through the following steps:

- (i) Define the Fourier Series with n terms:

$$q_n(t) = a_0 + \sum_{j=1}^n \left(a_j \cos\left(\frac{j2\pi}{T}t\right) + b_j \sin\left(\frac{j2\pi}{T}t\right) \right); \tag{4.10}$$

- (ii) Determine the real roots of the first derivative:

$$t_1, t_2, \dots, t_k \text{ such that } q'(t_1) = 0, \dots, q'(t_k) = 0; \tag{4.11}$$

- (iii) The segmented trend estimate is set to:

$$T(t) = \alpha_j + \beta_j(t), \tag{4.12}$$

for $t_{j-1} < t < t_j, t_0 = 0$ and where $j = 1, \dots, k + 1$.

4.3.2 Real Life Examples

The results obtained in the SSG are presented in Table 4.5 with intervals of 30 s and for the entire 3 min in Table 4.6.

Table 4.5 Values obtained for the Time Delay between teams' movements in an SSG, for periods of 30 s

Period of time (s)	x axis (s)	y axis (s)
[0; 30[3	3
[30; 60[2	3
[60; 90[3	3
[90; 120[2	3
[120; 150[3	3
[150; 180]	3	3

Table 4.6 Values obtained for the Time Delay between teams' movements in an SSG, in the entire period of time of 3 min

Period of time (s)	x axis (s)	y axis (s)
[0; 180]	3	3

No results are presented for the match because this metric required the existence of data from both teams, which is not available in the evaluated match.

4.3.3 General Interpretation

The Time Delay can be measured in longitudinal (goal-to-goal) and lateral (side-to-side) directions. This measure assesses the existing Time Delay between both teams' movements using the Geometrical Center [8]. The measure quantifies the delay of teams in adjusting to each other's movements in both axes [8]. In the study that proposed this measure it was found that in the majority ($\approx 80\%$) of the cases the teams adjusted the geometrical centers in less than 0.5 s in the longitudinal axis [8]. More time was verified in lateral axis [8].

This measure can be used to identify longer periods (>0.5 s) of no-synchronization that can result in critical moments (e.g., shots, goals, counter-attacks). The association of shots/goals suffered and made can be performed to improve the usefulness of this measure to anticipate critical events. Moreover, this measure can be used to identify the evolution of synchronism in different periods of the match. Finally, coaches may use this information to design games that promote smaller or longer periods of delay and to adjust the team with real playing scenarios.

4.4 Coupling Strength

4.4.1 Basic Concepts

The Coupling Strength is calculated as the cross-correlation between the movement of the two teams, according to their Geometrical Centers.

Definition 4.10 [2, 8] Given a time-series with N measurements, where gm_{t1} and gm_{t2} sets of points representing the Geometrical Center of each team across the time-series, a window size w_{max} , a time lag τ on the integer interval $-\tau_{max} \leq \tau \leq \tau_{max}$ and a time index i , a pair of time windows, W_x and W_y can be selected from the time-series, for each coordinate of the Geometrical Center. The time windows are given by:

$$W_x = \begin{cases} \{x_i, x_{i+1}, x_{i+2}, \dots, x_{i+w_{max}}\} & \text{if } \tau \leq 0 \\ \{x_{i-\tau}, x_{i+1-\tau}, x_{i+2-\tau}, \dots, x_{i+w_{max}-\tau}\} & \text{if } \tau > 0 \end{cases} \quad (4.13)$$

$$W_y = \begin{cases} \{y_{i+\tau}, y_{i+1+\tau}, y_{i+2+\tau}, \dots, y_{i+w_{max}+\tau}\} & \text{if } \tau < 0 \\ \{y_i, y_{i+1}, y_{i+2}, \dots, y_{i+w_{max}}\} & \text{if } \tau > 0 \end{cases} \quad (4.14)$$

Definition 4.11 [2, 8] From the time windows, the cross-correlation between the time windows can be defined as:

$$r(Wx, Wy) = \frac{1}{w_{max}} \sum_{i=1}^{w_{max}} \frac{(Wx_i - \mu(Wx))(Wy_i - \mu(Wy))}{sd(Wx)sd(Wy)}, \quad (4.15)$$

where $\mu(Wx)$ and $\mu(Wy)$ are the mean of Wx and Wy , and $sd(Wx)$ and $sd(Wy)$ are the standard deviation of Wx and Wy . The Coupling Strength between the teams is the cross-correlation value at zero-lags, when $\tau = 0$.

4.4.2 Real Life Examples

The results obtained in the SSG are presented in Table 4.7 with intervals of 30 s and for the entire 3 min in Table 4.8.

No results are presented for the match because this metric required the existence of data from both teams, which is not available in the evaluated match.

4.4.3 General Interpretation

The Coupling Strength was proposed to quantify the degree of coordination/synchronization of both teams' movements in both axes (longitudinal and lateral) [8]. This measure represents an association between the Geometrical Centers of both teams. In the unique study conducted with this measure, 80% of the time it was verified a

Table 4.7 Values obtained for the Coupling Strength in an SSG, for periods of 30 s

Period of time (s)	x axis (s)	y axis (s)
[0; 30[0.8573	0.5026
[30; 60[0.9769	0.8284
[60; 90[0.8948	0.8316
[90; 120[0.8810	0.6271
[120; 150[0.1766	0.5366
[150; 180]	0.2692	0.5425

Table 4.8 Values obtained for the Coupling Strength in an SSG, in the entire period of time of 3 min

Period of time (s)	x axis (s)	y axis (s)
[0; 180]	0.6673	0.6419

Coupling Strength between 0.9 and 1 s, in longitudinal axis [8]. Smaller percentage of the time was verified in the lateral axis. Therefore, the Coupling Strength showed that both teams were highly synchronous for most of the match time [8].

This measure can be used to identify the time spent in synchronization between teams and to characterize the capacity of a team to adjust to the other. Moreover, can be used to identify critical moments of the match that may contribute to increase or decrease the Coupling Strength of the teams. Finally, different small-sided and conditioned games lead to different Coupling Strengths and for that reason coaches can use such information to work the capacity of the team to synchronize with other or only to test the development of Coupling Strength of two teams during the match.

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