Report on the Broué–Malle–Rouquier Conjectures

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Abstract This paper is a short survey on the state-of-the-art concerning the main 1998 Broué–Malle–Rouquier conjectures about 'Complex reflection groups, Braid groups, Hecke algebras'.

Keywords Braid groups • Complex reflection groups • Hecke algebras

1 Introduction

About two decades ago, M. Broué, G. Malle and R. Rouquier published a programmatic paper [14] entitled *Complex reflection groups, Braid groups, Hecke algebras* (see also [13]). Motivated by earlier prospections on generalizations of reductive groups, they managed to associate to every *complex* reflection group two objects which were classically associated to *real* reflexion groups (a.k.a. finite Coxeter groups): a generalized braid group and a Iwahori-Hecke algebra. Moreover, they put forward good reasons to believe that the nice properties of these objects in the classical case could be extended to the general one. This paper was followed by a couple of others (most notably [15]) adding precisions on what could be expected. The present paper aims at reporting on the progression of this program. However, it is not possible to explore, in a short text like this one, all the ramifications of the program, because it is connected to a whole area in representation theory (Cherednik algebras and related topics). Therefore, one has to make a choice in order to provide a potentially useful review of it.

In this paper, we made the following choice. We decided to focus on what we regard as the most fundamental properties of *the objects* at the core of [14], that is braid groups and Hecke algebras, disregarding the context in which these objects have been first introduced (an attempt to generalize reductive groups and related objects), and disregarding as well specific properties that might be of use for specific representation-theoretic problems.

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This focus implies that we are going to emphasize what is now actually *proven* of the 'main' (in the above sense) conjectures concerning the objects appearing in [14], and that we shall try to provide a hopefully handy text for people interested in these objects, who would probably appreciate pointers to the literature concerning practical aspects (homology groups, matrix models for representations, etc.).

After reading this text, one impression could be that the progresses have not been that spectacular in the past twenty years. After all, among the exceptional complex reflection groups, the smaller one (G_4) is the only one for which *all* the natural questions mentioned below are settled by now ! Moreover, *all* the results below, when they are proved for every reflection group, need to use the classification of the complex reflection groups in their proof. One should keep in mind however that all these exceptional complex reflection groups are the fundamental symmetric groups arising in low-dimensional phenomena, and therefore, although one might spend time dreaming at a 'general, conceptual proof' (if it exists), ad-hoc proofs should not be regarded as a waste of time. Not only do they provide the basis for applying the conjectures to the particular phenomenon controlled by a given reflection group, but they usually provide additional results on the specific group that are sometimes crucial for applications.

As an example of the first aspect, we mention that the case of the Hecke algebra of the smallest reflection group G_4 itself was successfully applied in studying a potentially new invariant of knots [29], improving our understanding of the Links-Gould invariant and of the Birman-Wenzl-Murakami algebra [42, 43]. Similarly, the cases of G_4 and G_5 were used in [39] and [37] to identify up to isomorphism two different constructions of the same representations of the usual braid groups, while the cases of G_8 and G_{16} are used in [18] to recover and explain a classification due to Tuba-Wenzl of small-dimensional irreducible representations of the braid groups.

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2 Complex Reflection Groups

Recall that a complex reflection group is a finite subgroup of some general linear group over the complex numbers $\mathrm{GL}_r(\mathbb{C})$, which has the property of being generated by (pseudo)-reflections, namely endomorphisms whose invariant subspace is an hyperplane. If W is such a group, we denote by $\mathcal{R} \subset W$ the set of all pseudo-reflections belonging to W, and call r the *rank* of W. A recent reference on such objects is [31]. It is convenient to introduce a subset $\mathcal{R}^* \subset \mathcal{R}$ of so-called *distinguished reflections*. If $s \in \mathcal{R}$, the set of elements in W fixing $\mathrm{Ker}(s-1)$ is a cyclic group of some order m, and there is only one element in this set with eigenvalue $\exp(-2\pi \mathrm{i}/m)$. This is the distinguished pseudo-reflection attached to $\mathrm{Ker}(s-1)$, and $\mathcal{R}^*$ is the collection of all such elements.

A basic property of a complex reflection group is that it can be canonically decomposed as a direct product of *irreducible* ones—meaning that $W \subset \mathrm{GL}_r(\mathbb{C})$ acts irreducibly on $\mathbb{C}^r$. By Schur's lemma, such groups have cyclic center. Moreover, by a change of basis one can always assume $W \subset \mathrm{GL}_r(K)$, where K is the subfield of $\mathbb{C}$ generated by the traces of elements of W. Finally, a fundamental result of Steinberg says that, if S is any subspace of $\mathbb{C}^r$, then the subgroup W_S of W made of all the elements fixing S is a reflection subgroup, generated by $\mathcal{R} \cap W_S$. Such subgroups are called parabolic subgroups.

Completing a quest of several decades, irreducible complex reflection groups were classified by Shephard and Todd, in [47]. They either belong to an infinite series $G(de, e, r) < \mathrm{GL}_r(\mathbb{C})$ of groups of monomial matrices, or to a finite set of 34 exceptions. These 34 exceptions were labelled $G_4, G_5, \dots$ up to G_{37}. Using this classification, a general result due to M. Benard (see [5]) is that all the irreducible representations of W can be defined over K.

Inside this list of exceptions, some of the groups can be realized as *real* reflection groups. Since they are the geometric realization of finite Coxeter groups, they are pretty well understood. In the Coxeter-Dynkin classification, the correspondance is $G_{23} = H_3$, $G_{28} = F_4$, $G_{30} = H_4$, $G_{35} = E_6$, $G_{36} = E_7$, $G_{37} = E_8$. In this case, the rank of W is equal to the minimum number of reflections which are necessary to generate the group. In the general case, it can be shown that this number is either r or $r + 1$, where r is the rank of W. In the former case W is called *well-generated*, in the latter *badly generated*. Among the (exceptional) groups of rank at least 3, only G_{31} is badly generated. We denote $n(W) \in \{r, r + 1\}$ the minimal number of reflections needed to generate W.

Since most of the conjectures that we are interested in have been proven early enough for the general series of the $G(de, e, r)$ (see Ariki-Koike [3], Broué-Malle [12], Ariki [2], Bremke-Malle [10]) we will concentrate on the exceptional groups. While the details of the classification are cumbersome, its general scheme is clear enough. We recall it because the proof of many results on the BMR conjectures uses separation of cases in families that originate from the proof of the classification. It proceeds as follows:

1. If the action of the group on $\mathbb{C}^r$ is imprimitive, one proves that W has to belong to the infinite series of the $G(de, e, r)$
2. If it is primitive of rank 2, then $W/Z(W)$ can be identified through the isomorphism $\mathrm{PSU}_2 \simeq \mathrm{SO}_3(\mathbb{R}_{\mathrm{red}})$ with the group of rotations of the tetrahedron, of the octahedron or of the icosaedron, thus splitting this case in three families. All such W are then obtained as a cyclic central extension of such a group.
3. If it is primitive of higher rank, then a theorem of Blichtfeld asserts that the pseudo-reflections of W can have order only 2 or 3. The rest of the proof is based on this result, and needs a lot of careful analysis to be completed. Actually it turns out that only three groups, G_{25}, G_{26} and G_{32} admits pseudo-reflections of order 3. Moreover, only one of these groups (G_{31}) is badly generated.

A convenient software for dealing with these exceptional groups is the (development version of the) CHEVIE package for GAP3 (see [20, 44]). In particular it contains an increasing number of matrix models for irreducible representations of the Hecke algebras (an therefore of the associated braid group).

3 Braid Groups

A crucial consequence of Steinberg's theorem is that the action of W on the complement $\mathbb{C}^r \setminus \bigcup \mathcal{A}$ of the attached reflection arrangement $\mathcal{A} = \{\mathrm{Ker}(s-1); s \in \mathcal{R}\}$ is free. Broué, Malle and Rouquier defined a (generalized) braid group attached to $W \subset \mathrm{GL}_r(\mathbb{C})$ as $B = \pi_1(X/W)$. It fits into a short exact sequence $1 \to P \to B \to W \to 1$, where $P = \pi_1(X)$ is the fundamental group of the hyperplane complement. They defined (conjugacy) classes of distinguished generators for B, called *braided reflections*, which map onto (pseudo-)reflections, and which generate the group. More generally, they proved that every parabolic subgroup W_S of W can be lifted to a 'parabolic' subgroup B_S of B, isomorphic to the braid group of W_S, in a way which is well-defined up to P-conjugacy.

When W is a real reflection group, there are distinguished 'base-points', namely the (contractible) components of the real hyperplane complement, and distinguished generators attached to a choice of such a component (so-called Weyl chamber), namely the straight loops around the walls of the Weyl chamber inside the orbit space X/W (see [11]). The corresponding braid group is known as an Artin group of finite Coxeter type, and these groups are well-understood: there is a nice presentation mimicking the Coxeter presentation for W, and all the conjectures that we are exploring in this paper are known for them. Therefore, the list of exceptional groups which need to be taken care of actually ends at G_{34}.

One thing which is useful to keep in mind when studying B is that, up to (abstract) group isomorphisms, the correspondence $W \mapsto B$ is not 1–1. Actually, every B can be obtained by only considering the 2-reflections groups (that is, complex reflection groups whose pseudo-reflections all have order 2). Therefore, when proving purely group-theoretic properties of B (not of P !), one can assume that W is a 2-reflections group.

3.1 Center

The center of P obviously contains the homotopy class $\boldsymbol{\pi}$ of the loop $t \mapsto e^{2\pi i t} x_0$, where $x_0 \in X$ is the chosen base-point. Since $e^{\frac{2\pi i}{|Z(W)|}}$ is a generator of the cyclic group $Z(W)$, Broué, Malle and Rouquier defined a central element $\boldsymbol{\beta}$ of B as the homotopy class of the loop $t \mapsto W.(e^{\frac{2\pi i t}{|Z(W)|}} x_0)$. They conjectured

1. $Z(P)$ is infinite cyclic, generated by $\boldsymbol{\pi}$.
2. $Z(B)$ is infinite cyclic, generated by $\boldsymbol{\beta}$.

3. The exact sequence $1 \to P \to B \to W \to 1$ induces a short exact sequence $1 \to Z(P) \to Z(B) \to Z(W) \to 1$.

Item (ii) was proven by Bessis in [7]. Items (i) and (iii) were proven by Digne, Marin and Michel in [26].

3.2 Presentations

It was conjectured in [14] that there exists 'nice' finite presentations, similar to the Artin presentations in the Coxeter case, and in particular satisfying the following requirements:

- the generators are $n(W)$ braided reflexions, where $n(W)$ is the minimal number of reflections needed to generated W
- the relations are homogeneous and positive.

A theorem of Bessis states that such a presentation exists as soon as the highest degree of W is regular in the sense of Springer (see [6]). This applies to all exceptional reflection groups except G_{15}, for which such a presentation was already known (see [14]). The case of the general series had also been established in [14], and therefore the result is known in all cases.

For practical purposes however, one needs more precise results, specific to each of the exceptional cases. To this end, Bessis and Michel manufactured a GAP3 package, additional to CHEVIE, called VKCURVE (see [8]). This software computes presentations for complements of algebraic curves and can be useful also in other contexts. Experimental presentations for all the exceptional groups were presented in [8], their rigorousness were subsequently justified in [7], and additional presentations are given in [35].

3.3 Additional Properties

We gather here a few results obtained on these generalized braid groups since [14].

3.3.1 Word Problem, Conjugacy Problem

It is known how to solve the word problem and the conjugacy problem for these groups. One major tool for this is the construction by Bessis in [7] of so-called Garside monoids, as introduced in [23], for all well-generated groups. This proves that the corresponding braid groups are the groups of fractions of such a monoid, and therefore are torsion-free, and have decidable word and conjugacy problem. For the general series, these consequences were clear from the description of B in

[14], while for the exceptional groups of rank 2 they can be easily deduced from the description in [4] of the braid group (see also e.g. [26, 46] for some Garside monoids aspects in these cases, too). In the case of G_{31}, one needs more involved tools: the group B, viewed as groupoid, is equivalent to a Garside groupoid in the sense of [24] (see again [7]) and therefore has a solvable word and conjugacy problem, too.

3.3.2 Homology

It was known earlier that, for the general series (see [45]), the Coxeter groups and the groups of rank 2, the spaces X and X/W are $K(\pi, 1)$. It was proved by Bessis in [7], that this result is also true for all exceptional groups. This reproves that B is always torsion-free, and also provides a way to compute the homology of B from X/W. The introduction of Garside monoids in [7] also provides, using the work of Dehornoy-Lafont in [22], several complexes from which the homology of B can be in principle computed. From this, the homology of B for all exceptional groups but the higher homology groups of G_{34} was computed in [16]. The integral homology for the general series still remains a bit mysterious (see [16] for partial results).

3.3.3 Linearity?

The proof of the linearity of the usual braid group [9, 30], and its subsequent extension to the Artin groups of finite Coxeter type [21, 25], has been a major breakthrough. It was achieved using a linear representation that we will call the Krammer representation. A detailed study of this representation provided additional properties of the group: that they can be seen as Zariski-dense subgroups of the general linear group (and therefore essentially cannot be decomposed as direct products), and that the pure braid groups are residually torsion-free nilpotent [36, 37].

A natural question is then whether the similar properties hold in the general case. This was conjectured in [39], where a generalization of the Krammer representation has been constructed. It is shown there that the faithfulness of this representation would have the same consequences on the structure of the group as in the real case. The construction of [39] focus on 2-reflection groups. It has been generalized by Chen in [19] to arbitrary reflection groups.

4 Hecke Algebras

One can attach to W a Laurent polynomial ring $R = \mathbb{Z}[u_{i,c}^{\pm}]$, where c runs among the conjugacy classes of distinguished pseudo-reflections, and $0 \leqslant i < e_c$ where e_c is the order of an arbitrary pseudo-reflection inside the conjugacy class c. There is a natural specialization morphism $\theta : R \to K$, $u_{j,c} \mapsto \exp(-2\pi \mathrm{i} j/e_c)$, and a useful ring automorphism of R induced by $u_{i,c} \mapsto u_{i,c}^{-1}$, that we denote $z \mapsto \bar{z}$.

The Hecke algebra associated to W is defined as the quotient of the group algebra RB of B by the relations $\prod_{0 \leqslant i < e_c} (\sigma - u_{i,c}) = 0$, where c runs among the conjugacy classes of pseudo-reflections, and σ runs among the braid reflections mapping to a pseudo-reflection in c under the natural mapping $B \to W$. Since two braided reflections mapping to the same pseudo-reflection are conjugated, it is enough to impose only one relation per conjugacy class. We have a natural isomorphism $H \otimes_\theta K \simeq KW$, and therefore H can be seen as a deformation of KW.

The automorphism $z \mapsto \bar{z}$ of R can be extended to an anti-automorphism of RB by putting $\bar{b} = b^{-1}$ for all $b \in B$, and this induces an anti-automorphism of H, since the defining ideal of H is easily checked to be invariant under this anti-automorphism.

4.1 Freeness Conjecture

The basic conjecture about H is that it should be a free module over R, of rank $|W|$. It has been proven in [14] that it is enough to show that H is spanned over R by $|W|$ elements. A weak version of this conjecture states, as an important first step, that H should be at least finitely generated as a R-module. By general arguments based on Tits' deformation theorem, this weak version is strong enough to imply that, after extension of scalars to an algebraic closure F of the field of fractions of R, there exists an isomorphism $H \otimes_R F \simeq FW$ (see e.g. [40]). A more difficult result due to Losev (see [32]) states that it also implies that every specialization $\varphi : R \to \mathbb{C}$ of H to the complex numbers has the same dimension: $\dim H \otimes_\varphi \mathbb{C} = |W|$. An account on the earlier works on this conjecture can be found in the introduction of [40]. We just recall from there that the case of the general series (strong version) is proved in [1, 3, 12]. We focus here on the most recent and inclusive results on exceptional groups.

The weak version is now known for every group, thanks to results of Etingof-Rains ([27]; see also [40]) for the groups of rank 2 (from G_4 to G_{22}), Marin [38, 40] for the groups G_{25}, G_{26}, G_{32}, and Marin-Pfeiffer (see [41]) for the remaining groups $G_{24}, G_{27}, G_{29}, G_{31}, G_{33}, G_{34}$.

The full version is, for now, known for all the groups of rank at least 3 [38, 40, 41], and all the groups belonging to the first two families of groups of rank 2 (from G_4 to G_{15}) by work of E. Chavli (in the course of writing, see [17]) plus the groups G_{16} (Chavli, see [18]) and G_{22} (Marin-Pfeiffer, see [41]). The remaining groups are G_{17}, G_{18}, G_{19}, G_{20}, G_{21}. For these five groups, the freeness conjecture is still open, but seems now to be within reach. In all the cases for which this version is proved, one can actually find a basis originating from the braid group itself, as expected in [15, 1.17].

4.2 Trace Conjecture(s)

It is conjectured that H admits the structure of a *symmetric algebra* over R. This means that there should exist a *symetrizing trace* $t : H \to R$, that is a R-linear form satisfying $t(ab) = t(ba)$, such that the associated map $H \to \mathrm{Hom}_R(H, R)$, $x \mapsto (y \mapsto t(xy))$ is an isomorphism. It was proved in [10] that H satisfies this conjecture for the general series.

This property is important in particular in order to understand the possible specializations. A computational understanding of such a trace is related to the knowledge of the so-called *Schur elements* associated to it. These elements are essentially (the inverse of) the coefficients of the decomposition of such a trace as a linear combination of the matrix traces associated to the irreductible representations of $H \otimes_R F$, here assumed by the freeness conjecture to be isomorphic to FW.

It was proved in [15] that, if the freeness conjecture is true, then there exists at most one trace satisfying the following properties.

1. t is a symmetrizing trace
2. $t_0 = \theta \otimes t : KW \simeq H \otimes_\theta K \to R \otimes_\theta K \simeq K$ is the usual symmetrizing trace on KW, defined by $t_0(w) = 0$ if $w \in W \setminus \{1\}$, $t_0(1) = 1$.
3. for all $b \in B$, we have $t(\boldsymbol{\pi})\overline{t(b^{-1})} = t(b\boldsymbol{\pi})$.

If there is such a trace, these conditions define a canonical trace on H. However, the fact that the trace constructed in [10] for the general series satisfies this condition is apparently still conjectural and this is an exception to the usual motto that 'everything is known for the infinite series'. So far, the only (non-Coxeter) exceptional groups for which this conjecture has been proved are G_4, G_{12}, G_{22} and G_{24}, in [35] (the case of G_4 was later independently checked by the author in [43]), under the freeness assumption. Moreover, in these three cases, the trace used satisfies the characterization above. Since the freeness conjecture is now known to hold for these three groups, this solves the trace conjectures for these cases.

We also mention that, under the validity of the freeness conjecture, Malle constructed the Schur elements associated to a potential trace satisfying similarly nice properties, for every exceptional complex reflection groups in [33, 34].

4.3 Additional (Conjectural) Properties

In the case of Coxeter groups, much more structural properties of the Hecke algebras are known. A natural and widely unanswered question is whether these properties can be extended to the general case. Among these, two probably deserve a natural interest. Before stating them, we recall that, as a corollary of the lifting of parabolic subgroups to the braid group, the choice of a parabolic subgroup W_0 of W endows H with the structure of a H_0-module, where H_0 denotes the Hecke algebra. This structure is well-defined up to P-conjugacy.

The two properties in question, for Coxeter groups, are the following ones:

1. if W_0 is a parabolic subgroup of W, then H is a free H_0-module of rank $|W|/|W_0|$
2. if S is an R-algebra, then the center of $S \otimes_R H$ is a free S-module.

Among the exceptional groups, the first property has been proved only for a couple of inclusions (W, W_0), in the course of proving the freeness conjecture. These are: (G_{32}, G_{25}), (G_{25}, G_4), $(G_4, \mathbb{Z}_3)$, $(G_{25}, \mathbb{Z}_3 \times \mathbb{Z}_3)$, (G_{26}, G_4) (see [38, 40]), $(G_8, \mathbb{Z}_4)$, $(G_{16}, \mathbb{Z}_5)$ (see [18]), $(G_{12}, \mathbb{Z}_2)$, $(G_{22}, \mathbb{Z}_2)$, (G_{24}, B_2), (G_{27}, B_2), (G_{29}, B_3), (G_{31}, A_3), (G_{33}, A_4), (G_{33}, D_4) (G_{34}, G_{33}) (see [41]). Concerning the second property, it has been proven for $S = R$ by Francis [28] for the groups G_4 and $G(4, 1, 2)$. We are not aware of any other result in this direction.

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