

# Chapter 9

## The Conceptual Elements of Multiple Representations: A Study of Textbooks' Representations of Electric Current

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### 9.1 Introduction

Research findings showed that students did not always understand the role of multiple representations despite the efforts of a science teacher (Treagust et al. 2003). For instance, an argument offered in physics education for enhanced student understanding is to present physical concepts or problems using multiple representations in the form of words, diagrams, graphs, tables, or bar charts (Rosengrant et al. 2009; Van Heuvelen and Zou 2001). Moreover, the focus of representations could be on the use of animations, colour coding, icons, or simulations (Dancy and Beicher 2006; Homer and Plass 2010). Nevertheless, to promote active learning of students, physics teachers should use inquiry-related activities and analyse the elementary features of physical concepts which students may have difficulty in learning (Duit et al. 2012).

A framework of multiple representations of physical concepts could be succinctly based on Ainsworth's (1999) three main functions of multiple external representations: complementary information, constrain interpretation, and construct deeper understanding. Firstly, physics teachers should use representations that contain complementary information and support complementary cognitive processes. Secondly, there should be additional representations to constrain students' interpretation of an unfamiliar representation of a physical concept. Thirdly, multiple representations can be used to construct an abstract concept, and establish relations

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among representations such that there is a deeper understanding of a physical concept. However, the functions of multiple representations can be enhanced with a more structured approach in learning.

On the first function in providing complementary information, there could be at least three different levels of representation of concepts – macroscopic, symbolic (pictorial, algebraic, and physical forms such as graphs and analogies), and sub microscopic (Treagust et al. 2003). Additionally, in supporting complementary cognitive processes, there should be no gap in the sequence of representations. For example, the presentation of mathematical equations from  $PV/T = \text{constant}$  to  $PV = nRT$  without including either  $P \propto n$  or  $V \propto n$  may cause a cognitive gap for students in learning (De Berg and Treagust 1993). However, physics teachers could plan a *deliberate gap* in the sequence of representations as an inquiry-related activity. Students could also be guided to identify gaps in the three different levels of representations, and they may have a more meaningful learning experience when they help each other to bridge the gaps between representations.

To have active learning and deeper understanding of physical concepts, the ‘Educational Reconstruction approach’ of Duit et al.’s (2012) can enhance the multiple representations framework of Ainsworth’s (1999). Essentially, the framework of multiple representations could include two thinking and learning processes: elementarization and reconstruction. Firstly, the analysis of the conceptual *elements* of multiple representations can help to constrain interpretation. Secondly, students should *reconstruct* the physical concept in order to understand deeply the reasoning among its representations. In short, the constructivist conceptual change approach of Duit et al. (2012) can help to enhance the second and third functions of multiple representations.

Nevertheless, Ainsworth’s multiple representations framework (Ainsworth 1999) does not necessarily help to define a physical concept and thus it does not always constrain interpretation. To constrain interpretation of physical concepts, we propose to include the following five conceptual elements of multiple representations: *object*, *nature*, *cause*, *mathematical equation*, and *condition* (Wong 2014). Furthermore, alternative conceptions of students could be related to these five conceptual elements (Wong et al. 2016). For instance, electric current is an important physical concept in which many students were found to have alternative conceptions (e.g. Duit 1985; Sanger and Greenbowe 2000; Tsai et al. 2007). In a study conducted by de Posada (1997), students were confused with three conceptual elements of electric current: *object*, *nature* and *cause*. Some students used the term ‘atoms’ (*object*) inaccurately, failed to convey the *nature* of electric current, and were unable to provide the correct *cause* of electric current. Thus, we should incorporate conceptual elements of multiple representations to constrain interpretation of a physical concept such that students’ alternative conceptions could be significantly reduced.

In this chapter, we focus on five conceptual elements of multiple representations pertaining to electric current: *object*, *nature*, *cause*, *mathematical equation*, and *condition*. Utilising Duit and colleagues’ theory of *educational reconstruction* (Duit et al. 2012), firstly, we discuss how physics teachers could analyse these conceptual

elements of electric current and its representations – namely, elementarization – and secondly, we provide inquiry-related activities and suggest how students could be guided to reconstruct the concepts of electric current and its representations – namely reconstruction.

## 9.2 Elementarization (Analysing Five Conceptual Elements of Electric Current)

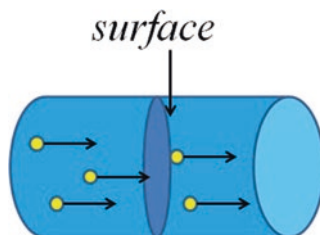
Based on our preliminary textbook analysis, five conceptual elements were identified as *object* (charge-carriers), *nature* (characteristics), *cause* (or effect), *mathematical equation*, and *circuit condition*. In this study, we analyse five conceptual elements of electric current that could be found in definitions and diagrams of 40 introductory physics textbooks. The selected textbooks were published in the United States (US) and United Kingdom (UK) because they are influential in the learning of students and teachers. We analysed definitions of electric current that are written in the form of “a rate of flow of charge” and diagrams that illustrate the definition within the same section of a textbook. These diagrams usually provided a definition of electric current or were labelled ‘electric current’. The findings are summarised in Table 9.1 as shown below.

The concept of electric current was not expressed consistently in words and in diagrams among these textbooks. For example, the diagrams illustrating the concept of electric current are usually drawn as either Fig. 9.1 or Fig. 9.2 with varying

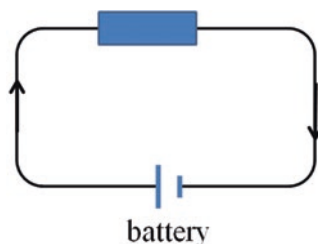
**Table 9.1** Percentage of conceptual elements of electric current in definition and diagram of introductory physics textbooks (n = 40, percentage in parenthesis)

Conceptual elements	Examples of description or representation	Definition	Diagram
Objects / Charge carriers	Electric Charge (+ve and -ve) / Net charge	36 (90)	7 (18)
	Electrons (-ve)	2 (5)	7 (18)
	Positive charge (+ve)	0 (0)	5 (13)
Nature / characteristics	Rate of flow of electric charge	16 (40)	–
	Movement of charge / flow of electrons	20 (50)	19 (48)
	Conservation of electric current	–	2 (5)
Cause	Potential difference	1 (3)	3 (8)
	Electric field	1 (3)	4 (10)
Mathematical Equations	$I = dq/dt$ or $I = \Delta q/\Delta t$	26 (65)	1 (3)
	$I = q/t$	8 (20)	1 (3)
	Graph involving current, charge and time	–	4 (10)
Circuit Condition / Conduction Medium	Metal / Conductor / Wire	14 (35)	13 (33)
	Area / Surface / Point	10 (25)	5 (13)
	Complete Circuit	5 (13)	5 (13)

**Fig. 9.1** Textbook A's diagram of electric current



**Fig. 9.2** Textbook B's diagram of electric current



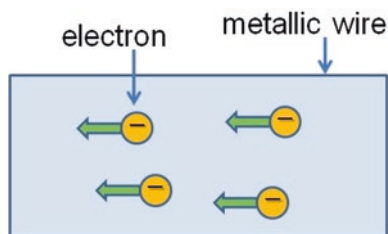
details. They showed that electric current is a rate of flow of charge through a cross-sectional area (13%), metallic wire (33%), or in a complete circuit (13%) (See Table 9.1). Some textbook authors also specified charge carriers (object), conservation of electric current (nature), or electric field (cause) in the diagrams. In addition, the mathematical equation,  $I = dq/dt$ , could be illustrated graphically by relating electric current, electric charge, and time (10%).

Pertaining to the five conceptual elements, we first discuss possible alternative conceptions of students which can be related to problems of representations that are expressed in words, diagrams, or symbols, as found in current textbooks. Next, we provide suggestions how the concept of electric current can be presented verbally, diagrammatically, symbolically, and graphically for students in the secondary schools, colleges and universities.

### 9.2.1 Object

In a study conducted by Garnett and Treagust (1992), some students who studied both physics and chemistry had difficulty in understanding the concept of electric current as compared to students who only studied chemistry. For example, some dual-discipline students considered electric current as a flow of protons through metals or a flow of electrons through electrolytic solutions. Garnett and Treagust (1992) propose that the physics syllabus should adopt the electron flow model of electric current in metallic conductors. It is possible that different conventions of electric current used in chemistry and physics posed conceptual problems for some students. However, the problem of convention pertaining to electric current has not

**Fig. 9.3** Electric current as the flow of *electrons*



been resolved. For instance, Arons (1990) argues that the positive current convention<sup>1</sup> should be maintained in physics.

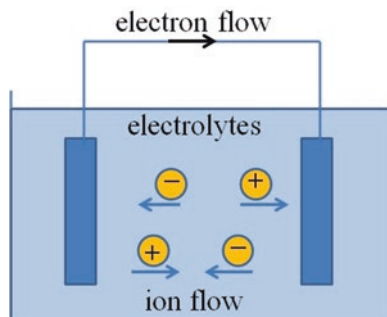
Among the textbooks, electric current was commonly defined as a flow of electric charge (90%) or electrons (5%). On the other hand, the diagrams that illustrate the definition of electric current specified the charge-carriers as electric charge (18%), electrons (18%), or positive charge (13%). Firstly, a better term for the *object* of electric current can be charged particles, charge-carriers, or electrons. The term electric charge should be used as an attribute of charge-carriers. Secondly, the charge-carriers could be specified as electric charge in a general definition of electric current, and they were drawn as electrons in a diagram as a specific definition. In other words, the verbal and diagrammatic representations of electric current may provide complementary information in the general and specific sense respectively, or vice versa.

As another example, a textbook definition of electric current could be specified as a flow of electric charge, but the diagram shows a complete circuit with a metallic wire and positive charge-carriers. In this case, the textbook author used the diagram to provide complementary information by showing a positive current convention. However, the charge-carriers in the metallic wire are electrons, and they are negatively charged. Thus, the flow of positive charge-carriers in the metallic wire is an idealization and it does not accurately represent the flow of charge-carriers in the real world.

In secondary schools, physics teachers should compare and contrast the two conventions of electric current with their students. When electric current is defined as a flow of electrons, it could be represented in Fig. 9.3 to show the positive current convention. The representations of electric current should be consistent diagrammatically, verbally, and symbolically. (The direction of electric current is symbolically represented by  $I$  with an arrow.) When electric current is defined as a rate of flow of charge-carriers, it could be diagrammatically represented as a flow of ions in electrolytes (Fig. 9.4). The charge-carriers as specified in the definition and its diagram should be complementary and consistent.

<sup>1</sup>Arons (1990) provides four reasons for maintaining the positive current convention in physics: (1) it underlies the definitions of electric field strength and potential difference; (2) the treatment of capacitive and inductive circuit elements; (3) the standard mnemonics of electromagnetism; and (4) the common notations in diagrams of electrical circuits.

**Fig. 9.4** Electric current as the flow of *ions*



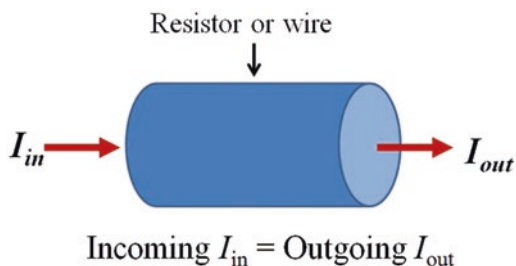
At the college or university level, there could be more details in the Fig. 9.3 such as the copper atoms and the collision between an electron and a copper atom. It is possible that students still have misunderstandings of the figure that represent electric current in textbooks. For instance, electrons are sometimes drawn as balls of comparable size to the copper atoms (de Posada 1997). Physics teachers should explain that a representation of electric current may be idealised or exaggerated. An explanatory note could be included beside the figure to clarify the size of an electron and atom.

### 9.2.2 *Nature (or Characteristics)*

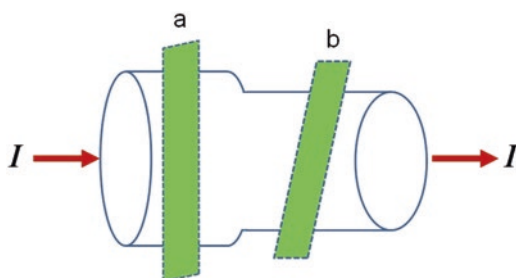
One common alternative conception of electric current is that it could be *used up* in an electrical circuit (Osborne 1983; Shipstone 1984). Students may have difficulty understanding the abstract nature of electric current or the idea of current conservation (Stocklmayer and Treagust 1996). On the contrary, students could be considered correct if they described the kinetic energy of electrons *being reduced* instead. This is because the speed of an electron may be reduced during collisions with the copper atoms in a metallic wire. Therefore, students are not completely incorrect when they conceptualised something is being used up in the electrical circuit. However, textbooks' definition of electric current does not specify a *constant* rate of flow of charge-carriers.

The nature of electric current was described in textbooks as either 'rate of flow of electric charge (40%) or 'movement of charge / flow of electrons' (50%). To be precise, physics textbooks written for secondary schools and colleges should include the phrase 'rate of flow' instead of simply 'flow'. That is, it should refer to the rate of flow of charge-carriers through a cross sectional area per unit time. However, the nature of electric current could be represented in diagrams as movement of charge / flow of electrons (48%) or conservation of electric current (5%). It should be noted that a diagram is a static form of representation and it mainly shows the direction of

**Fig. 9.5** The constant nature of electric current



**Fig. 9.6** The current  $I$  through the conductor has the same value at imaginary planes  $a$  and  $b$



movement of charge-carriers. To show the rate of flow of charge-carriers, physics teachers should use an animation.

In secondary schools, the nature of electric current can be verbally represented as constant or conserved. Additionally, there could be a diagrammatic representation in which the incoming electric current before passing through a segment of wire (or resistor) is the same as the outgoing electric current after passing through the segment of wire (See Fig. 9.5). We should explain that a battery maintains the potential difference and electric current of an electrical circuit. Although an electron may slow down after a collision with the copper atom, it can gain back kinetic energy due to the presence of a potential difference. In other words, the flow of electrons is not constant from a sub microscopic perspective.

At the college or university level, the constant nature of electric current may be illustrated by a metallic conductor that has different cross-sectional areas (See Fig. 9.6). In short, the electric current has the same value through different *imaginary planes* that cut across a metallic conductor. However, the diagram may be considered as 'inconsistent' with a definition if it states that an electric current is the rate of flow of charge-carriers moving past *a point* per unit time. Essentially, the electric current through an imaginary plane or a point is an idealisation. In the real world, electrons may move haphazardly by colliding with the copper atoms in a metallic wire instead of travelling in a straight line as commonly shown in the diagrams of most textbooks. The random flow of charge-carriers could also be clearly illustrated by using an animation.

### 9.2.3 Cause

In de Posada's study, students were asked, "Why do metals conduct electric current? (Posada 1997, p. 453)" The common identified causes were coded as macroscopic and atomic. For macroscopic causes, students' responses included 'confluence', 'charge of battery', and 'temperature difference'. For atomic (sub-microscopic) causes, they could be 'atomic regularity', 'atomic movement', and 'atomic disorder'. Some students expressed that their difficulty in learning electric current is due to two different perspectives: macroscopic-energy (physics) and atomic world (chemistry). These students felt that the concept of electric current is inconsistently presented in chemistry and physics (de Posada 1997).

In general, the cause or effect of electric current is less explicitly specified in textbook definitions. Some definitions of electric current in physics textbooks included the following *cause*: potential difference (3%) or electric field (3%). For example, electric current in a metallic conductor is a flow of 'free' electrons due to a potential difference across the ends of the conductor (Breithaupt 2008). Similarly, diagrams of textbooks also include the same cause, potential difference (8%) and electric field (10%), when they are used to illustrate the definition of electric current. However, the term 'potential difference' may have different definitions when they are being used in different textbooks (Mulhal et al. 2001; Gunstone et al. 2009). It was reported that students were unable to clearly distinguish electric current and potential difference (McDermott and Shaffer 1992). Some students also interpreted *potential difference* as "possible difference" or "different ability" (Ryan 1985).

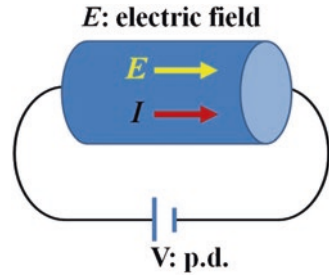
The term *potential difference* could still remain as a mysterious notion to many students (Dupin and Joshua 1987, 1989). It is an abstract concept which has not been adequately represented in many textbooks. In secondary schools, some physics textbooks provided diagrams such as "pressure difference causes the flow of water" to explain how the concept of a potential difference results in an electric current. Interestingly, only one physics textbook introduced the term *electric potential difference* to distinguish it from gravitational potential difference (Wilson et al. 2007). The term potential difference by itself is imprecise, and thus it can be misleading to students.

It has been reported that high school students had the alternative conception in which electric current results in electric potential difference rather than vice versa (Cohen et al. 1983; Dupin and Joshua 1987). Thus, physics teachers should distinguish the *cause* and *effect* of electric current. We may diagrammatically represent electric current in terms of cause and effect as shown below with short explanatory notes (See Figs. 9.7 and 9.8). The cause of electric current can be specified as an electric field or electrical potential difference, and the effect of electric current can be specified as a magnetic field. However, the concepts of electrical potential difference, electric field, and magnetic field are abstract and they should be clarified by using more verbal explanations.

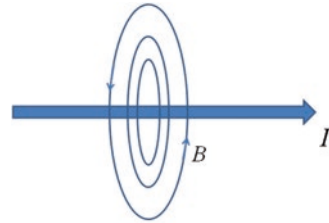
In college textbooks, the *cause* of electric current can be more comprehensively specified as an electrical potential difference  $V$  volts supplied by a battery, and



**Fig. 9.7** The *cause* of electric current: electric potential difference or electric field



**Fig. 9.8** The *effect* of electric current: magnetic field



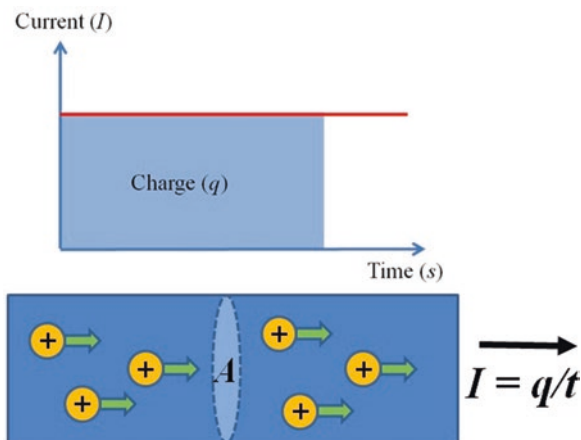
sub-microscopically the *cause* is the electric field  $E$  in the metallic wire (Reh fuss 2004). Furthermore, electric current can be operationally defined and it is measurable by its *effect*. When electric current and magnetic field are represented in a diagram, some textbooks had shown only a circular magnetic field line or three equally spaced magnetic field lines around the wire. Generally speaking, the magnetic field strength of electric current can be illustrated by different spacing of the magnetic field lines (See Fig. 9.8). The spacing between the circular magnetic field lines is wider when it is further away from the electric current. This convention of drawing magnetic field strength can be related to the closeness of magnetic field lines about a bar magnet (Hewitt 2006).

### 9.2.4 Mathematical Equation

A physical concept is sometimes quantitatively represented by a mathematical definition or equation. However, it is possible to have an inaccurate understanding of the equation. The form of the equation could be represented with a diagram and its symbols should be clearly defined or explained. For example, the symbol  $I$  is used to represent electric current for historical reasons; Ampère (1822) used the symbol  $i$  to represent the *electric current intensity*.<sup>2</sup> Also importantly, the form of an equation whether it is written as  $I = V/R$  or  $V = IR$  may suggest different meanings. It is possible that students misinterpreted the equation if it is written as  $V = IR$ . This

<sup>2</sup>It is based on the French word *intensité*. Historically, André-Marie Ampère used the symbol  $i$  in formulating the eponymous Ampère’s force law.

**Fig. 9.9** Representing  $I = q/t$



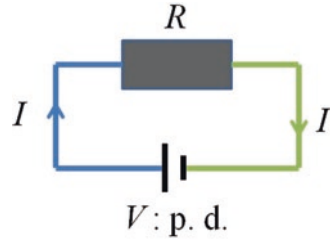
equation may suggest that voltage is a consequence of an electric current because students commonly conceptualise voltage as an attribute of an electric current (Afra et al. 2009; Cohen et al. 1983; Psillos et al. 1988).

The mathematical definition of electric current was stated as  $I = dq/dt$ ,  $I = \Delta q/\Delta t$  (65%), or  $I = q/t$  (20%) among the textbooks. If the equation is  $I = dq/dt$ , it means an instantaneous rate of flow of charge-carriers. If the equation is  $I = q/t$ , it means an average rate of flow of charge-carriers. Two textbooks included both equations,  $I = dq/dt$  and  $I = q/t$ , and explain that they are applicable to electric current that is changing or steady respectively. Similarly, the equation in the diagrams could be specified as  $I = dq/dt$  (3%) or  $I = q/t$  (3%). Textbook authors may consider it redundant to state the same equation in both definition and diagram. Alternatively, 10% of textbooks relate electric current, electric charge, and time by using a graph. One textbook states that the gradient of tangent line ( $dq/dt$ ) of ‘electric charge-time graph’ is the electric current (Adams and Allday 2000). However, when a symbol such as  $q$  is not clearly defined pertaining to the graph, it can cause a cognitive gap in learning.

In short, the equation  $I = q/t$  is related to the *nature* of electric current and  $I = V/R$  is related to the *cause* of electric current. In secondary schools, physics teachers can represent the equation  $I = q/t$  with a definition, a diagram, and a graph as shown in Fig. 9.9. Verbally, we can provide a definition such as “electric current ( $I$ ) is the rate of flow of net charge ( $q$ ) through a cross-sectional area of a wire per unit time ( $t$ )”. This definition also provides the meaning of the symbols,  $I$ ,  $q$  and  $t$ , used in the equation. In addition, students should be able to relate the mathematical definition of electric current,  $I = q/t$ , to a graph. For example, in an ‘electric current-time graph’, the area under the *curve* is the ‘flowing electric charge’,  $q$ . To be precise, the symbol  $q$  can be defined as the total amount of electric charge that flows through a cross-sectional area of a metallic wire within a time period,  $t$ .

In secondary schools, we propose physics teachers to represent the equation  $I = V/R$  verbally and diagrammatically in Fig. 9.10. We can explain that *electric current ( $I$ ) in a resistor with electrical resistance ( $R$ ) is due to the potential difference ( $V$ ) of*

**Fig. 9.10** Representing  $I = V/R$



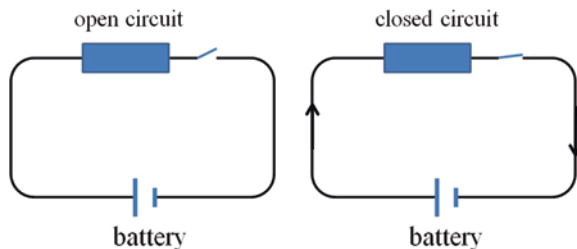
a battery. We should be aware of a limitation of this representation: what were drawn in the Fig. 9.10 are the wire, battery, and resistor. It is not always straightforward for students to conceptualise an *electric current* that flows in a wire, *potential difference* of a battery, and *electrical resistance* of the resistor. However, we can explain the concepts by using two colours in drawing the wire: any point in the blue segment has the same electric potential and any point in the green segment also has the same electric potential. If the battery has a potential difference of 1.5 volts, any point in the blue segment is higher than any point in the green segment by 1.5 volts. Thus, the electric current,  $I$ , through the resistor with electrical resistance,  $R$ , can be calculated by using the equation,  $I = V/R$ , if the resistance  $R$  remains constant. Physics teachers should explain that there is an idealisation in this figure in which the metallic wire has no electrical resistance.

At the college and university level, physics teachers should consider using the equation  $I = \Delta V/R$  instead of  $I = V/R$ . This is because students might associate  $V$  with voltage or electric potential instead of potential difference. Therefore, it is important to emphasise the form of the equation for electric current as  $I = \Delta V/R$  by comparing it with  $V = IR$  and  $I = V/R$ . Furthermore, physics teachers should introduce the equation  $I = dq/dt$  instead of  $I = q/t$  for students who have some knowledge of calculus. However, some textbook authors may essentially state that “For constant electric current:  $I = q/t$ . For variable current:  $I = dq/dt$ ”. This may cause confusion to students because it does not seem consistent with the earlier notion of electric current which is conserved or constant. Physics teachers should explain that the equation  $I = dq/dt$  is more precise than  $I = q/t$  and it is applicable to constant electric current and variable electric current. In other words, electric current defined in terms of the equation  $I = dq/dt$  may still be constant under ideal circuit conditions.

### 9.2.5 Condition

Circuit condition is an important consideration in determining electric current of an electrical circuit. In a study conducted by Dupin and Johsua (1987), they identified alternative conceptions of electric current in the contexts of circuit conditions. For example, some students had the alternative conception that an electric current could present in an open circuit. It was also reported that some elementary and middle

**Fig. 9.11** open circuit / closed circuit



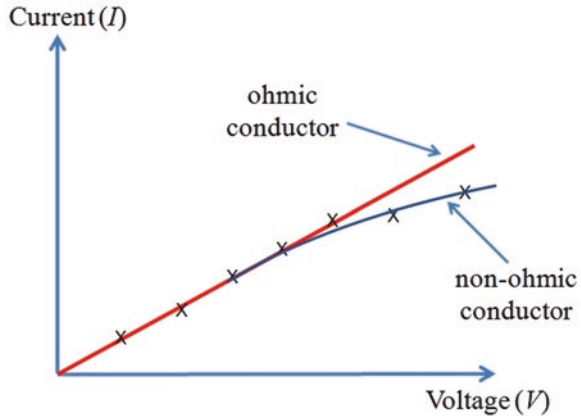
school teachers did not know how long a copper wire has to be such that there is a measurable effect on the electric current (Heller and Finley 1992). Physics teachers should explain that the electrical resistance of a copper wire may vary with the electrical potential difference of a battery. The electrical resistance of a copper wire can increase due to a heating effect of electric current, and depends on the temperature of the copper wire. Strictly speaking, the copper wire is not an ohmic conductor in which the ratio of electric current to its electrical potential difference remains constant, irrespective of applied voltage.

Many textbook authors do not explicitly specify any condition for the concept of electric current. Some textbook definitions specified circuit *condition* such as ‘metal, conductor, wire’ (35%), ‘area, surface, point’ (25%), or simply circuit (13%). In addition, the diagrams showed circuit condition such as ‘metal, conductor, wire’ (33%), ‘area, surface, point’ (13%), and a complete circuit (13%). Alternatively, a textbook states that “under the steady-state conditions assumed here, an electron must pass through plane *aa* for every electron that passes through plane *cc*” (Halliday et al. 2005, p. 684). Physics teachers should clarify that when an electric current is not constant, it can result in an induced electromotive force. Thus, when a switch is closed to form a complete circuit, it will take a short while for the electric current to reach a steady state or constant value.

In secondary schools, physics teachers may emphasise a simple circuit *condition* for electric current: closed circuit. We may compare a closed circuit and open circuit by using a Fig. 9.11. It shows that there is no electric current when the circuit is open, and vice versa. However, it has been reported that students could have different definitions of a complete circuit or closed circuit (Fredette and Lockhead 1980). Thus, we can define a complete circuit (or closed circuit) as a condition in which there is a continuous conducting path for an electric current to flow through an electrical circuit. Nevertheless, there is a limitation of diagrams in textbooks and in Fig. 9.11: it does not show how an electric current reaches a steady state in a complete circuit. If time permits, there should be an animation to show the flow of charge-carriers in reaching the steady-state condition, and there could be a graph to show how the average speed of charge-carriers varies with time.

In colleges or universities, physics teachers should explain that the constant nature of electric current is subjected to the conditions of circuit elements such as a copper wire. We can specify three circuit conditions as follows: (1) low constant

**Fig. 9.12** current-voltage characteristic of ohmic and non-ohmic conductor



voltage; (2) constant temperature; and (3) constant size. If the potential difference across a copper wire is relatively high, there will be a heating effect and its electrical resistance can be significantly increased. Thus, the electric current through the copper wire can have a lower expected value depending on its temperature. In other words, the copper wire can behave like a non-ohmic conductor rather than an ohmic conductor, at a higher applied voltage. Experimentally, the ratio of an electric current to potential difference of a non-ohmic conductor can remain constant at a lower applied voltage (See Fig. 9.12). The current-voltage characteristic of an ohmic conductor is a straight line graph in contrast to a curve for non-ohmic conductor. Thus, the electric current through the copper wire does not always remain constant.

Generally speaking, the conditions pertaining to electric current may not be consistently expressed in words, diagrams, equations, and graphs. Students' problem solving skills on questions related to electric current could be weakened by the inconsistent representations of electrical resistance of a metallic wire. For example, in determining the electric current in a simple circuit, students should idealise the metallic wire as having *no* electrical resistance (as compared to a filament). In analysing the current-voltage characteristic of a metallic wire in a graph, students are expected to idealise the metallic wire as an ohmic conductor which has a constant electrical resistance. In answering a qualitative question, students may need to explain that the metallic wire is a non-ohmic conductor in the real world, and thus the electric current is not necessarily constant. The concept of electric current in relating to the metallic wire may vary depending on the context.

To summarise, textbook authors should improve the use of multiple representations of electric current effectively. The concept of electric current is multifaceted and may not be comprehensively presented among the textbooks pertaining to the five conceptual elements. In fact, each conceptual element may not be consistently presented in words, diagram, equation, and graph, among the textbooks or even within a textbook. In addition, the concept of electric current presented is usually idealised and does not accurately reflect the real world. Fundamentally speaking,

there are problems of representations in which a diagram, for example, only provides a limited perspective of the concept of electric current. To provide a dynamic aspect of electric current, physics teachers should use animations to help students in visualising the concept.

### 9.3 Reconstruction

Students should be guided to reconstruct multiple representations of electric current. The concept of electric current can be unpacked as five conceptual elements as shown in Table 9.2. These conceptual elements of electric current are closely related and can be individually represented in a separate diagram or combined together with varying details as complementary information. Some suggestions on how to reconstruct the concept of electric current are presented as follows.

#### 9.3.1 Reconstructing the Concept of Electric Current: Inquiry-Based Representation

Physics teachers should design an *inquiry-based representation* which can trigger students to think about a physical concept (Fig. 9.13). Students may be asked to explain which part of the electrical circuit has a higher electric current or whether the electric current is constant throughout the circuit. There could be discussion questions focusing on each conceptual element as shown below:

1. Object/charge carriers: What are the charge-carriers in an electrolyte? What are the charge- carriers in a copper wire? Why are there free electrons in a metal?

**Table 9.2** Five conceptual elements of electric current

Conceptual element	Electric current
Object	Electrons Charge-carriers (positive)
Nature (or characteristic)	Conserved or constant
Cause (or effect)	Potential difference, electric field (cause) Magnetic field, heat (effect)
Mathematical equation	$I = V/R$ or $I = \Delta V / R$ $I = Q/t$ or $I = dQ/dt$
Condition	Closed circuit (or complete circuit)
	Circuit condition:
	(1) Low constant voltage
	(2) Constant temperature
	(3) Constant size

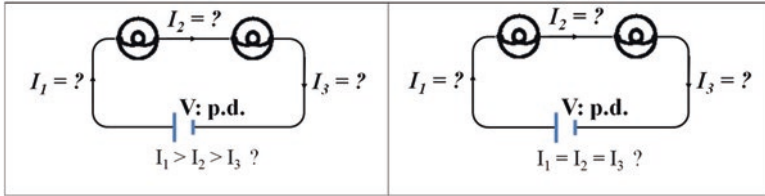


Fig. 9.13 An inquiry-based representation

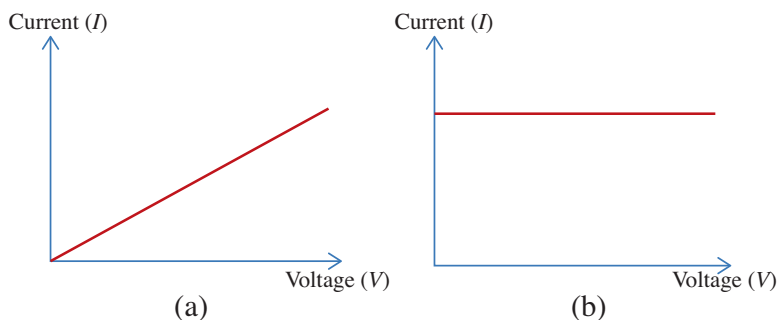
2. Nature/characteristics: What is used up or reduced in an electrical circuit? Would you define electric current as a *constant* flow of charge? Is the constant nature of electric current an idealisation? Why?
3. Cause: Why do metals conduct electric current? How does an electron lose or gain kinetic energy in a copper wire or electrical circuit?
4. Mathematical equation: How do you determine the electric current in this electrical circuit? By using  $I = q/t$ ? Or  $I = V/R$ ?
5. Circuit Condition: What are the conditions for the use of this mathematical equation? What are the conditions for the constant flow of charge-carriers?

As a result of students’ discussions and physics teachers’ interventions, students should have a deeper understanding of electric current whether it attenuates after passing through a light bulb.

In this thinking activity, students could be requested to present their knowledge of electric current in words, diagram and mathematical equation. It is possible that students’ multiple representations of electric current are inconsistent with each other. For example, their definitions and drawings pertaining to the *object* of electric current could be inconsistent. Students may also recall the concept of electric current as presented by a textbook which is inconsistent in multiple representations. Physics teachers can later use Fig. 9.3 to Fig. 9.12 to guide students to conceptualise the constant nature of electric current verbally, diagrammatically, and mathematically.

### 9.3.2 Reconstructing the Graphs of Electric Current

In his Nobel lecture, Wilczek (2004) mentioned that ‘Ohm’s first law is  $V = IR$ . Ohm’s second law is  $I = V/R$ . I’ll leave it to you to reconstruct Ohm’s third law (p. 413)’. He clarified that different forms of the equation can suggest different meanings. Historically, there were also two versions of Ohm’s Law: ‘the law for a part of a circuit’ and ‘the law for a whole circuit’ (Ashford and Kempson 1908; Kipnis 2009). The law for a part of a circuit is ‘electric current through a conductor is directly proportional to potential difference at its ends, and the resistance of the conductor is constant’. The law for a whole circuit is ‘electric current through a



**Fig. 9.14** Graph of electric current versus voltage

conductor is directly proportional to the potential difference at its ends and inversely proportional to its resistance'. These two laws could be represented by the following two equations respectively:  $I = \Delta V/R$  and  $I = E/(R + r)$ . ( $E$  is the electromotive force of a battery and  $r$  is the internal resistance of the battery.)

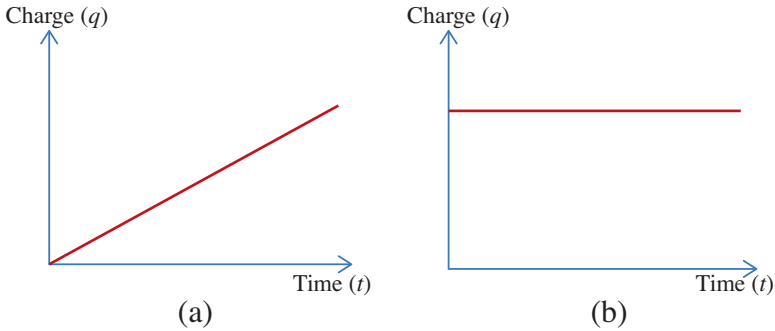
Generally speaking, students tend to apply a mathematical algorithm or write down several possible equations, in order to analyse an electrical circuit. It is possible that they do not clearly understand the reasoning behind Ohm's law and its equation. Thus, students may be asked to explain which of the following graphs (electric current versus voltage) below correctly represents Ohm's law (Fig. 9.14). Similarly, students should be requested to justify their answers in words, diagram (submicroscopic and macroscopic perspective), and mathematical equation.<sup>3</sup>

Importantly, multiple representations cannot replace the experiment in which students interact with the equipment in the laboratory. There should be an experiment for students to reconstruct Ohm's law in order to understand the operational meaning of electric current, potential difference and electric resistance in the real world. During the experiment, potential difference is the manipulated (independent) variable and electric current is the measured (dependent) variable. In a current-voltage graph, the  $x$ -axis represents the potential difference and  $y$ -axis represents the electric current. The *correct* graph is a straight line which passes through the origin (Young and Freedman 2004). It can be represented as  $I \propto \Delta V$  which means that the electric current through the electrical circuit is directly proportional to the potential difference of an ideal battery. However, students and teachers may not fully understand the concept of direct proportionality which is embedded in the equation (Yap 1992).

We should also let students analyse the electric charge-time graph (See Fig. 9.15). There could be discussion questions such as "which of the following graph is correct?", "what is the meaning of gradient of electric charge-time graph?", "could

<sup>3</sup>Historically, Ohm used the equation  $x = a/(b + l)$  to model his experimental data. In universities, Ohm's Law may be symbolically represented as  $J = \sigma E$ , where  $J$  is the current density at a given location in a conductor,  $E$  is the electric field at that location, and  $\sigma$  is the conductivity of a material.





**Fig. 9.15** Graph of electric charge versus time

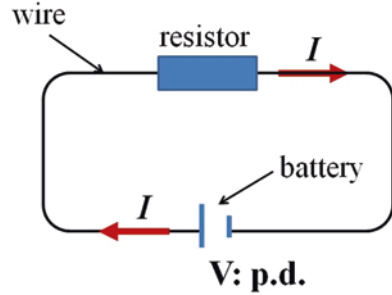
electric current be deduced by using the equation,  $I = q/t$  or  $I = dq/dt$ ?", "what is the meaning of the symbol  $q$ ?", "does  $q$  means total charge, net charge, an amount of charge?", "should the symbol  $\Delta q$  be used instead?" However, a textbook simply termed  $q$  and  $\Delta q$  as 'charge flowing' (Dobson et al. 2002). We can explain that the symbol  $\Delta q$  refers to a very small amount of electric charge that flows through a cross-sectional area during a very short time interval,  $\Delta t$ .

### 9.3.3 *Reconstruct the Concept of Electric Current: Macroscopic and Sub-microscopic Perspectives*

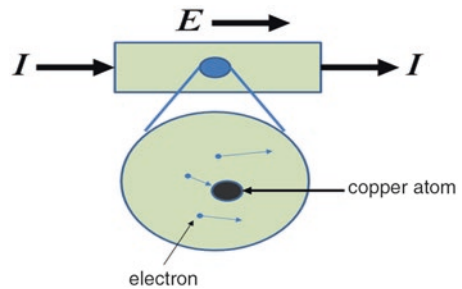
During a revision period, physics teachers could ask students to summarise the concept of electric current from the macroscopic and sub-microscopic perspectives. (Students may imagine that they take over the role of a physics lecturer.) As an example, the macroscopic and sub-microscopic perspectives of electric current can be represented by including four conceptual elements: *object*, *nature*, *cause*, and *condition*.

**Macroscopic Perspective** It is possible to have a macroscopic perspective of electric current with four conceptual elements as shown in Fig. 9.16. Physics teachers should guide students to provide four verbal explanations pertaining to this figure. Firstly, the *object* related to electric current may include a copper wire, resistor, and battery. Secondly, the *nature* of electric current is shown by two arrows with identical length in parallel to the wire. This suggests that the electric current remains constant after passing through a resistor. Thirdly, the macroscopic *cause* is the potential difference due to the presence of a battery. In other words, the battery maintains the potential difference and provides electrical energy for the constant flow of charge-carriers. Fourthly, the macroscopic *condition* may refer to a simple 'one loop' closed circuit. Because of this simple circuit condition without additional branches, there is no splitting of the electric current throughout the electrical circuit. Thus, the electric current is constant and having the same value everywhere in the copper wire.

**Fig. 9.16** A macroscopic perspective of electric current



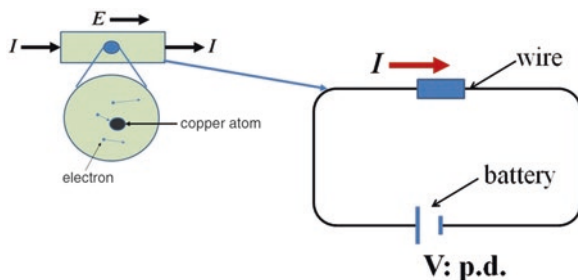
**Fig. 9.17** A sub-microscopic view of electric current



**Submicroscopic Perspective** It is also possible to have a sub-microscopic perspective of electric current with four conceptual elements as shown below (Fig. 9.17). Firstly, the object may refer to copper atoms and electrons. Secondly, the individual electron is not moving in a perfectly horizontal direction from a sub-microscopic perspective. There is randomness in the flow of electrons with different velocities as shown in Fig. 9.17; the length of an arrow represents the speed of an electron. Thirdly, the sub-microscopic cause is an electric field due to the presence of a battery. The electric field is the cause that results in the electric force on the electron, and the electric current. Fourthly, the circuit condition may refer to a copper wire which has a relatively low electrical resistance. Sub-microscopically speaking, there will be a heating effect due to the electric current, and it can increase the random motion of the lattice atoms, as well as the collision rate (between an atom and electron), and thus increase the electrical resistance. Hence, the electric current may not remain constant under real circuit conditions.

**Synthesising Five Conceptual Elements of Electric Current** Electric current in a metallic conductor can be defined as a constant rate of flow of ‘free’ electrons due to a potential difference across the ends of the conductor, under constant circuit conditions. Initially, the concept of electric current can be presented with a comprehensive definition, and elaborated in greater detail as deemed appropriate by physics teachers. Subsequently, students could be guided to translate the conceptual elements of electric current in different representations. As a summary, both macro-

**Fig. 9.18** A diagrammatic definition of electric current



scopic and sub-microscopic representations of electric current can be combined together as a ‘diagrammatic definition’ (Fig. 9.18). It provides a comprehensive representation of electric current diagrammatically. Based on this diagram, a student may explain that the electrons are moving past the *stationary* copper atoms due to an electric field. However, students should be guided to reconstruct the concept of electric current by providing a summary or designing their own diagrammatic definition.

Nevertheless, physicists may prefer a field model of electric current as compared to a fluid model (Stocklmayer and Treagust 1994). Thus, physics teachers could provide a brief historical development of the concept of electric current. By reconstructing the concept with a historical perspective, it may help students to distinguish an early scientist’s conception of electric current with the current scientific concept. Better still, some historical drawings could be used for analysis during classroom learning.

## 9.4 Conclusions and Limitations

In addition to focussing on the different forms of representations, physics teachers should analyse the conceptual elements of these multiple representations because they were inconsistently represented in textbooks, and thus possibly contribute to students’ alternative conceptions. For instance, the concept of electric current is multifaceted and it can be unpacked as involving the following elements: object, nature, cause, equation, and condition. These five conceptual elements of electric current should be consistently presented from the macroscopic and sub-microscopic perspectives to *constrain* students’ interpretation. Moreover, a conceptual element may be *complementarily* presented via a hybrid of verbal, diagrammatical, symbolical, and graphical representations. Most important, students should be guided to *reconstruct* the physical concept for a meaningful and deeper understanding. Physics teachers may present other physical concepts by using multiple representations with these five conceptual elements in mind.

One limitation of this chapter is that we only analysed definitions of electric current that are essentially written as “a flow of charge”. Most textbooks also included

statements to the effect of ‘a potential difference causes an electric current’. However, there could be disagreement whether this statement should be regarded as a definition. It depends on an educational researcher’s definition of *definition*. Similarly, we only analysed diagrams that were used to illustrate a definition of electric current. For the purpose of focus, we did not include diagrams that appeared in another section which illustrated the notion of potential difference or how pressure difference causes a flow of water. These diagrams could be included in another study if a researcher adopts a broader definition of *definition*.

Another issue is that physics teachers could have difficulties in using multiple representations. In Yap’s (1992) study, he assessed pre-service teachers’ conceptions of direct proportionality in identifying a graph, providing a mathematical relationship, identifying the pattern in a table, and expressing the concept in words. No one in this study was consistently correct across all of the designed activities in different representations. Many participants simply defined direct proportionality as ‘*Y* increases as *X* increases’, and they were inconsistent in providing the reason for the concept with respect to a graphical representation. Yap (1992) proposes a need in science teacher education to train preservice teachers and in-service teachers in integrating their knowledge from activities in different representations, thereby leading to a meaningful understanding. We should not assume physics teachers can master the use of multiple representations without difficulties.

Importantly, students and teachers should be warned of the problems of representations in physics. They should be shown the Belgian surrealist painter René *Magritte’s* painting of a pipe which has an inscription ‘this is not a pipe’ (Gilbert and Treagust 2009). However, students and teachers may not immediately realise that ‘this is only a *representation* of a pipe’. Some students could simply make sense of the painting by thinking that the word *pipe* has another definition such as a ‘metal tube that is used to convey water, gas or oil’. Thus, it is not a ‘narrow tube with a small bowl at one end for containing tobacco’ as shown. It is possible that students make sense of the painting for other different reasons that were not intended by the painter. Similarly, students may not always understand the multiple representations of a physical concept despite the best efforts of a physics teacher.

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