

# Chapter 1

## Nature of Multiphase Flows and Basic Concepts

George Yadigaroglu and Gad Hetsroni

### 1.1 The Nature of Multiphase Flows

Multiphase flows and heat transfer with phase change are one specialized discipline among many others in the much more general area combining thermodynamics, fluid mechanics and heat transfer; often this area is referred to as *thermal-hydraulics*. The study of multiphase flows requires basic knowledge in these three areas. Appendix I of this volume, a tutorial, provides some of the indispensable minimal background from these three areas for the reader that may need it.

The adjective *multiphase* characterizes situations where several different phases—liquids, gases, solids—are flowing simultaneously. In the case of liquids, the two flowing media can also be two chemically different ones, e.g. oil and water. *Two-phase flows* take place when only two phases are present, most often liquid and gas.

Another categorization of flows is according to the presence or absence of heat transfer: we speak of *adiabatic* or *diabatic* multiphase flows. In diabatic multiphase flows, we can have phase change: *vaporization* or *condensation*. Phase change can also take place, however, in adiabatic flows due to pressure changes; such vaporization is referred as *flashing*.

Another important distinction is according to the flow direction: *vertical*, *horizontal*, or *inclined*. Flow direction is more important in multiphase flows when gravity plays a more important role in presence of large differences in the density of

---

Gad Hetsroni—Deceased Author.

---

G. Yadigaroglu (✉)  
ETH-Zurich, Zurich, Switzerland  
e-mail: yadi@ethz.ch

G. Hetsroni  
Technion - Israel Institute of Technology, Haifa, Israel

the phases; such differences can easily be of the order of 1000 (e.g. air–water flow at atmospheric pressure).

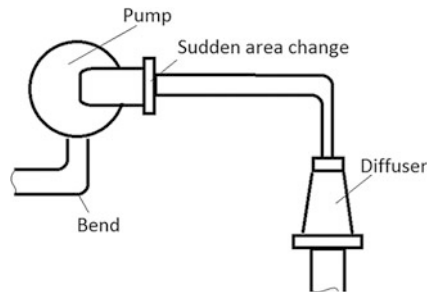
Finally, we speak of *parallel or co-current flow* when the two-phases flow in the same direction, and of *counter-current flow* when they flow in opposite directions (e.g. falling liquid and rising gas).

Multiphase flows are present in everyday life, in nature, in industrial processes, in power plants, in the oil and gas industry, etc. All phase-change processes such as boiling and condensation produce two-phase flows; these heat and mass transfer processes are “core” considerations in the multiphase flow area where many applications involve phase change or at least interactions between phases. Water boils in power plants to produce steam. In processing plants, different phases are mixed to react or are the products of chemical reactions. Thus, heat transfer with phase change (e.g. boiling or condensation) is intimately linked to two-phase flows.

After some reminders and formal definitions in Sect. 1.2, we will consider in Sect. 1.3 a few examples where the nature of multiphase flows is dominant, and controls the behaviour, the response and the operation of the whole system. In particular, we note that some peculiarities of multiphase flow produce unique responses that may have a profound effect on the dynamics and safe operation of the system; we touch here only two such phenomena: the Critical Heat Flux (CHF) phenomenon and a particular type of flow instability.

We will deal mainly with *one-dimensional* flows in ducts. In fluid mechanics, the notion of one-dimensional flow is applied in a broader sense. For example, the flow in a complex system of pipes that may have bends, enlargements, curved sections, etc., such as the one shown in Fig. 1.1 is, strictly speaking, certainly not one dimensional. However, as long as the three-dimensional details of the velocity and temperature distributions do not interest us and we deal only with the average velocity and the mass flux-weighted average fluid temperature, such systems can still be treated as “one dimensional”. Indeed, we are dealing in this volume mainly with one-dimensional two-phase flow, very much like in the excellent book by G.B. Wallis that has exactly this title (Wallis 1969).

**Fig. 1.1** A complex circuit with bends, enlargements, fittings, a pump, etc. that can be still considered as one dimensional when only cross-sectional-average variables are of interest



## 1.2 Phases, Components, Fields

A *phase* is a thermodynamic definition for the state of the matter, which can be either solid, liquid or gas; these can co-exist in a conduit. Examples of multiphase flows are abundant, e.g. when oil is produced, one normally gets oil, water, gas and sand flowing in the pipelines (three-phase flow). Normally, by two-phase (or by extension multiphase) flow we mean a mixture of two (or more) phases which is not extremely dilute and the phases have a distinguishable size. Thus, though clean city tap water is, strictly speaking, a two- (water–air) or three- (water–air–microscopic solids) phase mixture, for practical purposes, it is considered a single phase. However, city water carrying sand should, for filtration purposes at least, be considered a two-phase mixture.

The term *mixture* is most of the time used to denote the two (or more) phases flowing together and does not necessarily imply that these are intimately *mixed*. For example, in the case of annular flow that we will introduce below, we may still refer to the flow as *the two-phase mixture* in spite of the fact that the liquid film on the wall and the gaseous core are not at all “mixed”. The term “separated flow” is often used loosely to denote two-phase flows where the two phases have different average velocities. This distinguishes such flows from the *homogeneous* ones, where the phases have the same average velocity; again, such flows may strictly speaking not be homogeneous at all. For example, bubbly flow with fairly large bubbles can be considered as *homogeneous*.

A *component*, is a chemical species. So, the term two-component is used to describe the flow of two chemical species. A water–steam mixture is two-phase, one-component, while a water–air mixture is two-phase, two-component flow; a water–oil mixture is one-phase, two-component, etc. The approach in modelling of the two alternative two-phase configurations—with one or two components—is often the same or very similar, though the physical behaviour of different mixtures may be quite different.

The term *field* is used to denote a topologically distinct or clearly identifiable fraction of a phase. For example, in the so-called annular flow, the liquid can be present as either a film on the wall or as droplets in the core where the gas flows; the droplets and the film can be considered as different fields. In a closed vessel such as a pressure cooker containing boiling water, we may define a field of steam bubbles in the liquid and a field of steam in the space above the liquid surface as separate fields.

For simplicity of presentation, the approaches and various methods that will be covered here will be presented mainly for two-phase flows. However, their extension to multiphase and multi-field flows is most often possible, although such applications are less frequently encountered in practice.

In this volume, we mainly deal with liquid-gas mixtures. There are many other two-phase flows, e.g. gas-solid flows such as fluidized beds, conveying of granular materials by gas; liquid-solid flows where sedimentation and filtration may be phenomena of interest, etc. Such flows will not be treated in this volume and in this

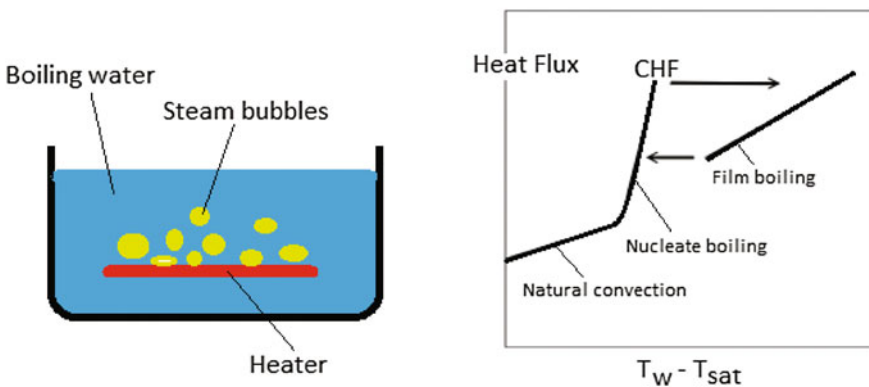
series of books which are mostly directed to two-phase gas-liquid flows of industrial interest: steam-water flows, oil and gas flows, two- or multiphase flows in processing plants, etc. There are other texts which deal with the other types of multiphase flows, namely the Handbook by Hetsroni (1982) and monographs such as Fan and Zhu (1998) for gas-solid flows and Brown and Heywood (1991) for liquid-solid flows (slurries).

### 1.3 Multiphase Flow Phenomena

We give examples in the following section of situations where peculiarities of multiphase flow produce unique responses not found in single-phase flows. The Critical Heat Flux (CHF) phenomenon and a particular type of flow instability will be briefly introduced. In Sect. 1.3.2, we discuss phenomena that are not unique to multiphase flows but become much more complex in the presence of several phases.

#### 1.3.1 Phenomena Unique to Multiphase Flows

The *critical heat flux (CHF)* phenomenon is a situation that may take place in heat transfer with boiling. To illustrate the situation, we consider a very simple experiment conducted with a heated tube immersed in a pool of liquid, Fig. 1.2. This is the situation referred to as *pool boiling*. The heat flux from the wall is plotted in the graph of Fig. 1.2 against the excess wall temperature (i.e. the wall temperature  $T_w$  minus the saturation temperature  $T_{sat}$  of the fluid in the pool). The heat flux is gradually increased; one can observe that once a certain heat flux is reached, the



**Fig. 1.2** A pool boiling experiment (left). The boiling curve (right)

CHF, a dramatic change takes place and a further increase in heat flux will cause a very large, sudden rise in wall temperature, resulting most likely in the failure of the tube. When the CHF is reached, the liquid can no longer wet the heater surface and cooling takes place through a vapour film surrounding the heater rather than by boiling of the liquid in contact with the wall. As the vapour has much reduced thermal conductivity and density, heat transfer deteriorates markedly and leads to the temperature excursion. This subject is of utmost practical importance and will be further treated in other volumes. It is a phenomenon taking place only in heat transfer with phase change.

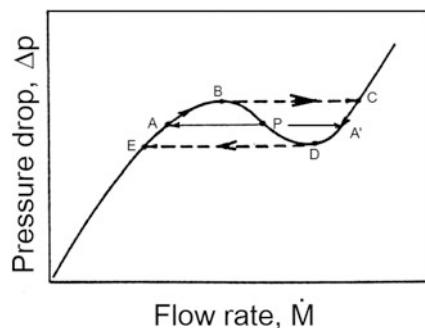
We also note here that as the heat flux is *reduced*, the operating point stays on the film boiling characteristic curve rather than going back at CHF to the nucleate-boiling curve. It appears that the film surrounding the heater is stable, keeping film boiling going on; it will take a further reduction of the heat flux to produce another jump back to the nucleate-boiling curve. This particular hysteresis behaviour during pool boiling was studied by Nukiyama (1934) who published the *boiling curve*.

Another example which is typical to two-phase flow is a flow excursion which can occur in a two-phase loop having certain flow characteristics. The situation is illustrated in Fig. 1.3. If the two-phase section of the loop has a negative slope in its pressure drop—flow-rate characteristic ( $\dot{M}$ ,  $\Delta p$ ), i.e. if  $\partial p / \partial \dot{M} < 0$  something that happens often in two-phase flow (and cannot happen in single-phase flow), a flow excursion will occur. This situation was described first by Ledinegg (1938). Ledinegg instabilities can be of great practical importance regarding the stability of two-phase equipment such as steam generators. Flow instabilities will be dealt with in another volume; some additional information is included in the following section.

### 1.3.2 Phenomena Complicated by the Presence of Many Phases

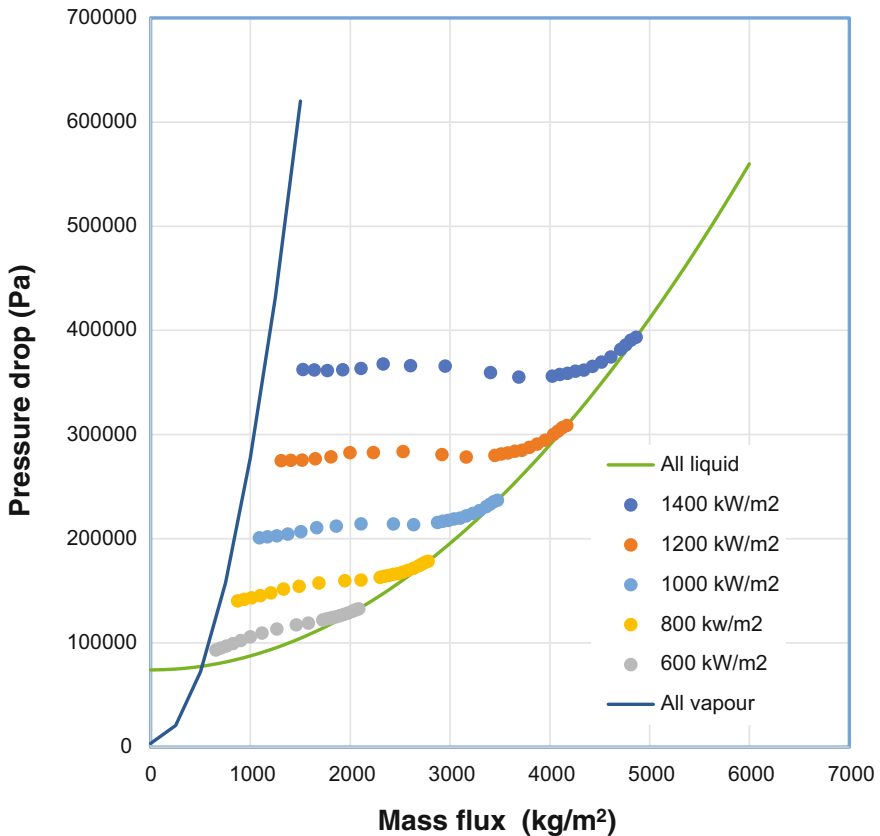
A number of single-phase flow phenomena take also place in single-phase flows, but the presence of two or more phases complicates greatly the situation.

**Fig. 1.3** The flow-rate versus pressure-drop characteristic of a boiling channel. The Ledinegg instability occurs if the slope of the characteristic is locally negative, as in the segment BD. Point P is unstable and the operating point will migrate to either A or A'.



For example, the *pressure drop* is a factor of importance in the design of any hydraulic system. Its estimation is rather straightforward in single-phase systems, but it becomes much more difficult—and inaccurate in fact—in two-phase flows. Figure 1.4 shows the pressure drop—flow rate characteristic of a heated pipe. The all-liquid and all-vapour characteristics are nearly parabolic, but, in between the heated pipe exhibits a much more complex behaviour in the presence of two-phase flow. As the heat flux increases, a negative-slope part of the characteristic emerges; it is such behaviour that is responsible for the Ledinegg instability just mentioned.

*Critical flows* are another case where the multiphase nature of the problem greatly complicates the situation. We will use this phenomenon to show the importance of non-equilibria in two- or multiphase flows. In the simpler case of



**Fig. 1.4** The flow rate—pressure drop characteristics of a heated pipe at different heat fluxes and with all-liquid and all-vapour flow (with no heat addition). Pipe length  $L = 10$  m, diameter  $D = 0.02$  m, friction factor  $f = 0.005$ . Pressure  $p = 69$  bar. Inlet non-dimensional subcooling  $\Delta h_{in}/h_{LG} = 0.45$ . The computations were performed with Thom's charts given as an Appendix to Chap. 6

two-phase flow, the gas and the liquid may have different average velocities and temperatures; this is what we denote as *non-equilibrium*.

In the case of single-phase flow, the flow becomes choked as the critical flow condition is reached when, essentially, the flow velocity reaches the sonic velocity of the fluid; the latter is determined as a thermodynamic fluid property depending only on temperature and pressure (e.g. Shapiro 1953). The assumption is made that the properties of the fluid are given along its thermodynamic path by its state equation. The same approach can be taken for two-fluid mixtures but difficulties appear: the state equation of the mixture depends now on whether the gas and liquid velocities and temperatures are equal or not. As such equilibria are subject to all sorts of flow conditions and on the history of the fluid upstream of the choking point, there is no simple answer to this problem and the critical flow of a two-phase mixture depends on assumptions made about the state of the mixture.

## 1.4 Flow Regimes

One of the major difficulties in multiphase or two-phase flows is that the phases are distributed in the duct in particular ways; the various typical (topological) configurations that result are called *flow regimes*.

The boundaries between phases are called *interfaces*. The topology of the flow, i.e. the geometry of the interfaces, is not known and cannot be determined a priori, but is rather a part of the solution. In contrast, for example, in single-phase flow of a fluid in a tube, knowing the geometry we can determine, either experimentally or analytically, the velocity distributions, the shear stress distribution, the pressure drop, etc. When two phases flow in a conduit, we cannot tell a priori how the phases are going to distribute themselves: whether bubbles will be distributed uniformly throughout the liquid (we call this situation *bubbly flow*) or whether the bubbles will coalesce and gas will flow in the centre of the pipe, while the liquid will form a film on the wall (this is called *annular flow*). *Flow regime determination* is needed to answer such questions; this is treated in Chap. 4.

The cross-sectional distribution of the phases in the pipe determines other parameters such as heat transfer, pressure drop and, without knowing this phase distribution, we cannot calculate these. Furthermore, it is not likely that the two phases will flow with the same average velocity. On the contrary, most likely, the gas will flow at a higher average axial velocity than the liquid, which will cause a change of the *void fraction* or *volume fraction* of the gas phase in the channel (i.e. the fraction of area occupied by gas in the cross section). Often, until we have a way to determine the flow regime, there is really no way we can accurately model and

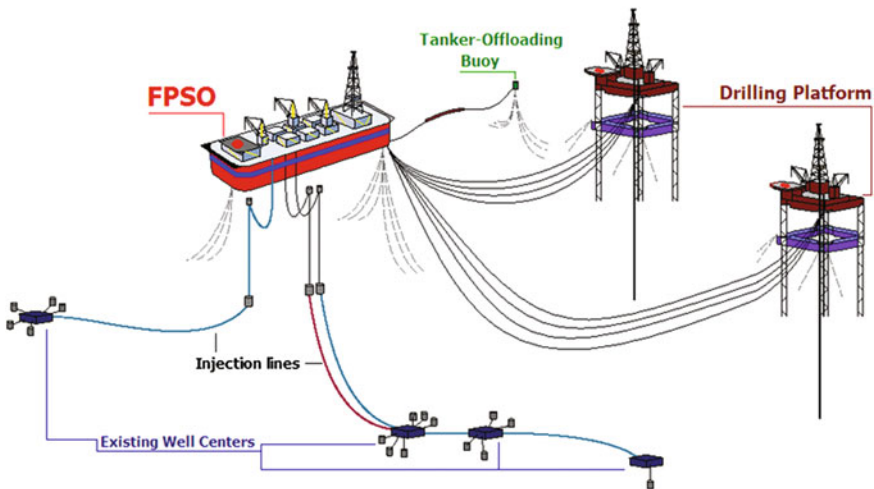
calculate other parameters of engineering significance. Therefore, one of the first problems we will address in this volume is the study and determination of *flow patterns* or *flow regimes*. The reader should be aware that flow patterns depend on many parameters, such as the inclination of the conduit, the geometry, pressure, type of fluid, etc.

## 1.5 Some Important Multiphase Flow Systems

We site now some examples where multiphase flows play a dominant effect in the design and operation of very large engineering systems.

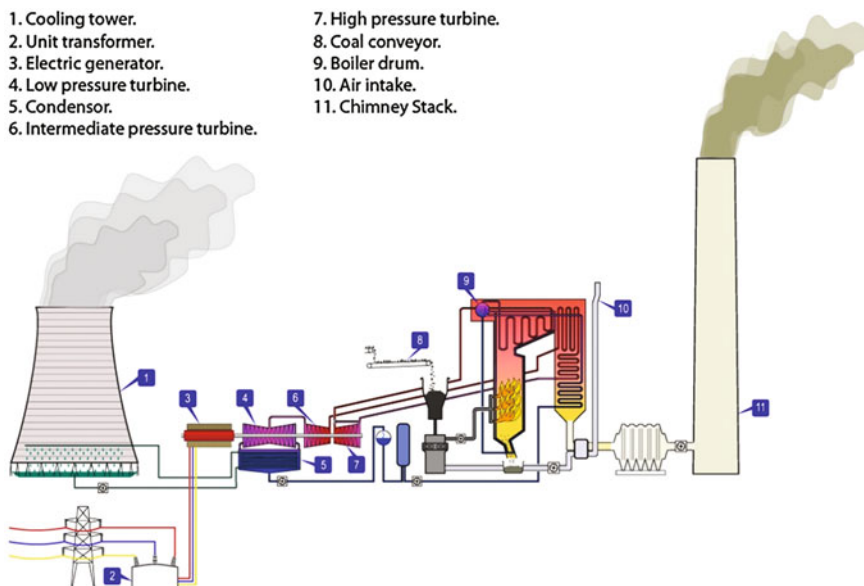
*Offshore production* has been important for the extraction of hydrocarbons since the 1950s. It usually involves a drilling platform which operates a number of wells to produce crude oil. The oil comes out of the ground usually as a mixture of crude oil, sea water, sand and gas—a multi-component, multiphase mixture which needs to be handled carefully because of various hazards and because of the extremely costly equipment involved. Figure 1.5 shows production wells, drilling platforms, and a floating production, storage and offloading vessel.

*Electric power production from oil, coal or gas:* A majority of power plants use a steam cycle where water is the working fluid. Figure 1.6 shows the steam cycle of a modern plant; water gets vaporized in the high-pressure steam generator section and the low-pressure steam exiting from the turbine gets condensed in the condenser. Again, two-phase flows and phase change processes are present throughout the steam plant.



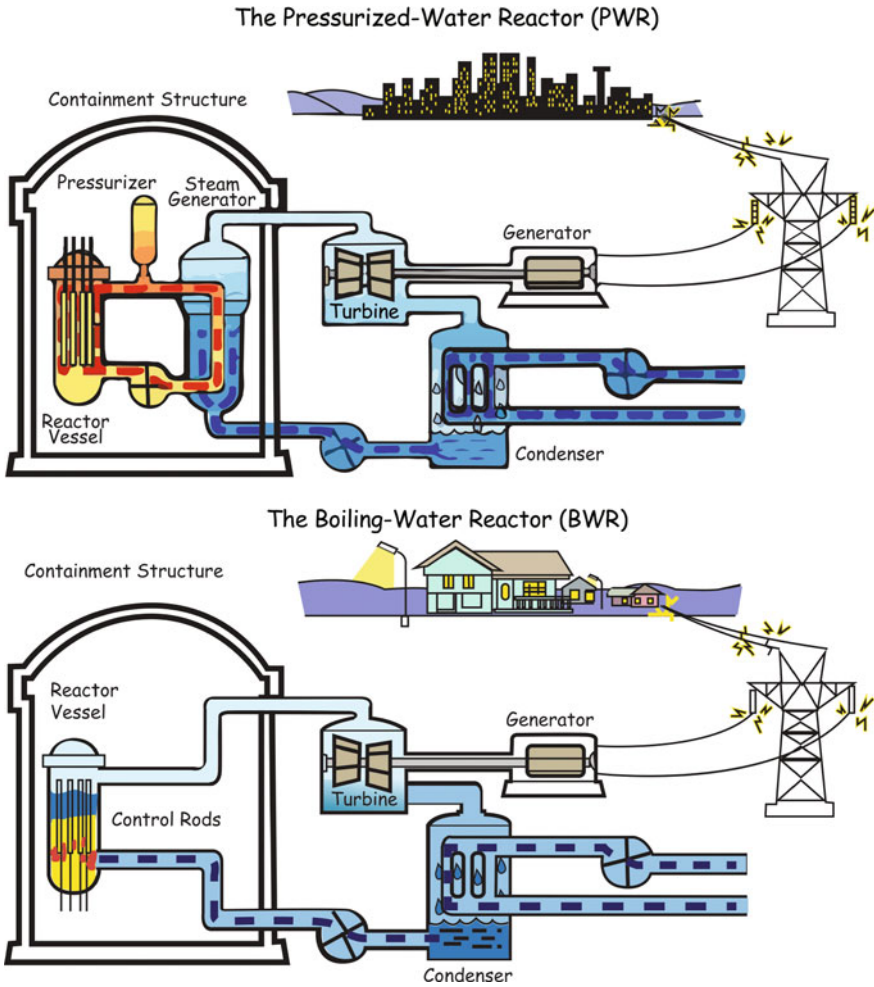
**Fig. 1.5** FPSO, floating, production, storage and offloading for offshore production of oil





**Fig. 1.6** A modern steam power plant. Boiling and two-phase flows take place in the steam generator while evaporation of droplets in the cooling tower (from <http://www.zeroco2.no>)

In a nuclear power plant (NPP), the *nuclear steam supply system* (NSSS or “en-triple-ess”) is the part of the plant where water at high pressure is heated in the reactor vessel by the nuclear chain reaction taking place in the fuel rods. Steam is then produced (either in the reactor vessel, as in a boiling water reactor, Fig. 1.7 (bottom), or in a steam generator, as in a pressurized water reactor, Fig. 1.7 (top), and used in the turbine-generator to produce electric power. The multiphase flow of water and steam takes place in very costly equipment. To operate the system safely and efficiently for the lifetime of the equipment, one needs detailed knowledge of multiphase flow and heat transfer. Understanding two-phase flow situations and phenomena and being able to predict the outcomes becomes even more important in case of an accident in a NPP when unusual situations are encountered. In fact, the disciplines of two-phase flow and heat transfer with phase change progressed tremendously the last three or four decades of the twentieth century driven by the need to fully understand, model and simulate the complex phenomena taking place during thermal-hydraulic accidents in NPPs. Other books in this series will be devoted to two-phase flows in nuclear power plants.



**Fig. 1.7** Pressurized Water Reactor (*top*) and Boiling Water Reactor (*bottom*). Figures from [www.nrc.gov](http://www.nrc.gov). The conventional power production systems of these two main types on NPPs are quite similar and operate under very similar thermodynamic conditions. In the BWR, direct steam generation takes place in the reactor core while, in the PWR, it takes place in the intermediate steam generator

## 1.6 Averaging in Two-Phase Flows

It is often necessary to perform averaging in multiphase flows and in particular averaging over the flow cross section in one-dimensional, two-phase flows. This is in particular necessary when dealing with the conservation equations where one starts with instantaneous local equations that must be integrated in time and space to arrive at usable forms.

Averaging will be applied either to two-phase *mixture* variables (variables for both the liquid and the gas flowing together) or to each phase separately. We will use averages of *mixture* properties such as the mixture density as well as averages of *phase* variables such as the channel-cross-sectional-average liquid or gas velocity. This section briefly introduces the concepts.

Generally, we will add the subscript  $k$ , ( $k = L, G$ ), where  $L$  denotes the liquid and  $G$  the gas, to the phase variables to differentiate them from mixture variables that are usually written without a subscript. Thus,  $f_k$  is a local instantaneous variable pertaining to phase  $k$ , e.g. the local, instantaneous velocity or enthalpy,  $u_k$  or  $h_k$ .

### 1.6.1 Space Averaging

We consider space averages of generally instantaneous values. The following averages can be defined:

Cross-sectional average of any variable  $f$

$$\langle f \rangle = \frac{1}{A} \int_A f dA, \quad (1.6.1)$$

where the angle brackets operator  $\langle \cdot \rangle$  denotes the cross-sectional averaging over the flow area  $A$ .

*Phase* cross-sectional average of a *phase* variable  $f_k$

$$\langle f_k \rangle_k = \frac{1}{A_k} \int_{A_k} f_k dA, \quad k = L, G \quad (1.6.2)$$

where the integration is now performed only over the area of the channel  $A_k$  occupied by phase  $k$  at a given moment. The second subscript outside the angle brackets in  $\langle f_k \rangle_k$  reminds us that the cross-sectional average of the local phase property  $f_k$  was performed over the area of the channel occupied by phase  $k$  only:  $\langle \cdot \rangle_k$ .

### 1.6.2 Time Averaging

The time or statistical average of any variable  $f$  is

$$\bar{f} = \frac{1}{T} \int_{[T]} f dt. \quad (1.6.3)$$

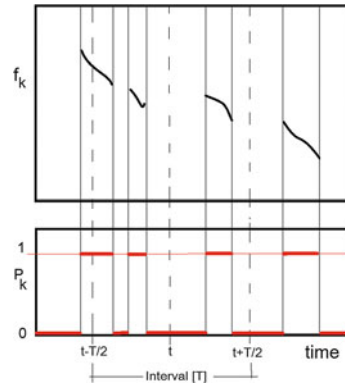
The integration interval  $[T]$  must be chosen such that, for example, high frequency perturbations such as those created by turbulence are averaged out while the transient nature of the process is still well represented.

The *phase* time or statistical average of a *phase* variable  $f_k$  is

$$\bar{f}_k^k = \frac{1}{T_k} \int_{[T_k]} f_k dt, \quad (1.6.4)$$

where  $[T_k]$  is the subset of residence time intervals where phase  $k$  is present at a given point and  $T_k$  is the sum of the presence times of phase  $k$  at that given point.  $T$  is the total averaging time interval. The rather heavy notation for the interval  $[T_k]$  is necessary to remind us that it is discontinuous, Fig. 1.8.

**Fig. 1.8** Measurements of a piecewise continuous variable  $f_k$ , such as phase velocity, pertaining to phase  $k$  (*above*) and phase indicator function  $P_k$  (*below*). Such signals could be actually measured, e.g. by a hot-wire anemometer (phase velocity); the signal of the anemometer could be treated to detected the phase present



## 1.7 Void Fractions and Their Measurement

The term *void fraction* (denoted in general by  $\varepsilon_G$ , where the subscript  $G$  denotes the gas, dimensionless) generically describes in space or time the fraction occupied by the gas phase. The notion is specialized as needed below. For each definition of the void fraction, an experimental method that may produce such a measurement is indicated; some additional information on void fraction measurements is given in Chap. 5.

### 1.7.1 The Local Void Fraction

The *local void fraction* is defined as the fraction of time in which the gas phase occupies a given point in space  $\mathbf{r}$ . We may characterize the presence (or absence) of

phase  $k$  ( $k = G, L$ ) at a given point  $\mathbf{r}$  and at a given time  $t$  by the unit or zero value of the *phase density* or *phase presence* function  $P_k(\mathbf{r}, t)$

$$P_k(\mathbf{r}, t) = \left\{ \begin{array}{l} 1 \text{ if } \mathbf{r} \text{ is in phase } k \\ 0 \text{ if } \mathbf{r} \text{ is in the other phase} \end{array} \right\}.$$

The instantaneous value is usually integrated over a time period  $T$  to give a time-averaged value. Thus the *time-averaged local  $k$ -phase fraction* is the time-averaged phase density function

$$\bar{\varepsilon}_k(\mathbf{r}, t) = \frac{1}{T} \int_T P_k(\mathbf{r}, t) dt = \overline{P_k(\mathbf{r}, t)} \quad (1.7.1)$$

according to the definition of the time-averaging operator. In the case of gas,  $k = G$ , this becomes the *local statistical or time-average void fraction*:

$$\bar{\varepsilon}_G(\mathbf{r}, t) = \frac{1}{T} \int_T P_G(\mathbf{r}, t) dt = \overline{P_G(\mathbf{r}, t)}.$$

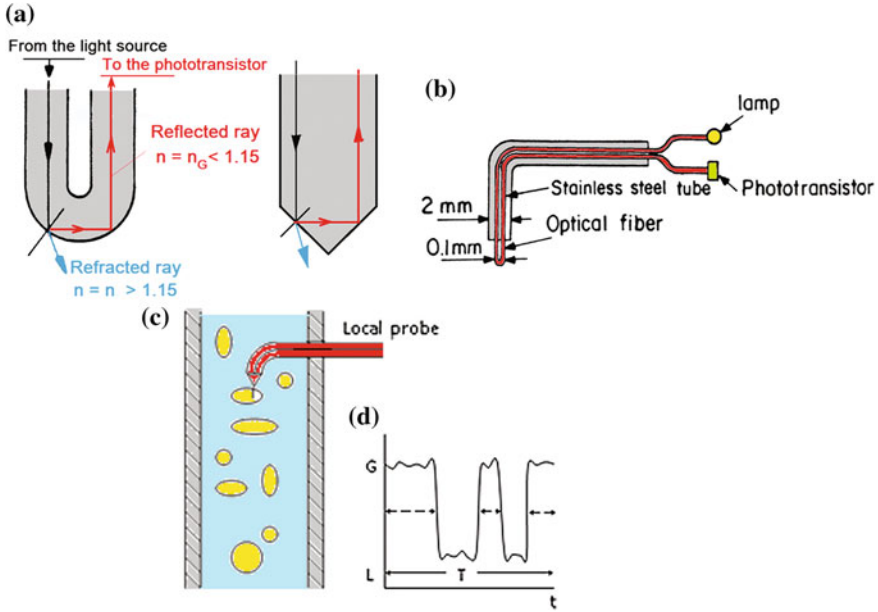
Alternatively, we can define the local statistical or time-average void fraction as the fractional presence time of the gas phase at a given point:

$$\bar{\varepsilon}_G \equiv \frac{T_G}{T} \quad \text{and also} \quad 1 - \bar{\varepsilon}_G \equiv \bar{\varepsilon}_L \equiv \frac{T_L}{T} \quad (1.7.2)$$

where  $T_k$  ( $k = L, G$ ) is the summation of all the times during which phase  $k$  was present at the measuring point, and  $T$  is the total period of observation. The bar over  $\varepsilon$  denotes time or statistical averaging. The overbar will be dropped for simplicity of notation when it is implicit that we are dealing with time-average values. Also, if no subscript is added to  $\varepsilon$ , it is assumed that the meaning is  $\varepsilon_G$  and the liquid local time-averaged fraction becomes  $1 - \varepsilon = \varepsilon_L$ . Clearly,  $\varepsilon_G + \varepsilon_L = 1$ .

The local void fraction can be measured by a miniature resistive probe, a U-shaped fibre-optical sensor or a hot-wire anemometer. All these devices detect the presence of a phase at their sensing tip.

An optical probe is sensitive to the change in the refractive index of the surrounding medium enabling measurements of local void fraction. It can also record interfacial passages and interface passage frequencies. Optical probes can operate in conducting or non-conducting liquids. The tiny optical probe, which is frequently used, Fig. 1.9, consists, e.g. of a single optical fibre, 40  $\mu\text{m}$  in diameter, which may be bent in a U shape. A light shines from a source through one arm of the U bend. This light, according to Snell's law, is either refracted (if the surrounding medium is liquid) or reflected to the other side of the U bend (if the surrounding fluid is gas), Fig. 1.9a. By analysing the signal of the phototransistor, one can infer the *local* void fraction, Fig. 1.9d.



**Fig. 1.9** a Active parts of fibre-optical sensors showing the refraction or reflection of the light. b U-shaped fibre-optical sensor assembly (Danel and Delhaye 1971). c Optical probe positioned inside the channel in bubbly flow. d Probe signal indicating the presence of gas or liquid

### 1.7.2 The Chordal Void Fraction

The *chordal void fraction* is defined as the fraction of a chord or ray occupied at a given instant by gas:

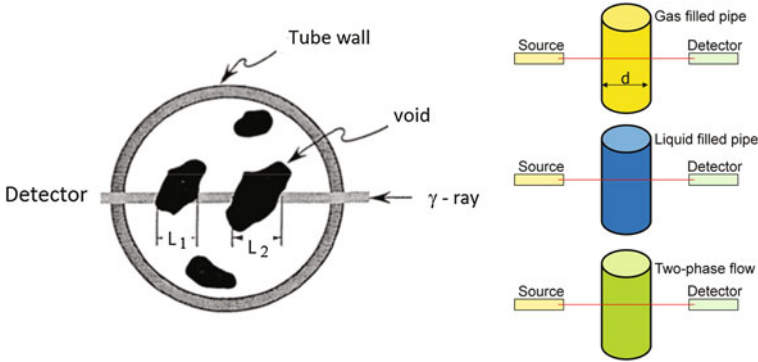
$$\varepsilon_{G1} = \frac{L_G}{L_L + L_G},$$

where  $L_k$ ,  $k = L, G$  is the instantaneous cumulative length on the chord occupied by phase  $k$ . This is best illustrated in Fig. 1.10 where  $L_G = L_1 + L_2$ .

The chordal-average void fraction is typically measured by means of radiation absorption methods. These methods use  $\gamma$  or X-ray beams which are attenuated by the material through which they pass. The intensity after absorption of a collimated beam of initial intensity  $I_0$  (photons/m<sup>2</sup>s) is given by

$$I = I_0 e^{-\mu z},$$

where  $\mu$  is the linear absorption coefficient, which is a property of the absorbing material and the type of radiation and  $z$  is the distance travelled through a



**Fig. 1.10** Chordal void fraction. *Left* The sum of the segments  $L_1$  and  $L_2$  over the diameter define the chordal void fraction. *Right* Calibration method

homogeneous absorbing medium. In applying this technique to the measurement of void fraction, a collimated beam is passed through the channel walls and through the two-phase mixture to a detector. First one measures the intensity of the beam at the detector when the channel is full of liquid,  $I_L$  and then full of gas,  $I_G$ . The void fraction is then related to the intensity  $I$  measured during two-phase flow by

$$\varepsilon_{G1} = \ln\left(\frac{I}{I_L}\right) / \ln\left(\frac{I_G}{I_L}\right).$$

The chordal void fraction can be averaged in time if necessary. An arrangement to measure chordal-average void fractions is depicted in Fig. 1.11.

### 1.7.3 The Cross-Sectional Void Fraction

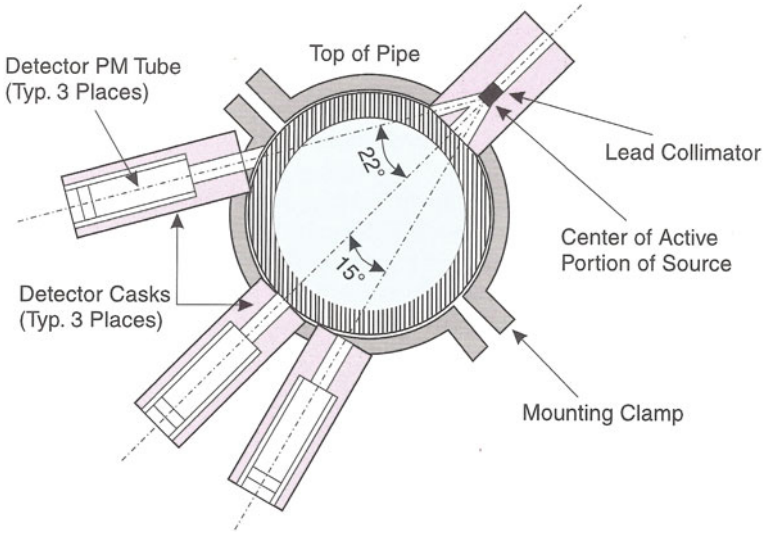
The *cross-sectional-average instantaneous void fraction* is the fraction of the cross section occupied at a given instant by the gas:

$$R_G \equiv \frac{A_G}{A}, \tag{1.7.3}$$

where  $A_G$  is the sum of the areas which are occupied by voids in the cross section (the white areas in Fig. 1.12),  $A_L$  is the sum of the areas which are occupied by the liquid, and  $A$  is the total cross-sectional area. Similarly, for the liquid:

$$R_L \equiv \frac{A_L}{A} = 1 - R_G.$$

In terms of the *phase presence* function  $P_k(\mathbf{r}, t)$ , we can write



**Fig. 1.11** An early realization of a three-beam gamma densitometer measuring three chordal-average void fractions that can be combined to obtain the cross-sectional-average value (Lassahn 1977; redrawn figure courtesy of EPRI)

$$R_k(t) = \frac{1}{A} \int_A P_k(\mathbf{r}, t) dA$$

where we have ignored the theoretical difficulties present in the integration of a discontinuous function. The instantaneous cross-sectional void fraction can be integrated in time to provide a time-averaged value over a time integral [T]

$$\bar{R}_G = \frac{1}{T} \int_{[T]} R_G(t) dt = \frac{1}{T} \int_{[T]} \frac{1}{A} \int_A P_G(\mathbf{r}, t) dA dt.$$

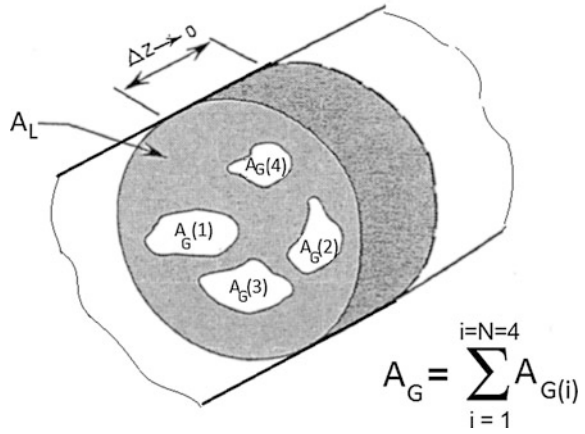
Similarly, we can space average the local statistical void fraction

$$\langle \bar{\varepsilon}_G \rangle = \frac{1}{A} \int_A \bar{\varepsilon}_G(\mathbf{r}) dA = \frac{1}{A} \int_A \frac{1}{T} \int_{[T]} P_G(\mathbf{r}, t) dt dA.$$

We realize that thanks to the *commutativity* of the space ( $\langle \cdot \rangle$ ) and time ( $\bar{\cdot}$ ) averaging operators the two expressions above are identical and we have, indeed for either phase



**Fig. 1.12** The cross-sectional void fraction is obtained by summing all the areas occupied by gas in the cross section



$$\langle \bar{\epsilon}_k \rangle = \bar{R}_k. \tag{1.7.4}$$

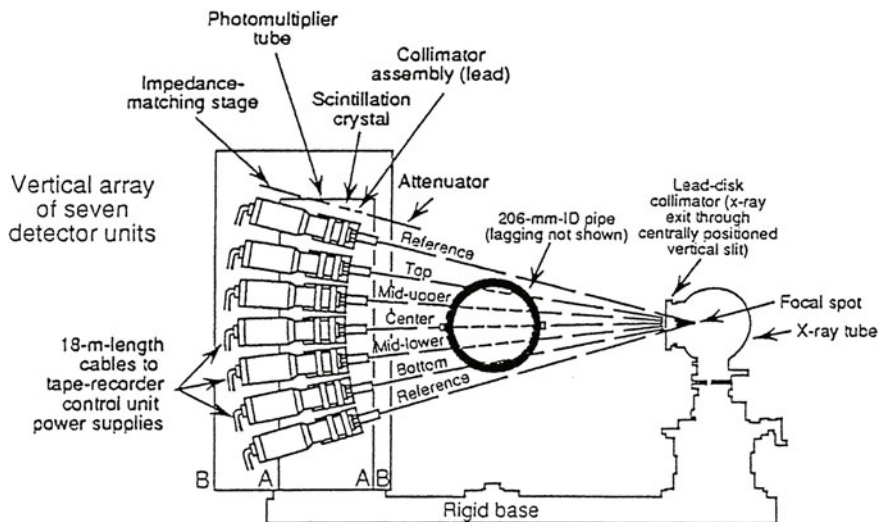
The cross-sectional void fraction can be obtained by integrating chordal-average measurements over the cross section as shown in Fig. 1.11, or by using the “one-shot” technique, or by a neutron scattering technique. Integration of chordal-average measurements can be done either by traversing a collimated beam across the channel (this obviously can be used only for steady-state measurements) or by using a multi-beam arrangement, as illustrated in Fig. 1.13 that shows multi-beam *gamma densitometers* that produce an estimate of the cross-sectional-average void fraction by weighting appropriately the chordal void fractions measured by many beams.

The “one-shot” method uses a broad radiation beam (as wide as the channel) with special collimators to adjust for the different path length and wall absorption.

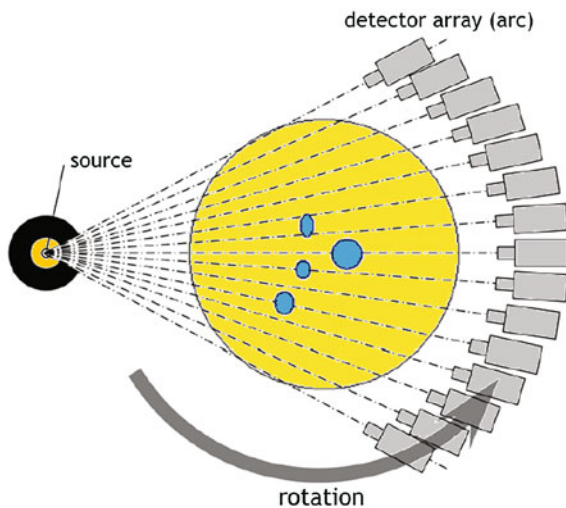
In the remainder of this volume, we will practically always use the cross sectional and time-average void fraction  $\langle \epsilon_G \rangle$  or  $\langle \epsilon \rangle$  (where we dropped the overbar to alleviate the notation).<sup>1</sup>

---

<sup>1</sup>Mathematically, the use of a cross-sectional-average void fraction may lead to some difficulties. Therefore, it should really be understood as a volume-average void fraction defined over a length  $\delta z$ , as  $\delta z \rightarrow 0$ .



Outline A represents rigid detector support plate  
 Outline B represents lead shielding box (cut away for beam entry)



**Fig. 1.13** Multi-beam x-ray systems for determination of multiple chordal-mean void fraction and hence cross-sectional-average void fractions. *Top* One of the first instruments (Smith 1975). *Bottom* Source and an array of rotating detectors can produce tomographic images (courtesy HM Prasser)

### 1.7.4 The Volume-Average Void Fraction

The channel (volume) average void fraction is defined as:

$$\varepsilon_{G3} = \frac{V_L}{V_L + V_G},$$

where  $V_G$  is the volume occupied by the gas and  $V_L$  is the volume occupied by the liquid in a certain volume  $V_G + V_L$  of the channel. The average void fraction over a full length of the channel can be obtained by integrating many local or cross-sectional averages or by the *quick-closing-valve technique*. In this method two valves which can be simultaneously and quickly operated are placed at the two ends of the test section. At the appropriate time, the two valves are actuated and the volume of liquid in the test section is captured. The liquid is left to settle and its volume measured.

### 1.7.5 Averages of Products

In integrating over the flow cross section or in time to obtain average quantities, we often encounter averages of products of variables. Difficulties will arise when such products, e.g. products of void fraction, velocity and enthalpy have to be cross-sectionally averaged.

Time averages

The time average of the product of a phase-related quantity  $f_k$  (such as the phase velocity) by the *local presence function*  $P_k$  can also be written as an integral in  $[T_k]$ , as  $P_k$  is zero outside this interval:

$$\frac{1}{T} \int_{[T]} f_k(\mathbf{r}, t) P_k(\mathbf{r}, t) dt = \frac{1}{T} \int_{[T_k]} f_k(\mathbf{r}, t) dt$$

that can be written as  $\overline{f_k P_k} = \frac{T_k}{T} \overline{f_k}^k$ .

We can verify that when  $f_k = 1$ , this equation produces the identity  $\frac{T_k}{T} = \frac{T_k}{T}$ .

We have already shown that thanks to the *commutativity* of the space and time averaging operators, Eq. (1.7.4), we have

$$\langle \bar{\varepsilon}_k \rangle = \bar{R}_k$$

Space averages

Let us consider now space averages of products of the void fraction and another phase variable  $f_k$ , e.g. velocity—such terms will appear when the momentum conservation equation is averaged in space:

$$\frac{1}{A} \int_A \bar{\varepsilon}_k \bar{f}_k dA.$$

We will derive first a fundamental relation (Delhaye 1981) that will be useful in dealing with such space averages of products. It is going to be a fairly long derivation; we will first show that

$$\overline{\bar{\varepsilon}_k f_k^k} = \overline{P_k f_k}. \quad (1.7.5)$$

The left-hand side, using Eq. (1.7.5), can be written as  $\overline{\bar{\varepsilon}_k f_k^k} = \frac{T_k}{T} \overline{f_k^k}$ . The right-hand side can be expanded using the definition of time averaging, Eq. (1.6.3), and considering the fact that a time average of  $P_k f_k$  over  $[T]$  is the same as a time averaging of  $f_k$  over  $[T_k]$  as

$$\overline{P_k f_k} \equiv \frac{1}{T} \int_{[T]} P_k f_k dt = \frac{1}{T} \int_{[T_k]} f_k dt = \frac{1}{T} T_k \overline{f_k^k}.$$

We have found identical expressions for both sides of Eq. (1.7.5) proving its validity.

We integrate now Eq. (1.7.5) over the cross-sectional area, i.e. we apply the operator  $\langle \cdot \rangle$  and expand the product using the definition of the time and space averaging operators:

$$\langle \overline{\bar{\varepsilon}_k f_k^k} \rangle = \langle \overline{P_k f_k} \rangle \equiv \frac{1}{A} \int_A dA \left( \frac{1}{T} \int_{[T]} P_k f_k dt \right).$$

Inverting the order of space/time integration, this expression becomes

$$\frac{1}{T} \int_{[T]} \left( \frac{1}{A} \int_A P_k f_k dA \right) dt = \frac{1}{T} \int_{[T]} \left( \frac{1}{A} \int_{[A_k]} f_k dA \right) dt = \frac{1}{T} \int_{[T]} \left( \frac{1}{A} A_k \langle \varepsilon_k \rangle_k \right) dt \equiv \overline{R_k \langle f_k \rangle_k}.$$

We have found the *fundamental relationship*:

$$\langle \overline{\bar{\varepsilon}_k f_k^k} \rangle = \overline{R_k \langle f_k \rangle_k} \quad (1.7.6)$$

meaning that the time average of the instantaneous cross-sectional void fraction  $R_k$  times the space average of a phase property is equal to the space average of the local void fraction times the true time average of the phase property. In particular, if  $f_k = 1$ ,

$$\langle \bar{\varepsilon}_k \rangle = \overline{R_k}$$

denoting that the space and time-averaged void fraction can be obtained either by space averaging local time-average values or by time averaging instantaneous space averages, as we have already seen above, Eq. (1.7.4).

We will be dealing usually with time-averaged quantities. If we neglect the difference between the time average of a product and the product of the time averages, i.e. if we write

$$\overline{R_k \langle f_k \rangle} = \overline{R_k} \cdot \overline{\langle f_k \rangle}$$

Equation (1.7.6) becomes

$$\langle \overline{\varepsilon_k} \overline{f_k^k} \rangle = \overline{R_k \langle f_k \rangle} = \overline{R_k} \cdot \overline{\langle f_k \rangle} = \langle \overline{\varepsilon_k} \rangle \overline{\langle f_k \rangle}_k$$

where we used the relation  $\langle \overline{\varepsilon_k} \rangle = \overline{R_k}$ , that is

$$\langle \overline{\varepsilon_k} \overline{f_k^k} \rangle = \langle \overline{\varepsilon_k} \rangle \overline{\langle f_k \rangle}_k. \quad (1.7.7)$$

From now on we will omit time-averaging bars and consider that we are dealing with properly time-averaged properties only. The notation  $\langle \varepsilon_k \rangle$ ,  $k = L, G$  denotes the time and cross-sectionally averaged liquid and gas fractions. Equation (1.7.7) is then simplified as

$$\langle \varepsilon_k f_k \rangle = \langle f_k \rangle_k \langle \varepsilon_k \rangle. \quad (1.7.8)$$

This relationship will allow us to “open” the angle brackets of products of the void fraction with another variable, for example, the cross-sectional-average phase velocity:

$$\langle \varepsilon_k u_k \rangle = \langle u_k \rangle_k \langle \varepsilon_k \rangle. \quad (1.7.9)$$

A similar relationship does *not* exist, however, for products of variables not containing the void fraction, e.g.  $uh$ . In this case, the angle brackets *cannot* be opened,  $\langle fg \rangle \neq \langle f \rangle \langle g \rangle$ . This fact will be again revisited during the derivation of the conservation equations.

Later, during the discussion of the drift-flux model in Chap. 5, we will have to make a distinction between local quantities and cross-sectional averages. For this reason we will maintain the practice of using angle brackets ( $\langle \cdot \rangle$ ) to denote cross-sectional-average quantities, in spite of the fact that it burdens considerably the notations.

## 1.8 Phase Flow Rates and Flow Quality

The *flow quality* is defined as the ratio of the *gas mass-flowrate*  $\dot{M}_G$  to the *total mass flowrate* (in kg/s):

$$x \equiv \frac{\dot{M}_G}{\dot{M}}, \quad \dot{M} = \dot{M}_L + \dot{M}_G. \quad (1.8.1)$$

This *mass flow rate-based* definition is different from the common thermodynamic definition of quality, usually also denoted by the symbol  $x$ , which is the ratio of steam *mass* to total mass. To be fully consistent and clear we should have used the symbol  $\dot{x}$  instead of simply  $x$  for the flow quality. As we will practically never use the mass-based definition, we are not going to do this to simplify the notation.

The flow rates of the phase  $k$  were denoted as  $\dot{M}_k$  (kg/s). The corresponding volumetric flow rates are  $\dot{Q}_k = \dot{M}_k / \rho_k$  ( $\text{m}^3/\text{s}$ ) where  $\rho_k$  is the cross-sectional-average density of phase  $k$ . The phase *mass fluxes* are the phase flow rates per unit flow area:

$$\dot{m} = \frac{\dot{M}}{A}, \quad \dot{m}_k = \frac{\dot{M}_k}{A}.$$

Similarly, we can define the *cross-sectional-average* volumetric fluxes; departing, however, from the rule of using the same lower-case symbol for the fluxes, we will follow the usual practice and denote these by  $\langle j \rangle$  (rather than  $\dot{q}$ ):

$$\langle j_k \rangle = \frac{\dot{Q}_k}{A} = \frac{\dot{m}_k}{\rho_k}, \quad \langle j_L \rangle = \frac{\dot{Q}_L}{A} = \frac{\dot{m}(1-x)}{\rho_L}, \quad \langle j_G \rangle = \frac{\dot{Q}_G}{A} = \frac{\dot{m}x}{\rho_G}. \quad (1.8.2)$$

Their sum is the *total volumetric flux*  $\langle j \rangle$ :

$$\langle j \rangle = \langle j_L \rangle + \langle j_G \rangle = \dot{m} \left( \frac{x}{\rho_G} + \frac{1-x}{\rho_L} \right). \quad (1.8.3)$$

The volumetric fluxes are cross-sectional-average quantities. They have units of velocity, ( $\text{m}^3/\text{s}$  per  $\text{m}^2$  or  $\text{m/s}$ ). For this reason, the channel-average volumetric fluxes are also called *superficial phase velocities*  $U_{sG}$  and  $U_{sL}$ :

$$\langle j_k \rangle = \frac{\dot{Q}_k}{A} \equiv U_{sk}. \quad (1.8.4)$$

The German term *Leerrohrgeschwindigkeit*, meaning “velocity in the empty pipe” explains well the situation: the superficial phase velocities are the ones that the phases would have had *if* they were flowing *alone* in the pipe.

We used  $\langle j_k \rangle$  to denote the *cross-sectional-average* value. The distinction between  $j_k$ , a *local* value and its cross-sectional average  $\langle j_k \rangle$  will become necessary later.<sup>2</sup>

### 1.8.1 Determination of the Flow Quality

In flows with no phase change—typically non-miscible, two-component flows such as air–water—we usually have ways of knowing the flow rates of the two phases, and the quality can be obtained from its basic definition, Eq. (1.8.1). For example, if the two phases are injected and mixed in the channel, they can be metered before injection. In this case the quality does not change along the channel.

When there is phase change in the channel, e.g. if the channel is heated and boiling takes place, then the quality increases along the channel. The inverse is true for condensation, the quality decreases along the channel in this case. In two-component flows, the quality can change also, if for example, the gas is getting dissolved in the liquid or the liquid evaporates into the gas.

More generally, in the case of multiple-component, same-phase flows, such as oil–water we may speak of *flowing mass fraction*. For example:

$$x_{oil} = \frac{\dot{M}_{oil}}{\dot{M}_{oil} + \dot{M}_{water}}, \quad x_{water} = \frac{\dot{M}_{water}}{\dot{M}_{oil} + \dot{M}_{water}}.$$

Petroleum engineers call  $x_{water}$  the “water cut” (the ratio of water produced to the volume of total liquids from an oil well).

For *single-component flows with phase change*, the determination of the quality is more complex. First, we need to calculate the enthalpy of the flowing mixture. At *steady state*, the enthalpy distribution along the channel  $h(z)$  can be determined using the enthalpy balance for the channel that will be discussed in Chap. 2 as:

$$h(z) = h_{in} + \frac{1}{\dot{M}} \int_0^z q'(z) dz \quad (1.8.5)$$

where  $h_{in}$  is the enthalpy at the inlet,  $\dot{M}$  the mass flow rate and  $q'$  the total “linear” energy input into the fluid per unit channel length. The total linear heat input term  $q'$  (W/m) may contain in addition to the heat flux from the wall  $q''$  (W/m<sup>2</sup>) other sources of “volumetric” heating of the fluid such as absorption of radiation or heat from chemical reactions that we denote by  $q'''$  (W/m<sup>3</sup>). The *total linear heat input* from all sources is given as:

---

<sup>2</sup>In Chap. 5, we will introduce the *local* volumetric fluxes.

$$q' = q''P_h + q'''A$$

where  $P_h$  is the heated perimeter of the channel and  $A$  its cross-sectional area. It is easy to understand the meaning of this equation, if its terms are multiplied by an elementary channel length  $dz$ :  $P_h \cdot dz$  is the heat transfer area for the heat flux  $q''$  and  $Adz$  is volume where the volumetric heating  $q'''$  takes place.

Thermodynamically, the enthalpy of a two-phase, single-component flowing mixture, e.g. steam-water flow in a steam generator, is given as a mass flow rate weighted quantity as we will see later:

$$h = xh_G + (1 - x)h_L \quad (1.8.6)$$

where, in simplified notation without the angle brackets,  $h_L$  and  $h_G$  are the (cross-sectional average) enthalpies of the two phases,  $\langle h_k \rangle_k$ . In general, the (*actual*) local quality of the mixture will be given from Eq. (1.8.6) as

$$x = \frac{h - h_L}{h_G - h_L}. \quad (1.8.7)$$

Since often there is no thermal equilibrium between the phases (this will be further discussed elsewhere), one cannot calculate easily the phase enthalpies and then the quality merely by knowing the local enthalpy from Eq. (1.8.5) and getting the quality from Eq. (1.8.6); we need to know the values of the average local phase enthalpies  $h_L$  and  $h_G$  and these may have evolved along the channel in complex ways. Consequently, it is difficult to calculate or even measure with high precision the phase enthalpies and the quality of the liquid–vapour mixture flowing in a channel where phase change takes place.

A very simple “asymptotic” or limiting case is when both phases are saturated, i.e. if they have both the thermodynamically given *saturation* enthalpy  $h_{sat}$  corresponding to the local pressure in the channel  $p(z)$

$$h_k(z) = h_{sat,k}(p(z)), \quad k = L, G.$$

The  $h_{sat,k}$  are the enthalpies we get from the so-called *Steam Tables* (e.g. Grigull et al. 2012) or more generally from fluid-property tables (Lemmon et al. 2010) for fluids other than water.

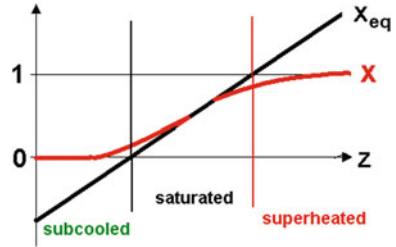
Notwithstanding the state of the fluid regarding thermal equilibrium, a *fictitious* quality, the so-called *thermal-equilibrium quality* can be calculated by *assuming* that both phases are saturated, i.e. that their temperatures are equal to the saturation temperature corresponding to their local pressure. The thermal-equilibrium quality is:

$$x_{eq} = \frac{h(z) - h_{L,sat}}{h_{LG,sat}}, \quad h_{LG,sat} \equiv h_{G,sat} - h_{L,sat}$$

where  $h_{LG,sat}$  is the latent heat of vaporization.



**Fig. 1.14** Variation of the actual quality  $x$  and of the equilibrium quality  $x_{eq}$  along the channel. The equilibrium quality varies linearly in case of a uniform heat input along the channel



The thermal-equilibrium quality is the asymptotic, limiting value that a flowing mixture may reach in a well-mixed flow. In many situations, e.g. in the long tubes of steam generators, it may be a good approximation to the actual quality.

The notion of the thermal-equilibrium quality is a very useful one as it characterizes the properties of the flowing mixture, even when the latter is not in equilibrium. It is widely used as a characteristic of the flow, because it is easily available and it determines the equivalent state of the fluid that would have existed in the presence of thermal equilibrium. Often the authors neglect even to mention that they are referring to the thermal-equilibrium quality in their formulations; they simply speak of “quality”. Note that, as defined above,  $x_{eq}$  can be negative or greater than one. A negative value of  $x_{eq}$  denotes that the mixture would have been subcooled liquid, while  $x_{eq} > 1$  means that it would have been superheated vapour, Fig. 1.14.

### 1.9 Velocities, the Triangular Relationship and Other Useful Relations

There are several velocities which one can define, and we must be clear about what the definitions mean. In general, the two phases will not have the same velocity and there will be a relative velocity between them, as discussed below.

The *true instantaneous, local* velocities of the phases  $u_G$  and  $u_L$  are the velocities by which the phases actually travel at a given instant and at a given point. Their *phase cross-sectional averages* can be obtained as:

$$\langle u_k \rangle_k = \frac{\dot{M}_k}{\rho_k A_k} = \frac{\dot{Q}_k}{A_k} = \frac{A}{A_k} \langle j_k \rangle \tag{1.9.1}$$

where  $\dot{M}_k$  is the mass flow rate of phase k,  $\rho_k$  its density and  $\dot{Q}_k = \dot{M}_k / \rho_k$  the volumetric flow rate.

The cross-sectional-average velocities calculated *over the entire flow area* (as if the phase were flowing alone in the entire flow area) already defined by Eq. (1.8.2) are also called *superficial velocities* (subscript s)

$$U_{sk} = \frac{\dot{M}_k}{\rho_k A} = \frac{\dot{Q}_k}{A} \equiv \langle j_k \rangle \quad (1.9.2)$$

These are also the *volumetric fluxes* that we defined earlier; this second name represents better the physical situation and is easily understood by looking at the last equality in Eq. (1.9.2). The sum of the volumetric fluxes is the *total volumetric flux*  $\langle j \rangle$ :

$$\langle j \rangle = \langle j_L \rangle + \langle j_G \rangle \text{ or } U_s = U_{sL} + U_{sG}$$

The total volumetric flux can also be called the *velocity of the centre of volume*; we will discuss this further in the following section.

Using Eqs. (1.9.1) and (1.9.2), one obtains the relationship between the true cross-sectional-average velocity and the superficial velocity of each phase:

$$\langle u_G \rangle_G = \frac{U_{sG}}{\langle \varepsilon_G \rangle} = \frac{\langle j_G \rangle}{\langle \varepsilon_G \rangle}, \quad \langle u_L \rangle_L = \frac{U_{sL}}{\langle \varepsilon_L \rangle} = \frac{\langle j_L \rangle}{\langle \varepsilon_L \rangle} \quad (1.9.3)$$

The *velocity ratio*  $S$  is the ratio of the cross-sectional average of the true velocities of the phases, i.e.

$$S \equiv \frac{\langle u_G \rangle_G}{\langle u_L \rangle_L}$$

This useful parameter is often misleadingly called the *slip ratio*. Clearly, however, it is not a “slip”, i.e. a velocity difference. Using Eq. (1.9.2),

$$S = \frac{\dot{Q}_G / A_G}{\dot{Q}_L / A_L} = \frac{\dot{Q}_G A_L}{\dot{Q}_L A_G} = \frac{\dot{Q}_G \langle \varepsilon_L \rangle}{\dot{Q}_L \langle \varepsilon_G \rangle} = \frac{\dot{Q}_G \langle 1 - \varepsilon_G \rangle}{\dot{Q}_L \langle \varepsilon_G \rangle} = \frac{\langle j_G \rangle \langle \varepsilon_L \rangle}{\langle j_L \rangle \langle \varepsilon_G \rangle} \quad (1.9.4)$$

Solving for the void and liquid fractions,

$$\langle \varepsilon_G \rangle = \frac{\langle j_G \rangle}{S \langle j_L \rangle + \langle j_G \rangle}, \quad \langle \varepsilon_L \rangle \equiv \langle 1 - \varepsilon_G \rangle = \frac{S \langle j_L \rangle}{S \langle j_L \rangle + \langle j_G \rangle} \quad (1.9.5)$$

To express the velocity ratio in terms of the quality, we recall that

$$\dot{M}_G = x \dot{M} = \rho_G \dot{Q}_G \quad \text{and} \quad \dot{M}_L = (1 - x) \dot{M} = \rho_L \dot{Q}_L$$

and substituting into Eq. (1.9.4) we readily obtain the *triangular relationship*:

$$S \equiv \frac{\langle u_G \rangle_G}{\langle u_L \rangle_L} = \frac{\rho_L}{\rho_G} \frac{x}{1 - x} \frac{\langle 1 - \varepsilon_G \rangle}{\langle \varepsilon_G \rangle} \quad (1.9.6)$$

As the phase densities are normally known, this relationship links the three variables,  $S$ ,  $x$  and  $\langle \varepsilon_G \rangle$ . Usually, we know or we can calculate the quality  $x$ . However, we need some additional information to get the void fraction such as  $S$  or a correlation for the void fraction in terms of  $x$ . We will spend later the entire Chap. 5 on methods to estimate the void fraction.

Solving Eq. (1.9.6) for the void and liquid fractions in terms of quality and velocity ratio:

$$\langle \varepsilon_G \rangle = \frac{\rho_G x}{S \rho_G (1-x) + \rho_L x}, \quad \langle 1 - \varepsilon_G \rangle = \frac{S \rho_G (1-x)}{S \rho_G (1-x) + \rho_L x} \quad (1.9.7)$$

Another two useful relationships between the quality and the volumetric fluxes can be obtained starting from the definition of the quality  $x$  and of  $1-x$  and expanding these

$$x \equiv \frac{\dot{M}_G}{\dot{M}} = \frac{A \langle \varepsilon_G \rangle \rho_G \langle u_G \rangle_G}{\dot{M}} = \frac{A \langle j_G \rangle \rho_G}{\dot{M}}$$

$$1-x \equiv \frac{\dot{M}_L}{\dot{M}} = \frac{A \langle \varepsilon_L \rangle \rho_L \langle u_L \rangle_L}{\dot{M}} = \frac{A \langle j_L \rangle \rho_L}{\dot{M}}$$

Dividing these expressions, member by member, we find the useful relationships,

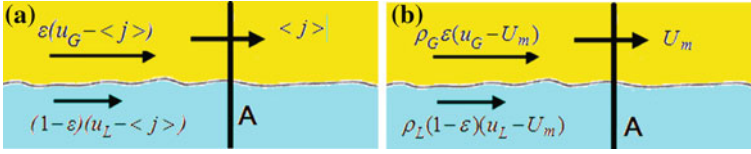
$$\frac{1-x}{x} = \frac{\langle \varepsilon_L \rangle \rho_L \langle u_L \rangle_L}{\langle \varepsilon_G \rangle \rho_G \langle u_G \rangle_G} = \frac{\langle j_L \rangle \rho_L}{\langle j_G \rangle \rho_G} \quad \text{or} \quad \frac{\langle j_L \rangle}{\langle j_G \rangle} = \frac{\rho_G (1-x)}{\rho_L x} \quad (1.9.8)$$

that can be solved for  $x$ :

$$x = \frac{\langle j_L \rangle \rho_G}{\langle j_L \rangle \rho_L + \langle j_G \rangle \rho_G}. \quad (1.9.9)$$

### 1.9.1 Velocities of the Centre of Volume and Mass

Figure 1.15a shows a two-phase flow past a plane having an area  $A$  and moving with the velocity  $\langle j \rangle$ . Let us assume that the flow is stratified as shown in the figure. Assuming that the gas flows faster, a volume of gas equal to  $\langle \varepsilon \rangle (\langle u_G \rangle_G - \langle j \rangle) A$  will cross the plane in the positive  $z$  direction, while a volume of liquid equal to  $\langle 1 - \varepsilon \rangle (\langle j_L \rangle - \langle j \rangle) A$  will cross it in the opposite direction (we use  $\varepsilon = \varepsilon_G$ ). Summing up the two:



**Fig. 1.15** Flows of volume and mass crossing plane A: **a** moving with velocity  $\langle j \rangle$ , **b** with the velocity  $U_m$

$$\begin{aligned} \langle \varepsilon \rangle (\langle u_G \rangle_G - \langle j \rangle) A + \langle 1 - \varepsilon \rangle (\langle u_L \rangle_L - \langle j \rangle) A \\ A (\langle j_G \rangle + \langle j_L \rangle) - \langle j \rangle (\langle \varepsilon \rangle - \langle 1 - \varepsilon \rangle) = A (\langle j \rangle - \langle j \rangle) = 0 \end{aligned}$$

proving that no net volume will cross this plane (Yadigaroglu and Lahey 1976). In a somewhat more sophisticated way we can show the same thing for the general case, starting from the local quantities shown in Fig. 1.15a; we obviously obtain the same result:

$$\begin{aligned} \int_A [\varepsilon(u_G - \langle j \rangle) + (1 - \varepsilon)(u_L - \langle j \rangle)] dA \\ = \langle \varepsilon u_G \rangle - \langle \varepsilon \langle j \rangle \rangle + \langle (1 - \varepsilon) u_L \rangle - \langle (1 - \varepsilon) \langle j \rangle \rangle \\ = \langle \varepsilon u_G \rangle + \langle (1 - \varepsilon) u_L \rangle - \langle \varepsilon \rangle \langle j \rangle - \langle 1 - \varepsilon \rangle \langle j \rangle \\ = \langle \varepsilon \rangle \langle u_G \rangle_G + \langle 1 - \varepsilon \rangle \langle u_L \rangle_L - \langle j \rangle = \langle j_G \rangle + \langle j_L \rangle - \langle j \rangle = 0. \end{aligned}$$

A second useful velocity is the *velocity of the centre of mass* defined as:

$$U_m \equiv \frac{\dot{m}}{\langle \rho \rangle} = \frac{\rho_L \langle u_L \rangle_L \langle \varepsilon_L \rangle + \rho_G \langle u_G \rangle_G \langle \varepsilon_G \rangle}{\rho_L \langle \varepsilon_L \rangle + \rho_G \langle \varepsilon_G \rangle}.$$

This one is the velocity of a plane traversed by zero net mass flux as:

$$\begin{aligned} \int_A [\rho_G \varepsilon (u_G - U_m) + \rho_L (1 - \varepsilon) (u_L - U_m)] dA \\ = \langle \rho_G \varepsilon u_G \rangle - \langle \rho_G \varepsilon U_m \rangle + \langle \rho_L (1 - \varepsilon) u_L \rangle - \langle \rho_L (1 - \varepsilon) U_m \rangle \\ = \rho_G \langle \varepsilon \rangle \langle u_G \rangle_G + \rho_L \langle 1 - \varepsilon \rangle \langle u_L \rangle_L - \rho_G \langle \varepsilon \rangle U_m - \rho_L \langle 1 - \varepsilon \rangle U_m \\ = \dot{m} - \langle \rho \rangle U_m = 0 \end{aligned}$$

by the definition of  $U_m$ .

## 1.9.2 Homogeneous Flow

Flows with equal phase velocities, i.e. if the velocity ratio  $S = 1$ , are called *homogeneous* flows. Our usage of the term does not imply any other “homogeneity” in the flow (e.g. well mixed). For *homogeneous flow*, setting  $S = 1$  in Eq. (1.9.7) we obtain the *homogeneous* void fraction, often denoted by  $\beta$ :

$$\langle \varepsilon_G \rangle_{\text{hom}} \equiv \beta = \frac{\rho_L x}{\rho_G(1-x) + \rho_L x} \quad (1.9.10)$$

and in terms of volumetric fluxes, Eqs. (1.9.5) yield

$$\beta = \frac{\langle j_G \rangle}{\langle j_L \rangle + \langle j_G \rangle}, \quad 1 - \beta = \frac{\langle j_L \rangle}{\langle j_L \rangle + \langle j_G \rangle} \quad (1.9.11)$$

Comparing Eqs. (1.9.7) and (1.9.10), one can immediately see that, as in most cases  $S > 1, \langle \varepsilon_G \rangle < \beta$ . Indeed, as the gas flows faster, it needs less cross-sectional flow area.

Another useful relationship is obtained for homogeneous flow from Eq. (1.9.3):

$$\langle u_G \rangle_G = \frac{\langle j_G \rangle}{\langle \varepsilon_G \rangle} = \langle u_L \rangle_L = \frac{\langle j_L \rangle}{\langle \varepsilon_L \rangle} \quad \text{or} \quad \frac{\langle j_G \rangle}{\langle \varepsilon_G \rangle} = \frac{\langle j_L \rangle}{\langle \varepsilon_L \rangle} \quad (1.9.12)$$

In the case of homogeneous flow, all mixture and phase velocities become equal:

$$\langle u_G \rangle_G = \langle u_L \rangle_L = \langle j \rangle = U_m = \frac{\dot{M}}{\langle \rho \rangle A}.$$

The equilibrium quality was considered a limiting or asymptotic situation for equal phase temperatures and used as an *index* characterizing the flow. Similarly, the homogeneous void fraction can also be considered as the limiting case of equal phase velocities and used as a useful two-phase flow index.

## 1.10 A Few Useful Non-dimensional Numbers

A few non-dimensional numbers or groups of variables that are very often used will be introduced in this section, in anticipation of their actual use in the following chapters.

The density ratio

One of the most commonly used non-dimensional parameter is the density ratio  $\rho_L/\rho_G$  that appears naturally on many occasions. It provides also a useful way of considering the effect of system pressure as, e.g. in boiling flows the ratio of saturated densities is uniquely linked to the pressure.

### The Martinelli X

Martinelli and co-workers suggested during the mid 40s (Martinelli et al. 1944, Lockhart and Martinelli 1949) a *separated flow* model for the frictional pressure drop, i.e. a model for flow with two distinct phase velocities (that will be discussed in Chap. 6). In the formulation of their model, they arrived at the parameter

$$X^2 = \frac{(dp/dz)_{frLP}}{(dp/dz)_{frGP}} \quad (1.10.1)$$

where  $(dp/dz)_{frLP}$  is the frictional pressure gradient of the liquid as if it were flowing *alone* in the pipe at the liquid flow rate, i.e. the frictional pressure gradient based on the superficial velocity of the liquid. The subscripts LP are used to denote this “flowing alone” condition.  $(dp/dz)_{frGP}$  is the corresponding gradient for the gas.

Note that the computation of the two frictional pressure gradients should be based on a Reynolds number and a friction factor consistent with the  $(dp/dz)_{frkP}$  assumptions, i.e. using the volumetric flux as the velocity, computing first

$$\begin{aligned} \text{Re}_{LP} &= \frac{\rho_L \langle j_L \rangle D}{\mu_L} = \frac{\langle j_L \rangle D}{v_L} = \frac{\dot{m} (1-x) D}{\mu_L} \\ \text{Re}_{GP} &= \frac{\rho_G \langle j_G \rangle D}{\mu_G} = \frac{\langle j_G \rangle D}{v_G} = \frac{\dot{m} x D}{\mu_G} \end{aligned} \quad (1.10.2)$$

and then the corresponding (Fanning) friction factors  $f_{kP}$ , according to whether the flow is laminar or turbulent, using the appropriate correlations, for example:

for laminar flow,  $f_{kP} = \frac{16}{\text{Re}_{kP}}$ ,  $k = L, G$

and for turbulent flow,  $f_{kP} = \frac{C}{\text{Re}_{kP}^m}$

where C and m are empirical constants. The pressure gradients are then given as

$$-\left[ \frac{dp}{dz} \right]_{frkP} = \frac{4f_{kP}}{D} \frac{\dot{m}^2 (x_k)^2}{2\rho_k} = \frac{4f_{kP}}{D} \frac{\rho_k \langle j_k \rangle^2}{2}.$$

The Martinelli parameter can be computed in terms of known quantities. For the case of turbulent flow in both phases ( $X$  with subscript  $tt$ ), and with the exponent of the friction factor correlation  $m = 0.2$ , it takes the simple form

$$X_{tt} = \left[ \frac{1-x}{x} \right]^{0.9} \left[ \frac{\rho_G}{\rho_L} \right]^{0.5} \left[ \frac{\mu_L}{\mu_G} \right]^{0.1} \quad (1.10.3)$$

This parameter involves the ratios of the main quantities determining the pressure gradient to the appropriate powers and is a quite useful reference parameter introduced in various correlations for void fraction, and pressure drop or even

boiling heat transfer.  $X^2$  is also a measure of the degree to which the two-phase mixture is closer to liquid, i.e.  $X^2 \gg 1$ , or to gas, i.e.  $X^2 \ll 1$ .

It is useful to express  $X_H$  in terms of volumetric fluxes; we use Eq. (1.9.8) to obtain

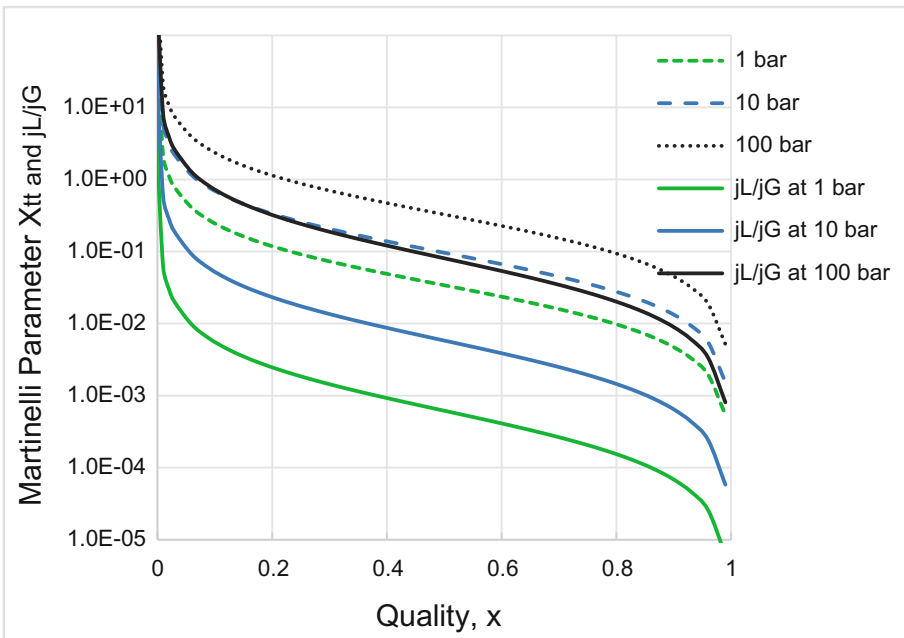
$$X_H = \left[ \frac{\langle j_L \rangle}{\langle j_G \rangle} \right]^{0.9} \left[ \frac{\rho_L}{\rho_G} \right]^{0.4} \left[ \frac{\mu_L}{\mu_G} \right]^{0.1} \tag{1.10.4}$$

The variation of  $X_H$  with  $x$  (for constant properties), together with the ratio of  $\langle j_G \rangle / \langle j_L \rangle$ , Eq. (1.9.8) vs  $x$ , are plotted in Fig. 1.16 to give the reader a feeling for these dependencies.

Figure 1.17 is a plot of the Martinelli parameter  $X_H$  and of the ratio of volumetric fluxes  $\langle j_L \rangle / \langle j_G \rangle$  versus quality of saturated steam/water flows. The purpose of the plot is to show that both vary widely with the quality. Even within the “more reasonable” quality range (0.05–0.95), the variations are of two to three orders of magnitude, mainly due to the wide variation of the density with pressure, as shown in Fig. 1.17 for saturated water/steam and air/water systems.

The Laplace wavelength

As we will see in Chap. 4 on interfacial instabilities, a non-dimensional length characterizing the balance between surface tension and gravity forces is the so-called Laplace length of wavelength:



**Fig. 1.16** Variation of the Martinelli parameter  $X_H$  and of the ratio of volumetric fluxes  $j_L/j_G$  with quality; saturated steam/water flow at three different pressures

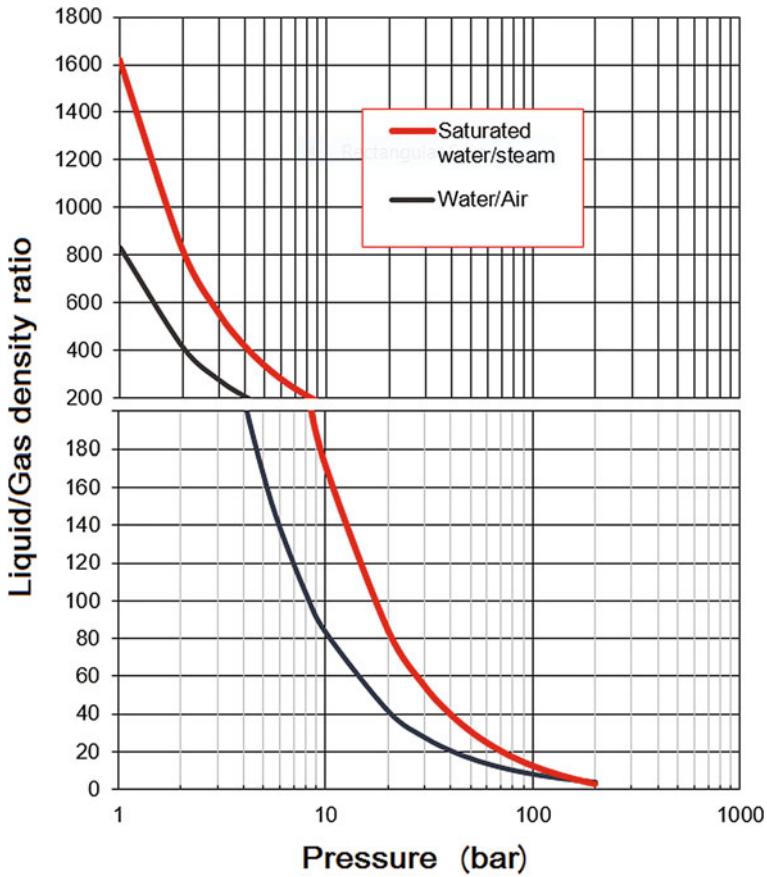


Fig. 1.17 Variation of the liquid/gas density ratio for water/air and saturated water/steam

$$L \equiv \sqrt{\frac{\sigma}{g(\rho_L - \rho_G)}}$$

Non-dimensional volumetric fluxes

Wallis (1969) introduced the non-dimensional volumetric fluxes that find many uses as

$$J_k^* \equiv \frac{\langle j_k \rangle}{\sqrt{\frac{gD(\rho_L - \rho_G)}{\rho_k}}}, \quad k = L, G$$

where  $D$  is a characteristic system dimension, usually the diameter. There are cases, however, where a characteristic system dimension is not available, and the



phenomena may depend on some intrinsic length scale such as the Laplace length. For such cases we would rather use the Kutateladze numbers<sup>3</sup>:

$$K_k \equiv \frac{\langle j_k \rangle}{\sqrt{4\sigma g(\rho_L - \rho_G)} \rho_k}, \quad k = L, G$$

where the Wallis diameter  $D$  was replaced by the Laplace length  $L$ .

## 1.11 System of Units

The internationally accepted SI system of units will be used, unless otherwise necessary. This is a self-consistent set of units, which makes it easy to check equations and eliminate ambiguities such as kg-mass or kg-force.

The basic units are: length: m (meter), mass: kg (kilogram), time: s (second), temperature: °C or K (by convention, degree Kelvin is written as K, not °K).

Other units are derived from these, such as force: N (Newton = kg m s<sup>-2</sup>); pressure: Pa (Pascal = N m<sup>-2</sup>). Unfortunately, Pa is a very small unit and MPa (= 10<sup>6</sup> Pa) or bar (= 10<sup>5</sup> Pa) are frequently used instead; energy: J (Joule = Nm = kg m<sup>2</sup> s<sup>-2</sup>); power: W (Watt = J s<sup>-1</sup>); thermal conductivity (W m<sup>-1</sup> °C<sup>-1</sup> = J s<sup>-1</sup> m<sup>-1</sup> °C<sup>-1</sup>), etc.

A large number of older texts and papers on multiphase flows were written using the British system of units. Appendix II gives the most often needed but more difficult to find conversion factors for the heat-transfer and fluid-mechanics areas.

Appendix III lists the general nomenclature used in this volume. Additional nomenclature is defined locally, as needed.

## 1.12 Sources of Information

There are several textbooks that have been written on the subject of two- or multiphase flows and heat transfer with phase change and several journals that publish regularly on these topics.

Two of the first books on two-phase flow are the classical Tong (1965) and the excellent book by Wallis (1969). A second edition by Tong and Tang (1997) has been published more recently. The monograph by Wallis (1969) has not “aged” and remains a very valuable source of information.

The early first edition by Collier and its third edition by Collier and Thome (1994) provide the basic information on boiling phenomena. The more recent book by Carey (1992) contains a wealth of information on phase change phenomena.

---

<sup>3</sup>There are other Kutateladze numbers in addition to the ones given here.

Ishii and Hibiki (2011) provide detailed discussions and derivations of the fundamental conservation equations and various models used for two-phase flows. Noticeable in the German speaking area are the books by Mayinger (1982) and Stephan (1988). Handbooks on multiphase systems have been published by Hetsroni (1982), Kandlikar et al. (1999), and Crowe (2005).

More specialized textbooks are those of Hewitt (2013) on annular flows, Levy (1999), Kleinstreuer (2003), Ghiaassiaan (2008) on “miniature systems”, Govier and Aziz (1972) on “complex mixtures in pipes” (petroleum engineering flows). Lahey and Moody (1977, 1993) and Tong and Weisman (1979) published books on Boiling and Pressurized Water Reactors, respectively, that contain a lot of two-phase flow related material. Kolev (2012a,b,c, 2015a,b) has published five volumes containing a lot of detailed information on modelling and computing two-phase flows. Computational methods for multiphase flows are presented in the books by Prosperetti and Tryggvason (Prosperetti and Tryggvason 2007) and by Tryggvason et al. (2011).

A monograph specialized on flow regimes is that of Cheng et al. (2008); Yarin et al. (2009) wrote on two-phase flows and boiling in microchannels. Finally, there are notable handbooks or specialized monographs sponsored by companies, such as those of Wolverine Tube, Inc. (2014–2010) and the two monographs by Chexal et al. (1997, 1999). The latter contain most recent models and methods for computing pressure drop and void fraction in two-phase flows as well as the computer tools for their implementation.

The interested reader will find some of the early seminal work in the proceedings of workshops that were organized and attracted prominent researchers. The proceedings of such a meeting that took place in Istanbul were published by Kakac and Mayinger (1977). The book by Delhay et al. (1980) contains a series of chapters on several topics of industrial interest. One could also mention here three Workshops organized by the US Department of Energy (DOE) (jointly with the US Electric Power Institute, EPRI, the third time) on “Two-Phase Flow Fundamentals” at the National Bureau of Standards, Gaithersburg, Maryland (1985), at the Rensselaer Polytechnic Institute, Troy, New York (1987), and for the last time at Imperial College in London (DOE/EPRI 1992). The objective of the first workshop was to identify, formulate and prioritize research problems in the field of multiphase flow and heat transfer. A selection of “numerical benchmark sets” and “physical benchmark exercises” was also chosen. These were then performed by various international participants who compared their findings at the second workshop. Presentations on the two-phase flow models and on interfacial relationships, and the selected Data Sets and Numerical Benchmark Tests are included in Volume 3 of *Multiphase Science and Technology* (Hewitt et al. 1987). There is also a report on the first meeting by Kestin and Podowski (1985). The results of the second workshop were published in Vols. 5, 6, and 8 of *Multiphase Science and Technology*.

Most of the articles on boiling heat transfer and two-phase flow can be found in the following journals:

International Journal of Multiphase Flow, IJMPF  
 International Journal of Heat and Mass Transfer, IJHMT  
 Journal of Heat Transfer, Transactions ASME, JHT  
 American Inst. of Chemical Engineers Journal, AIChE Journal  
 Chemical Engineering Progress, Symposium Series  
 Multiphase Science and Technology (reviews), etc.

In addition, applications to nuclear systems are found mostly in:

Nuclear Science and Engineering  
 Nuclear Technology  
 Nuclear Engineering and Design  
 Annals of Nuclear Energy, etc

Petroleum-industry related papers can be found in:

Petroleum Science and Technology  
 Journal of Petroleum Technology  
 Oil and Gas Journal, etc

## References

- Banerjee S, Hussein E, Meneley DA (1978) Simulation of a neutron scattering method for measuring void fraction in two-phase flow. *Nucl Eng Des* 53:393–405
- Brown NP, Heywood NI (1991) Slurry handling design of solid-liquid systems. Elsevier Applied Science, London and New York
- Carey VP (1992) Liquid-vapor phase-change phenomena. Hemisphere Publ. Corp, Washington
- Cheng L, Ribatski G, Thome JR (2008) Two-phase flow patterns and flow-pattern maps: fundamentals and applications. *Appl Mech Rev* 61(5). doi:050802
- Chehal B et al (1997) Void fraction technology for design and analysis. EPRI—Siemens. <http://www.epri.com/abstracts/Pages/ProductAbstract.aspx?ProductId=TR-106326>
- Chehal B et al (1999) Pressure drop technology for design and analysis. EPRI—Siemens. <http://www.epri.com/abstracts/Pages/ProductAbstract.aspx?ProductId=TR-113189>
- Collier JG, Thome JR (1994) Convective boiling and condensation, 3rd edn. Oxford University Press
- Crowe CT (2005) Multiphase flow handbook. Taylor & Francis
- Danel F, Delhay JM (1971) Optical probe for measuring the local voidage in two-phase flow systems. *Mes Regul Automat* 36(8–9):99–101
- Delhay JM (1981) Local instantaneous equations, local time-averaged equations, and composite-averaged equations. In: Delhay JM, Giot M, Riethmuller ML (ed) *Thermohydraulics of two-phase systems for industrial design and nuclear engineering*. Hemisphere, Washington
- Delhay JM, Giot M, Riethmuller ML (eds) (1980) *Thermodynamics of two-phase systems for industrial design and nuclear engineering*. Hemisphere Publishing Corp, Washington DC
- DOE/EPRI (1992) Proceedings of the third international workshop on two-phase flow fundamentals, 15–19 June 1992, London, US DOE and EPRI
- Fan L-S, Zhu C (1998) Principles of gas-solid flows. Cambridge University Press, New York
- Ghiaasiaan SM (2008) Two phase flow boiling and condensation in conventional and miniature Systems. Cambridge University Press
- Govier GW, Aziz K (1972) The flow of complex mixtures in pipes. Van Nostrand Reinhold, New York

- Grigull U, Straub J, Schiebener P (2012) Steam Tables in SI-Units/Wasserdampftafeln: concise steam tables in SI-Units (Student's Tables) Properties of ordinary water substance up to 1000° C and 1000 bar. Springer Science & Business Media, Third enlarged edition
- Hetsroni G (1982) Handbook of multiphase systems. Hemisphere. McGraw-Hill
- Hewitt G (2013) Annular two-phase flow. Elsevier
- Hewitt GF, Delhaye JM, Zuber N (1987, 1990, 1992, 1994) Multiphase science and technology, vols 3, 5, and 6, Hemisphere, Washington
- Ishii M, Hibiki T (2011) Thermo-fluid dynamics of two-phase flow, 2nd edn. Springer Science & Business Media, New York
- Kakac S, Mayinger F (1977) Two-phase flows and heat transfer, Proceedings NATO advanced study institute on two-phase flows and heat transfer, Istanbul, Turkey, 16 Aug 1976, vols 1–III. Hemisphere Publishing Corp, Washington, DC
- Kandlikar SG, Shoji M, Dhir VK (1999) Handbook of phase change: boiling and condensation. Taylor & Francis
- Kestin J, Podowski MZ (1985) Report on international workshop on two-phase flow fundamentals, held at the National Bureau of Standards, Gaithersburg, Maryland, 22–27 Sept 1985
- Kleinstreuer C (2003) Two-phase flow—Theory and application. Taylor & Francis
- Kolev NI (2012a) Multiphase Flow dynamics 2—Mechanical interactions. Springer, Berlin Heidelberg
- Kolev NI (2012b) Multiphase flow dynamics 3—Thermal interactions. Springer, Berlin, Heidelberg Kolev NI (2009)
- Kolev NI (2012c) Multiphase flow dynamics 4—Turbulence. Diesel fuel properties. Springer, Berlin Heidelberg, Gas Adsorption and Release
- Kolev NI (2015a) Multiphase flow dynamics 1 – Fundamentals, 5th edn. Springer, Berlin, Heidelberg
- Kolev NI (2015b) Multiphase flow dynamics 5—Nuclear Thermal Hydraulics, 3rd edn. Springer-Verlag, Berlin Heidelberg
- Lahey RT, Moody FJ (1977, 1993) The thermal-hydraulics of a boiling water nuclear reactor. Amer Nuclear Society
- Lassahn GD (1977) LOFT 3 Beam Densitometer Data Interpretation. US NRC report NUREG-1111
- Ledinegg M (1938) Instability of flow during natural and forced circulation. Die Wärme 61 (8):891–898 (English translation as report AEC-tr-1861 (1954))
- Lemmon EW, Huber ML, McLinden MO (2010) NIST Reference fluid thermodynamic and transport properties—REFPROP Version 9.0 User's guide thermophysical properties Division National Institute of Standards and Technology Boulder, Colorado 80305
- Levy S (1999) Two-phase flow in complex systems. John Wiley & Sons Inc
- Lockhart RW, Martinelli RC (1949) Proposed correlation of data for isothermal two-phase, two-component flow in pipes. Chem Eng Prog 45:39–48
- Martinelli RC, Boelter LK, Taylor TH, Thomsen EG, Morrin EH (1944) Isothermal pressure drop for two-phase two-component flow in a horizontal pipe. Trans ASME 66(2):139–151
- Mayinger (1982) Strömung und Wärmeübergang in Gas-Flüssigkeitsgemischen, Springer-Verlag, Wien
- Nukiyama S (1934) The maximum and minimum value of the heat transmitted from metal to boiling water under atmospheric pressure. J Japan Soc of Mech Engrs 37: 367–374. See also translation in Int J Heat Mass Transfer (1966) 9:1419-1433
- Prosperetti A, Tryggvason G (2007) Computational methods for multiphase flow. Cambridge University Press
- Rousseau JC, Riegel B (1978) SUPER CANON experiments. Transient two-phase flow. In: Proceedings of the second CSNI specialists meeting, Paris, 12–14 June 1978
- Shapiro AH (1953) The dynamics and thermodynamics of compressible fluid flow, Vol 1, Chap 4. Ronald Press, New York
- Smith AV (1975) Fast response multi-beam X-ray absorption technique for identifying phase distributions during steam-water blowdowns. J Br Nucl Energy Soc 14(3):227–235

- Stephan K (1988) Wärmeübergang beim Kondensieren und beim Sieden. Springer-Verlag, Berlin
- Tong LS (1965) Boiling heat transfer and two-phase flow. Wiley, New York
- Tong LS, Tang YS (1997) Boiling heat transfer and two-phase flow. CRC Press
- Tong LS, Weisman J (1979) Thermal analysis of pressurized water reactors
- Tryggvason G, Scardovelli R, Zaleski S (2011) Direct Numerical Simulations of gas-liquid multiphase flows, Cambridge University Press
- Wallis GB (1969) One dimensional two phase flow. McGraw-Hill
- Wolverine Tube, Inc. (2004–2010) Heat transfer data book. <http://www.wlv.com/heat-transfer-databook/>
- Yarin LP, Mosyak A, Hetsroni G (2009) Fluid flow, heat transfer and boiling in micro-channels. Springer Verlag