

# Hybridization of Multi-chaotic Dynamics and Adaptive Control Parameter Adjusting jDE Strategy

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**Abstract.** This research deals with the hybridization of several approaches for evolutionary algorithms, which are the adaptive control parameter adjusting strategy and multi-chaotic dynamics driving the selection of indices in Differential Evolution (DE). The novelty of the paper is given by the experiments with the multi-chaos-driven adaptive DE concept inside adaptive parameter adjusting DE strategies. These experiments are representing the investigations on the mutual influences of several different randomizations types together with adaptive DE strategies. The multi-chaotic concept is representing the adaptive switching between two different chaotic systems based on the progress of individuals within population. This paper is aimed at the embedding of discrete dissipative chaotic systems in the form of multi-chaotic pseudo random number generators for the jDE, which is the state of the art representative of simple adaptive control parameter adjusting strategy for DE. Repeated simulations for two different combinations of driving chaotic systems were performed on the IEEE CEC 13 benchmark set. Finally, the obtained results are compared with the canonical not-chaotic jDE.

**Keywords:** Differential Evolution · Deterministic chaos · jDE

## 1 Introduction

This research deals with the interconnection of the two recent techniques for evolutionary algorithms, which are the adaptive control parameter adjusting strategy and multi-chaotic dynamics driving the selection of indices in Differential Evolution. This paper is aimed at investigating the influence of complex multi-chaotic dynamics to the performance of simple adaptive Differential Evolution (DE) algorithm [1]. The adaptive control parameter adjusting strategy of interest within this paper is the state of the art representative jDE [2].

A number of DE variants have been recently developed with the emphasis on adaptivity/selfadaptivity [2], ensemble approach [3] or utilization for discrete domain problems. The importance of randomization as a compensation of limited amount of search moves is stated in the survey paper [4]. This idea has been carried out in subsequent studies describing various techniques to modify the randomization process [5, 6] and especially in [7], where the sampling of the points is tested from modified distribution. The importance and influence of randomization operations was also deeply experimentally tested in jDE strategy [8]. Together with this persistent development in such mainstream research topics, the basic concept of chaos driven DE have been introduced.

Recent research in chaos driven heuristics has been fueled with the predisposition that unlike stochastic approaches, a chaotic approach is able to bypass local optima stagnation. A chaotic approach generally uses the chaotic map in the place of a pseudo random number generator [8]. This causes the heuristic to map unique regions, since the chaotic map iterates to new regions. The task is then to select a very good chaotic map (or combination of chaotic maps) as the pseudo random number generator (PRNG).

The focus of our research is the direct embedding of chaotic dynamics in the form of chaos pseudo random number generator (CPRNG) for heuristic. The initial concept of embedding chaotic dynamics into the evolutionary/swarm algorithms is given in [9]. Later, the initial study [10] was focused on the simple embedding of chaotic systems for DE and Self Organizing Migration Algorithm (SOMA) [11]. Also the PSO (Particle Swarm Optimization) algorithm with elements of chaos was introduced as CPSO [12] followed by the introduction of chaos embedded PSO with inertia weigh strategy [13], further PSO strategy driven alternately by two chaotic systems [14] and finally PSO with ensemble of chaotic systems [15]. Recently the chaos driven heuristic concept has been utilized in ABC algorithm [16] and applications with DE [17].

Firstly, the motivation for this research is proposed. The next sections are focused on the description of evolutionary algorithm jDE, the concept of chaos driven jDE and the experiment description. Results and conclusion follow afterwards.

## 2 Related Work and Motivation

This research is an extension and continuation of the previous successful experiments with multi-chaos driven DE concept [18] and initial research with connection of jDE algorithm and chaotic dynamics [19]. The motivation and novelty for this research are based on following basis and assumptions:

- Recent advances in connection of complexity and heuristic [20] together with the research focused on selection of indices in DE [21] supported the previous very promising experimental results obtained through the utilization of different chaotic dynamics as CPRNGs driving the selection, mutation, crossover or other processes in particular heuristics.
- The idea was then to connect into the one complex concept two different influences on the performance of DE, which are control parameters adaptability and complex randomization framework, which is the multi-chaotic dynamics as complex CPRNGs utilizing several known different influences on the heuristics.

- The novelty of the paper is given by the aforementioned concept and the investigations on the mutual influences of several different randomizations types together with simple parameter adaptive DE strategies. The goal is to investigate whether the complex adaptive randomization is beneficial within adaptive strategies or is suppressed by the control parameter self-adjustment. The results may be used for the possibility of creating the multi-chaotic framework with adaptively driven pool of many chaotic systems for any adaptive modern DE strategy.

The adaptive jDE strategy is hybridized here with multi-chaotic CPRNGs. Furthermore we wanted to show, that it is possible to successfully implement as the plug-in the multi-chaotic CPRNGs not only to the canonical versions of DE/PSO/Firefly but also to the more advanced adaptive strategies.

### 3 The Concept of Chaotic jDE

DE is a population-based optimization method that works on real-number-coded individuals [1, 22]. DE is quite robust, fast, and effective, with global optimization ability. There are essentially five inputs to the heuristic.  $D$  is the size of the problem,  $Gmax$  is the maximum number of generations,  $NP$  is the total number of solutions,  $F$  is the scaling factor of the solution and  $CR$  is the factor for crossover.  $F$  and  $CR$  together make the internal tuning parameters for the heuristic.

A simple and very efficient adaptive DE strategy, known as jDE, was introduced by Brest et al. [2]. This adaptive strategy utilizes basic DE/rand/1/bin scheme [1] with a simple adaptive mechanism for mutation and crossover control parameters  $F$  and  $CR$ . The ensemble of these two control parameters is assigned to each individual of the population and survives if an individual is successful. The initialization of values of  $F$  and  $CR$  is fully random with uniform distribution for each solution in population. If the new generated solution is not successful, i.e. trial vector has worse fitness than compared original active individual; the new (possibly) mutated control parameters disappear together with not successful solution. The both DE control parameters can be randomly mutated with given probabilities  $\tau_1$  and  $\tau_2$ . If the mutation condition happens, new random value of  $CR \in [0, 1]$  is generated, together with new value of  $F$  which is mutated in  $[F_l, F_u + F_l]$ . These new control parameters are thereafter stored in the new population. Input parameters are typically set to  $F_l = 0.1$ ,  $F_u = 0.9$ ,  $\tau_1 = 0.1$ , and  $\tau_2 = 0.1$  as originally given in [2, 23].

The general idea of basic ChaosDE (Chaos\_jDE) and CPRNG is to replace the default pseudorandom number generator (PRNG) with the discrete chaotic map. As the discrete chaotic map is a set of equations with a static start position, we created a random start position of the map, in order to have different start position for different experiments (runs of EA's). This random position is initialized with the default PRNG, as a one-off randomizer. Once the start position of the chaotic map has been obtained, the map generates the next sequence using its current position.

Previous successful initial experiments with chaos driven PSO and DE algorithms [15, 18] have manifested that very promising experimental results were obtained through the utilization of Delayed Logistic, Lozi, Burgers, Lozi, Delayed Logistic map

and Tinkerbell maps. These chaotic maps have unique properties with connection to DE: Either strong progress towards global extreme, but weak overall statistical results, like average cost function value and std. dev., and tendency to premature stagnation; Or the continuously stable and very satisfactory performance. In this research, two representatives of these aforementioned different classes of influences were connected together and embedded into jDE, thus Multi-Chaos\_jDE concept was introduced and tested.

### 4 Chaotic Maps

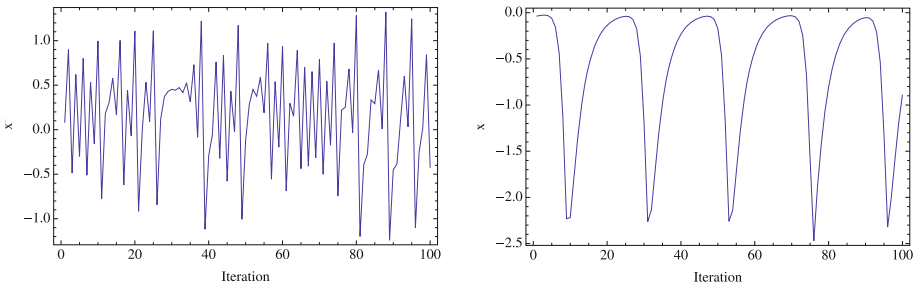
Following two discrete dissipative chaotic maps were used as the multi-chaotic pseudo random generators for jDE: Burgers (1), and Lozi map (2).

The Burgers mapping is a discretization of a pair of coupled differential equations which were used by Burgers [24] to illustrate the relevance of the concept of bifurcation to the study of hydrodynamics flows. The Lozi map is a discrete two-dimensional chaotic map. With the typical settings as in Table 1, systems exhibits typical chaotic behavior [25].

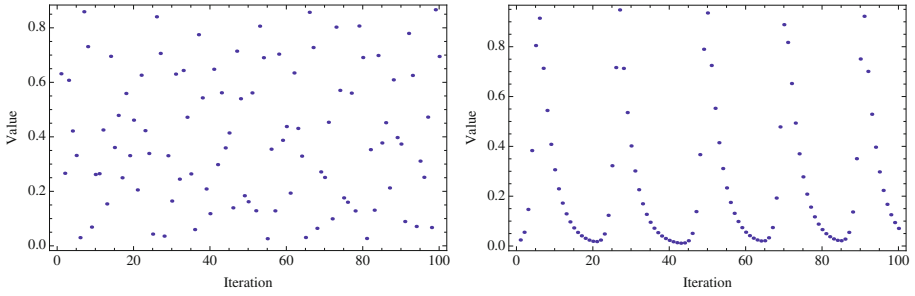
**Table 1.** Definition of chaotic systems used as CPRNGs

Chaotic maps equations	Parameter settings
$\begin{aligned} X_{n+1} &= aX_n - Y_n^2 \\ Y_{n+1} &= bY_n + X_n Y_n \end{aligned} \quad (1)$	$a = 0.75$ and $b = 1.75$
$\begin{aligned} X_{n+1} &= 1 - a X_n  + bY_n \\ Y_{n+1} &= X_n \end{aligned} \quad (2)$	$a = 1.7$ and $b = 0.5$

The illustrative simulation outputs of two chaotic systems used for obtaining of pseudo-random numbers are depicted in Fig. 1.



**Fig. 1.** Simulation outputs – chaotic series used for obtaining of pseudo random numbers and generated by means of the chaotic Lozi map (left) and Burgers map (right).



**Fig. 2.** Detailed sequencing and dynamics of real pseudo-random numbers transferred into the range  $<0-1>$  generated by means of the chaotic Lozi map (left) and Burgers map (right).

Furthermore Fig. 2 shows sequencing and dynamics of pseudo-random numbers transferred into the range  $<0-1>$  obtained from chaotic series (See Fig. 1).

## 5 Results

IEEE CEC 2013 benchmark set [26] was utilized within this experimental research for the purpose of performance comparison of two versions of multi-chaotic jDE and original (canonical) jDE,

Experiments were performed in the combined environments of Wolfram Mathematica and C language; canonical jDE for comparisons therefore used the built-in C language pseudo random number generator Mersenne Twister C representing traditional pseudorandom number generators in comparisons. All experiments used different initialization, i.e. different initial population was generated in each run.

Within this research, one type of experiment was performed. It utilizes the maximum number of generations fixed at 1500 generations, Population size of 75 and dimension  $dim = 30$ . This allowed the possibility to analyze the progress of all studied jDE variants within a limited number of generations and cost function evaluations.

To maximize the benefit from the influences of different chaotic dynamics, the simple adaptive control approach was used. The exact transition point is determined by following simple rule: If the change of global best value between two subsequent generations is less than 0.01 over more than 1% of total number of generations, the chaotic systems used as the CPRNGs are alternated.

To track the influence of adaptive multi-chaotic approach, an experiment encompasses three groups:

- *Multi-Chaos\_jDE LoziBur*: Initialized with Lozi map, subsequently the adaptive switching between Lozi and Burgers map is applied.
- *Multi-Chaos\_jDE BurLozi*: Initialized with Burgers map, subsequently the adaptive switching between Lozi and Burgers map is applied.
- State of the art adaptive representative canonical jDE (with default PRNG).

Results of experiments are shown in Tables 2 and 3. Table 2 contains the average cost function (CF) error values (CF – fmin), whereas Table 3 shows the minimum error values of the found CF values (CFmin – fmin) representing the best individual solution for all 50 repeated runs of two versions of Multi-Chaos\_jDE, and canonical jDE. Finally the Within Tables 2 and 3, the bold values represent the best performance, italic equal; based on summary characteristics. The significant differences (even for known extreme) for *f2-f4* are given by the high complexity of composite function.

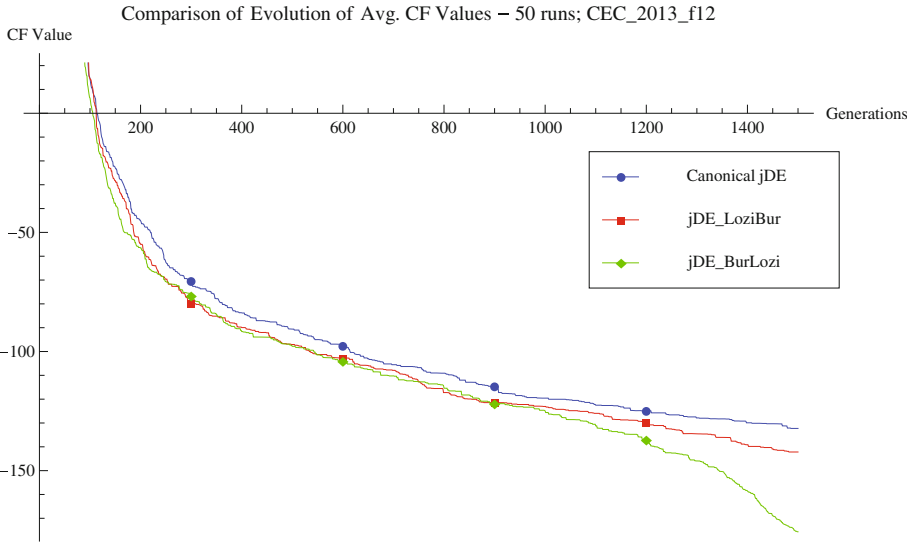
**Table 2.** Final average CF values: Performance comparison for two versions of Multi-Chaos\_jDE and canonical jDE on CEC 13 Benchmark Set, *dim* = 30, max. Generations = 1500

DE Version/Function	f min	jDE	Multi-Chaos_jDE LoziBur	Multi-Chaos_jDE BurLozi
f1	-1400	9,63E - 13	5,00E - 13	<b>1,67E - 13</b>
f2	-1300	1,09E + 07	3,26E + 06	<b>1,87E + 06</b>
f3	-1200	3,13E + 06	2,29E + 06	<b>1,93E + 06</b>
f4	-1100	2,39E + 04	<b>4,93E + 03</b>	4,98E + 03
f5	-1000	1,94E - 08	<b>1,23E - 08</b>	2,16E - 09
f6	-900	1,98E + 01	<b>1,97E + 01</b>	2,56E + 01
f7	-800	1,14E + 01	6,06E + 00	<b>5,75E + 00</b>
f8	-700	<i>2,10E + 01</i>	<i>2,10E + 01</i>	<i>2,10E + 01</i>
f9	-600	3,43E + 01	3,00E + 01	<b>1,66E + 01</b>
f10	-500	9,89E - 01	1,66E - 01	<b>3,80E - 02</b>
f11	-400	<b>3,66E + 01</b>	4,72E + 01	6,59E + 01
f12	-300	1,68E + 02	1,58E + 02	<b>1,24E + 02</b>
f13	-200	1,79E + 02	1,74E + 02	<b>1,65E + 02</b>
f14	-100	<b>2,26E + 03</b>	2,80E + 03	4,78E + 03
f15	100	<b>7,15E + 03</b>	7,22E + 03	7,38E + 03
f16	200	2,67E + 00	<b>2,66E + 00</b>	2,68E + 00
f17	300	<b>8,68E + 01</b>	9,75E + 01	1,46E + 02
f18	400	2,16E + 02	2,15E + 02	<b>2,12E + 02</b>
f19	500	<b>7,45E + 00</b>	8,31E + 00	1,11E + 01
f20	600	1,26E + 01	1,24E + 01	<b>1,23E + 01</b>
f21	700	3,12E + 02	<b>2,93E + 02</b>	3,12E + 02
f22	800	<b>2,76E + 03</b>	3,29E + 03	4,89E + 03
f23	900	<i>7,49E + 03</i>	<i>7,49E + 03</i>	7,63E + 03
f24	1000	2,06E + 02	<i>2,04E + 02</i>	<i>2,04E + 02</i>
f25	1100	2,92E + 02	2,64E + 02	<b>2,46E + 02</b>
f26	1200	<i>2,00E + 02</i>	<i>2,00E + 02</i>	<i>2,00E + 02</i>
f27	1300	6,06E + 02	4,10E + 02	<b>3,88E + 02</b>
f28	1400	<i>3,00E + 02</i>	<i>3,00E + 02</i>	<i>3,00E + 02</i>

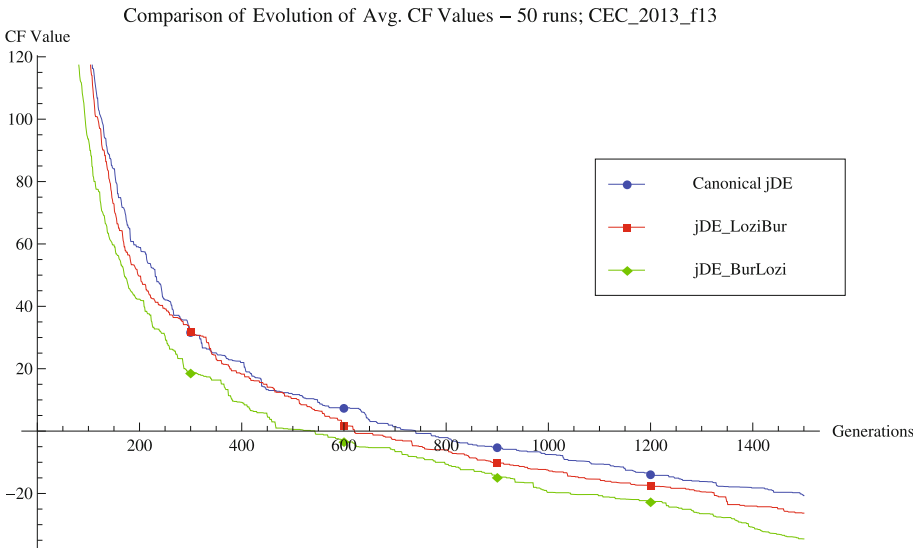
**Table 3.** Best solutions - minimal CF values: Performance comparison for two versions of Multi-Chaos\_jDE and canonical jDE on CEC 13 Benchmark Set,  $dim = 30$ , max. Gen. = 1500

DE Version/Function	f min	jDE	Multi-Chaos_jDE LoziBur	Multi-Chaos_jDE BurLozi
f1	-1400	2,27E - 13	2,27E - 13	<b>0,00E + 00</b>
f2	-1300	5,04E + 06	1,09E + 06	<b>3,95E + 05</b>
f3	-1200	3,44E + 04	4,30E + 04	<b>1,25E + 03</b>
f4	-1100	1,31E + 04	<b>1,84E + 03</b>	1,85E + 03
f5	-1000	5,48E - 09	4,15E - 09	<b>4,74E - 10</b>
f6	-900	1,54E + 01	1,46E + 01	<b>1,41E + 01</b>
f7	-800	3,32E + 00	1,86E + 00	<b>8,31E-01</b>
f8	-700	2,09E + 01	2,09E + 01	2,09E + 01
f9	-600	2,89E + 01	8,63E + 00	<b>5,85E + 00</b>
f10	-500	3,36E - 01	8,87E - 03	<b>7,40E - 03</b>
f11	-400	<b>2,67E + 01</b>	2,89E + 01	4,71E + 01
f12	-300	1,43E + 02	1,30E + 02	<b>2,79E + 01</b>
f13	-200	1,52E + 02	1,48E + 02	<b>7,02E + 01</b>
f14	-100	<b>1,66E + 03</b>	2,31E + 03	4,03E + 03
f15	100	<b>6,30E + 03</b>	6,57E + 03	6,63E + 03
f16	200	1,98E + 00	<b>1,78E + 00</b>	2,10E + 00
f17	300	<b>7,90E + 01</b>	8,30E + 01	1,20E + 02
f18	400	1,96E + 02	1,95E + 02	<b>1,80E + 02</b>
f19	500	<b>6,11E + 00</b>	6,62E + 00	8,03E + 00
f20	600	1,20E + 01	1,17E + 01	<b>1,16E + 01</b>
f21	700	2,00E + 02	2,00E + 02	2,00E + 02
f22	800	<b>2,10E + 03</b>	2,63E + 03	4,07E + 03
f23	900	7,07E + 03	<b>6,84E + 03</b>	6,97E + 03
f24	1000	2,02E + 02	2,01E + 02	<b>2,00E + 02</b>
f25	1100	2,59E + 02	2,01E + 02	<b>2,00E + 02</b>
f26	1200	2,00E + 02	2,00E + 02	2,00E + 02
f27	1300	3,87E + 02	3,23E + 02	<b>3,11E + 02</b>
f28	1400	3,00E + 02	3,00E + 02	3,00E + 02

The different influences of two CPRNGs around switching point are visible from the Figs. 3 and 4 (around generations No. 1350), which depicts for selected test functions the illustrative examples of time evolution of average CF values for all 50 runs of two versions of Multi-Chaotic\_jDE and canonical jDE.



**Fig. 3.** Comparison of the time evolution of avg. CF values for the all 50 runs of Canonical jDE, and all two versions of Multi-Chaos\_jDE, Visible switching point approx. gen. No. 1350; CEC2013\_f12,  $D = 30$ .



**Fig. 4.** Comparison of the time evolution of avg. CF values for the all 50 runs of Canonical jDE, and all two versions of Multi-Chaos\_jDE, Visible switching point approx. gen. No. 1350; CEC2013\_f13,  $D = 30$ .



## 6 Conclusion

The primary aim of this work is to use and test the hybridization of complex combination of natural chaotic dynamics as the chaotic pseudo random number generators with adaptive switching and self-adaptive mechanism for differential evolution algorithm. In this paper the novel concept of Multi-Chaos\_jDE strategy driven by two chaotic maps (systems) is introduced. Two different influences on the performance of DE were connected here into the one adaptive multi-chaotic concept. Repeated simulations were performed on the IEEE CEC 13 benchmark set. The obtained results were compared with the original state of the art adaptive representative canonical jDE. The findings can be summarized as follows:

- It is fully manifested here the high sensitivity of the jDE to the complex dynamics and sequencing of the different chaotic PRNGs driving the selection of indices (solutions) in jDE.
- When comparing the Multi-Chaos\_jDE and original jDE, one version of multi-chaos driven heuristic has outperformed the original jDE in both cases of final average CF values and min. CF values. Also the transition phases are visible as in Figs. 3 and 4. This means that the chaos driven indices selection keeps its unique properties also in multi-chaotic framework and adaptive DE strategy environment. The previously successful single\_chaos\_jDE [19] was not compared here, as the motivation for the paper is given by the investigations on the mutual influences of several different randomizations types together with simple parameter adaptive DE strategies. Thus to investigate whether the complex adaptive randomization is beneficial within adaptive strategies or it is suppressed by the control parameter self-adjustment. The findings may be used as the basis for the building of the multi-chaotic framework with adaptively driven pool of many chaotic systems for any adaptive modern DE strategy (EPSDE, SHADE...).
- Based on the obtained results, we can assume that in case of differential evolution, the sensitivity to the CPRNG driving the selection of individuals and crossover process may be very beneficial together with the influence of adaptive tuning of control parameters.
- Furthermore many previous implementations of chaotic dynamics into the evolutionary/swarm based algorithms (not-adaptive/adaptive/ensemble based) showed that it is advantageous, since it can be easily implemented into any existing algorithm as a plug-in module.

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