Trends in the History of Science

Chris Rorres Editor

Archimedes in the 21st Century

Proceedings of a World Conference at the Courant Institute of Mathematical Sciences

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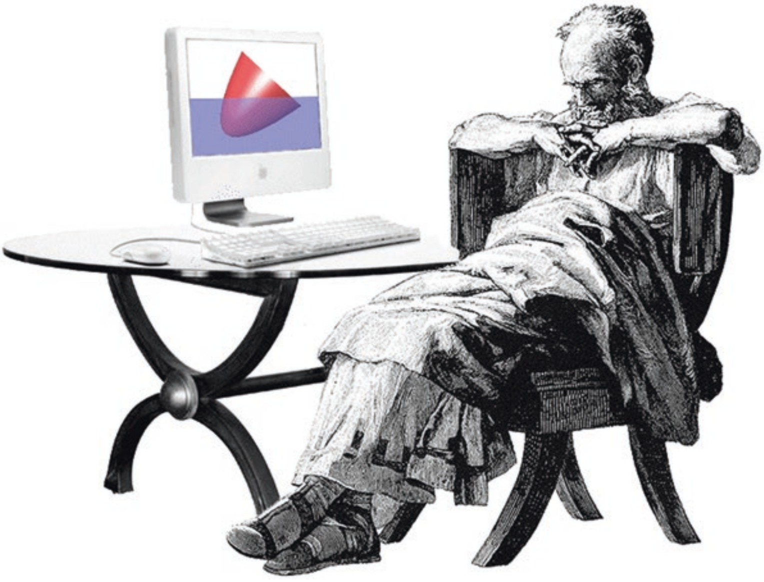

These Proceedings are dedicated to
Dennis L. Simms
(1926–2010)
Archimedes scholar, colleague, and friend—
who would have been a valuable contributor
to this conference.

Preface

Archimedes in the 21st Century, a two-day world conference held in Spring 2013 at the Courant Institute of Mathematical Sciences, New York University, took place during what is believed to have been Archimedes' 2300th birth year.

This conference was focused on the enduring and continuing influence of Archimedes in our modern world. Specifically, it celebrated his 23 centuries of influence on mathematics, science, and engineering.

Courant Institute Director Gérard Ben Arous opened the conference with this inspiring observation:

> I'm impressed by the idea that, 23 centuries after his lifetime, we're still looking at the influence of Archimedes in our modern world. It's as close a shot at immortality as one can imagine.

Eight invited talks presented during the first day of the conference are printed in this volume. The conference speakers were chosen because they are actively involved in fields whose origins trace back to Archimedes. As well, many of the presenters have conducted and published research that extends Archimedes' work in the twenty-first century.

The fields of mathematics, science, and engineering each claim Archimedes as one of their own. For that reason, the conference talks were divided into three categories: *Archimedes the Mathematician/Geometer*, *Archimedes the Scientist*, and *Archimedes the Engineer*.

Let me say a few words about what the conference *was not.*

Archimedes in the 21st Century was *not* a history of science conference, and therefore, no historians of science were invited to present. That was certainly no slight to the importance of the work of historians of science; many other conferences have emphasized that aspect of Archimedean scholarship (e.g., [1]).

An example surrounding this distinction is the *Archimedes Palimpsest*, which is mentioned only tangentially in these conference proceedings. This collection of works of Archimedes, hidden beneath the text of a prayer book sometime in the thirteenth century, was unveiled and edited by Heiberg in 1906 [2] and analyzed with modern technology in the last twenty years [3]. Its discovery was of immense value to historians, but its mathematical results had to be rediscovered independently in the sixteenth and seventeenth centuries by the pioneers of the mathematical Renaissance. Had its contents not been obscured in the thirteenth century, it could have had a significant influence on twenty-first-century mathematics and science. But the window of opportunity for the palimpsest came and went, and so humankind took a different path in the tree of possibilities.

Our conference drew an audience of mathematicians, scientists, and engineers who were primarily interested in learning about the latest twenty-first-century applications of Archimedes' works—along with their historical roots. Individuals already familiar with the history of Archimedes who were interested in learning what his works have led to in the twenty-first century also attended our conference.

As conference organizer and editor of these proceedings, I would now like to speak personally about our distinguished speakers:

Moshe Kam is the quintessential professional engineer and historian of science, a university dean of engineering, and a former president of IEEE (Institute of Electrical and Electronics Engineers). Moshe was our opening speaker, and he presented a complete and thoughtful timeline of engineering from ancient times to the twenty-first century, highlighting Archimedes' contributions to the field.

Larrie D. Ferreiro is an authority on military strategies and weaponry, a historian of naval architecture, a university professor of systems engineering, and an author of several books and many articles on these themes. Larrie's focus on *defense in depth* was astute and compelling, particularly when he applied Archimedean ideas to twenty-first-century military stratagems [4].

Mamikon Mnatsakanian is the developer of *Visual Calculus*, an ingenious approach to solving many problems in geometry and integral calculus. He is the coauthor of a recently published geometry work that provides fresh and powerful insights to that field [5]. His richly illustrated conference talk brought Archimedes' renowned *tombstone theorem* into the twenty-first century, extending it in numerous directions.

Horst Nowacki is a world authority on naval history and the author of *Archimedes and Ship Stability* [6]. Having extensively consulted with Horst during my own floating-bodies research [7, 8], I commend his encyclopedic knowledge of Archimedean laws as they have been applied to ship design over two millennia.

Dirk M. Nuernbergk is a world expert on the Archimedes Screw, particularly for the newest twenty-first-century application of operating Archimedes screws *in reverse* over rivers and streams in order to generate cheap electricity. This application is proving to be a boon all across Europe, Canada, and New Zealand. Dirk is at the center of this innovative research, both as a consultant engineer and as the author of numerous engineering journal articles; he has also written a handbook for setting up a hydropower screw [9].

Michael T. Wright is an ancient technology scholar and a mechanician who brought two of his handwrought models to our conference: the much-acclaimed Antikythera Mechanism and his newly created *Sphere of Archimedes*—which he unveiled in public for the first time at the conclusion of his talk. Michael's *Sphere* is the first model of an Archimedes' sphere that anyone in any century has ever attempted to reconstruct; it is a clear precursor of our modern-day planetarium and a marvel to behold.

Mary Jaeger was the only invited non-STEM presenter at our conference. She is the author of the scholarly volume, *Archimedes and the Roman Imagination* [10], and presented the classicist's perspective. In her well-researched talk, Mary chronicled events and people who have brought Archimedes into our modern world—surprisingly (and sometimes amusingly) perpetuating the anecdotes and myths about him.

At this conference, I presented the *Archimedes the Mathematician* segment along with Sylvain Cappell, Silver Professor of Mathematics at the Courant Institute of Mathematical Sciences.

You can view videos of all the Archimedes conference talks and learn more about the greatest mathematician and scientist of antiquity on my Archimedes website:

https://www.cs.drexel.edu/~crorres/Archimedes/AWC/

This site is the repository of my 50-year fascination with Archimedes. In it I've compiled knowledge about his inventions, the numerous fields of science and mathematics he engendered, discussions of many of his finished works, and my own research that extends and applies Archimedean principles to twenty-first-century problems. I created this website in 1995, and it has been under continual development and expansion since then.

Acknowledgments

I wish to thank all our speakers, both Friday and Saturday, for their thoughtful and inspiring presentations. Thanks, particularly, to my Drexel colleague, structural engineer Harry Harris, who transported his remarkable 1/60-scale working model of an Archimedes Claw/Iron Hand to the conference and spoke about it during our Saturday session.

I am very grateful to our sponsors who made the conference possible: First, the Courant Institute of Mathematical Sciences for hosting the conference and for providing guidance and support. Among my colleagues at Courant, I warmly thank Sylvain Cappell and Gérard Ben Arous.

Similarly, I want to thank the directors of the Institute of Electrical and Electronics Engineers (IEEE) for providing the lead grant for this conference; most especially, I thank Moshe Kam for helping us qualify for the IEEE grant.

Many thanks, as well, to Math for America and to MfA President John Ewing for their conference grant and enthusiastic support.

Also, a warm thank-you to professional staffers at NYU who helped plan, manage, and promote the conference, specifically James Devitt of NYU's Media Relations Department for disseminating communications about the Archimedes conference to the news media and to the greater NYU community, and Elizabeth

Rodriguez at the Courant Institute for her enormous help in organizing and managing the logistics of the conference.

My sincere and personal thanks to Susanna Davison and Rene Saxman for managing the conference registration and to Demos Vasiliou for providing sustenance for our conference reception.

And finally, I wish to thank my wife Billie who was co-organizer of the conference and coeditor of these proceedings, but who declined to be listed as such. (I should have insisted a little harder.)

Chris Rorres
April 2017

References

1. Paipetis, S.A. and Marco Ceccarelli (Editors), *The Genius of Archimedes—23 Centuries of Influence on Mathematics, Science and Engineering*, Proceedings of an International Conference held at Syracuse, Italy, June 8–10, 2010.
2. Netz, Reviel, William Noel, Natalie Tchenetska, and Nigel Wilson (eds), *The Archimedes Palimpsest*, Cambridge: Cambridge University Press, (2011).
3. Archimedes, *The Archimedes Palimpsest (10th century)* Private collection. Contents available on the web site "The Digital Archimedes Palimpsest", accessed March 2017: http://www.archimedespalimpsest.net.
4. Steel, Brett D., and Tamera Dorland (Editors), *The Heirs of Archimedes*, The MIT Press, Cambridge MA (2005).
5. Apostol, Tom M., and Mamikon Mnatsakanian, *New Horizons in Geometry*, Mathematical Association of America, Washington, DC, (2012).
6. Nowacki, Horst, *Archimedes and Ship Stability*, Max Planck Institute for the History of Science, Preprint 198, Berlin (2002).
7. Rorres, Chris, "Completing Book II of Archimedes's *On Floating Bodies*", *The Mathematical Intelligencer,* Volume 26, Number 3, pp. 32–42 (2004).
8. Rorres, Chris, "Archimedes' floating bodies on a spherical Earth", *American Journal of Physics,* Volume 84, Number 1, pp. 61–70 (2016).
9. Nuernbergk, Dirk M., *Wasserkraftschnecken: Berechnung und optimaler Entwurf von archimedischen Schnecken als Wasserkraftmaschine*, Verlag Moritz Schäfer, Delmond, Germany (2012).
10. Jaeger, Mary, *Archimedes and the Roman Imagination*, The University of Michigan Press, Ann Arbor, (2008).

Following is reprinted an article about our conference that appeared in *The New York Times*. It was written by science reporter Kenneth Chang, who attended both days of the conference. The article appeared in the June 25, 2013, *Science Times* section of the *NYT* and is reprinted here with permission.

The New York Times

SCIENCE JUNE 25, 2013

Archimedes: Separating Myth From Science

The Ancient Greek's work, though devoid of death rays, still inspires inventions.

By KENNETH CHANG

For the last time: Archimedes did not invent a death ray.

But more than 2,200 years after his death, his inventions are still driving technological innovations — so much so that experts from around the world gathered recently for a conference at New York University on his continuing influence.

The death ray legend has Archimedes using mirrors to concentrate sunlight to incinerate Roman ships attacking his home of Syracuse, the ancient city-state in the southeast Sicily. It has been debunked no fewer than three times on the television show "Mythbusters" (the third time at the behest of President Obama).

Rather, it is a mundane contraption attributed to the great Greek mathematician, inventor, engineer and military planner — the Archimedes screw, a corkscrew inside a cylinder — that has a new use in the 21st century. For thousands of years, farmers have used this simple machine for irrigation: Placed at an angle with one end submerged in a river or a lake, the screw is turned by a handle, lifting water upward and out at the other end.

A couple of decades ago, engineers found that running an Archimedes screw backward — that is, dropping water in at the top, causing the screw to turn as the water falls to the bottom — is a robust, economical and efficient way to generate electricity from small streams. The power output is modest, enough for a village, but with a small impact on the environment. Unlike the turbine blades that spin in huge hydropower plants like the Hoover Dam, an Archimedes screw permits fish to swim through it and emerge at the other end almost unscathed.

Such generators have been built in Europe, including one commissioned by Queen Elizabeth II of England to power Windsor Castle; the first in the United States could start operating next year.

And Archimedes' ideas are showing up in other fields as well.

"He just planted the seeds for so many seminal ideas that could grow over the ages," said Chris Rorres, an emeritus professor of mathematics at Drexel University, who organized the conference at N.Y.U.

A panoply of devices and ideas are named after Archimedes. Besides the Archimedes screw, there is the Archimedes principle, the law of buoyancy that states the upward force on a submerged object equals the weight of the liquid displaced. There is the Archimedes claw, a weapon that most likely did exist, grabbing onto Roman ships and tipping them over. And there is the Archimedes sphere, a forerunner of the planetarium—a hand-held globe that showed the constellations as well as the locations of the sun and the planets in the sky.

"Here was someone who just changed how we look at the universe," Dr. Rorres said.

Only a handful of Archimedes' writings survive, and much of what we think we know about him was written centuries after his death.

Some of the legends, like using mirrors to set the Roman ships afire, proved too good to be true. The same may go for the tale of Archimedes figuring out, while sitting in a bathtub, how to tell if the maker of a crown for the king had fraudulently mixed in some silver with the gold; according to this story Archimedes, too excited to put on clothes, ran naked through the streets of Syracuse shouting, "Eureka!"

As with the mirrors, the underlying principle works. But in practice, the tiny difference in volume between a crown made of pure gold and one made of a mixture of gold and silver is too small to be reliably measured.

Some of the talks at the conference were about using present-day ingenuity to figure out what Archimedes actually achieved in antiquity.

Michael Wright, a researcher at Imperial College London, has been trying to decipher how the Archimedes sphere showed the night sky. Although it is described in historical writings, no pieces or even drawings of it have survived. Others had already made celestial spheres, globes that show the positions of the constellations.

The Roman historian Cicero described the Archimedes sphere as uninteresting at first glance until it was explained. "There was a wonderful contrivance due to Archimedes inside," he wrote. "He had devised a way in which a single rotation would generate the several non-uniform motions."

If this description is taken literally, it would seem that Archimedes figured out the gearing needed to mimic the motion of the planets, including the retrograde motion where they appear to stop and reverse direction for a while before proceeding in their usual direction.

"This instrument was just like any other celestial sphere, except with the addition of indicators for the Sun, Moon, the planets moving over the sphere and a mechanism inside the sphere to move them," Mr. Wright said.

In the spring, he began building his version of the Archimedes sphere. He presented it in public for the first time at the conference.

"I can't guarantee that the original was like this," Mr. Wright said. "What I can say is this, in the simplest way that I can imagine it, fits the evidence we have. We've been talking for 2,000 years about this thing that Archimedes made, and nobody seems to have offered to show people what it was like. I had an idea. I thought it was worth making, even if it was so people could have an argument about it and disagree with it. That's a good way to get things going."

Dr. Rorres said the singular genius of Archimedes was that he not only was able to solve abstract mathematics problems, but also used mathematics to solve physics problems, and he then engineered devices to take advantage of the physics.

"He came up with fundamental laws of nature, proved them mathematically and then was able to apply them," Dr. Rorres said.

Archimedes oversaw the defenses of Syracuse, and while death ray mirrors and steam cannons (another supposed Archimedes invention debunked by "Mythbusters") were too fanciful, the Archimedes claw appears to have been a real weapon used against the Roman navy.

It is very likely that it took advantage of two scientific principles Archimedes discovered.

With his law of buoyancy, he was able to determine whether a paraboloid (a shape similar to the nose cone of a jetliner) would float upright or tip over, a principle of utmost importance to ship designers, and Archimedes probably realized that the Roman ships were vulnerable as they came close to the city walls.

"Archimedes knew about the stability of these kinds of ships," said Harry G. Harris, an emeritus professor of structural engineering at Drexel who has built a model of the claw. "When it is moving fast through the water, it is stable. Standing still or going very slow, it is very easy to tip over."

So using an Archimedean principle — the law of the lever, which enables a small force to lift a large weight, as in seesaws and pulleys — a claw at the end of a chain would be lowered and hooked into a Roman ship, then lifted to capsize the ship and crash it against the rocks.

Syracuse won the battle but was weakened under a long siege and fell three years later. And in 212 B.C., at the age of about 75, Archimedes was killed by a Roman soldier, supposedly furious that he refused to stop work on a mathematical drawing. His last words: "Do not disturb my circles!"

Of course, that bit about the circles is probably also a myth.

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Archimedes the Pragmatic Engineer

Moshe Kam

Introduction

Let me begin by looking at an engineering timeline that starts with the first evidence of counting (Figure 1). For the sake of this discussion, let us remind ourselves when the first humans settled in Greece, when the bow and arrow was developed, and when people first started trying to control the environment in an organized manner (such as attempting flood control on the Nile).

Getting a little closer in terms of relevant events, the lever is already used extensively around 2000 BCE, and we have waterwheels and catapults in use around 400 BCE (Figure 2). Let us not forget Pythagoras (c. 570–c. 495 BCE) and Thales (c. 624–c. 546 BCE) and, of course, the two major philosophers before Archimedes, namely, Aristotle (384–322 BCE) and Euclid (mid-fourth century to mid-third century BCE).

When we speak about Archimedes and his key contributions, we need to address the development of *statics* as a theoretical science, the science of *hydrostatics* (the principle of buoyancy and the stability of floating bodies), Archimedes' contribution to the theory and practice of simple machines (specifically the lever, the pulley, and the screw), and a collection of useful inventions, from the water screw to the improved water organ.

Just a quick reminder what happened *after* Archimedes: we need to remind ourselves that he lived before Ptolemy (90–168 CE) and that he lived before the invention of paper and certainly before the printing press:

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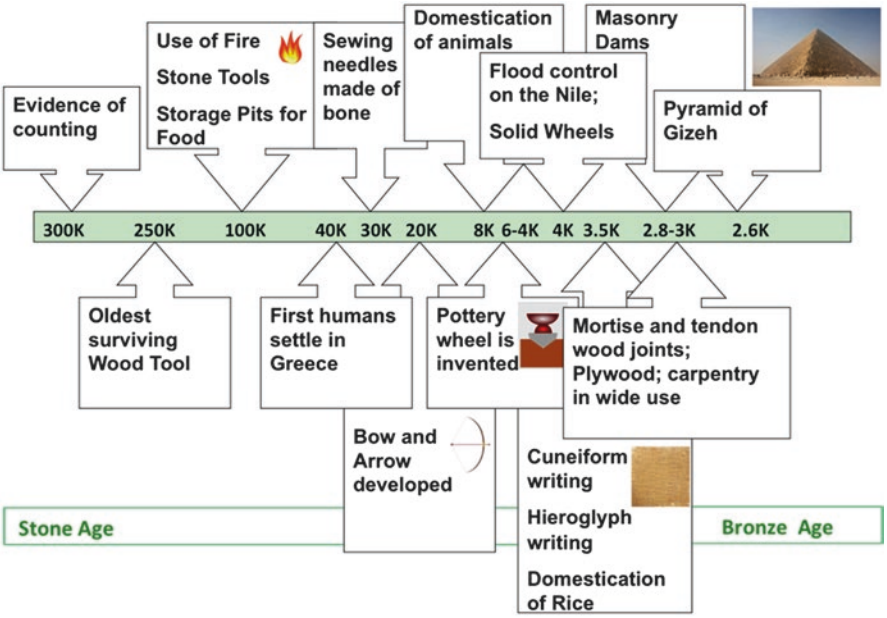

Figure 1 A quick orientation in time (BCE)

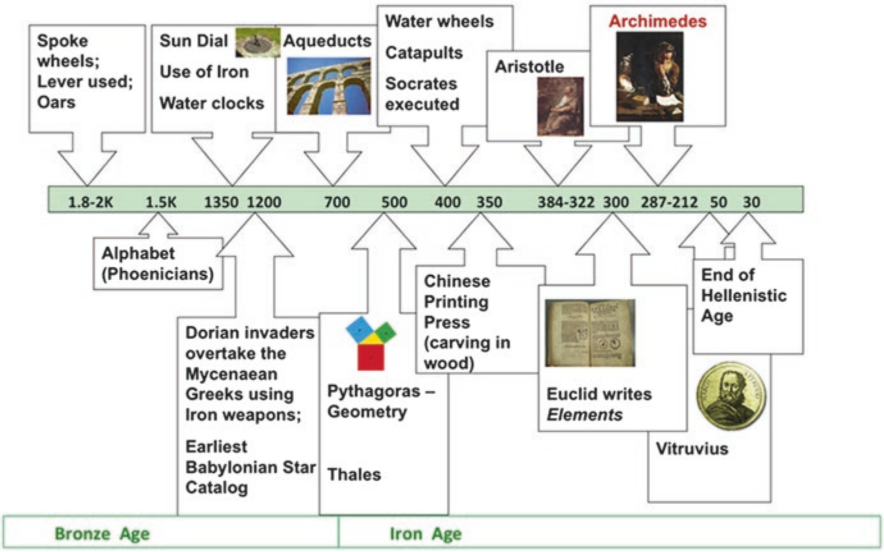

Figure 2 A quick orientation in time (BCE)

Post-Archimedes (CE)

- Ptolemy (90–168); *Almagest*
- Paper (Tsai Lun, 105)
- Diophantus of Alexandria (third century) – Algebra
- Windmill (China, 400)
- Porcelain (China, 700)
- Gun powder (China 1000; Roger Bacon 1242)
- Perspective in painting (1428)
- Printing press
  - Movable wooden blocks 1450
  - Movable metal blocks 1452 (Gutenberg)
- [Start of the Renaissance, 1500]

The Historical Record

We are lucky to have access to some of Archimedes' major work through early copies and, most importantly, through transcriptions from the Archimedes Palimpsest. These books are perhaps the best information we have about him. As we shall see in a moment, the historical record from the various historians who speak about him has a few fundamental problems. The books we have are:

- *On Plane Equilibriums* (two books)
- *On the Sphere and Cylinder* (two books)
- *On Spirals, Conoids and Spheroids*
- *On Floating Bodies* (two books)
- *Measurements of a Circle*
- *The Quadrature of a Parabola*
- *The Sand Reckoner*
- *Stomachion*
- *The Method of Mechanical Theorems*
- *The Cattle Problem*

In addition to the works we have, there are several that were lost. Pappus (c. 290–350 BCE) tells us about a work on semi-regular polyhedra. Archimedes himself mentions a work on the number system (now lost). There is a particularly interesting study on sphere-making that we will come back to. There is also a speculation about several other writings, now lost, on centers of gravity of solids, on plane figures, and on magnitudes.

Lost works:

- *On semi-regular polyhedra* (Pappus)
- *On the number system* proposed in the *Sand Reckoner*
- *On balances and levers* (Pappus)

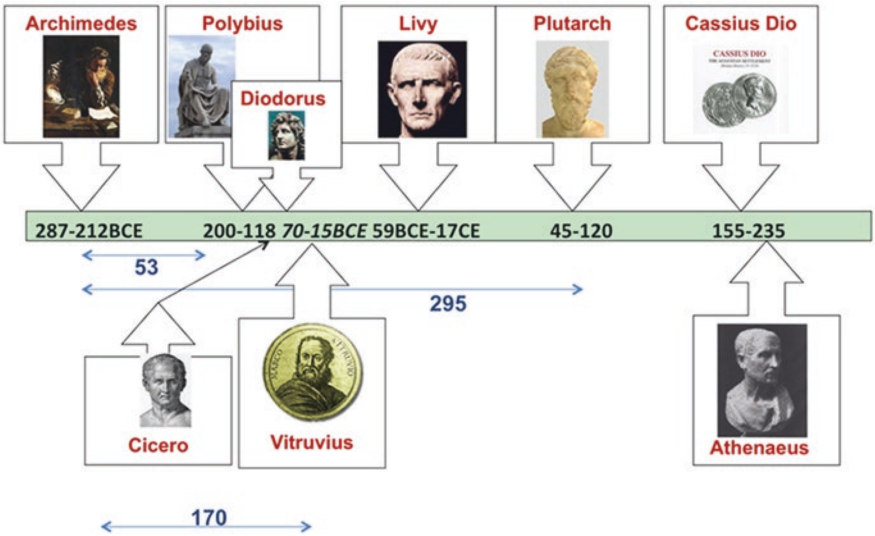

Figure 3 Archimedes and the historians

- *On sphere making* (Pappus)
- *About mirrors* (Theon)
- Evidence for further lost works is discussed in T. Sato [10]

As we look at the historical record, we need to also understand the sources that provide us with information about Archimedes (Figure 3). We are talking about historians like Polybius, Livy, and Plutarch. It is useful to look again at the timeline and to realize that many of these historians were far removed in time from Archimedes. In fact, the only historian who could have spoken to people who lived in Archimedes' time is Polybius (c. 200–118 BCE). Everyone else heard and learned from secondary and tertiary sources or read manuscripts of and about his works.

- Primary historians and writers:
  - Polybius (Greek, c. 200–118 BCE) *Universal History*
  - Livy (Roman, 59 BC–17 CE) *History of Rome from its Foundation*
  - Plutarch (Greek, c. 45–120 CE) *Parallel Lives: Marcellus*
  - Cassius Dio (Greek, c. 155–235 CE) *Roman History*
- Relevant technology is reviewed in *De Architectura* by Vitruvius (~15 BCE).
- A large number of apocryphal stories and texts were ascribed to Archimedes, including inventions that were not his.

An author who is very important to our discussion is Vitruvius (born c. 80–70 BC, died after c. 15 BC). He lived about 170 years after Archimedes and wrote a lot about Roman technology (including his multivolume work *De Architectura*). Vitruvius makes many references to Archimedes—to apocryphal stories, documents, texts, and inventions, some of which Archimedes probably

Figure 4 Wall painting from the Stanzino delle Matematiche in the Galleria degli Uffizi (Florence, Italy) (Painted by Giulio Parigi (1571–1635) in the years 1599–1600)

did not make. Some of these references are now in the public domain—everybody knows them. Still, the historical record is problematic.

One of the most famous and charming Archimedes stories is the one about the burning mirrors during the siege of Syracuse in the year 212 BCE. It describes how allegedly Archimedes used mirrors in order to focus the rays of the sun in order to burn the ships of the invading Romans (Figure 4).

There is no mention of this story by Plutarch, Polybius, and Livy. The earliest mention of fire in the context of the siege of Syracuse, I believe, is by Lucian (about 125 CE–after 180 CE) who writes in the second century. He refers to the use of fire in different ways in the battle, but he does not refer to mirrors. Mirrors are mentioned explicitly by the physician Galen, (129 CE–c. 216 CE) who also lived in the second century. Actually, it is not until the sixth century that we have a book on burning mirrors; that, of course, had happened very far removed from Archimedes' period. So in this case, and in other cases, we need to ask the question: What is the historical evidence and how convincing is it? As Simms [6] asks: Did Archimedes have the knowledge and skill, in this particular case, to design, build, and operate a burning mirror, specifically the kind of mirror that the commentators are describing? The most important question in this context, given what he had at his disposal, is "Would Archimedes have needed it, and would he have used it?" I let others who present their work at this conference to comment on this matter later.

The other comment that I want to make on the historical record comes from Plutarch. Of course it is Plutarch speaking, not Archimedes, but there is nevertheless something to his observations. Plutarch speaks about Archimedes' inventions, and he writes:

> *Archimedes possessed so high a spirit, so profound a soul, and such treasures of scientific knowledge, that though these inventions had now obtained him the renown of more than*

human sagacity, he yet would not deign to leave behind him any commentary or writing on such subjects;

What are "such subjects?" Plutarch further writes about Archimedes:

...but, repudiating as sordid and ignoble the whole trade of engineering, and every sort of art that lends itself to mere use and profit, he placed his whole affection and ambition in those purer speculations where there can be no reference to the vulgar needs of life;

In spite of the fact that this is Plutarch's, not Archimedes', opinion, one should remember that when you look at the works of Archimedes that we have today, they are of a more abstract mathematical nature, and we don't have works that describe specific devices (although there are mentions by Pappus that there was a book on spheres). Was Plutarch right when he described Archimedes as being focused on the "purer speculations" or was he, like us, deprived of works or writings on the "vulgar needs of life" that were lost over time?

Let us return to Archimedes the Engineer. We are talking about a rich set of contributions with long-term impact on engineering disciplines—the systematic study of statics; Archimedes' contribution to the use of three (out of five) simple machines known in antiquity, the winch, the lever, the pulley, the wedge, and the screw; the power of mechanical advantage; the introduction of hydrostatics (the equilibrium of fluids, the principle of buoyancy, and the stability of floating bodies); the introduction of the concepts of specific gravity and fluid pressure; and specific tools and devices (the Archimedean screw, the planetarium, and the water organ). Following Plutarch, let us, too, ask the question: What motivated Archimedes? This is not easy to discern. His inventions were, I think, motivated by his *desire to test theory*. I will demonstrate this claim with a quote or two. This matter is related to providing physical evidence for a geometric proof. Also, Archimedes was *testing the physical limits*; perhaps the ship Syracusia is a good example of that. Then there are *practical problems in the physical world* that he tried to solve. These focus on the defense of Syracuse and the needs of war. And finally, there were the *requests of the government*; there is no question that Archimedes was working for the government of the time.

The Sycarusia

Horst Nowacki's article in this conference addresses Archimedes and ship design in much greater detail, but let me just touch on the Syracusia. Figure 5 is an artist's concept of the glamorous Syracusia drawn in the 1800s.

We do know—or at least we speculate—from writings admittedly not very close to Archimedes' time that Syracusia was a very large ship. It was about 110 meters (360 feet) long. It is sometimes claimed to be the largest transport ship of antiquity. It had three decks. It could carry about 1700 tons, about 1900 passengers, 200 soldiers, as well as a catapult. The Syracusia sailed only once, to berth in Alexandria where it was given to Ptolemy (Ptolemaios) III Euergetes of Egypt (reigned 246–222 BCE) and was renamed the Alexandria. The reason that it did not continue

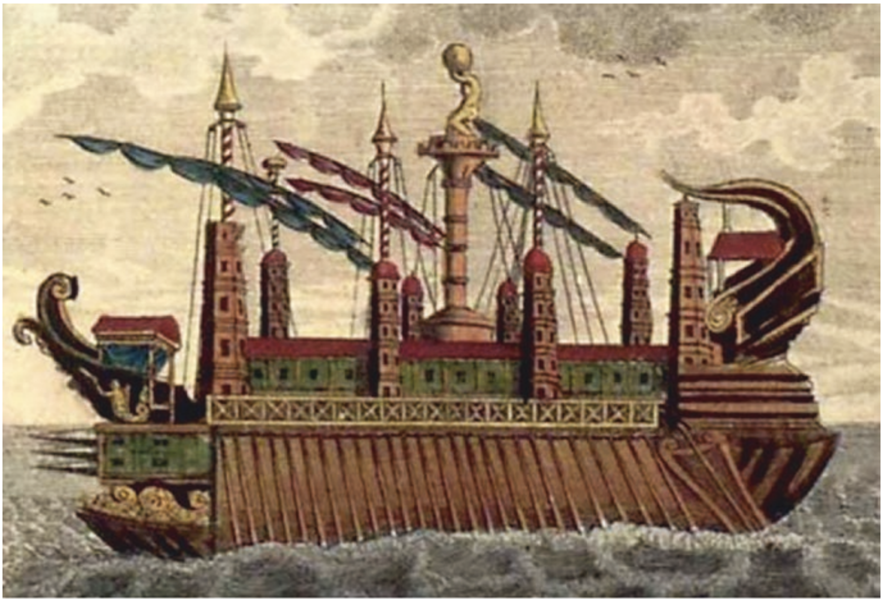

Figure 5 An 1798 exaggerated depiction of the Syracusia, a large cargo and passenger transport ship of Ancient Greece and one of the largest of antiquity

sailing was because it was gigantic—much too large for the time; there was not a harbor that could receive it. And it was too slow.

Nevertheless, there are several inventions that were connected directly or indirectly to the Syracusia, depending on how much you believe people who wrote about it. These writers describe complex systems of winches and pulleys that allowed the ship to be launched by only a few men or to be carried to land. The Archimedes screw allegedly was used to haul water from the ship's hulls, as a defense mechanism. Many years later, Athenaeus (who lived from the late second century to the early third century BCE) wrote: On these decks was placed a catapult which hurled a stone weighing what we would measure today to be about 75 kilograms and an arrow that was about 5 meters long; this engine was devised and built by Archimedes and enabled every arrow to be thrown about 180 meters.

The Archimedes Screw

We mentioned Archimedes screw—a solution to a practical problem which is still in use two millennia after its introduction (Figure 6). Dirk Nuernbergk will later discuss this device in depth. The Archimedes screw is a machine for raising water. It was used in Egypt for irrigation of fields which are not inundated directly by the water of the Nile. It was used in Spain to pump water out of mines. Allegedly it was used by

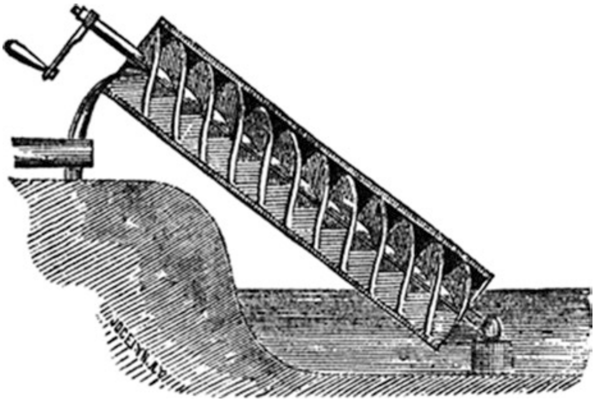

Figure 6 Archimedes screw

Archimedes to keep the hulls of the Syracusia dry. Although it is attributed to Archimedes, the correctness of this attribution has been disputed. Some writers say that Archimedes may have seen it when he was in Egypt and then adopted it. There is no attribution to Archimedes in the writings of Strabo, Philo of Byzantium, or Vitruvius (who wrote about the machine but did not attribute its design to Archimedes.)

The device is a screw inside a hollow pipe, usually turned by a windmill or by manual labor. As the shaft turns, the bottom end scoops up a volume of water. This water slides up in the spiral tube and pours out the other end. The contact surface between the screw and the pipe does not need to be perfectly watertight. Water from one section leaks into the next lower section so that a sort of mechanical equilibrium is achieved. Galileo was so impressed with this that he said, *This is not only marvelous, it is even miraculous (Non solo maravigliosa, ma miracolosa)*.

In 1983 when the Italian government wanted to commemorate Archimedes on a national stamp, it decided to use the Archimedean screw as one invention to put on the stamp (Figure 7).

Figure 8 is a picture from National Geographic magazine showing an Egyptian farmer using the Archimedes screw to irrigate a field, and Figure 9 is a picture of the Shipwreck Rapids water ride at SeaWorld Theme Park in San Diego.

A wastewater treatment plant outside of Memphis, Tennessee, uses seven huge Archimedean screws as part of its processing of debris-laden wastewater (Figure 10).

There are also descendants of the Archimedes screw, including the screw conveyor, that haul bulk materials with the pushing action of rotating blades (Figure 11). This device is used to convey powders, pellets, flakes, crystals, granules, and grains.

Several types of screw conveyors are built and used today in industrial plants, including a compact and totally enclosed design that is used in food processing plants; a 40 feet. long model that carries malt and rice from storage to a factory mill; and a helicoid screw conveyer that delivers 50 tons of coal per hour to a boiler-room bunker.

Figure 7 1983 Italian stamp

Figure 8 Archimedes screw (Helen and Frank Schreider, National Geographic)

Figure 9 Archimedes screw, SeaWorld Theme Park

Figure 10 Memphis TN Wastewater Treatment Plant [Manufactured by Lakeside Equipment Company of Bartlett, Illinois, USA]

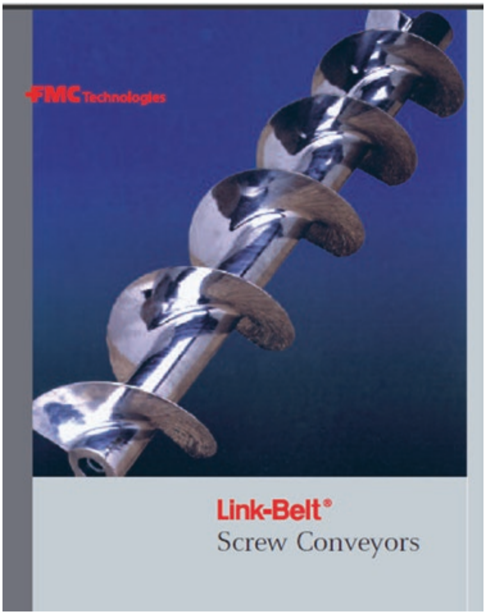

Figure 11 FMC Technologies recognizes the contribution of Archimedes in its commercial catalog; essentially Archimedes' original design has not changed in 23 centuries

Archimedean Astronomical Devices

On the other side of the spectrum of Archimedean inventions is an astronomical device that did not survive to modern times. Michael Wright is an expert on astronomical devices that date back to Archimedes' time, and he will speak about them later.

Archimedes' mechanical model showed the motions of the sun, moon, and planets as viewed from the earth. It is a complete, spherical, open planetarium in which one revolution of the sun, moon, and planets performs the same motions relative to the sphere of the fixed stars as they do in the sky in one day—and in which one can also see the successive phases and eclipses of the moon. Rowley's orrery from 1752 (Figure 12) may resemble Archimedes' device but shows the motions of the planets and moons of the solar system in a heliocentric model.

Much was said about the ingenuity of Archimedes' planetarium. It represents the mutually independent and widely different motions of the sun, moon, and planets by one mechanism simultaneously. It appears that Archimedes himself attached great significance to this particular construction. Pappus tells us (actually quoting others in *Collectio VIII*, 3; 1026) that there was a lost work of Archimedes devoted to sphere-making. The Archimedes' planetarium is praised by multiple authors. Cicero (106–43 BCE) writes that Archimedes must have been "endowed with greater genius than one would imagine it possible for a human being to possess" because he could build such a device. Claudius Claudianus (370–404 CE) writes a poem called "In sphaeram Archimedes." In the poem, Jupiter looks from the sky and tells others in the heavens how impressed he is of this sphere of glass that The Old Man of Syracuse has made.

Figure 12 Rowley's orrery

The Archimedes' planetarium found a role in philosophy and public debate—of course many years after Archimedes. Cicero uses it in order to argue against the Epicureans. In another context, Cicero uses the planetarium to claim that Archimedes was divine. Sextus Empiricus (160–210 CE) used it to claim the superiority of the creative intellectual principle against the material world. Lactantius (c. 250–c. 325 CE) used it against the atheists using an argument which was repeated many times: If a human can produce such a thing—if a human could make something that marvelous—could not then God have created the prototype of that which the intelligence of his creature was capable of imitating? Hence, the proof of the existence of God.

The Pulley, Multiple Pulley, and Catapult

We move now to much more practical Archimedean applications—the lever, the pulley, the multiple pulley, and the catapult. We all know the famous quote attributed to Archimedes: *Give me a place to stand and I will move the Earth.*

The lever and the wedge had been used in various forms for centuries prior to Archimedes. The lever appeared as early as 5000 BCE in the form of a simple balance scale. Several thousand years later, workers in the near east in India had built and used the *shaduf*, a crane-like lever that is used as an irrigation tool (Figure 13). It was first used in Mesopotamia about 3000 BCE. A shaduf is a long, wooden lever that pivoted on two upright posts. On one end, you have a counterweight and, on the other one, a pole with a bucket attached. To operate it, you push down on the pole to fill the bucket with water and then the counterweight helps lift the filled bucket. If you look up shaduf on YouTube, you can see some that are still in operation today.

Archimedes' law of the lever states: *Magnitudes are in equilibrium at distances reciprocally proportional to their weights*. (The debate continues about what exactly Archimedes proved and what he assumed when he made this statement.) The interesting thing for us, however, in addition to the great importance of the principle

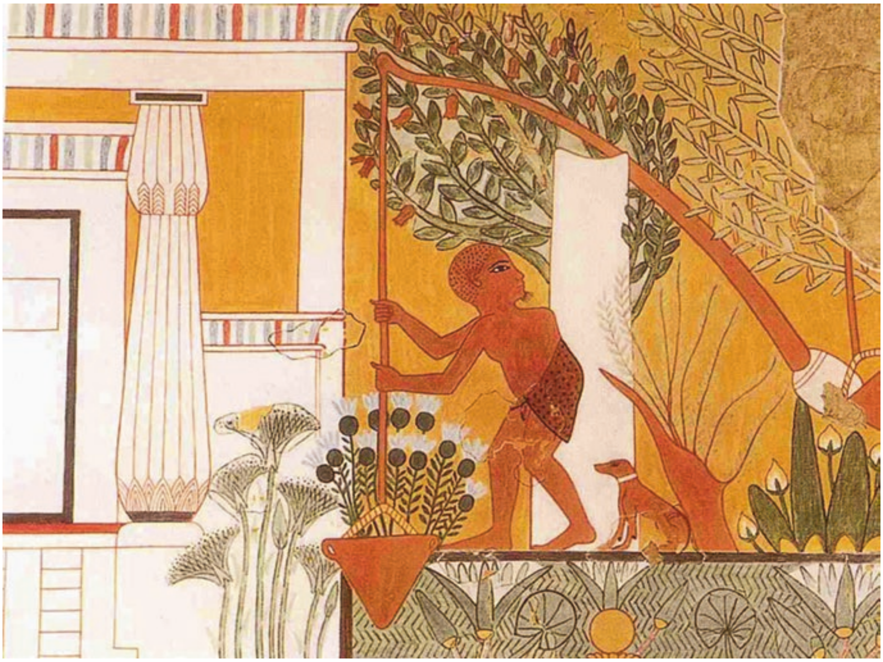

Figure 13 Painting of a shaduf in an ancient Egyptian tomb

itself, is the development of well-prescribed methodology. The problem that the lever solves is drawn from the physical world, but it is most illuminated when treated by an abstract mathematical approach. Archimedes advances the use of such treatment. Following in the footsteps of Euclid, he sets up a few axioms which are simple abstractions of everyday experience. From these axioms, he then derives, step by step, the less obvious properties.

I want to emphasize a point about the influence of physical world experimentation on Archimedes' more abstract work. In the method, Archimedes writes:

> *... certain things first became clear to me by a mechanical method, although they had to be proved by geometry afterwards because their investigation by the said method did not furnish an actual proof.*
>
> *But it is of course easier, when we have previously acquired, by the method, some knowledge of the questions, to supply the proof than it is to find it without any previous knowledge.*

This is, I think, a virtual statement of what engineering is.

Archimedes' contributions in this arena include a theoretical explanation considering that the pulley operates according to much the same principle as the lever. He introduces the principle of mechanical advantage—a measure for force amplification that is achieved by a tool, a mechanical device, or a machine system.

By 400 BCE, the Greeks were already using compound pulleys. Whereas a single pulley provides little mechanical advantage, compound pulleys that incorporate several wheels significantly reduce the effort needed to lift large weights. This compound pulley mechanism was crucial to the development of large cranes and artillery machines that interested Archimedes.

Archimedes appears to have perfected the existing technology, creating the first fully realized block-and-tackle system using compound pulleys and cranes to create a lifting machine. He demonstrated it, according to one story, by moving a fully loaded ship single-handedly while remaining seated some distance away. Whether or not this feat happened, we do not know, but the story amazed writers in antiquity. In our late modern era, compound pulley systems are widely used, for example, in everyday devices like elevators and escalators.

Plutarch describes a scene where Archimedes demonstrates his lifting machine for the King of Syracuse. When Archimedes made his proud boast about moving the earth, King Hiero, amazed by this claim, asked Archimedes for a practical demonstration. At the time, there was a ship in the dock that could not be drawn out except with a great deal of labor by many men. The ship was fully laden with passengers and freight. Plutarch then writes:

> *"[Archimedes] seated himself at a distance from her, and without any great effort, but quietly setting in motion with his hand a system of compound pulleys, drew her towards him smoothly and evenly, as though she were gliding through the water." (Plutarch's Lives, 'Marcellus' xiv. 8–9)*

This drawing from a book by Lazos [13], following Vitruvius, shows the kind of lifting machine ascribed to Archimedes (Figure 14). Lazos gives a step-by-step description on how to build such a system.

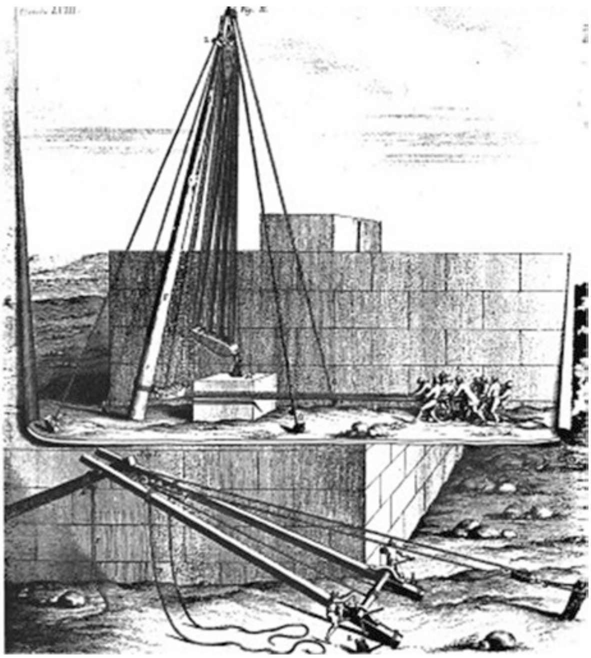

Figure 14 Lifting machine (trispaston) from Archimedes' times (Lazos [13])

Machines of War

Plutarch describes—admittedly many years after his time—several of Archimedes' war machines:

. . . when Archimedes began to ply his engine, he at once shot against the land forces all sorts of missile weapons, and immense masses of stone came down with incredible noise and violence against which no man could stand. For they knocked down those upon whom they fell in heaps, breaking all the rank and file.

Plutarch also describes the Archimedes' claw or iron hand:

...the most formidable of his war machines ... a dreadful thing to behold that lifted ships up into the air by an iron hand or beak, like a crane's beak. When they had drawn them up by the prow and set them on end, they plunged them to the bottom of the sea.

Models of the claw atop the walls of Syracuse and an approaching Roman quinquereme were built by my Drexel University colleague Harry G. Harris Figure 15). I believe this claw design is very close to the one Plutarch describes.

Figure 15 Archimedes claw lifting a Roman ship at the walls of Syracuse (Model by Harry G. Harris of Drexel University)

Archimedes' Principle

Another very important contribution made by Archimedes to the fields of mathematics, engineering, and science is Archimedes' principle; it states: *Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.* Of course we all know the Eureka story that allegedly gave rise to Archimedes' principle. Vitruvius tells it, describing how Archimedes allegedly developed a method for the precise measurement of the volume of irregular objects.

There was a crown. The crown was supposed to be made of gold. There was a suggestion that perhaps some silver had been poured in with the gold. Archimedes was given the task of finding out. While taking a bath, he gets the idea that by measuring the amount of water displaced by a submerged crown, he could determine its density and compare it to the density of pure gold. He immediately jumps out of the bath shouting, "Eureka! I have found it." He then he goes on to perform the experiment in order to determine whether or not there was a deception. According to the story, there was a deception.

This is a nice story, but did it happen? The story does not appear in the known works of Archimedes. The practicality of the method—which incidentally was the object of many writers and experimenters all the way back to Galileo—is questionable because if you write down the numbers, you find out that the accuracy that you need in order to determine the volume of the displaced water is not the one that Archimedes possessed. However, Archimedes could have applied his principle of the lever and what we now know as Archimedes' principle to do what you see in Figure 16, namely, balance an amount of pure gold given to the goldsmith with the crown in air and then submerge the gold and crown in water. If indeed the crown was pure gold, it would balance the pure gold in both air and water. But if there was some silver in it, it would not balance in water. This can be observed quite nicely,

Figure 16 Crown and gold nugget—balanced in air (*left*), but unbalanced in water (*right*)

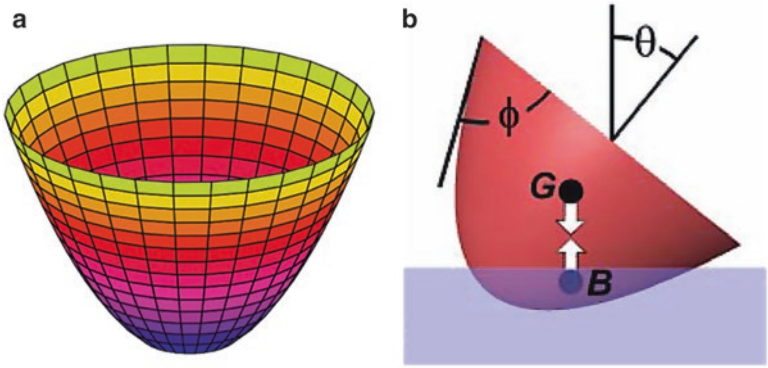

Figure 17 (**a**) A right paraboloid. (**b**) The tilt angle (θ) and base angle (ϕ) of a floating right paraboloid

without having to resort to any exotic ways of measuring. It is conceivable that Archimedes' principle was applied here.

Archimedes' related principle of flotation states that any floating object displaces its own weight of fluid. The most important implication here is in the design and stability of vessels. The stability of different shapes of floating bodies in water is, of course, critical to the design of ships and boats.

Archimedes wrote two books on floating bodies. In Book 1, he provides the development of what is known now as Archimedes' law of buoyancy. He gives a simple, elegant geometric proof that a floating segment of a homogeneous solid sphere is always in stable equilibrium when its base is parallel to the surface of the fluid—when it is either above or below the fluid surface. With this exposition, Archimedes initiates the science of hydrostatics and introduces the concept of fluid pressure. It is interesting that it took almost eighteen centuries until this work was continued (by Stevin, Galileo, Torricelli, Pascal, and Newton).

In Book 2, Archimedes extends stability analysis from a segment of a sphere to the right paraboloid. Additionally, he provides thorough studies of stability in equilibrium. There are many sophisticated ideas and complex geometric constructions in Book 2, but for many centuries, there was little interest in Book 2. Once the book was augmented with algebra and trigonometry (remember that Archimedes is essentially pre-algebra), plus analytical geometry, and after the field of mechanics reached maturity, there was renewed interest in Book 2. Book 2 is inspiring many new studies today.

Referring to Figure 17, in Book 2, Archimedes developed certain mathematical criteria connecting the paraboloid's tilt angle (θ) to its specific gravity (s) and its base angle (ϕ) when the paraboloid is floating stably. He did all that 1900 years before the invention of calculus. For example, in Proposition 8, Archimedes proved the following (we are using modern mathematical notation):

Archimedes' Proposition 8

A right paraboloid whose base angle ϕ satisfies $3 < \tan^2\phi < 15/2$ and whose relative density s satisfies $s < (1-\cot^2\phi)^2$, has precisely one stable equilibrium position with its base completely above the fluid surface.

The corresponding tilt angle θ is

$$\theta = \tan^{-1}\sqrt{\frac{2}{3}\left(1-\sqrt{s}\right)\tan^2\phi - 2}$$

Contemporary researchers—including our Conference Chair, Chris Rorres—continue to extend the conclusions of Archimedes' *On Floating Bodies* Book 2 (Rorres 2004; Girstmair & Kirchner 2008). They complete the book in the sense that they discuss cases that Archimedes most probably could not have worked on because he did not have the mathematical and computer tools we have today to apply to this problem.

Concluding Remarks

More than 2000 years after his death, Archimedes' work continues to be the focus of interest of engineers, mathematicians, and scientists. There is much ongoing use of his studies and inventions, and machines that use his ideas are designed, marketed, and sold. His writings continue to be relevant. Archimedes continues to provide us with a model of a person who seeks, finds, and tests solutions to some of the most important technical challenges of his day.

Archimedes moves seamlessly between theory and practice. He triumphs when he uses a systematic and thorough scientific methodology, which is coupled with empirical observation.

Even if we strip away many of the legends from his story, it is still obvious that Archimedes can serve not only as a model of a scientist and engineer with unparalleled analytical abilities but also as a pragmatist with solid grounding in practical needs and challenges. Like engineers of all eras, he is attuned to the needs of his consumers and the political and business climate. He uses the challenges that his clients and the times pose to him (and the funding he receives) to advance both theory and practice, often well beyond the immediate needs or specification of a client. His imagination responds to the needs he and the society are faced with far ahead of the problem statement he is given by funders.

All of these timeless qualities I would very much like to instill and observe in my own engineering students.

References

1. Assorted quotes from: Polybius (Greek, c. 200-118 BC) Universal History; Livy (Roman, 59 BC-17 AD) History of Rome from its Foundation; Plutarch (Greek, c. 45-120 AD) *Parallel Lives: Marcellus*
2. E. J. Dijksterhuis: *Archimedes*, translated by C. Dikshoorn, Princeton University Press, 1956
3. Vitruvius: *De Architectura* (as quoted by Dijksterhuis and others)
4. Athenaeus: *Deipnosophists*, Book 5, http://www.attalus.org/old/athenaeus5b.html#c40
5. Ph. Fleury, La mécanique de Vitruve, Caen, Presses Universitaires de Caen, 1993, p. 113
6. D. L. Simms: "Archimedes and the Burning Mirrors of Syracuse", *Technology and Culture*, Vol. 18, No. 1 (Jan., 1977), pp. 1-24.
7. C. Rorres: "Completing book II of Archimedes's on floating bodies," *The Mathematical Intelligencer* 26(3) (2004), pp. 32–42.
8. C. Rorres and H. G. Harris, "A Formidable War Machine: Construction and Operation of Archimedes' Iron Hand", *Proceedings of the Symposium on Extraordinary Machines and Structures in Antiquity, August 19-24, 2001, Olympia, Greece.*
9. K. Girstmair and G. Kirchner: "Towards a completion of Archimedes' treatise on floating bodies," *Espositiones Mathim*aticae, 26 (2008), pp. 219-236
10. T. Sato: "Archimedes' lost works on the centers of gravity of solids, plane figures and magnitudes", *Historia Scientiarum*, *Japan. Stud. Hist. Sci.* 20 (1981), pp, 1-41.
11. Claudian, Vol 2. Claudianus, Claudius. Maurice Platnaeur. William Heinemann; G.P. Putnam's Sons. London; New York. 1922.
12. T. Chandras: "Archimedes' life, works, and machines", *Mechanism and Machine Theory*, Volume 45, Issue 11, 2010, pp. 1766-1775.
13. C.D. Lazos: *Archimedes:* Th*e Ingenious Engineer*, Aiolos Publishers, Athens (1995.) in Greek.
14. J. G. Landels, *Engineering in the Ancient World*, University of California Press, Berkeley & Los Angeles, 1978.

Web Resources

15. Chris Rorres: Archimedes Home Page. https://www.cs.drexel.edu/~crorres/Archimedes/contents.html
16. J. J. O'Connor and E F Robertson: Archimedes Biography. http://www-history.mcs.st-and.ac.uk/Biographies/Archimedes.html
17. Engineering Timeline: http://www.preengineering.com/resources/timeline.htm
18. Gravity, Buoyancy, and the Stability of a Floating Objects: http://stemak.org/sites/default/files/Buoyancy%20Experiment.pdf
19. Stability and Buoyancy: http://user.engineering.uiowa.edu/~me_160/lecture_notes/lec04_2012.pdf

Archimedes the Military Engineer

23 Centuries of Defense-in-Depth, from 213 BCE to 2013 CE

Larrie D. Ferreiro

Introduction

Archimedes was not the first engineer to apply the principles of defense-in-depth to a military campaign, but his unique and systematic approach to the problem caught the attention of his contemporaries as well as that of future generations. Indeed, David Lane, historian of operations research, cites Archimedes' defense of Syracuse as a precursor to the systems approach to military operations, 23 centuries before the term was even invented.

Archimedes was far ahead of his time in other ways, as well. His carefully thought-out defensive approach suggests that he understood and followed the maxim later expressed by the great Prussian strategist Carl von Clausewitz in 1832: *War is the continuation of politics by other means*. Moreover, he saw that engineering was also the continuation of politics by other means, in that he developed his engineering approach to serve the political and military strategies. It was a clear demonstration that military and engineering strategies are most effective when they serve clear political goals.

Archimedes' chosen strategy to counter the Roman siege of Syracuse is today known as defense-in-depth. This is more than merely a layered defensive system. *Defense-in-depth* acknowledges that no single line of defense is foolproof against an attack. Instead, a series of multiple, layered defenses cause an attack to lose momentum, often by exploiting different weaknesses that make it harder for the enemy to develop a set of countermeasures. An example from biology is the human body's system of defense against infectious diseases, which consists of:

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- A physical barrier (skin) against infection entering the body
- An innate immune system (e.g., white blood cells) that provides immediate defense against any infection
- An adaptive immune system (e.g., antibodies) that "remembers" the body's response to an infection and prepares itself for subsequent encounters with that same infection

This chapter examines four examples of defense-in-depth, from Archimedes' time to the twenty-first century.

213 BCE: Archimedes and the Defense-in-Depth of Syracuse Against the Romans

Archimedes (c. 287 BCE–212 BCE) lived his life while Syracuse was generally at peace. Syracuse was one of the several Greek city-states on the island of Sicily. At the beginning of the third century BC, the Mediterranean basin was controlled by the Carthaginians in the west and the Greeks in the east. The Romans controlled most of the Italian peninsula and looked to expand its control over Carthaginian territory.

The beginning of the First Punic War (264 BCE–241 BCE) between Rome and Carthage was marked by Syracuse's King Hiero II attacking and laying siege to the rival city to the north, Messana. Archimedes, a member of Hiero's family who was just 23 at the time, may have witnessed this siege, though from the attacker's point of view. During the siege, Hiero aligned himself with Carthage, but after the Romans landed in Sicily, he switched his allegiance to Rome and thus avoided a costly battle. Hiero would hold the peace for the next half-century, until his death in 215 BCE.

During the long peace, Hiero built up his city and called upon Archimedes to oversee the defensive works against future sieges. Archimedes presumably spent much of the next five decades directing the extension and erection of walls on both the landward seaward sides, as well as incorporating a series of "engines" (machines) with varying ranges and capabilities to repel attacks "to any distance." During this time, he apparently traveled at least once to Egypt, where he was reputed to have invented the water screw (Archimedes screw) for irrigation (Figure 1).

The Second Punic War (there would be three Punic Wars, at roughly 50-year intervals) began in 218 BCE. The first phase of the war was marked by Carthaginian attacks against Roman lands, including Hannibal's famed crossing of the Alps with elephants and the Battle of Cannae. Hiero died during the conflict, and the next leader, his grandson Hieronymus, was assassinated after he tried to make peace with Carthage. The ruling faction that succeeded him was strongly pro-Carthaginian. Syracuse's strategic position between the Roman and Carthaginian empires (Figure 2) led Rome to dispatch its general Marcus Claudius Marcellus to bring Syracuse to its side.

Marcellus began his siege on Syracuse in 213 BCE, when Archimedes was about 74 years old. His defensive works had been untested since he began them, many

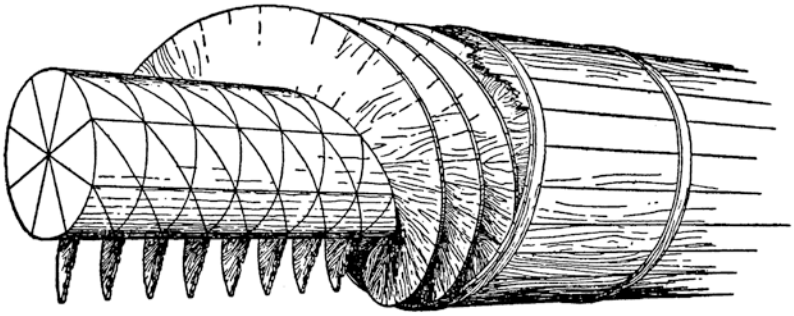


Figure 1 Archimedes' screw

Figure 2 Rome and Carthage. Note Syracuse's strategic position between them

decades before. He had, however, thought meticulously about the Roman modes of attack and where to place his strongest defenses. The city-state was ringed on the landward side by a defensive wall built atop steep cliffs and crags, almost impossible to climb opposed. On the seaward side, a line of tall cliffs to the north provided similar protection from amphibious landings. The main harbor was protected by the island of Ortygia, which provided clear lines of fire and permitted a chain to be drawn across the harbor mouth to prevent incursions. The only accessible part of the city was therefore a thousand-yard stretch of seawall at Achradina, which was

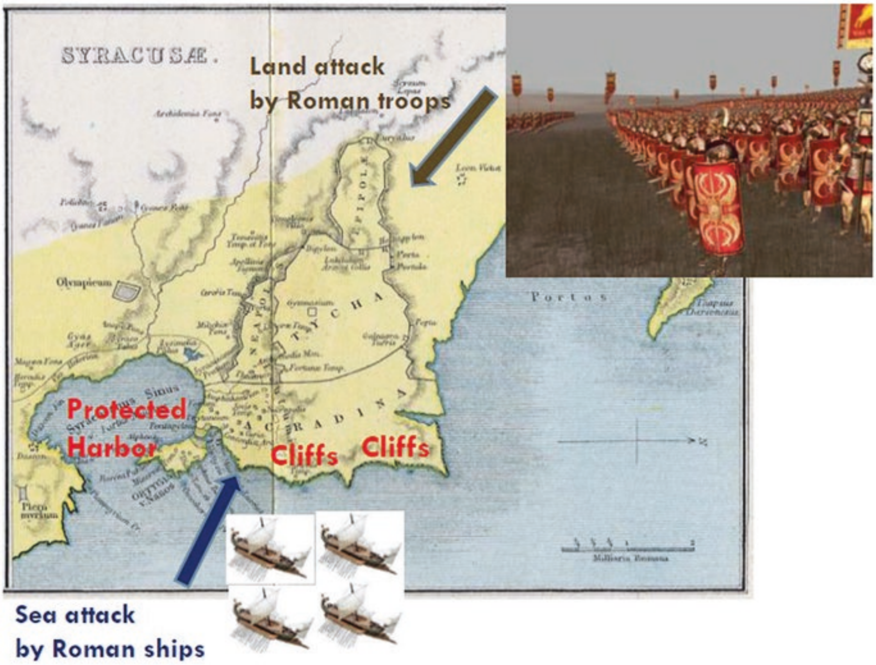


Figure 3 Syracuse under assault

fronted by a shallow and rocky coast. Archimedes carefully used the natural defenses of the terrain to concentrate his defensive forces at the vulnerable points, so that, according to the historian Polybius, "that there was no need for the defenders to busy themselves with improvisations; instead they would have everything ready to hand, and could respond to any attack by the enemy with a counter-move."

The Roman assault force consisted of both naval and army elements. Marcellus commanded a fleet of Roman ships that would attack the city-state at Achradina. At the same time, his joint commander Appius Claudius Pulcher brought troops with scaling ladders and towers to assault the north gate (Figure 3). Marcellus and Appius evidently expected that a quick siege would avoid more costly, time-consuming war by attrition. They were soon disabused of this by Archimedes' defenses.

Polybius, writing almost 70 years after the siege, focused his narrative on the naval assault at Achradina. His sources were apparently particularly impressed by Archimedes' system of defense-in-depth. In fact, Marcellus' attacking force was only lightly armed, which greatly contributed to his being stymied by Archimedes' superior firepower. Marcellus' naval force consisted of 60 quinqueremes, which were oared, ram-equipped warships similar to the more famous triremes, but larger and with five rowers per tier of oars instead of three. The ram bows were of course useless against a seawall, so the actual assault was conducted using four sambucae ("harps"), each of which consisted of two quinqueremes lashed together side by side for stability, with a large scaling ladder mounted to the decks. These sambucae

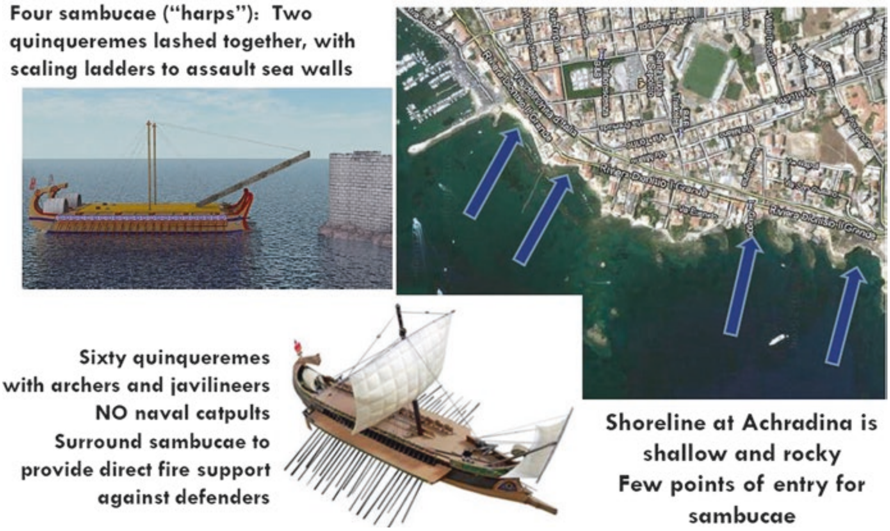


Figure 4 A Rome's naval siege at Achradina

would only be able to attack where the rocky coast was deep enough to allow it to come right up to the seawall, so that they could lean their ladders against the wall, allowing Roman troops to assault by escalade. Surrounding the sambucae were the quinqueremes with archers and javelineers on deck, who would provide fire support against the defending Syracusians (Figure 4). Marcellus followed standard Roman practice of the time, which relied heavily on massed troops, and did not employ siege artillery such as stone launchers, catapults, and crossbows. Archimedes, by contrast, had amassed a large, integrated arsenal of these weapons, plus one of his own design. For many months, this would render the Roman siege impotent.

Archimedes had developed a system of defense-in-depth that took advantage of Rome's inherent weakness in artillery and dependence on massed troops. The assaulting troops were massed against the city walls, and with javelineers throwing their spears upwards against defending troops, their effective range was about 15–20 yards. Archers could fire at ranges up to 60–80 yards. Not only were these weapons relatively short-ranged, but their throw weight and penetrating power against shields and armor were quite limited. Against this, Archimedes had arrayed multiple layers of defense with greater range, throw weight and capabilities. Rome's naval assault may have been doomed before it began (Figure 5).

Archimedes, by contrast, used a number of well-known Greek weapons to counter the Roman siege, one of which, the catapult, had been developed in Syracuse a century before his birth (Figure 6). At very short distances, he deployed cranes to drop large stones on the attacking sambucae and any quinquereme which happened to be just below the ramparts. Those ramparts would also (presumably) be lined with archers and javelineers whose height gave them a distinct advantage against the Romans in the ships below. At medium range and long ranges, a variety of stone

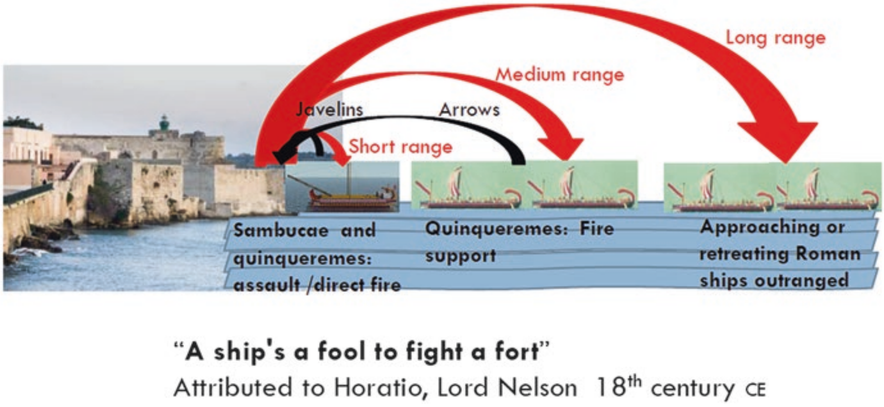


Figure 5 Archimedes' system of defense-in-depth

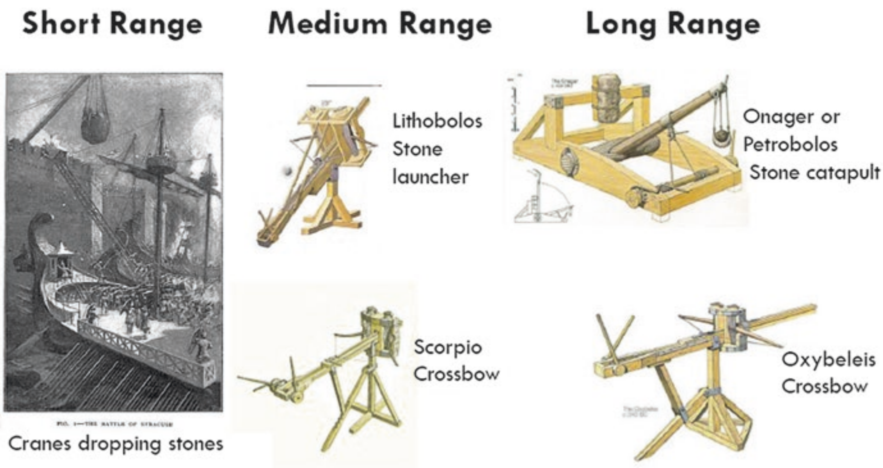


Figure 6 Well-known weapons used by Archimedes in the siege of Syracuse

launchers, catapults, and crossbows, powered by twisted animal sinews that provided the requisite torque, hurled rocks and bolts at high speed with great penetrating force. The largest weapons, such as the petrobolos and oxybeleis, could hurl heavy artillery out to several hundred yards, far outranging the Roman archers. This not only allowed the Syracusians to begin attriting Marcellus' shipborne troops long before they got into bow range but also denied them safe refuge upon retreat until they were far offshore. On the landward side, similar engines in a layered defensive array wreaked havoc on Appius' soldiers.

Polybius made note of one unusual weapon, invented by Archimedes, which caught Marcellus by surprise:

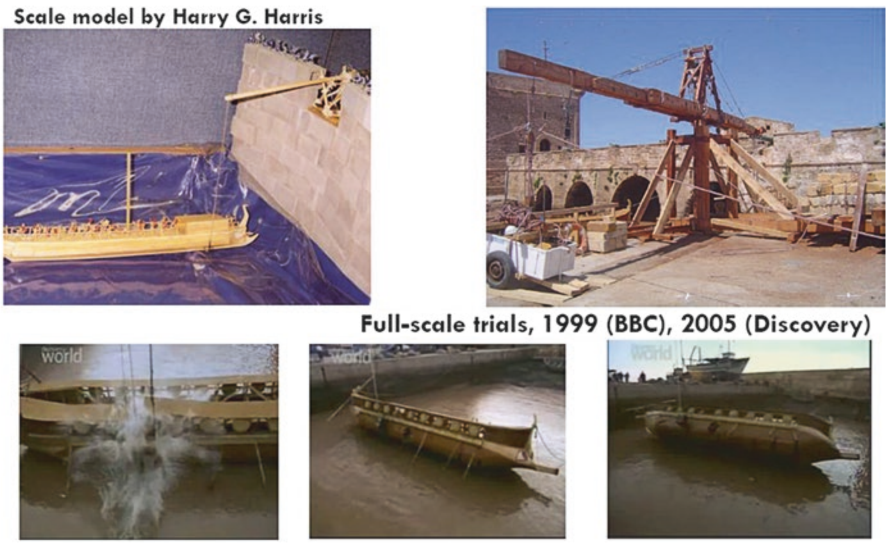


Figure 7 Archimedes' claw

> *A grappling-iron attached to a chain would be let down, and with this the man controlling the beam would clutch at the ship. As soon as the bow was securely gripped, the lever of the machine inside the wall would be pressed down. When the operator had lifted up the ship's bow in this way and made her stand on her stern, he made fast the lower parts of the machine, so that they would not move, and finally by means of a rope and pulley suddenly slackened the grappling-iron and the chain. The result was that some of the vessels heeled over and fell on the sides, and others capsized, while the majority when their bows were let fall from a height plunged under water and filled, and thus threw all into confusion.*

Historians have argued endlessly over whether "Archimedes' claw" ever existed. The description certainly appears fanciful in some parts: a fully laden quinquereme displaced around 100 tonnes, far greater than could be "lifted up by the bow" and which would certainly break the supporting beams. However, in 1999 and 2005, BBC and Discovery Channel (Figure 7) sponsored model and full-scale trials of a similar device and found that it could, conceivably, tip over such a vessel. (It should be noted that in 2010, the Discovery Channel's show *Mythbusters* was "directed" by President Barack Obama to determine whether the legend of Archimedes using burning mirrors to destroy the Roman fleet was plausible. It was not.)

As Polybius noted, *Marcellus' operations were thus completely frustrated by these inventions of Archimedes*. The system of defense-in-depth, with a mix of weapons and defenses that could protect the city at various ranges, made it impossible for the Romans to develop any systematic means of counterattack. This did not, however, spell victory for Syracuse. Marcellus and Appius settled on a protracted siege that would starve out the inhabitants. Though Carthage was able to occasionally break through the Roman blockade, supplies soon dwindled. After a stalemate that lasted almost a year, in 212 BCE, Appius' troops were able to breach the walls when they were left unguarded during a festival. Though Marcellus gave

Figure 8 Ram bow of *Olympias*, a modern replica of a Greek trireme

strict orders to take Archimedes alive, he/Archimedes was killed by a soldier who apparently did not recognize him. Another 8 months of siege against the interior citadel starved the population into submission. Syracuse and the rest of Sicily fell under Roman rule. Ten years later, the Second Punic War ended with a Roman victory over Carthage. Rome now dominated the Western Mediterranean and was set firmly on the path to becoming the world's greatest and most influential empire.

1866: The Screw Propeller and the Ram: Naval Defense-in-Depth

Archimedes would have been familiar with the many varieties of oared warships of his age—triremes, quadriremes, and quinqueremes—distinguished by the number of rowers in each tier of oars. The navies of Greece, Rome, and Carthage maintained fleets of these warships to fight for dominance of the Mediterranean. They were all equipped with the same weapon—a massive bronze ram, sturdily fixed to the keel structure, which would hole the enemy ship by breaking apart hull timbers (Figure 8). Fleets would face head to head and execute complex maneuvers in order to ram the enemy. For a thousand years, from 500 BCE until 1500 CE, the oared ram galley was the primary maritime weapon of the Mediterranean.

The introduction of naval artillery in the 1500s vaulted the sailing warship to primacy. For almost three centuries, the sailing ship of the line, equipped between 60 and 120 cannon that could devastate an enemy's hull and rigging, became the symbol and reality of maritime power (Figure 9). Naval battles were no longer

Figure 9 How battles were fought before steam: French and British ships of the line at the Battle of Chesapeake Capes, September 5, 1781

fought head to head, as in the day of the ram, but in long, parallel lines of battle where opposing fleets would sail side by side, pounding each other in hours-long gun duels.

Steam power began to take the place of sail by the middle of the nineteenth century. At first, steam engines drove paddlewheels, a well-understood technology derived from the waterwheels which dotted every nation's countryside. Steam power was soon adopted by navies, as it gave the advantage of not being reliant on the fickle wind. Steam came at a price; the massive propulsion machinery (boilers, pistons, and coal bunkers) demanded considerable volume and manpower, paddlewheels were vulnerable to being destroyed by gunfire, and the paddlewheel boxes took up valuable real estate in the center of the gun deck. This considerably reduced the firepower available. For example, a 3000 tonne sailing warship might carry 74 guns, but a comparably sized steam warship would have just 20 guns (Figure 10).

The invention of the screw propeller, claimed by dozens of individuals from the 1820s to the 1840s, solved many of these problems when it was adopted. The most influential inventor was a British farmer named Francis Pettit Smith, who was inspired by the Archimedes screw to develop a screw-shaped propulsion device for ships. Over several years, he refined his device from an elongated screw with several turns that resembled Archimedes' original device, to a screw with two turns. Smith even dubbed his test ship for the screw propeller SS *Archimedes*, which was launched in 1839. Based on their experiences, Smith and others further refined the idea to a propeller with a single turn and multiple blades. The fact that the screw could be located below the waterline both reduced propulsion vulnerability to

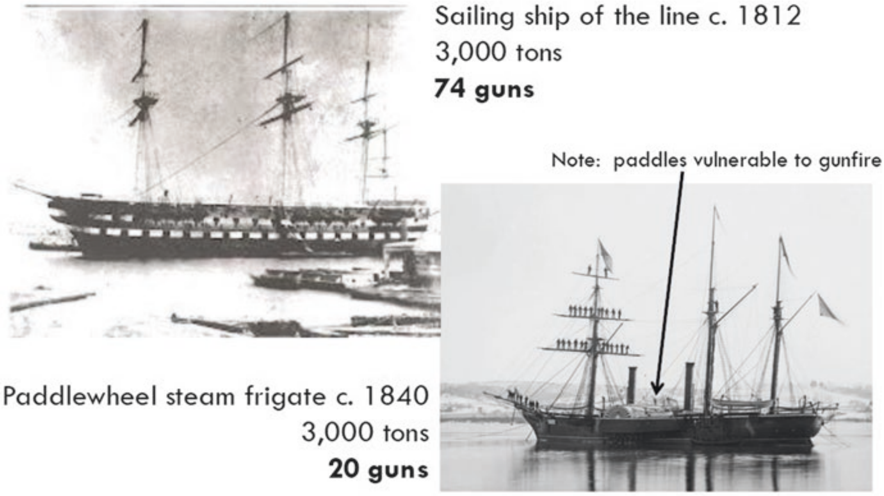


Figure 10 The problem with steam paddlewheel ships—lack of firepower

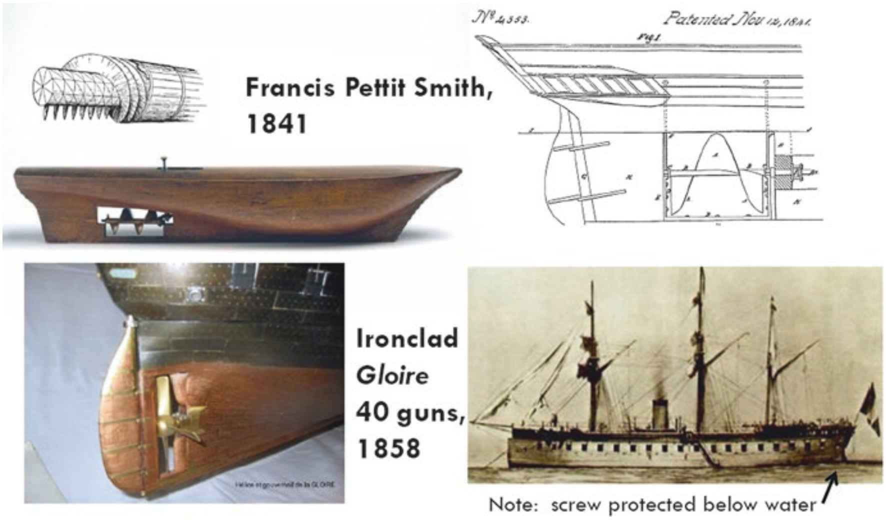


Figure 11 Screw propulsion comes of age

gunfire and freed up deck space for weaponry, often doubling of the number of guns that could be carried by a warship, though still fewer than the older sailing ships of the line (Figure 11).

As steam propulsion was coming of age, iron was adopted over wood as the shipbuilding material of choice. For warship builders, this presented both a great advantage and a serious problem. On the defensive side, iron was stronger than wood and more resistant to damage, but on the offensive side, it was far more

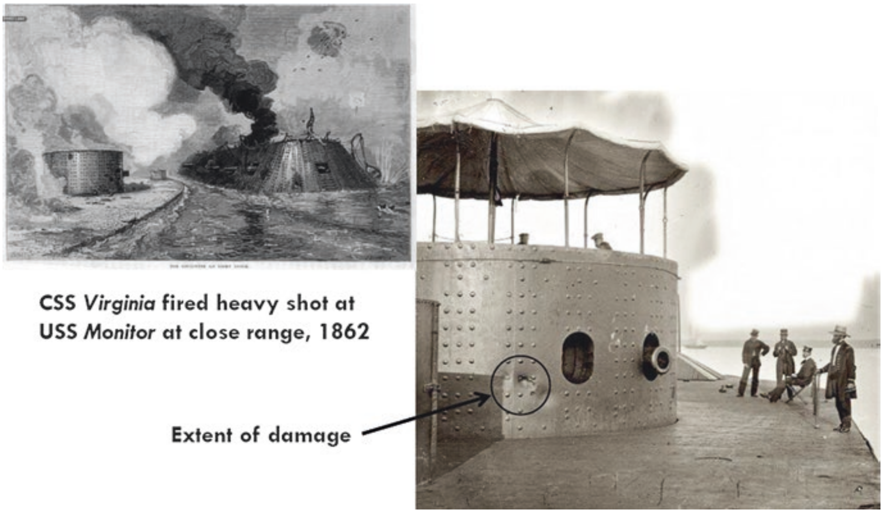


Figure 12 The problem with iron ships—protection

difficult for naval artillery to damage an enemy ship. During the 1862 Battle of Hampton Roads between the Confederate and Union ironclads CSS *Virginia* and USS *Monitor*, shot bounced off the ships' hulls and turrets even though they were firing at point-blank range (Figure 12).

The French navy had been the first to understand this problem and develop a solution that would provide the needed defense-in-depth. Just as the newfangled screw propeller hearkened back to the Age of Archimedes, its newest weapon, the ram, came from the same era and was, in fact, inspired by the Archimedes screw. Nicolas-Hippolyte Labrousse was a brilliant 22-year-old lieutenant in the French navy when in 1839 he witnessed the trials of Smith's screw-propelled SS *Archimedes*. He quickly realized that the screw could be harnessed to turn the ship into a ram like the Greek and Roman galleys and overcome the numerical advantage of the British navy. Labrousse drafted his idea the following year in a memorandum that argued for the "absolute" combat of ramming: … *as in Rome, the ram will re-establish equilibrium in favor of courage, and diminish superiority founded on greater numbers*. The idea was widely discussed and even tested over the next 20, but it was not until French industry had advanced sufficiently that the ironclad warship *Solferino* with a heavy, pointed ram could be built (Figure 13). As its constructor Dupuy de Lôme said, the warship … *could rip open by the shock, at even a moderate speed, any armored ship it attacked*. The British navy responded by constructing its own ram-equipped warship. The idea spread quickly; within months, navies around the world were ordering ram-equipped warships. Even the aforementioned CSS *Virginia* was built with a ram.

The combination of iron, screw, and ram provided a novel system of defense-in-depth that navies quickly adopted (Figure 14). First, the protection afforded by iron hulls and underwater screw propulsion allowed a ship to approach the gun-firing

Figure 13 The ram is introduced to iron warships: French ironclad *Solferino*, 1861

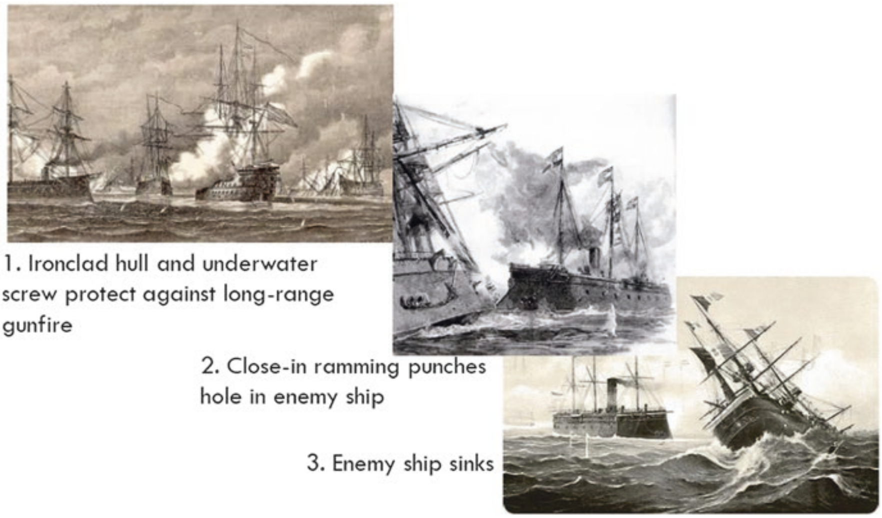
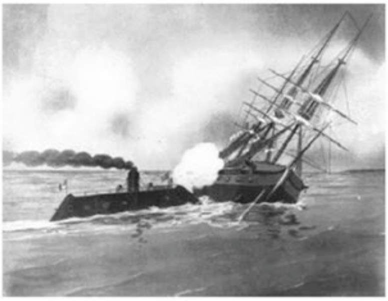
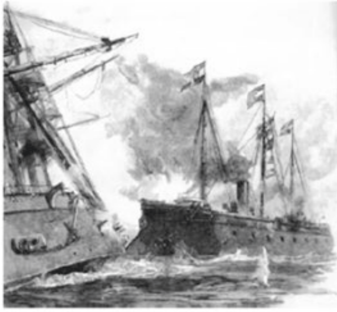


Figure 14 Screw and ram create defense-in-depth



CSS *Virginia* rams USS *Cumberland*
Battle of Hampton Roads,, 1862



Ferdinand Max* rams *Re d'Italia
Battle of Lissa, 1866

Huáscar* rams *Esmeralda
Battle of Iquique, 1872

Figure 15 Screw and ram in action

enemy vessel with relative impunity. Though the exchange of fire would likely cause superficial damage to both ships, the mortal wound would be inflicted when the ramming ship closed within striking distance and punched a hole in the enemy ship. The enemy vessel would immediately begin flooding as the ram was withdrawn, causing it to sink rapidly.

This system of naval defense-in-depth saw widespread usage in the 1860s and 1870s. The day before the ironclad duel during the aforementioned battle of Hampton Roads, CSS *Virginia* rammed and sank the Union frigate USS *Cumberland*. During the Third War of Italian Independence, the Austrian navy defeated the larger Italian navy at the Battle of Lissa (in the Adriatic) on July 20, 1866, in part because the Austrian flagship *Erzherzog Ferdinand Max* rammed and sank the Italian flagship *Re d'Italia*. On May 21, 1879, in the Battle of Iquique during the War of the Pacific, the Peruvian ironclad monitor *Huáscar* rammed and sank the Chilean corvette *Esmeralda* (Figure 15).

Thus, the Archimedes screw and the ram from antiquity combined to create a system of defense-in-depth that dominated naval thinking for several decades during the late nineteenth century. However, technological developments in the form of underwater torpedo and improvements in artillery were already changing the nature of naval warfare. Torpedoes now allowed a ship to hole and sink an enemy vessel at long distances without ramming. At the same time, more powerful guns, larger exploding shells, and improved gunfire control made naval artillery more deadly against even well-armored ships. By the turn of the twentieth century, ramming had become the equivalent of the bayonet charge against machine guns and was quickly dropped.

1940: The Maginot Line: A Case of Failure of Defense-in-Depth

The twentieth century ushered in more than just improvements to artillery and gunfire control. Warfare itself became industrialized, with revolutionary developments in communication, transportation, and weapon technologies that fundamentally altered the conduct of war. The World War I saw these elements brought together in devastating ways. The aftermath was staggering: 16 million dead, 21 million wounded or missing, and hundreds of billions of dollars in destruction. The effect upon France was shattering. Five percent of its population was killed outright—higher than any other major power—and one person in ten was wounded. The national birth rate plummeted. By 1930, France had just 40 million inhabitants compared with 70 million in Germany, on top of which there was a widespread shortage of military-aged men.

War planners knew that static defenses required fewer troops, so they carefully planned a large-scale system of defense-in-depth that would ensure the limited number of soldiers would be placed in the most advantageous positions (Figure 16). The primary defense, known as the Maginot Line, was directly along the French-German border. Its purpose was to greatly slow down any German advance, attriting those forces and buying time for French troops to be mobilized to meet the onslaught. Further north, the planners assumed that the Ardennes Forest along the borders with Luxembourg and Belgium were too dense and rugged to permit large-scale mechanized assault (tanks and mobile artillery) to cross. The region between the Ardennes and the English Channel would see the largest concentration of French troops to face down the anticipated German "right hook" through Belgium. At the same time, French planners anticipated that Britain would come in on their side if Belgian neutrality were violated.

The Maginot Line was built at enormous cost between 1930 and 1940. It was not a single barrier but rather a large-scale system of defense-in-depth (15–20 miles deep) that relied on a number of systems, tactics, and technologies to slow down and weaken the enemy (Figure 17). At the front was a series of antitank barricades to slow down tanks and other heavy vehicles, making them susceptible to counterfire. Behind those, blockhouses and strong houses, often camouflaged as residential homes, housed troops and antitank batteries to "sound the alarm" and provide

Figure 16 Maginot Line: part of French defense-in-depth

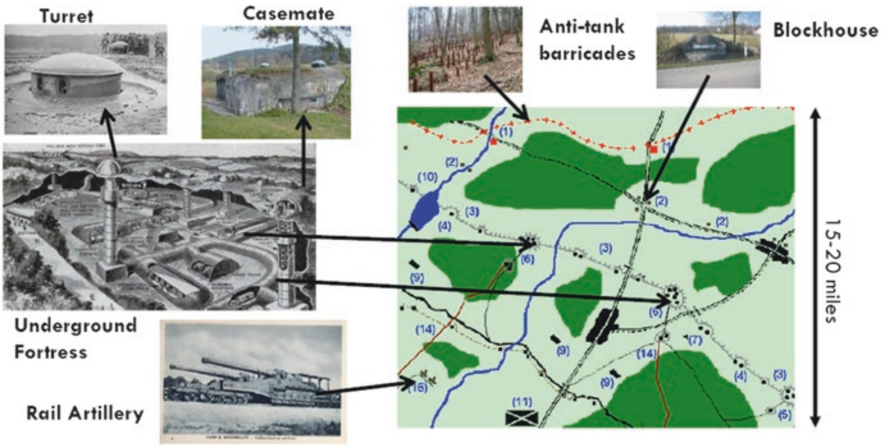


Figure 17 Defense-in-depth at the Maginot Line

counterfire. About 6–10 miles behind the front was the principal line of resistance, a network of underground fortifications, aboveground turrets and casemates, outposts, and shelters. These self-contained works housed the primary artillery and infantry forces to attack the (presumably depleted) enemy advance. Behind the

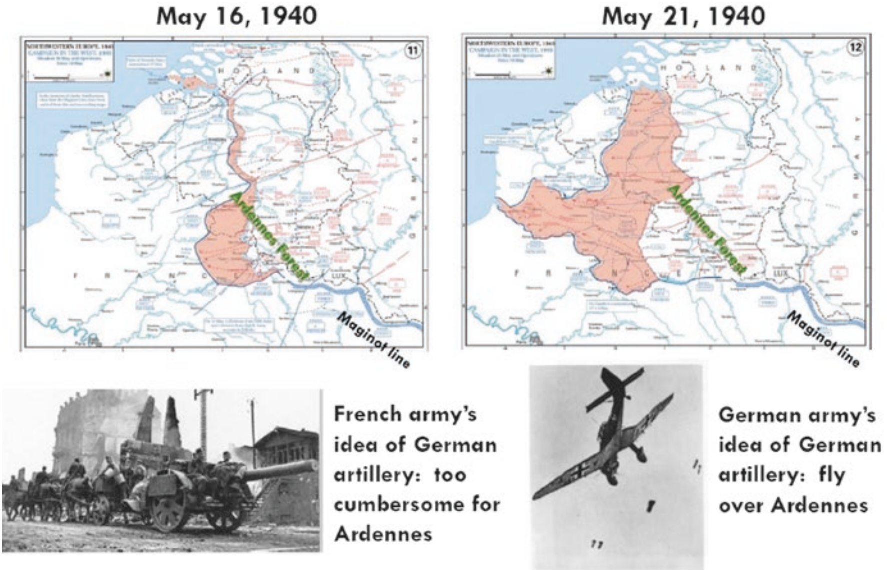


Figure 18 German forces bypass the Maginot Line and advance through Ardennes

principal line of resistance were additional defenses, including rail artillery, which could be quickly moved to different zones to destroy German troops that had broken through the principal line of defense. Behind the Maginot Line, masses of French troops would mop up the remaining German forces.

The German invasion plan of France was designed to deal with the Maginot Line. The heart of its strategy was to avoid attacking the Line head-on, instead relying on a flanking attack through the Ardennes Forest. The French planners considered the forest too dense to allow tanks and heavy artillery to pass and only lightly defended the line opposite. The German army, in a stunning display of what is today called “asymmetrical warfare,” did not send tanks through the Ardennes but instead relied on the Luftwaffe (notably its JU 87 Stuka dive bombers) for fire support.

Germany first deployed a decoy force in front of the Maginot Line to draw off French forces from the main line of attack. On May 10, 1940, the German army invaded the Low Countries and crossed them in 2 days. As the army began invading France, the Luftwaffe flew over the Ardennes and destroyed French emplacements and army positions, paving the way for German troops to advance. German forces were well into France by May 16, reaching the Channel on May 21 (Figure 18). A month later, all resistance had collapsed and France signed an armistice with Germany, beginning a 4-year long occupation.

Though Maginot Line was, in fact, a well thought-out system of defense-in-depth, it ultimately failed in its purpose to protect France from a German invasion. When the German army did attack the Line directly head-on, the French defenders were generally able to repel the attack. On the larger scale, however, advances in technology and tactics—notably the employment of air power—allowed the invading army to simply bypass the fortifications, surround them from the rear, and cut them off. In the face of these changes, the Maginot Line had simply become irrelevant to modern maneuver warfare.

2013: Cyber Defense-in-Depth for the Twenty-First Century

Defense-in-depth in the twenty-first century is increasingly focused on protecting against "cyberattacks" on the software elements of key systems. This goes well beyond the military. Critical infrastructure is increasingly software-driven: power plants, electrical grids, oil rigs, chemical factories, etc. each requires millions of lines of code to operate. Malicious code (viruses, worms, etc.) can be stealthily inserted by an attacker, which can lie dormant and unseen for long periods before wreaking havoc on a system.

Such attacks are becoming more prevalent as both corporations and governments increasingly connect their computer systems via the Internet and other networks and store information in vast third-party systems known as "the cloud." Cyberattacks can come from both state and non-state actors, and it is not easy to identify the source. In August 2012, Saudi Aramco, the largest oil production company in the world, was attacked by a virus which shut down most of its computers, causing the company to spend a week and many millions of dollars restoring services. Though a group of computer hackers claimed responsibility, suspicion also fell on rival companies and even the Iranian government. In March 2013, several South Korean TV stations and banks were attacked and their computer terminals shut down. Though some evidence pointed to North Korea as the source, a "hacktivist" attack has not been ruled out.

Cyberattacks have increased in both frequency and sophistication. Not only are the number of computer systems and users increasing almost exponentially, but the cyberattack tools available are also multiplying. Many of these tools are being offered as turnkey packages, which require little in the way of direct software skills and knowledge by a potential attacker (Figure 19).

Cyber defense, the deployment and use of systems and tactics against software attacks, has also grown more sophisticated, including the important realization that some attacks will get through any system of defense and to plan for recovery afterwards. Defense-in-depth against cyberattacks now includes the following diverse elements:

- Intelligence: understanding the threat.
- Passive defense: antivirus, firewalls, etc. known to most computer users
- Active defense and offense: using intelligence to create preemptive attacks against intruders and developing rapid retaliation and response
- Redundancy and separation: avoiding single points of vulnerability, for example, having multiple software types across critical systems
- Resilience: the ability to restore services after an attack

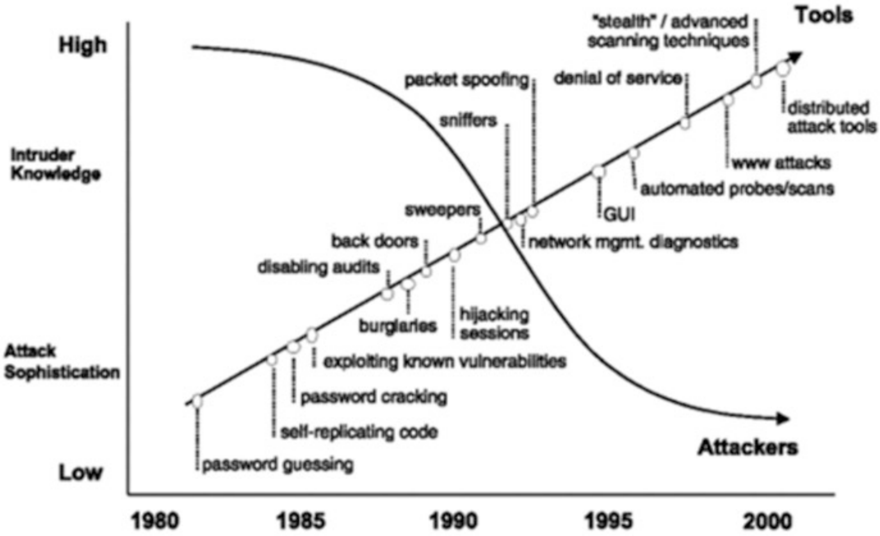


Figure 19 Cyberattacks are becoming more sophisticated, yet require less skill by intruders

Conclusions

Archimedes was one of the influential pioneers of defense-in-depth, which has proven to be a useful military and engineering strategy to meet varied political goals. The defense of Syracuse and the employment of the ram and screw showed how multiple, layered technical and tactical systems can prove effective in slowing down and countering an attack. However, even the best defense can be overcome; the Maginot Line was outflanked by rapid, unanticipated advances in technology and tactics, and the defenders were unable to recover from the assault when their carefully prepared protective works were simply bypassed.

The current generation of cyberwarfare planners is using lessons learned from these and other examples to create appropriate defense-in-depth for the modern era, especially in understanding that some attacks will get through, and provide acceptable levels of operations after attack. As cyber defense continues to evolve, the lessons from Archimedes will continue to be relevant in the twenty-first century.

Bibliography

Archimedes and the Siege of Syracuse

1. Mary Jaeger, *Archimedes and the Roman Imagination*. Ann Arbor: University of Michigan Press, 2008.
2. David C. Lane, "High Leverage Interventions: Three Cases of Defensive Action and Their Lessons for OR/MS Today", *Operations Research* 56/6 pp. 1535–1547, 2010.

3. William M. Murray, *The Age of Titans: The Rise and Fall of the Great Hellenistic Navies*. Oxford: Oxford University Press, 2012.
4. Mythbusters, *Presidential Challenge: Archimedes*, Discovery Channel http://dsc.discovery.com/tv-shows/mythbusters, accessed May 2013.
5. Stephanos A. Paipetis and Marco Ceccarelli (eds.). *The Genius of Archimedes: 23 Centuries of Influence on Mathematics, Science and Engineering*. Dordrecht: Springer, 2010.
6. Chris Rorres, *Archimedes*, http://www.math.nyu.edu/~crorres/Archimedes/contents.html accessed May 2013.

The Screw Propeller and the Ram

7. James P. Baxter, *The introduction of the ironclad warship*. Cambridge: Harvard University Press, 1933.
8. David K. Brown, *From Warrior to Dreadnought*. London: Chatham, 1997.
9. Larrie D. Ferreiro, "The Social History of the Bulbous Bow", *Technology and Culture* 52/2 pp. 335–359, 2011.
10. Jack Greene and Alessandro Massignani, *Ironclads at War: the Origin and Development of the Armored Warship, 1854-1891*. Pennsylvania: Combined Publishing, 1998.
11. John Morrison, *The age of the galley: Mediterranean oared vessels since pre-classical times*. London: Conway Maritime, 2003.

The Maginot Line

12. Robert Boyce, *French Foreign and Defence Policy, 1918-1940*. Abingdon: Routledge, 1998.
13. J.E. Kaufmann and H. W. Kaufmann, *Fortress France: The Maginot Line and French Defenses in World War II*. Mechanicsburg: Stackpole, 2007.
14. Judith M. Hughes, *To the Maginot Line: The Politics of French Military Preparation in the 1920's*. Cambridge: Harvard University Press, 1971.
15. Jean-Yves Mary, Alain Hohnadel and Jacques Sicard, *Hommes et Ouvrages de la Ligne Maginot*, 4 vols. Paris: Histoire & Collections, 2000–2009.

Cyberdefense

16. *Myriam D. Cavelty, et al. (eds.), Power and Security in the Information Age: Investigating the Role of the State in Cyberspace*. Surrey: Ashgate, 2008.
17. Paul Cornish, et al., *On Cyber Warfare*. London: Chatham House, 2010.
18. Martin C. Libicki, *Brandishing Cyberattack Capabilities*. Santa Monica: RAND, 2013.
19. Howard F. Lipson , *Tracking and Tracing Cyber-Attacks: Technical Challenges and Global Policy*. Pittsburgh: CMI/SEI, 2002.

Archimedes the Geometer

Extending His Tombstone Theorem into the Twenty-First Century

Mamikon Mnatsakanian

Archimedes was a great civilization all by himself. [1]

Archimedes' Tombstone Result

Archimedes was a physicist, engineer, mathematician, and astronomer—but above all he was a geometer. This is affirmed by the particular result that he wished to have inscribed on his tombstone (Figure 1).

This purely geometric result is described in his work *On the Sphere and Cylinder* [2]. Archimedes was obviously most excited with this amazing and simple "double ratio."

We present here some results of ours extending Archimedes' tombstone result. Our primary reference is *The Works of Archimedes* [2]. Many of our results were separately published in mathematical journals from 1998 to 2012, coauthored with my colleague Tom Apostol, and also appear in our book *New Horizons in Geometry* [3].

Solids Besides Cylinders with the Double Ratio 3:2

In his *Method* [2], Archimedes investigates the solid determined by the intersection of two cylinders (Figure 2 left), called a *bicylinder*. He constructs this solid by combining eight congruent cylindrical wedges shown in Figure 2 right. Archimedes finds the volume of this wedge and concludes that the volume of the bicylinder equals 2/3 that of its circumscribing cube. This is same volume ratio as in his tombstone result for the sphere and cylinder. He was excited with this special result because, as he explains in *The Method*, never before was the volume of a solid with curved surfaces reduced to that of a solid bounded exclusively by planes, here a cube.

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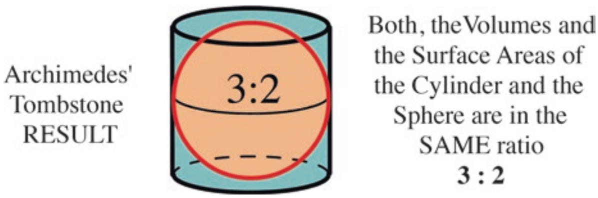


Figure 1

Figure 2

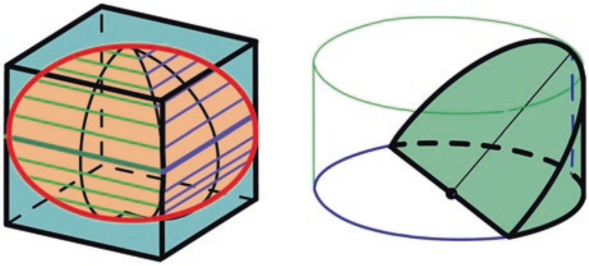
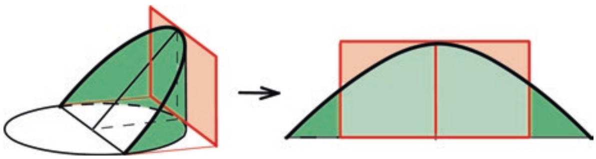


Figure 3



In this connection, we should note that in Figure 1, the volume of the solid bounded by the more curved surface of the sphere is reduced to that of a solid with a less curved surface, the cylinder. The surface curvature is “reduced” in one direction.

What Archimedes didn’t know is that the *surface area* of the bicylinder is also equal to 2/3 of the total surface area of the cube. This can be seen by unwrapping the cylindrical surface of the wedge onto a plane (Figure 3). In this way, it turns into the region under a sine curve whose area is easy to find in an elementary way. Its area is that of the two small squares in Figure 3 circumscribing the base semicircle. From this it follows that the total surface area of the cube is 3/2 times as large as that of the bicylinder.

Now that we have two similar examples, it is easier to find others. Let’s combine not eight but another even number of cylindrical wedges, equal in pairs, so that they have a common inscribed sphere. These are actually intersections of semicylinders, which we call *Archimedean pillows* or *globes*. Figure 4 shows two such solids made with six wedges and eight wedges (the bicylinder).

In Figure 5 the top views of more Archimedean globes, starting with a triangular base, are shown.

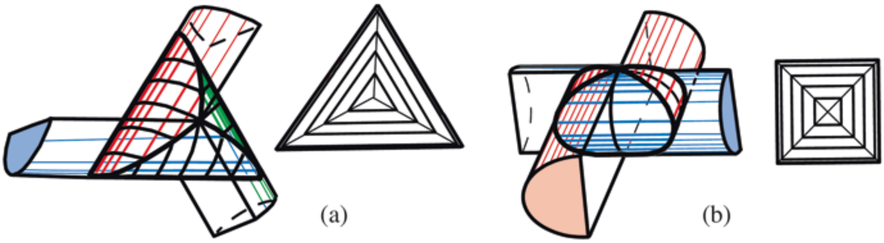


Figure 4

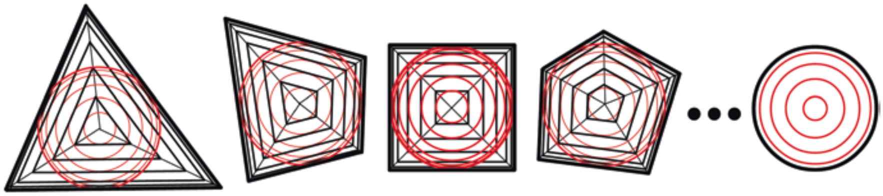


Figure 5

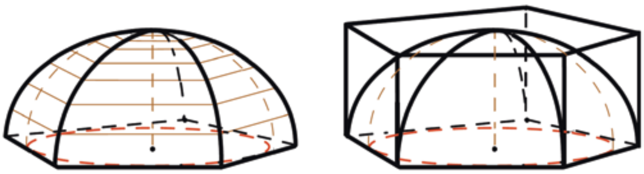


Figure 6

There are infinitely many Archimedean globes, differing in the number of wedges and the shapes. Each of these globes can be surrounded by a prismatic box, whose base is the equatorial polygon of the globe and whose height is the diameter of the cylinders. An example is given in Figure 6, where only one symmetric half is shown. The sphere is the limiting case of an Archimedean globe, and its circumscribing cylinder is the limiting case of its prismatic box.

All these solids have the same property: both the ratio of their volume and surface area to those of the circumscribing prismatic box are the same, namely, 2:3, just as for the sphere and cylinder.

Other Solids with Double Ratios, But Not Necessarily 3:2

Here in Figure 7, three solids circumscribing a sphere are shown besides the cylinder circumscribing a sphere. Does any have the same ratio of both volume and surface area to that of its inscribed sphere?

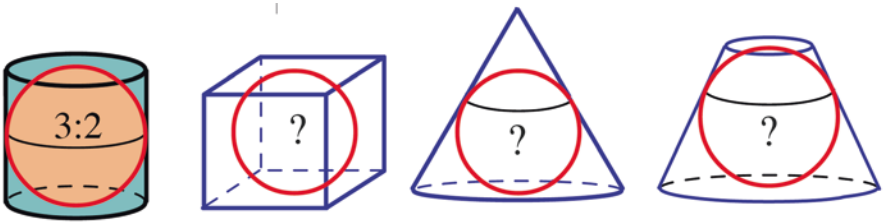


Figure 7

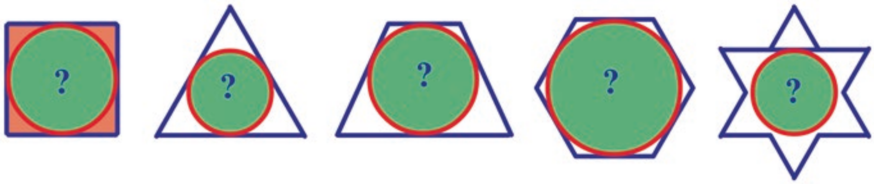


Figure 8

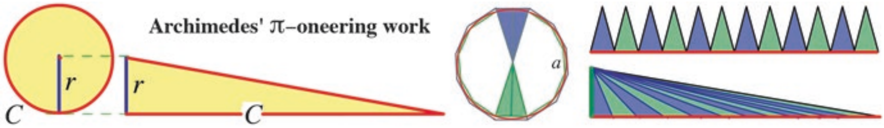


Figure 9

Figure 8 shows even simpler two-dimensional pictures. Which polygons circumscribing a circle have the same ratio of area and perimeter to the area and circumference of the circle?

Examples of such figures are given in Archimedes' π-oneering work *Measurements of the Circle*. These are regular polygons circumscribing a circle. He uses the property that such a polygon has area equal to half its perimeter times the radius of the in-circle. In Figure 9 he imagines how a circle equals a triangle whose base is the circle's circumference and whose height is its radius.

In fact, the traditional globes of Earth are Archimedean, usually made of 12 flat-printed pieces, each of a double sine arch, bent as a cylindrical wedge surface, and then pressed to a sphere.

This "triangle property" is equivalent to the claim that the ratio of the area of the polygon to the area of the circle is the same as the ratio of the perimeter of the polygon to the perimeter of the circle. It may be somewhat surprising, but this "triangle property" is common not only for regular polygons but for any polygon circumscribing a circle; moreover, it holds for any union of sectors of a circle and the corresponding union of the triangles determined by the sectors, each with one vertex at the center of the circle and one side tangent to the circle (Figure 10).

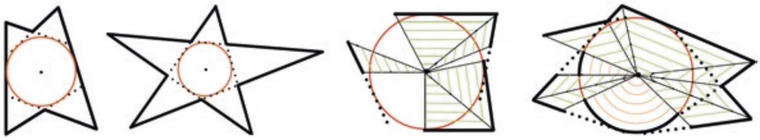


Figure 10

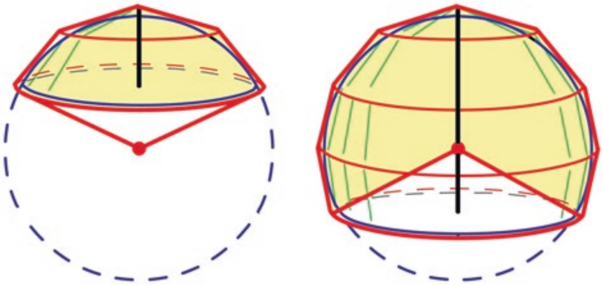


Figure 11

We call such figures *circum-gons*. The pentagram in Figure 10 has no side touching the circle, but if extended, they all are tangent to the circle; i.e., the pentagram circumscribes the circle inside it.

Samples of corresponding three-dimensional solids are found in Archimedes' work *On the Sphere and Cylinder*. He says: *The sphere equals a cone whose base is the surface and height is the radius of the sphere*. Of course he means that the volume is a third of base area times height. The same property has any sector of the sphere having its vertex at the center of the sphere, which is the limiting case of the solid bounded by truncated cones circumscribing the sphere.

Archimedes constructs such conical surfaces by rotating a regular polygon with the number of sides being a multiple of four or its discrete portions (Figure 11).

Archimedes does not mention anywhere that a similar property holds not only for conical sectors but for a cylindrical sector as well. This would simplify the derivation of his tombstone result. A more general solid having such property, called a *circum-solid*, is built from building blocks whose outer surfaces, tangent to the same inscribed sphere, are flat, or conical, or cylindrical or spherical (Figure 12). Each of them is combined from tiny cones, whose tiny bases are nearly flat and whose heights are equal to the same radius of the circle. They all have the "cone property"; the volume is a third of the base times the height. Flat-faced polyhedra circumscribing a sphere are the simplest such examples.

More examples of circum-solids are shown in Figure 13. A circum-solid need not be closed; it can be any sector with outer surface tangent to and vertex at the center of the in-sphere.

Also, circum-solids need not be convex; they can be starlike objects as in Figure 14. Using the double-equality property, we find the volume of the regular stellated dodecahedron by finding its surface area, which in turn can be reduced to

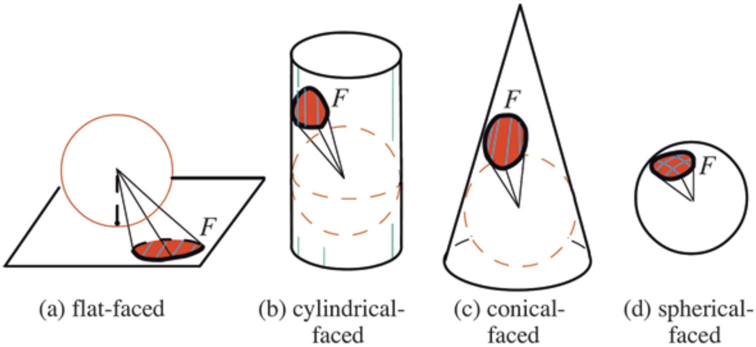


Figure 12

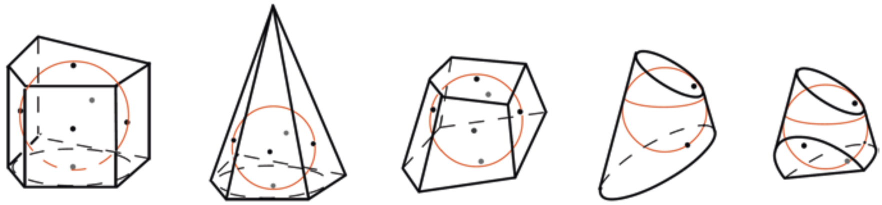


Figure 13

Figure 14

finding the perimeter of the regular pentagram. Here, a three-dimensional volume is reduced to a two-dimensional pentagon's perimeter.

The double-equality property also allows us to find the volume of the intersection of a cylinder with a cone (Figure 15). This is a more general solid than Archimedes' solid of intersection of two cylinders and much harder to treat using ordinary methods. Note that if the cylinder touches the surface of the cone internally, there is a common sphere inscribed in both of them. Thus, our solid of intersection is a closed circum-solid, and finding its volume can be reduced to finding its total surface area first, then multiplying it by the radius of the in-sphere, and dividing by 3. The surface areas, in turn, can be found in an elementary manner because they represent certain regions bounded by sine curves when unwrapped onto a plane.

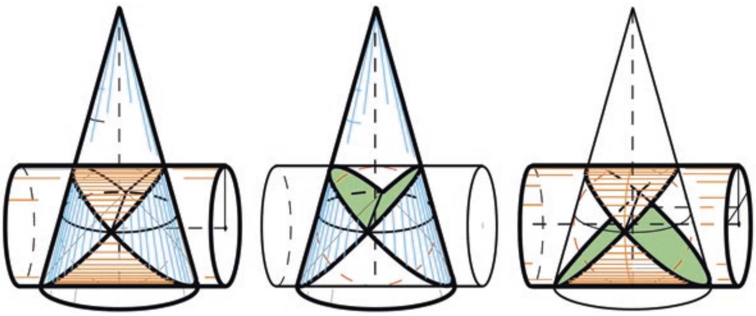


Figure 15

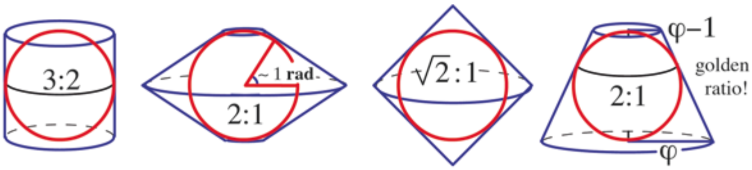


Figure 16

Simple Solids with a Common Double Ratio Other Than 3:2

We have just learned that there are infinitely many variations of solids circumscribing a sphere that have the same ratio of their volume and surface area to those of the inscribed sphere. Thus the Archimedes' tombstone result is not so surprising now as it was at first glance. Archimedes may not have known these facts, or he was excited with the simplicity of the cylinder and of the ratio 3:2.

Can we surround a sphere with a simple object that has a simpler double ratio, say 2:1?

In Figure 16 the Archimedean sphere and cylinder are shown, together with three symmetric circum-solids made of conical surfaces circumscribing a sphere. The first one has a double ratio 2:1, and its point of tangency with the in-sphere is at an angle almost equal to 1 radian, but its exact measure is 0.9955… radian (too bad). The second solid is simpler; it's a double cone with right vertex and intersection angles. The double ratio for it is "square root of 2." The third solid is a truncated cone with a beautiful property: one base has radius equal to the famous "golden ratio," if the sphere has unit radius, and the other one is shorter by 1. The base angle is the corner of a regular pentagram, with 72 degrees. Although the double ratio here is simpler, 2:1, the solid does not look as simple and nice as the cylinder does. By the way, as the angle changes, the double ratio takes its smallest value 3/2 when the truncated cone becomes a cylinder.

It seems that the answer to the question here is: there is no simpler result in 3D.

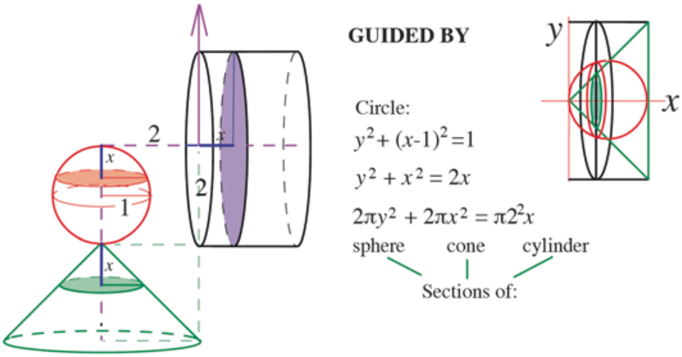


Figure 17

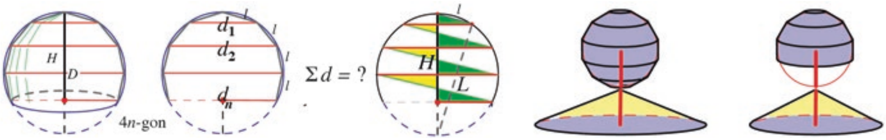


Figure 18

Circum-Solids with the Double Ratio 1:1

There are circum-solids with a double ratio of 1:1, but they are not bounded by closed surfaces because the double ratio would exceed 1. These must be sectors, but how simple can the sector be? Archimedes discovered his tombstone result first by balancing the sphere with a twice-larger (by base) cylinder and inscribed a cone in it. He was guided by geometric algebra or method of proportions, as shown in Figure 17. Later, looking for a rigorous proof, he came up with a nicer solution using the sphere's volume and surface area cone property (Figure 18). He finds this area by exhausting it by cones of revolution of inscribed and circumscribed polygons.

From here he finds that the volume of the sphere is twice that of the inscribed double cone, and only then does he turn from a cone to a cylinder, formulating his tombstone result in terms of a cylinder. He didn't look back again at his wished tombstone picture to see an even simpler proof of it. Here it is Figure 19: the sphere in the circum-cylinder that is punctured by an inscribed double cone.

The surface area of the sphere simply equals the lateral surface area of the punctured cylinder, and the volume of the sphere simply equals the volume of that punctured circum-cylinder.

In other words, the double ratio for the circumscribing punctured cylinder is perfectly 1:1. The same, perfect 1:1 ratio holds for infinitely many Archimedean globes and circumscribing punctured prisms (Figure 19). This can be seen by noticing that the horizontal cross-sectional areas of the two solids are equal (Figure 20).

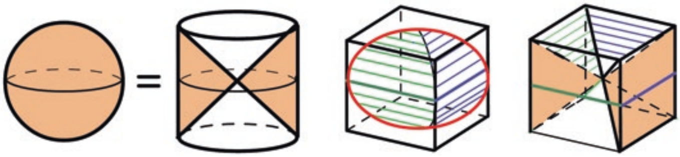


Figure 19

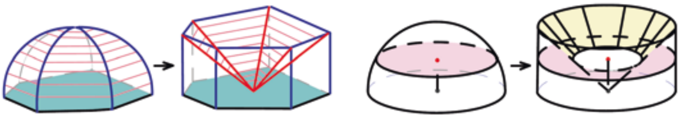


Figure 20

One is the Archimedean pillow shown with its punctured prismatic box. The sphere and punctured cylinder are the limiting case.

Analogies of Archimedes' Tombstone Result in Higher Spaces

Is there an analogy of Archimedes' tombstone result in 2D with simple figures and a simple double ratio? Look at the circum-gons in Figure 8. Is the analogy a circle and a circumscribing square? This picture is simple, but the double ratio in this case is $\pi/4$, not a simple ratio, and fuzzy in Archimedes' time. We have π involved in each regular polygon case, plus some trigonometric expression in general, leading to square roots. So, it looks like the answer is: there is no simple analogy in 2D.

Now we know that there are infinitely many circum-gons and circum-solids each having the "same" double ratio of "volumes" and "surface areas" with the inscribed "sphere," but our analysis showed no simple ratio in 2D and in 3D, except for the cylinder. The concept of circum-solids is easily extended to higher dimensions as well. In higher dimensions, there are richer varieties, because there are more types of building blocks circumscribing n-sphere in higher spaces. Among them are also "familiar" n-cube, n-cone, n-cylinder, and so on, plus other types of n-solids having no analogies in 3-space. Which of these may have a simple rational double ratio?

For the n-cylinder, the ratio of its volume to that of the inscribed n-sphere is not simple for any n except for $n = 3$. In general, the volume of an n-cylinder is that of an $(n - 1)$-sphere times the height (the diameter of the inscribed n-sphere in our case). And there is no simple volume relation between n-spheres in consecutive dimensions.

What about Archimedean globes? In 3D, finding the volume and surface area of the wedge is very simple. Take a tiny spherical wedge {21} with a very small angle, and simply stretch it vertically. Everything increases linearly with height: the

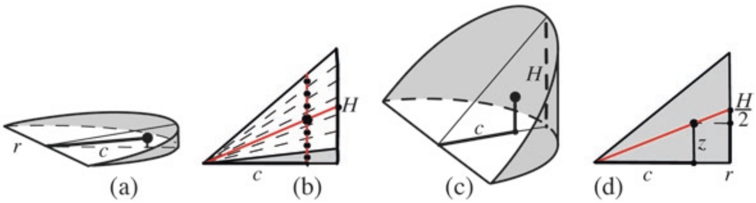


Figure 21

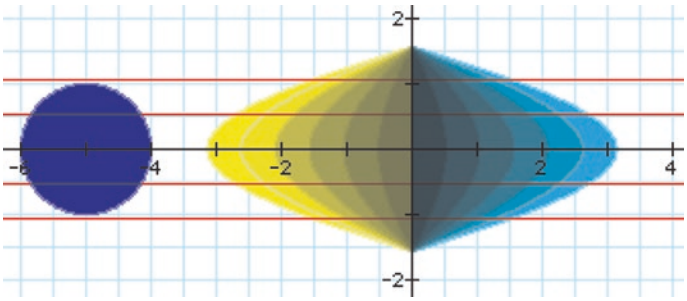


Figure 22

volume, the surface area, and even the height of the center of gravity of the cylindrical wedge. As for the horizontal position of the center of gravity, it is exactly above that of the tiny spherical wedge. On the other hand, if we stretch this tiny spherical wedge angularly, along the circle with the base radius, we will get a spherical wedge, and the volume and the surface area of it will increase linearly with the stretched angle. So, the spherical and the cylindrical wedges are very closely related objects. For instance, the surface of the cylindrical wedge in Figures 3 and 21 is an arch of a sine curve, and so is the surface of the spherical wedge. Shown in Figure 22 is the largest sine arch corresponding to the full spherical wedge, that is, a sphere. We see such maps of the earth globe and of the universe from outside. If we stretch angularly, along a circular arc of a given length, we get the same volume and surface area of the cylindrical wedge, the height being that arc length.

In a similar way, the spherical and cylindrical wedges are intrinsically related in all other dimensions, starting with 2D. The wedge in 2D is a right triangle, so combining them into a closed object leads to the square in Figure 8. No help. Because the said volumes ratio is that of the spheres in two consecutive dimensions, this ratio involves at least a π in it, exactly like in 2D compared to 3D. Perhaps we should not look for very familiar circum-solid objects in n-space. But we definitely want to have the n-sphere inside of an n-circum-solid.

Let's presume for now that the answer is that there are no analogies in dimensions more than three. After all, for 2,000 years, nobody found a simple analogy to Archimedes' tombstone result in any other dimension, despite all the efforts. Let's look into Archimedes' works even if we know that he didn't look into higher dimensions.

In fact, there are even no pictures of 3D solids in Archimedes' drawings; all figures are planar.

Here is what we find interesting. In his *Method* Archimedes writes: *I apprehend that some of my contemporaries or successors with the use of the method once it is established will discover new theorems in geometry that didn't occur to me*. Here we go. His balancing method does really reduce the integrals of a power function to that with one lesser power. Archimedes uses his balancing method in a dozen problems and foresees a big future for the applications.

How Archimedes Balanced the Volume of His Cylindrical Wedge

As we said above, finding the volume and surface area of a wedge is a simple stretching exercise. But how does Archimedes do it? He takes a cylinder, cuts it in half, and then chops out the wedge from one half (Figure 23). Now he makes an unexpected move: he says, if we slide the wedge and shift it back so that its center of gravity is exactly above the base circle of the initial cylinder, the wedge will be balanced by the semicylinder on the left. He doesn't even know where the center of gravity of the wedge is. Apparently, he does not need to know, and he never finds it, even after the balancing. His proof of this balance is as simple as x*1 = 1*x, and now he can find the volume of the wedge for he knows the volume of the semicylinder and its center of gravity. But where does Archimedes find the center of gravity of the semicylinder? We know it today; it's the center of gravity of a semicircular disk. But how does Archimedes know it? Search again.

Finally, here it is in Figure 24: an ignored Archimedes' exercise, lost at the very end of the book, at the end of *The Method*. It's absolutely fantastic, his balance of the semi-disk with a triangle.

Archimedes doesn't express any excitement with this balancing, though it is charming and very unexpected. Here Archimedes didn't know that there is another balance in this picture: the arc length of the semicircle also is balanced with the vertical base of the triangle!

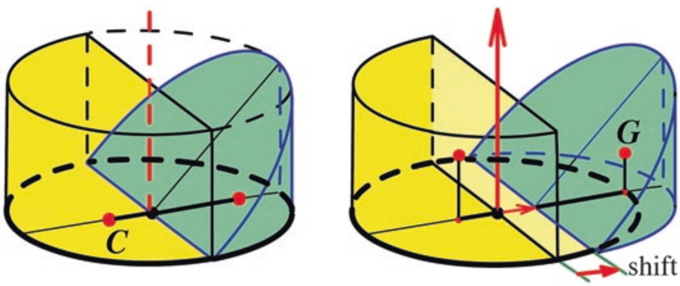


Figure 23

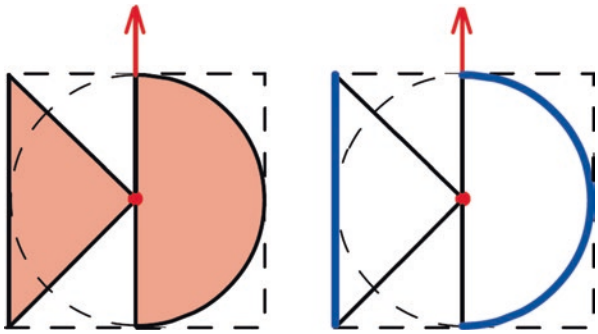


Figure 24

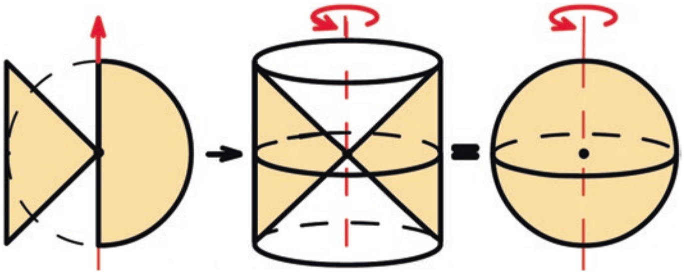


Figure 25

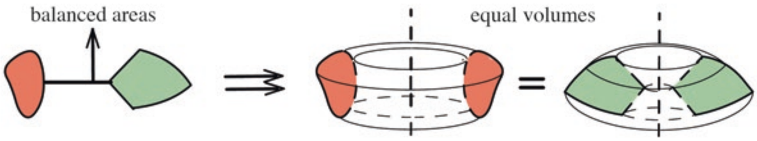


Figure 26

This double balance picture leads directly to the Archimedean tombstone double ratio (Figure 25).

Rotate this picture around the balancing axis to obtain two solids: a sphere produced by the semicircle and a punctured cylinder (by a double cone) produced by the triangle. Because the areas are balanced, the volumes of the two solids are equal, by the Pappus' rule.

Just multiply, in general, the equality of moments by 2π to get equal volumes of revolution (Figure 26).

And because the arc lengths are in balance, the surface area of the sphere is equal to the lateral surface area of the cylinder, by the second rule of Pappus (and multiplying by 2π). Note that the cylinder produced here circumscribes the sphere.

Archimedes proves the balance of areas by proving first the chord-by-chord balance with respect to the central axis. By the way, using another Pappus' rule

Figure 27

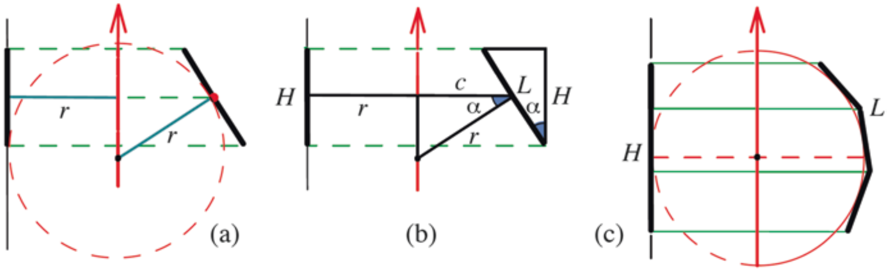


Figure 28

(multiplying by 2π) (Figure 27), we conclude that the horizontal cross-sectional areas of the two solids are equal, from which again follows the Archimedean tombstone result for the volumes.

To prove the chord-by-chord balance, Archimedes uses the Pythagorean theorem (PT). PT is a very powerful but very specialized tool, and any use of it may result in big limitations.

It is good for circles if the Cartesian equation is used because it is actually the PT relation.

How can we prove the double balance in some other way that is more revealing? Look. Figure 28 shows a circle and a line segment tangent at its midpoint to the circle.

On the other side of the vertical axis of the circle, a vertical tangent line to the circle is drawn. From similar triangles, it follows immediately that the line segment on the right is in balance with its projection on the vertical line on the left. The line segment is arbitrary, so we can combine several such segments, each tangent at its midpoint to the circle, and obtain a regular polygonal arc balanced on the right with its combined projection on the vertical tangent line on the left. In the limit, as we see, any arc of the circle itself is in balance with its vertical projection segment (Figure 29). Therefore, by rotating the picture around the balancing axis using Pappus' rule, (and multiplying by 2π), we conclude that any horizontal spherical zone has an area equal to that of the corresponding horizontal zone of the lateral surface of the cylinder circumscribing the sphere (Figure 30).

In particular this leads to the balance of the semicircular arc with the base of the triangle as claimed above, which in turn leads to the equality of the surface areas of the sphere and cylinder in the Archimedes tombstone result.

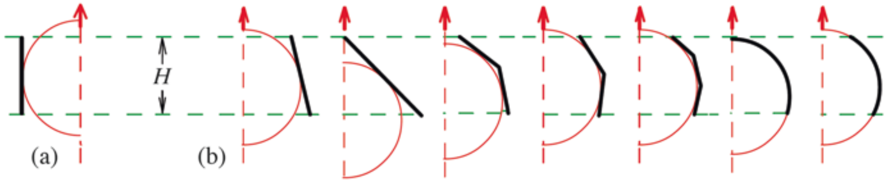


Figure 29

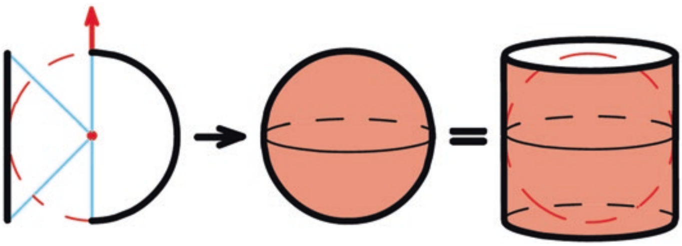


Figure 30

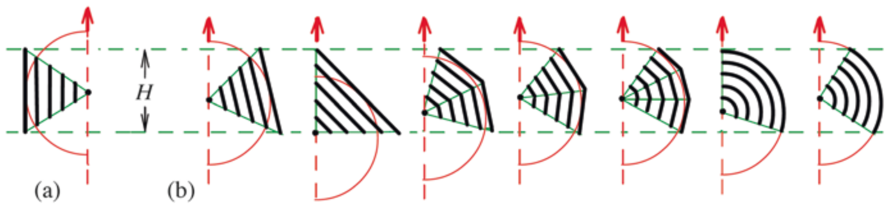


Figure 31

Discovering New Results Using the Double Balance

Now we can consider various concentric circles and tangent lines to them. By combining them, we can continuously fill sectoral areas as shown in Figure 31 and conclude about the areal balance of the polygonal trapezoidal sectors on the right and their simpler projection trapezoids on the left. In particular, we can fill the circular sector in the right that will be in balance with its projection triangle on the left. In the case of a semicircular disk, we obtain from here the original Archimedean areal balance in Figure 32. So by using simple similarity instead of the Pythagorean theorem, we have obtained richer balancing options for both areas and arc lengths, whereas the Archimedean semi-disk areal balance is a special case of these balancing pictures.

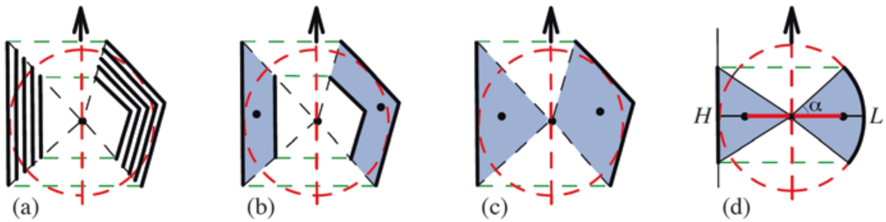


Figure 32

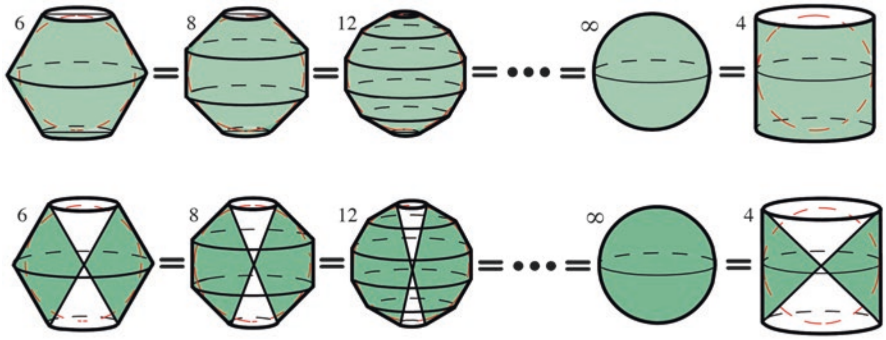


Figure 33

The families of solids in Figure 33 are rotated polygons circumscribing a sphere. They resemble Archimedean circumscribing conical surfaces. But these are $2n$-gons and are punctured by the central vertical double cone whose bases have the horizontal sides of the polygons as the diameters. The family exhibits double equality: the volumes of the solids are equal and the surface areas are equal. The sphere and the cylinder are the limiting, special cases. Again we see Archimedes' tombstone result as a special case here.

Because these solids are circum-solids, they have the same double ratio, but it changes from solid to solid, approaching the simplest value 3:2 for the cylinder.

A similar family can be obtained for polygonal prismatic wedges (Figure 34). The bases are the same even-sided polygons as above, and they are also punctured on the opposite sides of the polygons. The square and the sine arch are the special and limiting cases. They represent the Archimedean cylindrical wedge and the circumscribing cube. All solids in this family have equal volumes and equal lateral surface areas. The polygonal prismatic wedges circumscribe the Archimedean cylindrical wedge; therefore the ratio of their volumes and surface areas to those of the inscribed wedge is the same. This double ratio gets the value 3:2 for the limiting Archimedean cylindrical wedge.

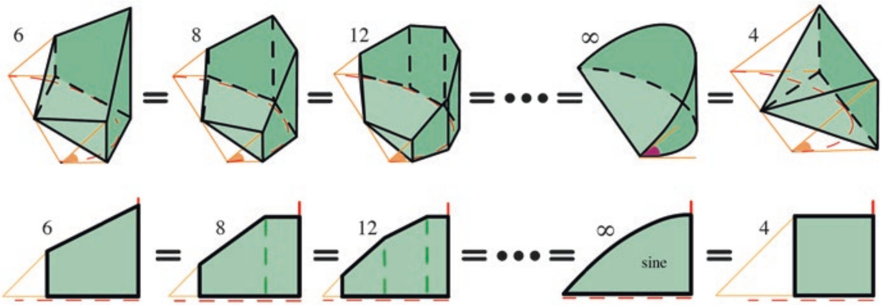


Figure 34

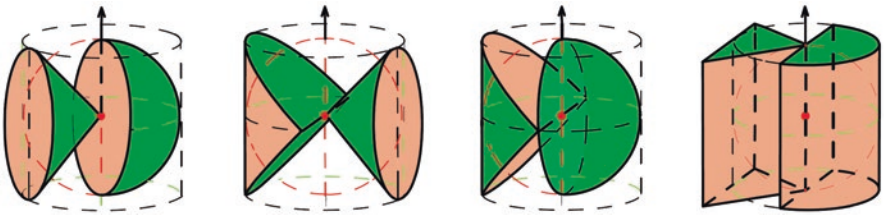


Figure 35

Not only can the double-equality results be easily generalized for infinite families of 3D solids, but using area-and-arc balancing in the plane, the balancing picture itself can be extended from 2D to 3D space, which then will lead to double-equality results in 4D, etc. Here is how easily this can be done.

Take the double balancing picture in Figure 24 and rotate it around the horizontal axis. The result is the first picture in Figure 35: the hemisphere here (obtained from a semicircle) is balanced with the cone (obtained from a triangle). Moreover, these two solids are in double balance, meaning both are in balance: the volumes and the outer surface areas, that is, the hemispherical surface is in balance with the base of the cone. For more partial double balancing pictures related to these solids and some others, see [3].

The second picture represents the double balancing of the cone with the cylindrical wedge. The third double balance follows from the first two. Interestingly these three balanced solids provide orthogonal projections exactly matching the one in Archimedes balancing of the semicylinder with the triangular prism that led him to discover the volume of the wedge.

Figure 36 shows one more interesting double balance of the cylindrical and spherical wedges of the same height. Note that the side view of these balanced solids presents two 2D figures, a circular sector and the corresponding projection triangle (of the same height). As we have seen before, they are in double balance with each other: and the areas are balanced and the arc lengths are balanced. The double balance in Figure 24 is a particular case of this one.

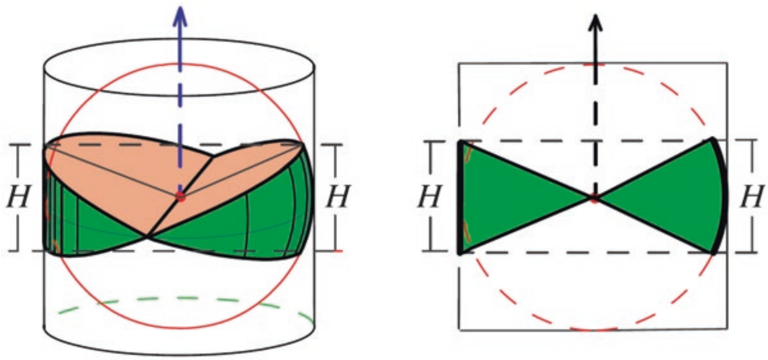


Figure 36

It should be noted that the type of polygonal balancing in Figures 33 and 34 does not allow simple extensions to higher dimensions.

How "2:N-Grave" ("To Engrave") Archimedes' Hyper-Tombstone

Now we are going to extend Archimedes' tombstone result to higher-dimensional spaces. We already know that a solid having the same double ratio of volumes and surface areas is not a problem. Every circum-solid satisfies that condition. And the variety of such solids grows with n quadratically. In 2D we had two building blocks; in 3D we had four of them. Of course there are more, but we should select, for their beauty, the simplest ones in their hyper-descriptions. The key to hyperspace extension is the punctured cylinder, for it has the simplest double ratio 1:1. But we have already mentioned that the cylinder (even punctured) has no simple double ratio in any other dimension but 3. The secret to our solution is that the 3-cylinder is in fact not a cylinder but a tricky appearance of another object that we call a *cylindroid*. This new object is as simple as the cylinder, but it's different from the cylinder. Only in 3D does it coincide with the cylinder. What is the n-cylindroid?

Figure 37 shows 2D figures obtained from a simple 1D sphere and cylinder. If we translate a 1-sphere, we get a 2-cylinder (rectangle), but if we tumble a 1-cylinder about its end, we get a 2-cylindroid, which is actually a 2-sphere or a circular disk. So a 2-cylindroid and its inscribed 2-sphere are exactly the same objects with a perfect double ratio, 1:1. As we can see, the 2-cylindroid is also a punctured 2-cylindroid so there is no contradiction in having double ratio 1:1. Recall that the 2-cylinder discussed previously was a square with double ratio $4/\pi$. Thus, we have resolved the main paradoxical issue: why, seemingly, is there no analogy of Archimedes' tombstone result in 2D? Here it is, a circle and its circumscribing 2-cylindroid (Figure 38).

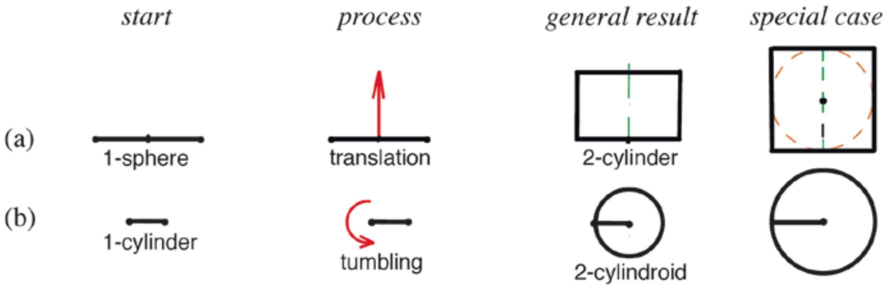


Figure 37

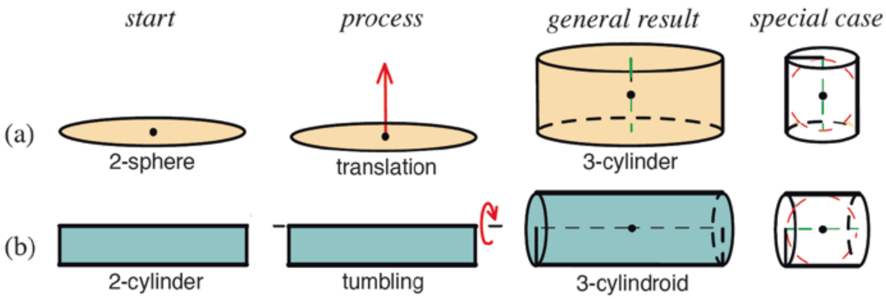


Figure 38

Now we move from 2D to 3D (Figure 38). As in the previous picture, when we translate a 2-sphere, we get a 3-cylinder, and when we tumble a 2-cylinder, we get a 3-cylindroid. In this case and only in this case, the 3-cylindroid is also a 3-cylinder; their directions are just exchanged. This coincidence will not happen in any other-dimensional space, I promise.

Now we'll construct a double balance and double equality of an n-hemisphere and an n-cone. Start with a 2D double balance and rotate it around the horizontal axis of symmetry (Figure 39). This produces a double balance of a hemisphere and its projection cone in 3D. Now rotate this balanced 3D picture through 4D space to get double balance of a four-dimensional hemisphere and its projection cone. It will help to visualize this process to flatten the 3D space into a hyperplane and look for its analogy with the previous 2D picture. Continuing in this way, we obtain a family of double balanced n-hemispheres and their projected n-cones in n-dimensional space starting with 2.

Now, take an $(n-1)$-double balanced picture and revolve it around the vertical balancing axis. As a result, the n-hemisphere will produce an n-hemisphere, and the $(n-1)$-cone will produce a punctured n-cylindroid, as in pictures in the second row in Figure 39. These two objects have equal volumes and equal surface areas (outer or lateral). And the double ratio of them is exactly 1:1 for every n starting with 2.

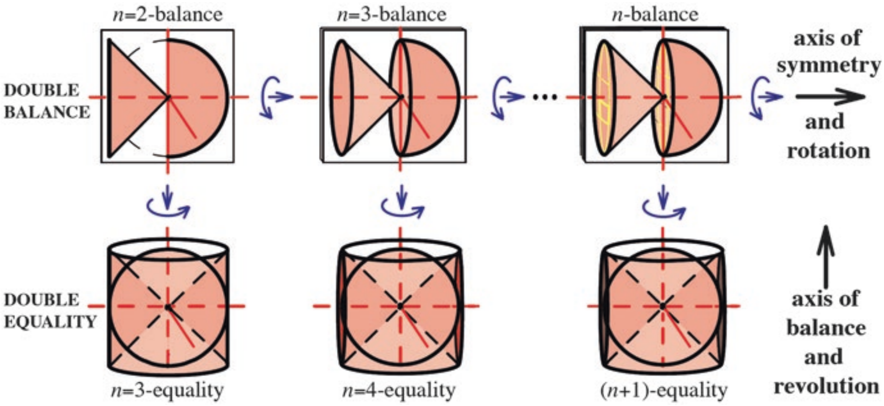


Figure 39

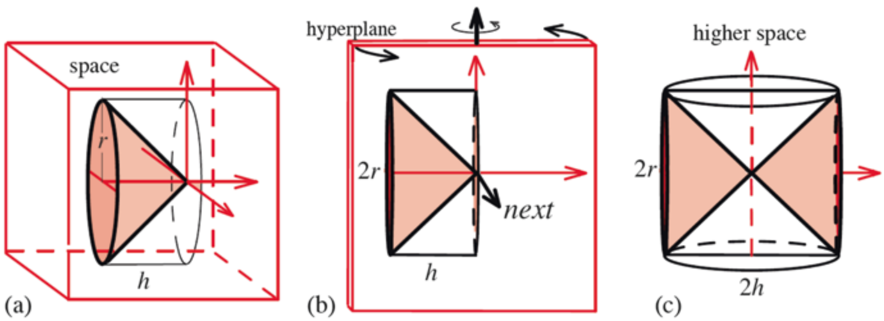


Figure 40

If we calculate the volume of the full, un-punctured n-cylindroid, we will get for its double ratio to the inscribed n-sphere, namely, n:2.

In the special case $n = 2$ and the double ratio 1:1, this 2:n-graving is already explained above (Figure 40).

Other Archimedean Discoveries About Spheres

In his work *On the Sphere and Cylinder*, Archimedes discovered the seven fundamental properties of the sphere shown in Figure 41 by comparing it to simpler cones and cylinders.

All these seven properties hold exactly the same way for all Archimedean pillows. For instance, the first one on the area of a spherical cap (segment) is generalized as follows: The area of a cap (segment) of an Archimedean globe equals that of

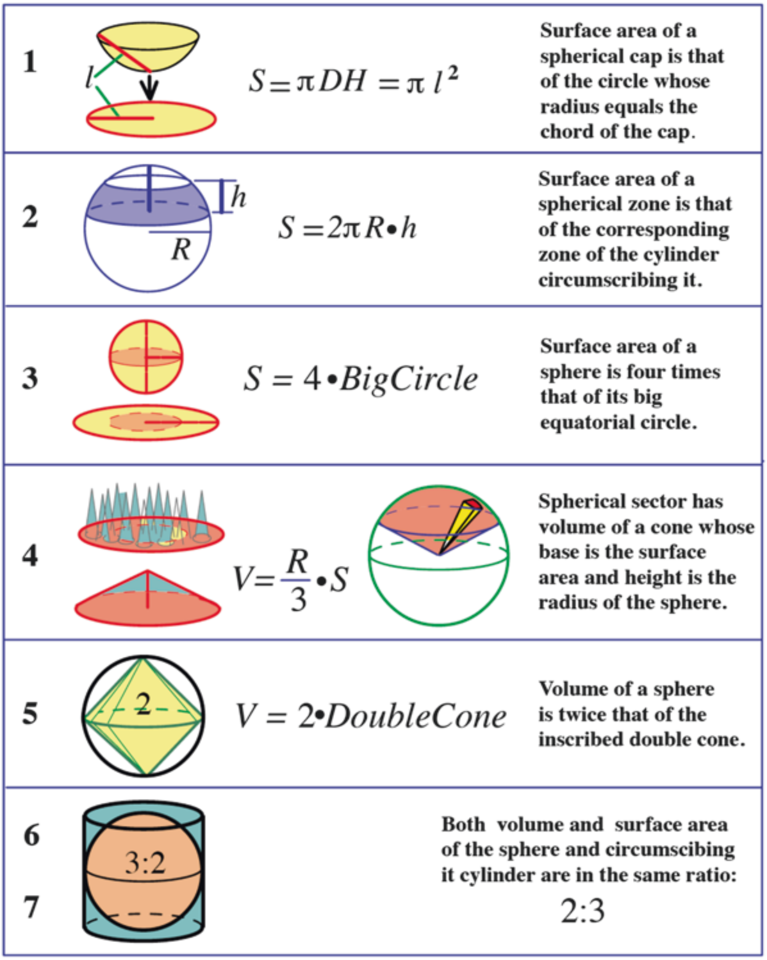


Figure 41

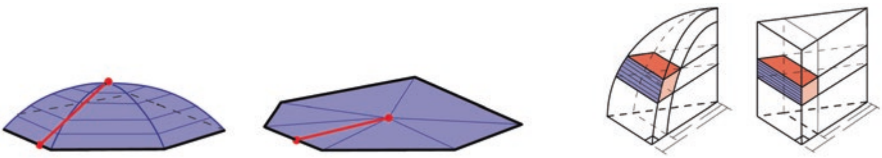


Figure 42

a polygon similar to its equatorial circum-gon that has the same apothem l as the cap (Figure 42). The second property also holds, because it holds for each of the wedges forming the globe (Figure 42). Property 3 is generalized as follows: the total surface area of any Archimedean globe is four times as large as its equatorial polygon.

We have already discussed properties 4 and 6–7 above. As for 5, it is also true for any globe: its volume is twice that of the inscribed double pyramid whose base is the equatorial circum-gon.

Thus, the Archimedean globe is a generalized sphere with all its properties, the sphere being the limiting case. More and interesting properties of the Archimedean globe, the sphere in particular, can be found in comparison with the punctured circumscribing box, related to centers of gravity of their segments, shells, and slices, as well as the corresponding volumes and inner and outer surface areas of such "washers" [3]. These are the properties of a cylindrical wedge as in Figure 42 that makes a peel of all Archimedean globes.

By the way, other properties of the sphere hold for the hyper-tombstone. For instance, the zone areas of the n-sphere and n-cylindroid are equal, also their cross-sectional hyper-areas are equal, and the n-sphere has volume $(n - 1)$ times as large as the inscribed in it double n-cone. Also the centroidal properties are similar. Interestingly, the areal centroid of an n-hemispherical surface is at the same altitude as the volume centroid of an $(n - 2)$-hemisphere. In particular, the areal centroid of a 3D hemisphere's surface is at the middle of its height because the volume centroid of an1D hemisphere, that is, of a line segment being the radius, is at its midpoint.

Concluding Remarks

We have discussed and extended those discoveries that directly relate to the tombstone of Archimedes. Archimedes' tombstone was lost and found and then lost again but never found again. However, his discoveries will live forever, to excite us with his vision, logic, and imagination and to still leave some room for doubt.

Bibliography

1. George F. Simmons, *Calculus Gems*, p. 43, McGraw-Hill, 1992.
2. Thomas L. Heath, *The Works of Archimedes*, Dover Publications, 2002.
3. Tom M. Apostol and Mamikon A. Mnatsakanian, *New Horizons in Geometry*, MAA, 2012.

Archimedes the Mathematician

Chris Rorres

> *"Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not."*
>
> *— G. H. Hardy [1]*

I would like to discuss some of Archimedes' mathematical ideas that are being used and applied twenty-three centuries after his lifetime. Although Archimedes' fame among the general populace in antiquity was based on his military machines and other inventions, he earned his immortality through his mathematical works. Let me begin with a mathematical word problem that Archimedes posed which is still being discussed today.

Archimedes' Cattle Problem

A simple precursor of this problem appeared in *The Odyssey* where Homer implicitly challenged his audience to determine the number of cattle that belonged to Helios, the god of the Sun. Homer's problem basically boiled down to computing the product 7×50. In Archimedes' more difficult version, Helios's cattle were divided into two genders (cows and bulls) and also into four different colors (white, black, dappled, and yellow). Archimedes gave certain mathematical relationships among the numbers of cattle in each of the resulting eight classes (white cows, dappled bulls, etc.) and challenged his colleague Eratosthenes to determine the total number of the cattle of the Sun. Here is the problem as it has been handed down to us [2]:

> A Problem
>
> *which Archimedes solved in epigrams, and which he communicated to students of such matters at Alexandria in a letter to Eratosthenes of Cyrene.*
>
> If thou art diligent and wise, O stranger, compute the number of cattle of the Sun, who once upon a time grazed on the fields of the Thrinacian isle of Sicily, divided into four herds

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> of different colours, one milk white, another a glossy black, a third yellow and the last dappled. In each herd were bulls, mighty in number according to these proportions: Understand, stranger, that the white bulls were equal to a half and a third of the black together with the whole of the yellow, while the black were equal to the fourth part of the dappled and a fifth, together with, once more, the whole of the yellow. Observe further that the remaining bulls, the dappled, were equal to a sixth part of the white and a seventh, together with all of the yellow. These were the proportions of the cows: The white were precisely equal to the third part and a fourth of the whole herd of the black; while the black were equal to the fourth part once more of the dappled and with it a fifth part, when all, including the bulls, went to pasture together. Now the dappled in four parts were equal in number to a fifth part and a sixth of the yellow herd. Finally the yellow were in number equal to a sixth part and a seventh of the white herd.
>
> If thou canst accurately tell, O stranger, the number of cattle of the Sun, giving separately the number of well-fed bulls and again the number of females according to each colour, thou wouldst not be called unskilled or ignorant of numbers, but not yet shalt thou be numbered among the wise.
>
> But come, understand also all these conditions regarding the cattle of the Sun. When the white bulls mingled their number with the black, they stood firm, equal in depth and breadth, and the plains of Thrinacia, stretching far in all ways, were filled with their multitude. Again, when the yellow and the dappled bulls were gathered into one herd they stood in such a manner that their number, beginning from one, grew slowly greater till it completed a triangular figure, there being no bulls of other colours in their midst nor none of them lacking. If thou art able, O stranger, to find out all these things and gather them together in your mind, giving all the relations, thou shalt depart crowned with glory and knowing that thou hast been adjudged perfect in this species of wisdom.

The described relationships among the numbers of the various cattle can be expressed as nine equations in ten unknowns. In the following equations, W represents the number of white bulls, B the number of black bulls, w the number of white cows, and so forth:

$$
\begin{aligned}
W &= (1/2 + 1/3)B + Y\\
B &= (1/4 + 1/5)D + Y\\
D &= (1/6 + 1/7)W + Y\\
w &= (1/3 + 1/4)(B + b)\\
b &= (1/4 + 1/5)(D + d)\\
d &= (1/5 + 1/6)(Y + y)\\
y &= (1/6 + 1/7)(W + w)\\
W + B &= s^2\\
Y + D &= m(m + 1)/2.
\end{aligned}
$$

It is impossible for Archimedes to have solved these equations, even though he is regarded as the greatest mathematician of antiquity. To believe he could is like believing that the strongest man in antiquity could lift 100 tons. Just as some kind of mechanical machine is needed to lift 100 tons, some kind of computing machine is needed to solve Archimedes' cattle problem, and such computing machines were not developed until the mid-twentieth century.

The first seven equations above are linear equations. The eighth equation expresses the fact that the white bulls plus the black bulls when joined together form a square herd; that is, a herd whose number is the square of some integer s. The last equation says that when the yellow bulls plus the dappled bulls are joined together, they form a triangular herd; that is, a herd whose number is a triangular number: one that can be expressed as $m(m+1)/2$ for some integer m.

So we have this underdetermined system of nine equations in ten unknowns and we seek nonnegative integer solutions. This is a classic Diophantine equation, one of the most famous. Now without too much difficulty, we can first solve the seven linear equations. Archimedes could have solved them, although it is very unlikely that he did since the resulting numbers involved are enormous. With their solution we can then eventually boil down all nine equations to just the following equation:

$$x^2 = 410,286,423,278,424y^2 + 1$$

for which positive integers x and y are sought. If we can find such x and y, then y determines the total number of cattle T through the expression

$$T = 224,571,490,814,418y^2$$

This equation for x and y is an example of a Pell equation, an equation that has been studied for hundreds of years going back to seventh-century India. The general form is $x^2 = Ay^2 + 1$ where A is some positive integer.

In 1880, a German mathematician, A. Amthor, was able to solve Archimedes' Pell equation [3]. He came up with the following explicit horrendous expression for the total number of cattle T:

$$T = \frac{25194541}{184119152}(w^{4658} - w^{-4658})^2$$

where

$$w = 300426607914281713365\sqrt{609} + 84129507677858393258\sqrt{7766}$$

This expression can be considered the solution to Archimedes' cattle problem in that it can be evaluated using the basic arithmetical operations. Amthor was able to prove that the first three digits in T were 776 and that the total number of digits was 206,545.

To determine all 206,545 digits, however, requires a computer. In 1965 three Canadian mathematicians/computer scientists found all 206,545 digits using an IBM 7040 computer [4]. It took them about eight hours of computing time. (Today it would take a few milliseconds on a computer with the power of an iPhone.) They published their results in the journal *Mathematics of Computation*, although they did not include the actual value of T in their paper. They simply stated that they had found it and had put it in the journal's archives for anyone interested in seeing all 206,545 digits.

The first publication that contained all the digits was by Harry Nelson in 1980 [5], who wanted to test his brand new Cray I computer. He generated all of the digits and published them in the *Journal of Recreational Mathematics*. In fact, he found several other solutions. These Pell equations have either no solution or infinitely many. The one that Nelson displayed was the smallest solution among the infinitely many that satisfy Archimedes' Pell equation.

Entering the twenty-first century, a Finnish mathematician in 2001 came up with a simpler formula than Amthor's [6]. His formula involved only integer arithmetic. No square roots are involved, and it can be calculated it in a matter of milliseconds on a laptop computer:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 109931986732829734979866232821433543901088649 & 392567302329690546856394748066206816187916440 \\ 30784636507697855142356992218944109072681060 & 109931986732829734979866232821433543901088649 \end{bmatrix}^{1164} \begin{bmatrix} 300426607914281713365 \\ 84129507677858393258 \end{bmatrix}$$

$$T = \frac{48222351474}{4657}(uv)^2$$

Following are the first and last fifty digits of the 206,545 digits of T obtained using the above formula:

77602714064868182695302328332138866642323224059233

…

05994630144292500354883118973723406626719455081800

(All 206,545 digits were printed as a handout and memento of this conference.)

Stomachion Puzzle

Another puzzle/challenge that Archimedes is credited with is called the Stomachion. It's a tangram-type puzzle with 14 pieces that originally form a square (Figure 1, left). It was thought that the objective of the puzzle was to reassemble the pieces, as in tangram, to form interesting shapes, like the elephants in Figure 1, right. But in this century *The New York Times* ran a story on its front page titled, "In Archimedes' Puzzle, a New Eureka Moment" written by Science Writer Gina Kolata [7]. The story reported that perhaps the purpose of the Stomachion was to rearrange the pieces in order to form the original square—as proposed by Stanford University professor, Reviel Netz [8].

Figure 2 exhibits two different ways that the fourteen pieces can be rearranged to form the original square. If the purpose of the Stomachion was to find such rearrangements, it leads to a problem in Combinatorial Mathematics of determining in how many ways the fourteen pieces can be rearranged.

The combinatorial problem was quickly solved by two (husband-and-wife) teams of mathematicians: Persi Diaconis and Susan Holmes at Stanford University

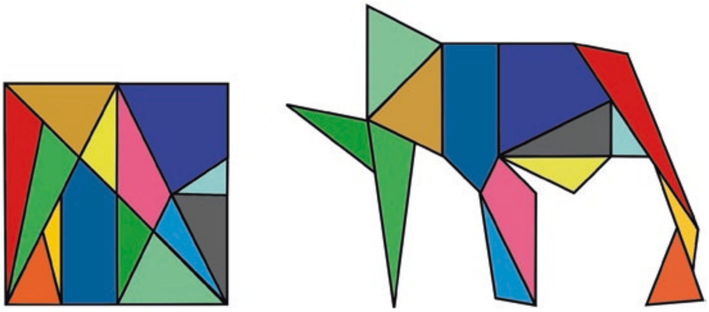

Figure 1 The fourteen pieces of the Stomachion originally forming a square (*left*) and the pieces rearranged to form an elephant (*right*)

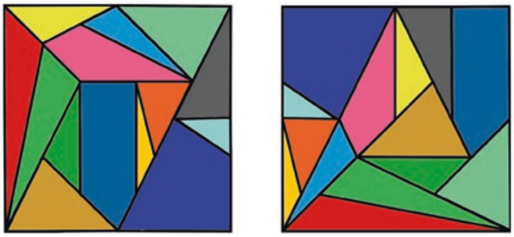

Figure 2 Two different arrangements of the Stomachion pieces that form a square

and Robert Graham and Fan Chung from the University of California, San Diego. And then independently a computer scientist, William Cutler, who simply used brute force rather than sophisticated mathematics, come up with the same solution, which is this: There are 17,152 ways to rearrange the pieces so that they form a square. We can reduce that number to 536 if we don't count rotations and reflections.

So here is a puzzle of Archimedes that has survived into the twenty-first century and that continues to challenge mathematicians and scientists.

Floating Bodies

Some of our speakers discussed their own research extending some of Archimedes' results. Let me mention some of my own research in this direction.

You might have noticed that the logo for this conference shows Archimedes seated at a computer (Figure 3). In fact, much of my research could be subtitled, *If Archimedes Had a Computer*. But he did not have a computer; he was restricted

Figure 3 Logo of the Archimedes World Conference (The seated image of Archimedes is a detail from an engraving of a painting by the nineteenth-century Italian artist Niccolò Barabino)

to compass-and-straightedge constructions, and this was a severe restriction when he was trying to solve problems in mathematical physics.

For example, in his work *On Floating Bodies*, Archimedes was interested in finding in how many different positions a paraboloid could float stably. He first found the obvious positions: the vertical up and vertical down positions. But for certain shapes and relative densities, a paraboloid can also float stably in a tilted position. Archimedes was able to determine those shapes and densities—but only when the base of the paraboloid was completely above or below the fluid level.

If the fluid level cuts the base, as shown on the computer screen in Figure 3, then we have a problem that cannot be solved by compass-and-straightedge construction. It's like trying to square a circle—it simply can't be done. Using a computer, however, I was able to analyze the cases when the fluid level cuts the base. The results appeared in *The Mathematical Intelligencer* in 2004 [9].

Figure 4 describes Archimedes' and my results in graphical form. I don't have space to fully explain this figure, but basically the left diagram depicts Archimedes' compass-and-straightedge determination of the stable equilibrium positions for various shapes and densities of a floating parabola when the base is not cut by the water level, while the right diagram is my completion of his graph using computer computations.

The complete surface has all sorts of interesting features—bifurcations, fold catastrophes, cusp catastrophes. Modern mathematics and computers are needed to understand the full extent of how complicated the behavior of a floating paraboloid is. And this behavior has some interesting applications. For example, a melting iceberg in the shape of a paraboloid may lose its stability and tumble over quite suddenly from a stable vertical equilibrium position to a stable tilted equilibrium position.

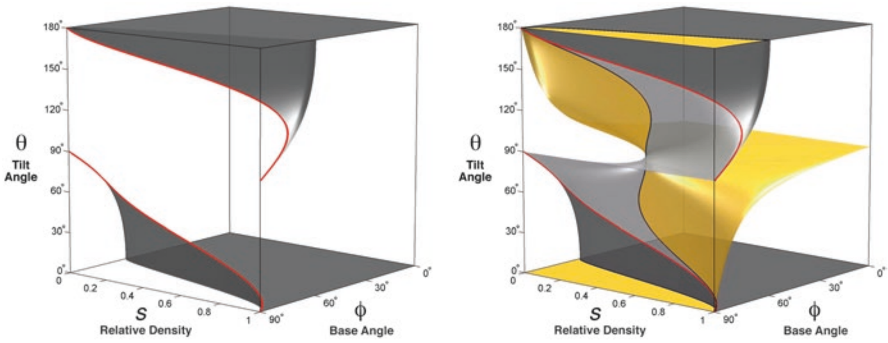

Figure 4 Archimedes' compass-and-straightedge results in graphical form (*left*) and my completion of the graph using a computer (*right*)

Today, ships are told to stay away from icebergs—not because they might hit them (it's not too difficult to avoid hitting an iceberg nowadays), but because a melting iceberg may suddenly, catastrophically tumble over without warning.

So this is what we can do with a computer to extend Archimedes' results. To Archimedes, a computer would have been something like a god coming down from Mount Olympus to sit next to him and perform his computations on command.

Archimedes Screw

Another discovery from antiquity that I have been interested in is the Archimedes screw. Figure 5 is an Italian stamp depicting an Archimedes screw, together with a supposed likeness of Archimedes.

The Archimedes screw has been in use for more than two millennia to raise water for irrigation and drainage purposes. Beginning in the twenty-first century, it is also being used to generate electricity by running in reverse; that is, by using the weight of falling water to turn the screw and drive an electric generator.

Through trial and error it is possible to arrive at a good estimate for the best design of an Archimedes screw—in particular, what should be the spacing of its blades and what should be its inner radius (the radius of its shaft) in order to raise or lower the most water with each turn of the screw. The ancients were pretty good at trial and error and came up with screw designs that were fairly efficient. But today with a computer, we can get precise optimal values of the screw parameters. Figure 6 is a computer-generated diagram from a paper of mine on the optimal design of an Archimedes screw published in the *Journal of Hydraulic Engineering* [10].

The location of the peak of the surface in Figure 6 determines the values of two screw parameters (the pitch or spacing of the blades and the radius of the shaft) that optimize the amount of water that a screw can raise or lower in one turn.

Incidentally, the likeness of Archimedes depicted in the Italian stamp in Figure 5 has also appeared on other stamps. Figure 7 shows this likeness on stamps from

Figure 5 Italian stamp (1983) depicting an Archimedes screw

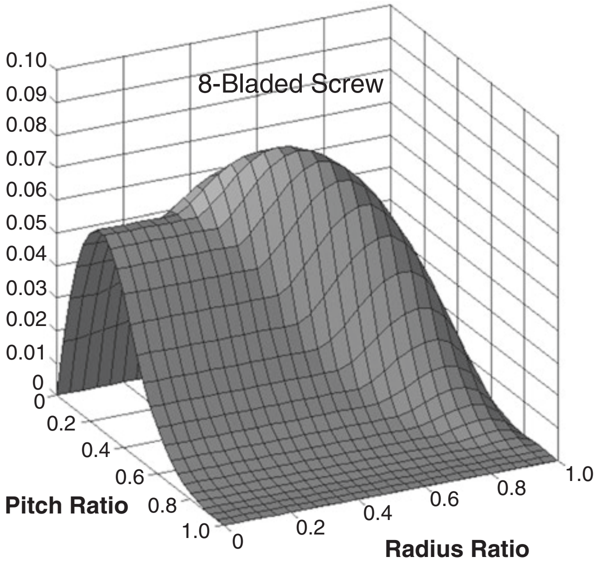

Figure 6 Diagram for the determination of optimal screw parameters

three other countries. This supposed likeness of Archimedes comes from the bust in Figure 7 that is found in the Archaeological Museum in Naples, Italy. Unfortunately, it is not a bust of Archimedes. It is a bust of Archidamos III, who was a fourth-century BC king of Sparta [11]. Somewhere along the way the two names were confused—and so now the best representation of Archimedes is that of an obscure fourth-century BC king of Sparta. In fact, no image of Archimedes has survived to modern times. We have no idea what he looked like.

Figure 7 Statue of Archidamos III together with three stamps from Greece (1983), Guinea-Bissau (2008), and San Marino (1982) that identify him as Archimedes

Centers of Gravity

Another concept that Archimedes is associated with is the center of gravity of a body. The idea of a center of gravity existed before Archimedes, as did the technique of locating it by suspending the body from various points and seeing where the suspension lines cross.

However, previous works on the center of gravity were mainly philosophical discussions. Archimedes was the first to actually do anything with the concept—that is to say, he was the first to determine the centers of gravity of various solid bodies using geometric methods.

Figure 8 illustrates Archimedes' technique for finding the center of gravity of a triangle. He first proved that if a triangle is suspended from one of its vertices, the suspension line must bisect the opposite side—that is, it must be a median of the triangle. Archimedes then concluded that the point common to the three medians is the center of gravity of the triangle.

Actually, all suspension lines of a body have a common intersection only if the gravitational field is uniform. In reading Archimedes' writings, I became interested in seeing what happens if the gravitational field is not uniform. Mathematics and physics textbooks tell us that in a nonuniform gravitational field, the lines of suspension do not cross at a single point, but they don't tell us what the lines of suspension do in this case.

So using a computer I came up with certain surfaces that non-crossing lines of suspension in a spherical gravitation field generate. These surfaces might be called "surfaces of gravity" and take the place of centers of gravity. Again, I don't have

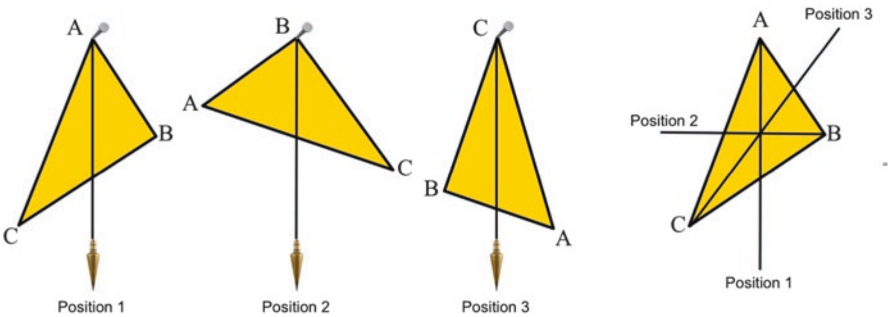

Figure 8 Finding the center of gravity of a triangle

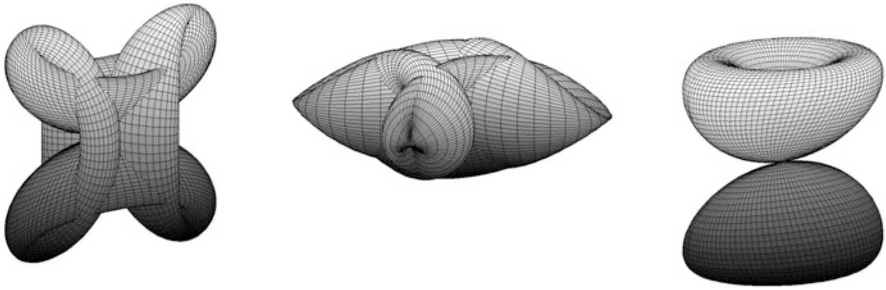

Figure 9 Three possible "surfaces of gravity"

space to describe my results except in very vague terms. Each of the three surfaces in Figure 9 is a surface of gravity for three particular rigid bodies in a spherical gravitational field. This is work that Archimedes could have done if he had had a computer, but without that opportunity, we had to wait twenty-three centuries to explore this problem.

The Archimedes Palimpsest

One final subject I want to discuss is the Archimedes Palimpsest. The rather severe looking gentleman in Figure 10 is Johan Heiberg, a famous Danish philologist who edited many of Archimedes' works. Perhaps his most notable contribution to Archimedean studies was his discovery and transcription of the Archimedes Palimpsest in Constantinople (modern day Istanbul) in 1906.

The Archimedes Palimpsest is a manuscript that contains writings of Archimedes long thought to be lost [12]. Its most valuable content is a particular work of Archimedes entitled *Geometrical Solutions Derived from Mechanics*, now simply called *The Method*. An English translation [13] of its contents first appeared in 1909.

Figure 10 Johan Heiberg (1854–1928)

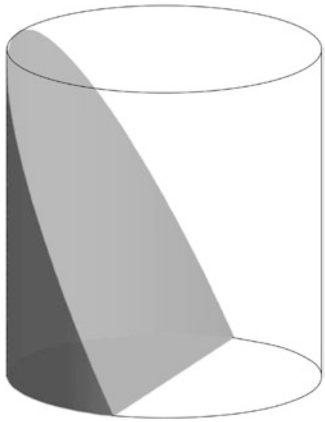

Figure 11 Circular wedge cut from a cylinder

Like Archimedes' cattle problem, *The Method* concerns a challenge from Archimedes to Eratosthenes. It begins with Archimedes reminding Eratosthenes that he/Archimedes had previously challenged him with finding the volumes of two particular solids—and since he had not heard back from Eratosthenes, he/Archimedes was now going to publish the results himself.

One of the solids whose volume Archimedes determined in *The Method* is the circular wedge shown in Figure 11. This wedge is formed by cutting off a piece of a cylinder by a plane that passes through a diameter of the cylinder's base. Archimedes wanted to compare the volume of that wedge with the rectilinear wedge that contains it (Figure 12). (The volume of the containing rectilinear wedge is the square of the radius R of the cylinder times the height H of the wedge, or R^2H.) So

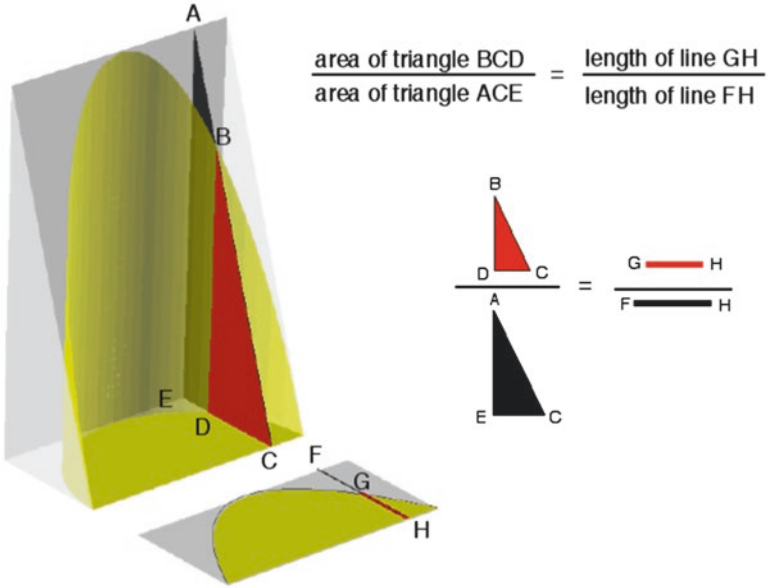

Figure 12 Determination of the volume of a circular wedge

Archimedes asked: What is the ratio of the volume of the circular wedge to that of the rectilinear wedge?

Archimedes found the ratio by what we would now say is Calculus. He considered the wedge to be made up of infinitely many triangular slices and then added the areas of the slices together. In Calculus terminology, he *integrated* the areas of the triangular slices. This and other similar proofs in his collected works are why he is called *The Father of the Integral Calculus.*

What was particularly brilliant about his proof was that he basically reverted to an earlier problem that he had solved: the determination of the area of a parabolic segment. He had earlier shown that the area of a parabolic segment is two-thirds the area of the rectangle that contains it. And from this he was able to show that the volume of a circular wedge was precisely two-thirds the volume of the rectilinear wedge that contains it.

Figure 12 illustrates his approach to the problem. He first showed that the ratio of the area of the triangle BCD to that of triangle ACE is the same as the ratio of the length of line GH to that of line FN. (G lies on the parabola enclosed by the rectangle). This is a purely geometric result relating certain ratios of areas to certain ratios of lengths.

Archimedes then summed both sides of the equations in Figure 12. He was vague about what he meant by *sum*—and he knew that he was being vague. He did not consider this a rigorous proof; he considered it a heuristic proof and later in *The Method* he gave a more rigorous proof.

But first the heuristic proof: If we sum all infinitely many areas of the triangle, then on the left-hand side of the equations in Figure 12, we will come up with the ratio of the volume of the circular wedge to the volume of the rectilinear wedge, and that's the same as the ratio of the parabolic segment to the circumscribing rectangle. And that previously was two-thirds:

$$\frac{\text{Volume of Circular Wedge}}{\text{Volume of Rectilineal Wedge}} = \frac{\text{Area of Parabolic Segment}}{\text{Area of Circumscribing Rectangle}} = \frac{2}{3}$$

Archimedes' final formula is this: the volume of a circular wedge is $(2/3)HR^2$, where H is the height of the wedge and R is its radius.

He found this to be a particularly nice result because, as we would say today, it doesn't involve pi. The circular wedge is thus a solid involving circular region that we can cube; that is, we can construct a cube using a compass-and-straightedge that has the same volume as a circular wedge. Although we cannot square a circle, we can cube this more complicated region because there is no pi, no transcendental number involved in the formula.

So here are some examples of Archimedes' mathematical investigations, the last one leading to the basic ideas behind the Integral Calculus. It's interesting that although Archimedes did several things in *The Method* using this idea of adding the areas of infinitely plane regions forming a solid, he did not regard his results as rigorous. He believed in the final results and he had tremendous intuition, but he knew that they did not fit into the logical scheme of things. Today we're still arguing about the best way to understand how zero times infinity can sometimes be a finite number.

Remark Many of the results I described in this paper are discussed in a website on Archimedes that I established in 1995 and continue to maintain [14].

References

1. Hardy, Godfrey Harold, *A Mathematician's Apology*, (Cambridge University Press, Cambridge, 1940), p. 81.
2. Lessing, G.E., *Zur Geschichte und Literatur, Ausv den Schatzen der Herzoglichen. Bibliothek zu Wolfenbüttel*, (Zweiter Beitrag, Braunschweig, 1773).
3. Amthor, A., "Das Problema bovinum des Archimedes," *Zeitschrift für Mathematik und Physik (Historisch-Literarische Abteilung),* Vol. 25 (1880), pp. 153-171.
4. Williams, H.C., German, R.A., and Zarnke, C.R., "Solution of the cattle problem of Archimedes," *Mathematics of Computation*, Vol. 19 (1965), pp. 671-674.
5. Nelson, Harry L., "A solution to Archimedes' cattle problem," *Journal of Recreational Mathematics,* Vo. 13 (1980-81), pp. 162-176.
6. Nygrén, Antti, "A simple solution to Archimedes' cattle problem," University of Oulu, Linnanmaa, Oulu, Finland, *Acta Universitatis Ouluensis, Scientiae Rerum Naturalium,* ISBN 951-42-5932-7 (March 2001).
7. Kolata, Gina, "In Archimedes' Puzzle, a New Eureka Moment," *New York Times* (December 14, 2003) p. 1.

8. Netz, Reviel, Fabio Acerbi, and Nigel Wilson, "Towards a Reconstruction of Archimedes' *Stomachion*," *SCIAMVS* 5 (2004), pp. 67-99.
9. Rorres, Chris, "Completing book II of Archimedes's *On Floating Bodies*," *Journal of Hydraulic Engineering*, Vol. 2 (2004), pp. 32–42.
10. Rorres, Chris, "The Turn of the Screw: Optimal Design of an Archimedes Screw," *ASME Journal of Hydraulic Engineering* 126 (2000), pp. 72-80.
11. Mattusch, Carol C., *The Villa dei Papiri at Herculaneum* (Getty Publications, Los Angeles, 2005). p. 172.
12. Wilson, Nigel, *The Archimedes Palimpsest* (Christie's Auction Catalog, New York, 1998).
13. *Geometrical Solutions Derived from Mechanics: A Treatise of Archimedes,* Edited by Johan Ludvig Heiberg, Translation into English by Lydia Gillingham (The Open Court Publishing Company, Chicago, 1909).
14. Rorres, Chris, Web site: *Archimedes Home Page* (accessed October, 2016) http://www.math.nyu.edu/~crorres/Archimedes/contents.html>

Archimedes and Ship Design

Horst Nowacki

Motto on Archimedes: All praise him, few read him, all admire him, few understand him. —A. Tacquet (1612–1660)

Introduction

Archimedes by many is regarded as the most eminent mathematician, mechanicist, and engineer in antiquity. He is also famous for his practical application of scientific knowledge in engineering design. His knowledge has influenced ship design for many centuries, either directly or indirectly. Yet it took two millennia before his basic insights were applied quantitatively in practice at the design stage of ships. Why this long delay? It appears worthwhile to trace the history of this tedious knowledge transfer from antiquity to modernity. How were these elements of knowledge created and justified by Archimedes, how were they passed on as his heritage, and how and when were they applied in ship design practice to this day?

Ship safety and stability considerations play a dominant role in the ship design decision process. One needs fundamental knowledge, based on physical principles, to design a safe ship. This kind of knowledge may be applied in two forms:

- As intuitive, qualitative knowledge, corroborated by observation and experience, augmented by a rational understanding of the mechanisms of stability. This sort of cause and effect feeling guides in numerous trade-off decisions on practical consequences of design measures.
- As a quantitative knowledge, based on calculations, to predict the stability performance of a new design.

Archimedes' principles have been exerting a strong influence in both categories, at least among those properly initiated. But it was a long and arduous road from his first creation and brilliant justification of the basic concepts of hydrostatics, which

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lay buried in rare copies of scrolls in scientific libraries in antiquity, to their broader spreading after rediscovery during the Middle Ages and their wider circulation and gradual acceptance by printed media and at last to their practical application in design calculations. They have turned into routine design tools today. It is almost a miracle that we benefit today from these long buried principles for the safety of our maritime designs. They have turned into routine design tools today.

To recount this history and to understand how this happened, the following questions will be addressed in this article:

- How did Archimedes create the fundamental knowledge and justify the laws of hydrostatics which until today form the foundation for judging ship floatability and stability in ship design?
- How might Archimedes himself have applied these insights to contemporary practical projects in ship design in his era?
- On what circuitous routes did Archimedes' insights arrive in modernity?
- How and when were the laws of Archimedes, supplemented by other fundamental methods, apt to be applied quantitatively and numerically to practical ship design calculations?
- What role do Archimedes' basic laws play today in modern ship design methodology?

It is the purpose of this paper to trace the history of the knowledge in ship floatability, stability, and design created by Archimedes and passed down through many centuries to its current relevance in modern ship design.

Archimedes

Precursors

Human knowledge on the risks of seafaring is ancient. The first human experiences in oceangoing navigation by waterborne vehicles date back well into prehistoric times. There is indirect, though conclusive, evidence of the first human settlers on the continent of then contiguous Australia/New Guinea arriving from mainland Southeast Asia between about 30000 and 40000 years ago by crossing a deep ocean water gap of about 100 miles using watercraft capable of carrying humans, animals, and cargo (Diamond [1]). In the Mediterranean Sea traces of waterborne navigation date back to about 10000 BC, notably in Egypt, Babylonia, later in Phoenicia, Crete, Cyprus, and Greece. Early ocean voyages, some of them across considerable distances, required seaworthy ships, safe against all hazards of the sea. Details are documented in the literature (Kemp [2], Johnson, Nurminen [3]).

Thus at the time of Archimedes during the third century BC, Greek shipbuilding and ship design, as he knew it, had already reached an advanced level of construction technology and design complexity for ships to be used in trade, cargo transport,

Table 1 Chronology of precursors and contemporaries of Archimedes

Thales of Milet (624–544 BC)
Pythagoras (580–496 BC) Demokritus (ca. 460-ca. 360 BC)
Plato (427–347 BC)
Eudoxus (410–356 BC)
Aristotle (384–322 BC)
Alexandria founded: 332 BC
Euclid (325 BC–ca. 265 BC?)
Mouseion in Alexandria: 286–47 BC
Archimedes (ca. 287–212 BC)
Eratosthenes (284–204 BC)

and even warfare. Safer and ever larger ships had evolved during many centuries. The safety of ships was judged, as today, by their ability to survive the risks of sinking and capsizing, i.e., by their floatability and upright stability. These properties had to be assessed largely intuitively on the basis of experience, observation, and comparison with similar designs. It was difficult to predict the performance of new designs prior to building them for lack of physical insights as well as analytical and numerical methods of design evaluation. The rational foundations for design decisions and for safe ship operations had not yet been laid.

It was Archimedes, the eminent mathematician, mechanicist, and engineer, who in a stroke of genius was able to combine these various viewpoints and to develop a rational theory of hydrostatics of floating objects, which was directly applicable to the issues of ship floatability and stability. Thus the physical principles of hydrostatics were then well understood although the practical application to ship design was long delayed, actually by about two millennia, before numerical calculations of these important elements of ship safety could be performed routinely by numerical methods at the design stage. We will discuss the reasons for this long delay in a later section. Nevertheless Archimedes and his contemporaries on the basis of his involvement in ship hydrostatics contributed much to the physical understanding of ship stability and thereby to judging the potential effectiveness of certain design measures.

On the mathematical side and in the logic of rigorous proofs, Archimedes had several predecessors, too, in the tradition of earlier Greek philosophy and mathematics. Table 1 gives an overview of some important precursors and contemporaries who influenced his work. It is important to single out Eudoxus [4], a pupil of Plato, who not only established rules and set standards for the rigor demanded in Greek proofs, but also worked out the method of exhaustion, an approximation scheme for evaluating integration hypotheses for curves and surfaces by successive refinement of inscribed or circumscribed polygons (or polyhedra). Archimedes made much use of this approximation technique, which was his tool for area, volume, and centroid evaluation of simple geometric figures, while integral calculus was not yet conceived in antiquity. Archimedes was well trained in the contemporary methods of

Greek mathematics, both by his early education received in Syracuse, reportedly also by his father, an astronomer, and by his contacts with and probable visit to Alexandria, an ancient center of excellence in mathematics in this era. Details on the knowledge thus available to him are found in Heath [5].

Thus Archimedes was able to build his deductions on a solid tradition in Greek mathematics and logical rigor, but he also was very creative in developing his own mathematical tools when he needed new, original ones.

On Floating Bodies

Preparations

Archimedes of Syracuse laid the foundations for the hydrostatics of floating objects in his famous treatise *On Floating Bodies* (OFB) (περὶ ὀχουμένων) [6]. In this treatise he was the first one single-handedly to establish the laws of equilibrium for a body at rest in a fluid, floating on top or submerged or even grounded, on a scientific basis by deduction from a few axioms or first principles. Thus although he never wrote about applications to ships, he did develop the physical foundation for judging the force and moment equilibrium of floating objects, including ships, i.e., their floatability (force equilibrium) and stability (moment equilibrium).

The Principle of Archimedes, based on force equilibrium of buoyancy and gravity forces, holds for objects of any shape. The criterion of stability was first pronounced by Archimedes for the special case of homogeneous solids of simple shape, a semisphere and a paraboloid of revolution. These results form the cornerstones of ship hydrostatics to this day.

We are fortunate that many, though not all, of Archimedes' treatises have survived from antiquity to this day, essentially all in copies, some in Latin translation, some in Arabic, and a few even in the Greek language of the original. Many were originally lost in late antiquity and were often luckily rediscovered much later, which will be addressed in a later section. But from those which are preserved we are able to reconstruct the train of thought that Archimedes took to arrive from the principles of geometry and engineering mechanics at his scientific foundation of hydrostatics.

Table 2 gives an overview of the essential preserved treatises by Archimedes. We cannot precisely date the first appearance of these works, but there is sufficient evidence in their contents to suggest their sequence of publication. Compilations of Archimedes' works exist in several classical and modern languages (Heath [5], Heiberg [6, 7], Dijksterhuis [8], Van Eecke [9], Czwalina-Allenstein [10, 11], etc.). They are essentially in agreement on the chronology of appearance. Thus it is in essence undisputed that OFB was preceded by a few other fundamental treatises, which we will briefly address here.

Table 2 Chronology of Archimedes' preserved treatises

Item	Title	Probable sequence
1	*On the Sphere and Cylinder*, Books I and II	(5)
2	*Measurement of a Circle*	(9)
3	*On Conoids and Spheroids*	(7)
4	*On Spirals*	(6)
5	*On the Equilibrium of Planes*, Books I and II	(1) and (3)
6	*The Sandreckoner*	(10)
7	*The Quadrature of the Parabola*	(2)
8	*On Floating Bodies*, Books I and II	(8)
9	*Stomachion*	
10	*The Method of Mechanical Theorems*	(4)
11	*Book of Lemmas*	
12	*The Cattle Problem*	

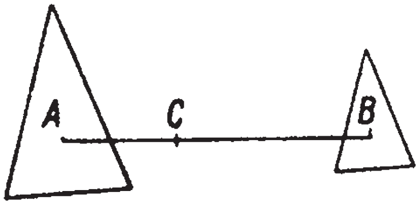


Figure 1 Lever system. Balance with unequal arms (From Czwalina-Allenstein [10])

The Law of the Lever

In his treatise *On the Equilibrium of Planes*, Books I and II, Archimedes concerns himself with the "moment"equilibrium of objects on a lever system like a balance (Figure 1). The objects might be homogeneous solids or elements of thin planar areas of constant thickness, and homogeneous gravity distribution, which can be regarded as solids so that the same lever laws can be applied in dealing with their equilibrium and centroids.

The principle of the lever and especially of the balance was certainly known since prehistoric times, e.g., in ancient Babylonia, Egypt, and China (see Sprague de Camp [12], Renn and Schemmel [13]). Archimedes presumably was the first to pronounce the physical law of the lever and to apply it to many mechanical and geometric systems. The Greeks did not know the concept and terminology of moments, so he spoke of "the law of the lever" for the same purpose.

In this treatise Archimedes derives the following conclusions from the law of the lever:

– The equilibrium of unequal weights on a balance of unequal arms (Figure 1) so that the weights are inversely proportional to their lever arms ("moment equilibrium")

- How to lump two or more objects into a single compound object with a compound centroid so that the statical moment of the compound object remains the same as for the sum of the separate objects
- Removing, adding, or shifting objects in a system and finding the new centroid

To summarize, in this treatise Archimedes pronounces the following principles, to which he can later resort for the stability of floating bodies:

- The law of the lever (moment equilibrium)
- Lumping objects into their centroids, thus forming resultants
- Finding compound centroids for a set of system components
- Removing, adding, or shifting objects in a system and its effect on the centroid

The Method of Exhaustion

In evaluating the area of planar figures or the volume of geometric solids, the ancient Greeks met with two major obstacles:

- They had no concept of real numbers, let alone irrational or transcendental numbers, nor did they have a consistent system of units for measuring length, area, and volume. They circumnavigated this difficulty by restricting themselves to finding only the ratio of a figure area or volume to that of a known figure, e.g., a square or a triangle whose size is known. In this context real numbers were expressed by ratios of integers. Irrational or transcendental numbers were approximated as such ratios. For example, finding the area of a circle relative to a circumscribed square led to the famous "quadrature of the circle" type of problem.
- They had no method to evaluate areas and volumes of figures equivalent to modern calculus. This is why they resorted to approximation methods based on a finite number of successive subdivisions of the given figure by simple shapes whose area and volume are known. The method of exhaustion, as it was named much later in the seventeenth century, is such an approximation method based on successive refinement of the result by means of a polygonal approximant whose deviation from a given figure shall be made as small as desired after a sufficient, finite number of subdivision steps. The method is not equivalent to integral calculus since a limiting process to infinitesimal step size is not performed. But for geometrically well-defined figures of simple shapes, very accurate approximations can be obtained after a finite number of steps.

The method of exhaustion then proceeds as follows (Figure 2): Two additional triangles AZB and BHC are constructed between the parabola and the original triangle by drawing parallels to the center line BD through the quarter points E and K of the baseline. For the parabola it can be shown that each of the new triangles has an area of (1/8) of triangle ABC, hence both together of (1/4) of the original triangle.

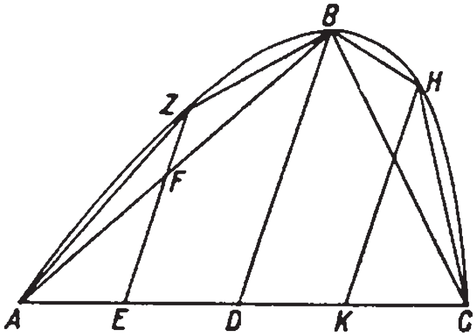


Figure 2 Paraboloid segment AZBHC and inscribed triangles (From Czwalina-Allenstein [10])

In a process of successive refinement in each step, new triangles are added between the polygon sides and the parabolic arc, always halving the intervals along the baseline. It can be shown that for the parabola each new set of triangles adds (1/4) of the area of the preceding set of triangles. Thus in a process of continuing refinement, the terms of triangle areas are added to a geometric progression whose quotient is (1/4). The sum of this progression is either known on arithmetical grounds, viz., for the parabola, or estimated after a finite number of steps when the truncation error appears small enough. For the parabola the end result is (4/3) times the area of the first triangle ABC. Such a result is first asserted inductively and then proven rigorously by reductio ad absurdum of any deviating results. Details of this proof can be found in Archimedes' original treatise (cf., e.g., Heath [5] or Nowacki [14]).

Archimedes, who seems to have adopted this method from Eudoxus, frequently used it for the evaluation of areas and volumes of simple shapes, i.e., for certain applications where modern analysis would use integral calculus. This explains why certain of his results were limited to simple geometric shapes, e.g., for the hydrostatic stability of floating objects, and were not extended to objects of arbitrary shapes, e.g., ships.

The Method of Mechanical Theorems

In certain cases Archimedes also had another fast and efficient method available to derive hypotheses for geometric results from mechanical analogies. This technique is described in his treatise, *The Method of Mechanical Theorems*, which was long lost, but then was rediscovered first by J. L. Heiberg in 1906 in a Greek monastery, the Metochion, in Constantinople in an old twelfth-century palimpsest. The Archimedes text had been rinsed off, and a Greek prayer book (euchologion) was written on the same vellum sheets. But Heiberg was able to decipher most of the original Archimedes text under a magnifying glass, to document, transcribe, and translate it into German [15]. The interesting history of this palimpsest is described in more detail in sections "The Manuscripts" and "Codex C".

In this treatise Archimedes explains to Eratosthenes how he applies principles of mechanics in geometric reasoning to obtain inductive conclusions on geometric facts. Mechanical theorems are based on observation, hence inductively founded. Archimedes therefore uses these methods only to help propose hypotheses on geometrical facts. Archimedes does not regard these results as validly proven, but a strict, deductive, purely geometric proof must follow to confirm the conjecture. Some of these subsequent geometric proofs are preserved; others are lost.

In his treatise *The Method…* Archimedes deals with the example of a solid paraboloid of revolution. He asserts:

> *The centroid of a right-handed conoid (here a paraboloid of revolution) cut off by a plane at right angles to its axis lies on the straight line which is the axis of the segment, and divides the said straight line in such a way that the portion of it adjacent to the vertex is double of the remaining portion.*

The proof, based on the law of the lever and *The Method …*, is presented in detail in the treatise *The Method…* (Heath [5]), proposition 5, and is also fully explained by Nowacki [14]. This confirms that the centroid of the paraboloid segment is located at 1/3 of segment height above the baseline or 2/3 below the summit. Thus Archimedes was able to demonstrate geometric facts without needing to resort to calculus. Archimedes later uses this result in the context of his study on hydrostatic stability of a paraboloid in OFB.

The Principle of Archimedes

The fundamental law of hydrostatics for a body at rest within or on top of a homogeneous liquid is pronounced and justified by Archimedes in his treatise OFB, Book I. It is deduced here strictly by an experiment of thought, i.e., without any experimental observation or other empirical basis. It holds for a body of arbitrary shape.

The liquid is assumed to be homogenous and such that any liquid particle is pressed downward vertically by all particles in the vertical line above it (OFB, Book I, § 1). We would call this a hydrostatic pressure distribution in modern terminology, but the Greeks did not know the concept of "pressure" in antiquity. In Book 1, §5, Archimedes asserts for a body in a liquid at rest:

> *A body submerges in a specifically heavier liquid to the extent that the volume of the liquid displaced by it weighs as much as the whole body.*

In modern terminology this *Principle of Archimedes* can be stated as:

$\Delta = \gamma V$, where
Δ = displacement = weight of body
V = displaced volume
γ = specific weight of liquid
γV = weight of displaced liquid volume
= buoyancy force

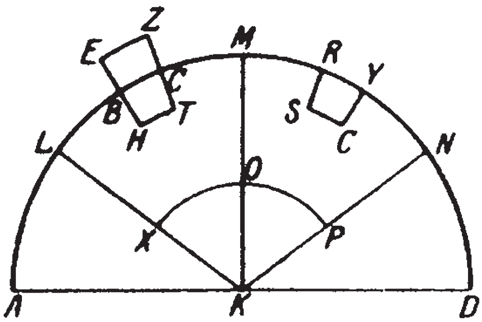


Figure 3 Proof of Archimedes' Principle (From Czwalina-Allenstein [10])

Archimedes' proof in OFB, Book 1, §5, is brilliantly elegant and brief (Figure 3):

- The surface of any liquid at rest is a spherical surface whose center point is the center of the earth (section ALMND).
- The body EZTH be specifically lighter than the liquid; hence it floats in the surface.
- We consider two neighboring equal sectors of the sphere, bounded by the surfaces LM and MN. The first sector contains the floating body whose submerged part is BHTC. The second sector instead has an equal volume RYCS filled with the liquid.
- The liquid is at rest. Thus the surfaces between XO and OP experience identical pressing loads. Thus the weights of the volumes on top of these surfaces are equal. But the weight of the liquid in the first sector, aside from the space BHTC, equals the liquid weight in the second sector, aside from the space RYCS. Therefore clearly the weight of the body EZTH equals the weight of the liquid volume RYCS. It follows that the liquid volume displaced by the body weighs as much as the whole body.

Note that this concise proof of the Principle of Archimedes is strictly deductive and is based entirely on an experiment of thought and very few axiomatic premises. It holds for arbitrary body shapes in an arbitrary type of liquid. It was derived for liquids and bodies at rest without knowledge of the pressure distribution anywhere.

The Eureka Legend

The famous Eureka legend about Archimedes' discovery of a hydrostatic law in the bathtub, which goes back to Vitruvius ([16], Book IX.3, published around the birth of Christ), is often misinterpreted and has led to confusion regarding this discovery. According to Vitruvius Archimedes was challenged by King Hieron to determine whether a wreath, made for the king by a goldsmith for a votive offering to the gods, was of pure gold or fraudulently made of gold mixed with silver. Archimedes is said to have observed how the water displaced by his body in a brimful bathtub was a measure of the displaced volume, which he regarded as a breakthrough in solving the king's wreath problem. Archimedes was apparently so elated by this discovery that he jumped up from the bathtub and ran naked through the streets of Syracuse shouting "Eureka" ("I have found it")! Vitruvius goes on to report how Archimedes then demonstrated the fraud. The wreath and two equally heavy pieces of pure gold and silver successively were each sunk in a bowl full to the brim of liquid. After removing the object its volume was determined by refilling the bowl to the brim and measuring the weight of the replacement liquid. This indeed gives a clue as to the relative densities of the two metals and the wreath, viz., the weight of the object related to the weight of the displaced water.

Thus Archimedes had discovered a method for measuring the *volume* of a submerged object and thus, if the weight of the object is known, its specific weight (weight per unit volume). This is sufficient to prove the fraud of using a lighter, false metal in the wreath. The specific weight of the objects is used as a criterion of comparison.

But the Principle of Archimedes cannot be proven by a human sitting in the bathtub. The human body in this scenario is usually supported in part by a ground force so that the equality of body weight and buoyancy force does not hold. Thus we must credit Archimedes for deducing his famous Principle strictly by experiment of thought, as explained. Is it not another sign of his brilliance that he was able to deduce this Principle without resorting to a physical experiment, let alone an observation in a bathtub?

The legend of the wreath has attracted many critical reviews debating the practical difficulties in realizing sufficiently accurate results. Recent reviews and experimental reconstructions of the wreath experiment are given by Costanti [17] and by Hidetaka [18]. There seems to be agreement among these scholars that it is feasible to reconstruct the experiments described by Vitruvius in a small enough bowl, though not in a large bathtub, in order to solve King Hieron's fraud problem and to measure specific weights of solids, though not in order to find the Principle of Archimedes. The bathtub, if anything, might have served as an inspiration.

Hydrostatic Stability of Floating Objects

In his treatise OFB Archimedes also deals with the stability of bodies floating on the surface of a liquid, especially of homogeneous solids of simple shape. His basic ideas are already shown in Book I, §§8-9, for a segment of a sphere. In Book II, §§ 2 ff. the example of an axisymmetric paraboloid segment as shown in Figure 4 is even more illustrative and will be discussed here.

The stability criterion for hydrostatic equilibrium is based on the experiment of thought to incline the body from its upright condition and to determine whether the resultant gravity and buoyancy forces acting on the body in this condition will tend to restore it to its upright condition. The angle of inclination (heeling angle) is finite but so that the base of the paraboloid segment does not get wetted. For the stability of this body, Archimedes asserts in Proposition II.2:

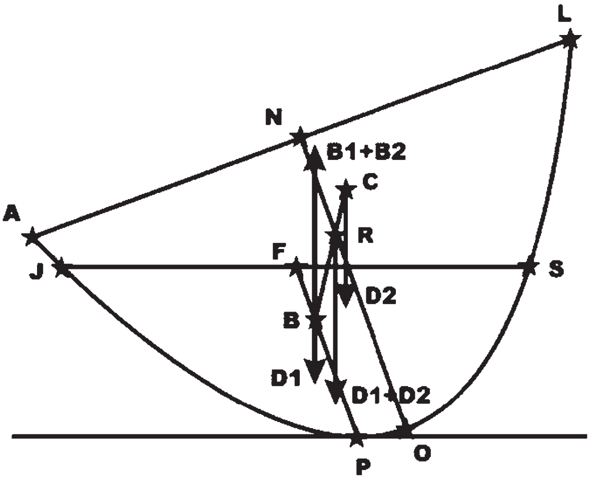


Figure 4 Inclined paraboloid segment (From Nowacki [14])

> *A homogeneous solid paraboloid segment cut off perpendicularly to its diameter, whose axis is not greater than 1.5 times the paraboloid's halfparameter, whatever its specific weight, if it floats in a liquid so that its base does not touch the liquid surface, will not remain at rest unless its axis is vertically oriented, but will restore itself to the upright condition.*

The proof is based on finding the centroid R of the homogeneous solid, through which the weight resultant D1+D2 is acting downward, i.e., the center of gravity in our terminology, and the centroid B of the submerged volume, i.e., the center of buoyancy, through which the equal and opposite resultant buoyancy force B1+B2 is acting upward. The lever arm between these two forces must be of such orientation that a positive restoring tendency ("moment") results, which corresponds to our familiar positive righting arm requirement. However there is a subtle difference in Archimedes' demonstration for the homogeneous solid: For the submerged part of the inclined solid alone, its center of gravity and its center of buoyancy are in the same position B so that this submerged part produces no lever arms (unlike in a ship). Thus it is sufficient to show that the gravity force D2 of the above surface part of the solid and its equal but opposite counterpart, the incremental buoyancy force B2, which acts through the center of buoyancy B, have a positive restoring arm. One can show that, if this "incremental righting arm" of B2 and D2 is positive, then the conventional righting arm of the forces (B1+B2) and (D1+D2) is also positive.

In the actual proof Archimedes applies several mechanical and geometric principles which he deduced in this treatise or in earlier work. For more details see also Nowacki [14]. He proceeds in the following steps:

– The paraboloid is intersected by a vertical plane in the parabola APOL and is inclined by a finite angle. The horizontal plane of the liquid surface intersects the paraboloid in JS. A tangent parallel to JS touches the parabola in P.

- A parallel line to the axis NO, drawn through P, bisects the chord length JS at midpoint F (proven in earlier in *Quadrature of the Parabola*, §19). PF is thus the axis of the submerged part of the paraboloid.
- Archimedes now uses his theorem on the centroid of the paraboloid segment lying on its axis at a point 2/3 of the axis length above the summit, proven in his treatise on *The Method*…. Therefore PB = (2/3) PF. Thus B is both the center of buoyancy and the center of gravity of the immersed part. The immersed part therefore creates no moments and can be left out of consideration for moment equilibrium. Further let OR = (2/3) ON, so that R is the center of gravity of the whole solid.
- Using the centroid shift theorem from his treatise *On the equilibrium of planes*, Book 1, §8, Archimedes constructs the center of gravity C of the above surface part JALS: If the immersed part JPOS, centroid B, is removed from the whole solid, centroid R, then the remaining above surface part JALS has a centroid C on the straight line BR, extended beyond R, such that

$$\text{RC:RB} = (\text{immersed volume}) : (\text{above surface volume})$$

- Thus C lies on one side, B on the other side of R.
- Therefore the vertical gravity force D2 through C and the equal but opposite buoyancy force B2 through B are not acting in the same vertical line, hence not in equilibrium of moments, but will tend to restore the paraboloid to its upright condition. Thus the investigated paraboloid is hydrostatically stable.

Although this derivation holds only for the homogeneous solid paraboloid, it can be shown that a similar reasoning can be developed for a solid of any shape and with nonhomogeneous mass distribution, hence also for ships. The incremental righting arm of the paraboloid is thus the ancestor of the conventional righting arm used in modern stability analysis. Positive righting arms are a necessary condition of upright ship stability.

Achievements and Deficits

To summarize the state of knowledge in the hydrostatics of floating bodies achieved by Archimedes, as displayed in his treatise OFB, and to state the elements of information still missing for a complete analytical and numerical evaluation of the hydrostatic properties of a ship design, the following presents a synopsis of his achievements and remaining deficits.

Archimedes contributed the following fundamental insights and methods pertinent to the hydrostatics of ships:

- Archimedes defined the resultant gravity and buoyancy forces (displacement and buoyancy) acting on a floating body and pronounced the force equilibrium principle of their equality in the same line of action and in opposite directions (Principle of Archimedes).

– He devised methods for calculating the resultants of buoyancy and weight acting through the CB and CG, at least for solids of simple shape (method of exhaustion, *Method of Mechanical Theorems*, Lumping of System Components into Compound Systems).
– From the axiom of moment equilibrium, Archimedes deduced a criterion for hydrostatic stability by introducing the concept of righting moments or righting arms based on the couple of buoyancy and displacement forces.

With this information a qualitative understanding of which design measures have favorable or unfavorable influence on the design was at least possible.

To perform a complete, quantitative analysis of the hydrostatic properties of a ship at the design stage, the following elements were still missing for Archimedes:

– A complete, continuous hull form definition for arbitrary ship shapes in whatever medium (mold, model, drawing)
– A method to calculate the volume and volume centroid of the underwater hull form (center of buoyancy, CB) for an arbitrary ship shape
– A practical scheme to calculate the weight of all component parts of the ship and therefrom the compound center of gravity, CG, of the entire ship
– Data for the external heeling forces and moments acting by wind and seaway on the ship in order to assess the required safety margins in floatability and stability

In view of these deficits Archimedes and his contemporaries were not yet able at the design stage to quantitatively predict the design's draft, trim and heel angles, and stability measures for the hull in empty and loaded conditions. This remained a matter of experience, observation, and empirical judgment. In case of doubt ballast might be taken in the lower hull to improve a ship's initial stability.

Archimedes' Role in Practical Ship Design

Archimedes shared with traditional Greek philosophers the Platonic aristocratic disdain of publishing any results of practical projects so that we have no evidence in his own writing of his involvement in technical inventions and engineering design. But we know from other, often much later writers (Vitruvius [16], Diodorus of Sicily, Plutarch [24] and others) that he excelled in innovative engineering work and in creative inventions for very practical purposes. His conception and realization of many practical devices and unique, original machines are well documented (e.g., by Dijksterhuis [8]). This has often raised the question to what extent Archimedes might also have been involved in practical ship design and ship construction projects.

The most prestigious shipbuilding project in Syracuse during Archimedes' lifetime was without doubt the design and building of the Syrakosia (or Syracusia), a giant cargo ship, mainly for the transport of grain, an export commodity from Sicily.

King Hieron II of Syracuse had ordered the ship to be built around 235–230 BC in Syracuse. We are curious to know whether Archimedes was involved in this representative project and was able to apply his theoretical knowledge in practice.

The most detailed technical report about the Syrakosia stems from Athenaeus who documented many details of the design and the history of this ship in his treatise "*Deipnosophistai*" [19], published somewhat after AD 192, thus more than four centuries after Archimedes and the existence of this ship. The ship impresses by its size, cargo capacity, accommodation, and outfitting. A thorough reconstruction was recently performed by Bonino [20]. According to this source some of the main features of the ship were:

Length = 80.0 m	Beam = 15.5 m
Draft = 3.85 m	Depth = 5.6 m
Displacement = 3010 metric tons	Payload = 1940 metric tons
Block coefficient of submerged part of hull = 0.615	

The ship had three masts and three decks. The combat deck had 8 towers with war machines. The complement was 825 people on board. Twenty horses with stables were accommodated, too. The accommodation spaces were of high, comfortable standard, with luxurious tiling on the floors, walls, and ceilings. A model of the ship was built by Tumbiolo based on Bonino's reconstruction drawings. Further details on the design and the history of the ship can be found in Bonino [20].

King Hieron had recruited the prominent naval architect Archias from Korinth to take the overall responsibility for the design and building of the ship. Archimedes is mentioned by Athenaeus as a supervisor (ό γεωμέτρης ἐπόπτης, the geometer supervisor) or perhaps just advisor in this project. It is recorded that he played a decisive role in the launching of the ship. When the lower half of the hull had been assembled on the slipway with a lead sheathing already attached, she was ready to be launched in order to add the top half later when she was afloat. But she would not move down the slipway. Archimedes was called to help. He mounted a windlass with a multiple pulley system to the hull to drag her down into the water. He was able to crank the windlass alone and to launch the ship single-handedly. He was much admired for this ingenious solution. It is also conceivable that Archimedes designed some military equipment to be placed on board for the ship's self-defense, e.g., catapults to fire arrows at enemy or pirate ships or a stone-hurler to shoot a stone of 180 pounds or a javelin of 18 foot length some 600 foot distance. Thus he was certainly welcome to contribute such ideas in his area of expertise.

But it very unlikely that he was prepared to offer an explicit stability analysis for the ship. The main reasons were explained in the preceding section: the lack of a continuous hull form definition, the difficulty in calculating volumes and centroids for an object of arbitrary shape, as well as the missing experience and data for setting target values for safe stability margins. Rather practitioners would know how to use ballast and cargo deep in the hold to improve stability when needed.

Table 3 Historians and commentators of Archimedes

Polybius (201–120 BC)
Cicero at tomb in Syracuse: 75 BC
Livy (59 BC–AD 17)
Vitruvius (ca. birth of Chr.) [14]
Diodorus of Sicily (ca. birth of Chr.)
Plutarch (AD 46–120.) [24]
Pappus of Alexandria (AD 290–?)
Serapeion library burnt: AD 391
Proclus (AD 400–ca. 485)
Eutokios of Askalon (AD 530–600)
Tzetzes, Byzantine writer: 12th cent.

Recent discussions in the literature (Zevi [21], Pomey and Tchernia [22], Pomey [23], Bonino [20]) are in agreement with this cautious opinion on Archimedes' possible involvement in practical ship design.

The Heritage

The Commentators

Archimedes' scientific work is quite authentically, though not completely, preserved in copies of his manuscripts although some copying errors, translation flaws, and lacunae cannot be ruled out. Thus it is important to know the history of the manuscript copies and to examine their quality.

In the Hellenistic era and in late antiquity, most of his manuscripts were collected in the great library of scrolls in Alexandria, Egypt, at the Mouseion (286–47 BC) and later at the Serapeion (through AD 391).

Archimedes most likely had spent an extended study period in Alexandria around 240 BC, as mentioned by Diodorus of Sicily. At the Mouseion many famous scientists lived and worked there together. The Mouseion served as a center for collecting many thousands of scrolls but also for copying and distributing this body of knowledge within the ancient scientific world. There Archimedes had met many contemporary scientists, e.g., Konon, Dositheos, Aristarchos, and Eratosthenes, among others, with whom he established lifelong friendships and communications. He sent them his manuscripts and thus probably made them available to the ancient scientific world via Alexandria.

Unfortunately the libraries there fell victim to great fires and lootings at least twice, the Mouseion in 47 BC, the Serapeion later in AD 391. Yet copies of Archimedes essential works existed elsewhere in other libraries and scientific centers.

Table 3 shows the names of some of the main historians, biographers, and commentators on Archimedes and his work during the centuries following his death. In this period access to his manuscripts still comprised a few that are now lost. This interest, mainly in the mathematical and mechanical treatises, continued for several centuries. Unfortunately OFB is nowhere mentioned by these authors.

The Manuscripts

Archimedes' own manuscripts are all lost. But fortunately several copies were made in antiquity and the early Middle Ages so that a total of 12 treatises of his are preserved and currently known (Table 2). Some of these manuscripts have survived in the original Greek language, others in Latin translation, a few even in Arabic.

In Alexandria, from where many of the master copies originated, Caesar had confiscated many scrolls from the Mouseion as war booty in order to ship them to Rome but apparently most were burnt or lost during the uprisings in 47 BC. The majority of the remaining scrolls were lost when the Serapeion temple, which served as library, was set on fire by a Christian mob in AD 391 (Sprague de Camp [12]).

The Byzantine Empire was best positioned geographically and suited culturally to resurrect the classical Greek traditions. In fact, Leon of Thessaloniki, a Byzantine cleric, in the ninth century, undertook a collection of Archimedes' dispersed works to which we owe the existence of at least three later master copies, called Codices A, B, and C by Heiberg [7] later, which became the master sources for all later preserved copies and translations. The treatise OFB was contained in Codices B and C.

In Sicily during the Norman and Hohenstaufen rule in the eleventh and twelfth century, a blossoming of classical literature and science occurred, promoted in part by exiled Byzantine scholars, who apparently brought at least two sets of Archimedes manuscript copies with them to the West. When the Hohenstaufen empire collapsed after the battle of Benevento in AD 1266, these copies ended up in the papal libraries.

One of these sets, Codex A, in Greek without the OFB treatise, changed hands a few times, was however copied before it was irretrievably lost in 1564. For several centuries these copies of Codex A were the only available sources for Archimedes in Greek language.

Evidently a second set of manuscripts by Archimedes had existed in Sicily and then in the papal library, which contained OFB. This made it possible to Willem van Moerbeke, a Flemish Dominican monk, who worked as a translator at the papal court in Viterbo from 1268 to 1280, to produce a Latin translation of Archimedes' preserved works based on both earlier sources and including OFB, published in 1269. This Latin translation, later called Codex B by Heiberg [7], and its copies provided the principal access to Archimedes for the Latin reading community.

After about 1500 by means of the fast spreading of Gutenberg's printing press, several printed editions of Archimedes' work soon appeared in order to satisfy the growing interest in classical science. Among the first few we mention the editions by Tartaglia (Venice, 1543) and Curtius Trajanus (Venice, 1565). Both were Latin translations, based on Codex B, the former with Book I, the latter with both books of OFB. Commandinos Latin editions (Venice, 1558/1565) based on Codex B, the 1565 edition with OFB, were highly regarded for their quality. After 1600 many other editions have appeared in Greek, in Latin, and in modern languages (Dijksterhuis [8]).

Clagett [25] has presented a very thorough survey of the history of Archimedes' works during the Middle Ages and through the revival in the Renaissance.

Codex C

Miraculously in 1906 a third master copy of Archimedes' collected works in Greek was rediscovered by J. L. Heiberg [15], a Danish scholar of classical languages, in a Greek monastery, the Metochion in Constantinople, later called Codex C. This copy was found in a palimpsest where Archimedes' text of the tenth century had been rinsed off and the parchment pages were reused and reassembled in the thirteenth century for a Greek prayer book (euchologion). As Heiberg personally writes in the preface of his German translation of *The Method*, which appeared in 1907 [15]:

> *Last summer I have examined a manuscript at the Metochion (in Constantinople) of the Monastery of the Holy Sepulchre in Jerusalem that underneath a prayer book (euchologion) of the 13th c. contains treatises by Archimedes written in the beautiful minuscula of the 10th c., which is only rinsed off, not scraped off, and is reasonably legible with a magnifying glass. The manuscript, no. 355. 4to, stemming from the monastery of St. Sabba near Jerusalem, is described by Papadopoulos Kerameus, who gives a sample of the writing below. From this it was immediately clear to me that the old text was Archimedes…… It is however more important that the manuscript contains the very nearly complete text of* "On Floating Bodies*", from which in the past only the Latin translation by Willem van Moerbeke was available; its numerous lacunae and grave corruptions can now be healed completely*.

Heiberg's preface goes on to praise the value of the first ever discovery of *The Method*…. The new findings from this palimpsest were soon transcribed, documented in Greek, and used in subsequent translations [7].

The adventurous history of Codex C continues in style: During the upheavals of the Greek-Turkish war of 1920–1922, the collections of the Monastery Library in Constantinople are taken to Greece. There the palimpsest disappears. It seems to have been acquired by a French private owner. Its condition further deteriorated, also by failed attempts to sell it as a medieval prayer book with added illuminations. Incredibly enough it eventually resurfaced at an auction by Christie's in New York in 1998, where it was bought by an anonymous American buyer for 2.2 million dollars.

Since then in 1999 the new owner lent the manuscript to the Walters Art Museum in Baltimore for secure conservation and renewed scientific evaluation. The museum team has been applying the most modern techniques for reconstructing the ancient text as accurately as possible. Advanced optical and computer-based methods have been used (multispectral imaging and confocal microscopy a.o.) to increase the contrast for the rinsed-off text and to focus on layers beneath the parchment surface to reconstruct the text from the badly damaged palimpsest.

Important results of this painstaking work have been published by members of the Baltimore team (Netz, Noel [26], Noel, Netz, Tchernetska, Wilson [27]). From these results the excellent quality of the reconstruction is evident, and many new findings were reported that go well beyond the status reached by Heiberg, thus closing gaps in the text and removing lacunae. Thereby our understanding of OFB has been largely confirmed and improved in details. The first ever finding of *The Method* adds much substance to our insights into Archimedes' lines of thought.

The Treatisers

During the fifteenth to seventeenth centuries, a tradition developed in all major European seafaring nations to document the existing and evolving shipbuilding knowledge, whether practical or more theoretical, in more or less learned treatises for diverse purposes. The authors are often called treatisers. They came from practical shipbuilding or more theoretical background or sometimes both. The treatises served as technical notebooks, as introductory texts for the general public, or just as an opportunity to display scientific and technical expertise. Shipbuilding technology and design methodologies underwent major changes during this period in practical and scientific know-how, so the treatises captured valuable elements of contemporary background knowledge. The treatises can serve to monitor the changes that occurred in ship design and production.

We will take a quick survey of the more essential treatises, mainly in order to identify traces of Archimedean knowledge during this period when Archimedes' written work began to be more widely circulated in print. But before this knowledge in ship design could be applied, a few major prerequisites had to be met: A complete, continuous hull form definition and an accurate methodology for evaluating areas, volumes, and centroids were indispensable.

During the Middle Ages and Italian Renaissance, Venice was a leading sea power and shipbuilding center in the Mediterranean. Some of the earliest treatises from Venice document the shipbuilding methodology practiced there. It is found in the treatises by Michael of Rhodes [28], ca.1434/1435; Trombetta [29], 1445; and later with more textual elaboration by Drachio [30], 1599. The ships were built "skeleton first," i.e., a skeleton of transverse frames from the keel through the bottom and continued all the way up in the sides was erected first to which the hull planking was attached later. Thus the shape was predefined by the outer edges of the skeleton frames. The Venetians and other Mediterranean shipbuilders used a special "lofting" method to lay out the shape of the transverse frames by means of a "sesto," i.e., a planar template for the master cross section of the hull with several schemes of marking, from which the shape of any other section at other stations along ship length can be derived by unique rules. Thus geometrically the frames of the skeleton are uniquely deduced from the master section by a process of translation, rotation, and clipping. Thus the hull surface is fully defined by the sesto (except for the ends of the ship). See, e.g., Alertz [31] for more details. In these treatises no reference to design calculations and to Archimedes' OFB is found.

In France, Spain, in Genova/Genoa, and at other Mediterranean cities, very similar lofting methods were used. In French the “maître gabarit” is the equivalent of the “sesto.” The French approach, also called the Mediterranean method, is well documented by Rieth [32]. Design calculations and reference to OFB are also still absent in the pertinent treatises.

Portugal, a successful seafaring nation during the Age of Exploration, also held a strong position in shipbuilding. Early Portuguese treatisers are Oliveira [33] 1580, Lavanha [34] 1614/16, and Fernandes [35] 1616. They deal primarily with ship geometry, molding rules, and ship construction. Lavanha develops precise ship drawings; Fernandes already presents a rudimentary ship lines plan. These sources however do not contain hydrostatic calculations nor references to Archimedes.

In England William Bourne (“Treasure for Travaylers” [36]), 1578, one of the first treatisers there, already describes an approximate practical method to obtain a ship’s volume estimate by taking its offsets when on a dry ground using measuring rods relative to some suitable reference plane outside the hull and up to the desired waterline. The offsets are then linearly connected to estimate cross-sectional areas up to the desired draft. This is done in several transverse sectional planes along ship length. The volume of any segment between measured neighboring cross sections at any given draft is then approximated by linear interpolants. Thus a rough volume estimate is obtained for the ship, which is converted to the ship’s weight or displacement on that draft based on the Principle of Archimedes.

Other English treatisers (Mathew Baker/John Wells, 1570–1627, cf. R. Barker [37], R. Dudley [38], 1646, Bushnell [39], 1664) further evolved methods for designing and molding ship geometry leading up to first ship lines plans on paper. However Anthony Deane [40], 1670, undertook the next steps of section area planimetry from lines plans by circular arc or rectangular/triangular approximants as well as volume estimation, segment by segment, between cross sections. The Principle of Archimedes was again used to find the displacement for any given draft. Stability analysis was not attempted.

During the same period early French treatisers (Fournier, 1643, Pardies, 1673) were mainly interested in nautical matters for the practice of seamanship. It was the Jesuit Père Paul Hoste [41], 1697, who first took on the challenge of estimating from lines plans the displacement (not unlike Bourne and Deane) and a measure of stability. Unfortunately his stability analysis was flawed because he misinterpreted Archimedes and missed the effects of the volume shifts due to a heeling inclination.

In the Netherlands the first pioneering scientific work on ship hydrostatics was performed by the Flemish/Dutch scientist Simon Stevin (1548–1620) to be addressed in more detail in the next section. His work has contributed to an early intuitive physical understanding of the principles of hydrostatics. The Dutch mathematician Johannes Hudde (1628–1704) had proposed a method in 1652 [42], later called the difference in drafts method, for measuring the cargo payload capacity or tonnage as a basis for port fees and taxes by taking the difference between the displacement of the ship fully loaded and the displacement empty. Offsets were taken for the waterlines in both loading conditions, and the volume between the two waterlines was approximated numerically by means of trapezoids and triangles.

This volume was converted to weight by the Principle of Archimedes. Other treatisers, in particular Witsen [43], 1671, and Van Yk [44], 1697, pursued similar paths, especially for estimates of volume, displacement, and payload capacity.

This short survey has been confined to traces of increasing Archimedean influence in ship hydrostatics. Many more details on the work of the treatisers are found in Barker [45] and Ferreiro [46].

In summary it is fair to state that by 1700 Archimedes' text in OFB was known to scientists, but very little of his knowledge had found its way into ship design practice. However in this period the issues of a continuous hull form definition by ship lines plans and of volume and centroid estimates for ship hulls by numerical approximation had been brought to satisfactory practical solutions.

Stevin, Galileo, Huygens

The rebirth of hydrostatics in the seventeenth century, directly based on Archimedes' written work in OFB, and its extension to new foundations and applications is essentially owed to the work of three famous physicists, Simon Stevin (1548–1620), Galileo Galilei (1564–1642), and Christiaan Huygens (1629–1695). They thoroughly studied and understood Archimedes' work, especially his treatise OFB, and were able to apply and extend it to practical applications in ship design. Blaise Pascal (1623–1662) achieved the equivalent in aerostatics.

Simon Stevin, the famous mechanicist, astronomer, and hydraulic engineer, worked on several fundamental problems of mechanics and also reestablished hydrostatics. He introduced the concept of hydrostatic pressure, which the Greeks had not known. He axiomatically developed a body of propositions embracing the entire fundamentals of hydrostatics in his treatise *The Elements of Hydrostatics* [47] (1586 in Dutch, 1608 in Latin translation). His premises are tantamount to the Archimedean properties of the liquid. In a liquid at rest the hydrostatic pressure increases linearly with the depth in proportion to the specific weight of the liquid since the weight of the liquid column on top of a given point causes this pressure. This opens the door to treating hydrostatics as a special case of field theory in the context of continuum mechanics, as it would be regarded later. Where Archimedes had dealt only with force resultants, Stevin was able to discuss hydrostatic phenomena as the result of pressure distributions. For ships the results are the same, but the approach is different. Stevin in his work gave full credit to Archimedes whom he praised. Unfortunately Stevin had misunderstood Archimedes' criterion of stability. By disregarding the effects of the volume shift in the heeled position of the vessel from the emerging to the immersed side, he came to the erroneous conclusion that for a stable ship the center of buoyancy B must always lie above the center of gravity G. This is actually a sufficient, but not a necessary, condition for upright stability.

Galileo, famous as a physicist and astronomer, also occupied himself with hydrostatics, which is not so widely known. In Florence in 1612 he published a treatise *Discourse on Bodies in Water* [48], where he defends his Archimedean viewpoint on the cause and magnitude of the buoyancy force against an opposition of Jesuit clerics who held the Aristotelean scholastic position that bodies specifically heavier than a liquid need not sink in that liquid. According to Aristotle, whether a body specifically heavier than a liquid sinks or floats on the surface depends on its shape. Galileo refuted this view and defined buoyancy in the manner of Archimedes.

Christiaan Huygens in 1650, thus at the tender age of 21 years, fully understood Archimedes' treatise OFB and was able to reconfirm Archimedes' results for the stability of simple shapes (sphere and paraboloid) and to extend the stability criterion to other homogeneous solids, the cone, the cylinder, and the parallelepiped [49]. He systematically varied the specific weight and the aspect ratios of these shapes, the main parameters for stability properties, and tested the stability of several shapes for a full circle (360°) of initial positions. Many solids turned out to have more than one stable equilibrium position. Huygens used an approach based on the principle of virtual work to define equilibrium, which is equivalent to and thus reconfirms Archimedes' results derived from force and moment equilibrium. He did not apply his methods to ships because he did not have a suitable, continuous analytical hull form definition.

Huygens did not want his three-volume treatise to be published, for he did not consider his results to be complete, useful, and original compared with Archimedes. Rather he wanted his manuscript to be burnt. But it was much later found in his legacy and at last published in 1908 in his Collected Works.

Calculus

To approximate the areas, volumes, and centroids of simple shapes, Archimedes had used the method of exhaustion, usually attributed to Eudoxus. This method relies on small, but finite, not infinitesimal elements to represent curves and surfaces. The method is not directly applicable to ships as objects of arbitrary shape. It defines its geometries by a finite number of polygonal or polyhedral elements. Integration by infinitesimal calculus by contrast is based on a limiting process to an infinite number of polygonal elements and derives its results analytically, often by means of a summation of an infinite series. Thus calculus can be applied to any analytically defined shape, hence also to arbitrary ship shapes.

The invention of calculus has many precursors and contributors (cf. Boyer [50]). But consistent foundations and a well-defined methodology were at last developed by Newton and Leibniz in the late seventeenth century. These methods spread fast during the first few decades of the eighteenth century. Thus by about 1730, when two leading scientists, Pierre Bouguer (1698–1758) and Leonhard Euler (1707–1783), simultaneously and independently, embarked on addressing the problems of ship hydrostatics again in a modern way, they had the mathematical methods and computational tools available to reformulate the integral quantities in Archimedes' approach in terms of the elegant and precise notation of calculus. Analytical, graphical, and functional representation of ship hull shapes were now achievable so that a modern reformulation of Archimedean hydrostatics as an application of continuum mechanics to ships had become feasible.

Bouguer and Euler

In 1727 the Parisian Royal Academy of Sciences held a prize contest on the optimum placement of masts in a sailing ship. Both Pierre Bouguer, a member of the academy, and Leonhard Euler, a Swiss citizen, then 20 years old and under the tutelage of Johann Bernoulli, participated in the contest and submitted contributions. The subject is closely related to ship hydrostatics because the optimum placement of sail area is a direct function of the permissible heel angle of the ship, where heeling moment and restoring moment are in equilibrium. But in their papers submitted for the competition, neither Bouguer nor Euler indicated any knowledge of Archimedean hydrostatics. Bouguer won the award nonetheless, and Euler's treatise was acknowledged as noteworthy by an "Accessit" verdict.

Neither scientist seems to have been satisfied by this intermediate result, for both of them continued to work intensively on ship hydrostatics during the next decade and a half, separately and independently without knowledge of the other's results before they were published (Nowacki, Ferreiro [51]). Bouguer participated in a scientific expedition to the Andes in Peru, today Ecuador, for geodesic measurements near the equator from 1735 to 1744 (Ferreiro [52]). He worked on his fundamental ship hydrostatics treatise *Traité du Navire* [53] mainly during this period. It was published in 1746 soon after his return to France. Euler worked on the same subject as a member of the Russian Academy of Sciences in St. Petersburg from 1737 to 1741. This work resulted in his two volume treatise *Scientia Navalis* [54], which after a long delay in the publishing process appeared in 1749. Both scientists respectfully acknowledged the other's work, which had led to largely equivalent results. They confirmed they had not known the other's work prior to its publication.

Bouguer does not mention or give credit to Archimedes anywhere but treads firmly in Archimedes' footsteps everywhere. For example, in Book II, Section I, chapter 1 of [53], he introduces the buoyancy force with an explanation that is tantamount to the Principle of Archimedes:

> The principle of hydrostatics, which must serve as a rule in this whole matter and which one must always have in mind, is that a body that floats on top of a liquid is pushed upward by a force equal to the weight of the water or liquid whose space it occupies.

Bouguer in another section also reconfirms this result by integration of the hydrostatic pressure over the submerged surface of the hull.

Euler freely acknowledges the debt he owes to Archimedes for the fundamentals of hydrostatics in buoyancy and stability. Euler begins his treatment of ship hydrostatics with the following axiom in the spirit of Stevin and field theory:

> The pressure which the water exerts on a submerged body in specific points is normal to the body surface; and the force which any surface element sustains is equal to the weight of a vertical water column whose basis is equal to this element under the water surface.

All other results in ship hydrostatics can be derived from this axiom. Euler proceeds to deduce the Principle of Archimedes by pressure integration over the submerged part of the hull surface.

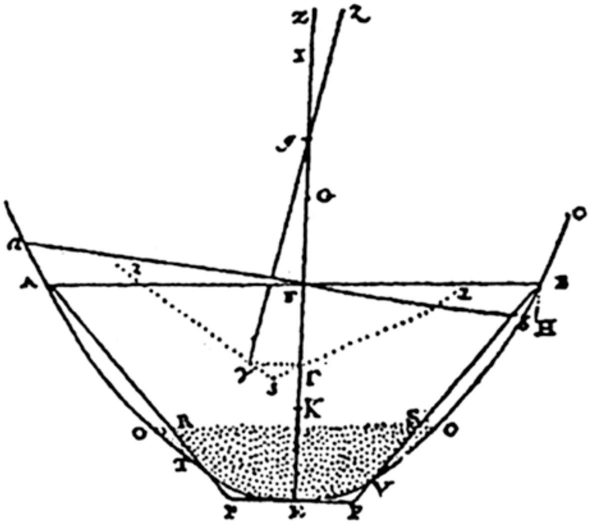


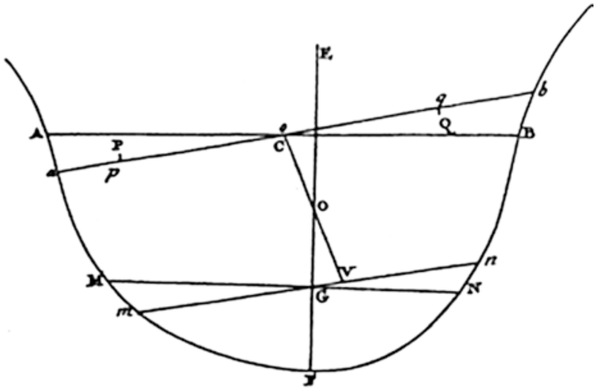
Figure 5 Bouguer's derivation of the metacenter (From Bouguer [53])

Regarding ship stability Bouguer and Euler went distinctly different ways, both departing from Archimedes' reasoning and both arriving at equivalent results, though expressed in different formulations.

For initial stability, i.e., the stability in the initial upright condition for very small (infinitesimal) angles of heel, Bouguer introduces the metacenter (point *g* in Figure 5) as the point of intersection of two infinitesimally neighboring buoyancy directions. In Figure 5 in the upright condition, the hydrostatic pressure resultant or buoyancy force acts through the volume centroid Γ of the submerged volume, also called center of buoyancy B in modern terminology. As the ship is slightly heeled, a slice of volume whose cross section is triangular in the figure is moved from the emerging side to the immersed side so that according to Archimedes' shift theorem (law of the lever) the center of buoyancy moves parallel to the shift from Γ to γ. The new buoyancy force runs normal to the new liquid surface and intersects the upright line of buoyancy in point g, the metacenter. The metacenter is at the same time the center of curvature of the curve, that is, the locus of the centers of buoyancy as the ship heels continuously. Bouguer constructs the metacenter by means of calculus and numerical integration (trapezoidal rule). His stability criterion is that the center of gravity of the ship (point G in Figure 5) must not lie above the metacenter g. The ship is stable for positive metacentric height Gg.

Euler likewise begins with an inclined, heeling ship (Figure 6) in analogy to Archimedes. He then applies the shift theorem to the emerging and immersed volume elements and constructs the new location of the center of buoyancy by integration of cross-sectional data over ship length using a calculus formulation. This enables him to compute the restoring moment, i.e., the couple formed by the buoyancy and gravity forces in the inclined position. If a positive restoring moment (or positive righting arm, similar to Archimedes) is acting, then the ship tends to restore itself to the upright condition after a small heeling disturbance; hence it is stable.

Figure 6 Euler's centroid shift in an inclined cross section (from Harris [55])



For infinitesimal angles of heel, the metacentric approach by Bouguer and the restoring moment criterion by Euler lead to identical expressions. For finite angles of heel, both scientists have proposed valid methods and criteria, though in different form. Both can claim that they have achieved a stability analysis based on either the metacentric curve or on the restoring moment/righting arm, which can be implemented by calculus and numerical integration. Both results are extensions to Archimedes' theory applicable to ships of arbitrary shape.

Both Bouguer and Euler have also addressed stability criteria for finite angles of heel, viz., the metacentric curve or the restoring moment in the inclined position, respectively. Further they both demonstrated how to deal with longitudinal stability, i.e., the response of the ship to trimming moments, viz., the longitudinal metacenter or the longitudinal restoring moment and the trim angle. They proceeded to show how many other applications in ship design and operations can be treated once the basic hydrostatic stability responses of the ship are known. This includes calculations for the determination of the draft, heel angle, and trim angle of the ship for any loading condition, e.g., during the loading and unloading of the ship, ship motions under wind load and in waves, maneuvering dynamics under sail, and much else. Calculus formulations and their numerical evaluations paved the way toward practical application of hydrostatic analysis of ship performance in general.

Bouguer's and Euler's published fundamental treatises in ship theory experienced quite contrasting reception and distribution. Bouguer's French text was readily understood and illustrated by many numerical examples. Textbooks for colleges were soon prepared in France on its basis. The French Navy soon made stability assessment by the metacenter criterion an official requirement for any new design. Euler's Scientia was written in Latin, lacked numerical examples, did not reach many practitioners, and remained relatively unknown in shipbuilding practice. But it was recognized as a valuable reference in future scientific work.

Chapman and Atwood

After the physical fundamentals of ship hydrostatics had been laid by Bouguer and Euler, the first attempts were made to apply this knowledge in practical ship design. Frederick Henrik Chapman (1745–1807) in Sweden and Thomas Atwood (1745–1807) in Britain were two outstanding engineering scientists and designing naval architects who took advantage of this new knowledge and adapted it to their practical needs. This brought the physical insights of Archimedes to fruition in practical design for the first time in full scope after a long delay of about two millennia.

Chapman, the son of an English shipbuilder and immigrant to Sweden, grew up in an atmosphere of practical shipbuilding orientation and scientific openness. As a young man he was both practically trained and mathematically oriented, so he picked up a broad basic education. He also spent a few years in England, France, and the Netherlands in a sort of self-paid shipyard traineeship and thus became familiar with not only the practical skills of the trade but also with the recent scientific know-how in those leading shipbuilding nations. He learned about the work of the Bernoullis, Bouguer, and Euler and knew how to apply it in his own ship design work. Thereby he was firmly entrenched in the tradition of Archimedes. He returned to Sweden in 1757 and soon acquired much responsibility in Swedish naval and merchant ship design, rose to high rank, and remained in a leading position in shipbuilding throughout his lifetime. He was thereby able to test his basic new insights and methods in practical design work and shipbuilding (cf. Harris [55] for detailed biographical information).

He also took pleasure and pride in publishing his insights and practical methods in treatises of technical and scientific orientation, very suitable as texts for ship design education. His *Treatise on Shipbuilding* [56], 1775, stands out as a textbook on his design methodology. He applied his knowledge of Bouguer's and Euler's work and implemented numerical quadratures by Simpson's rule, having taken private lessons from Thomas Simpson in England. Chapman also was an excellent designer of ship lines plans. Reportedly he drew some 2000 lines plans in his professional career. Many of these were documented in print and published. He made it a routine to calculate the displacement and a stability measure, the metacentric height, for every ship. Chapman also estimated the wind loads on the sails for critical operating conditions in order to provide sufficient safety margins in metacentric height for stability.

Chapman also knew how to influence hull shape and centroid location by geometric variation in order to set favorable stability indices (like metacentric height), viz., enough stability to be safe against extreme heeling moments but not too much in order to avoid abrupt ship motions in rough sea states. This illustrates how a stability analysis based on Archimedes' method was fully integrated into the design process.

George Atwood (1745–1807), an English mathematician and physicist, together with his French partner, the naval constructor Vial de Clairbois, recognized that the initial stability at small angles of heel was not sufficient to ensure a ship's safety, as

Bouguer and Euler had already pointed out. Thus they investigated the ship's righting moments at finite angles of heel, as Archimedes had done for the paraboloid. They used numerical quadrature rules to calculate the "righting arm" of the ship for a given draft, center of gravity, and heel angle over a wide range of heel angles (cf. [57]).

Thus by the end of the eighteenth century, all prerequisites were available to perform a complete displacement and stability analysis for a ship in its design and any other loading conditions. The application of Archimedes's knowledge was thereby extended to actual ships.

Archimedes in Modern Ship Design

Scope

The hydrostatic principles of Archimedes govern the floatability and stability of ships, two crucial elements of ship safety. Safety considerations pervade the entire ship design process. Thus the principles of Archimedes are also deeply embedded in the modern ship design process.

All areas of ship design are interdependent and thereby closely connected with each other (Figure 7). They all contribute to the overall efficiency and safety of ship performance. At the same time decisions in one area influence the others. The modern design process is viewed as an integrated decision process and is judged by the overall performance in all categories.

Safety is a design target in its own right. Safety is not confined to Archimedean floatability and stability requirements. Rather it encompasses all aspects of hazardous scenarios in the ship's lifetime. This must hold for all operating and loading conditions of the ship, also when the ship is damaged and perhaps partially flooded. Safety in ship operations aims essentially at the protection of human lives; at the prevention of damages to or loss of property objects, including the total loss of the ship; and at the protection of the natural environment of the ship. Safety hazards may jeopardize any or all of these vulnerable goods. Thus design considerations must account for all known hazards, evaluate their risks, strive for prevention measures, and seek an adequate safety margin in all categories.

In modern *Risk-Based Ship Design*, this is performed by aiming at the most adequate, achievable combined risk for all hazards in a most cost-effective manner. This will be discussed in more detail later.

Safety performance and shape design strongly interact. As for the principal dimensions, e.g., a shorter beamier ship of a given displacement tends to be more stable, though usually at the expense of a greater resistance, hence drawbacks in speed and power. Thus there are trade-offs between two important performance measures, a frequent situation in design. Therefore it is welcome that both of these effects can be quantified in design stage calculations, where Archimedes helps in modern design. The management of centroid locations and metacentric height by hull form changes is another example of interaction between safety and shape design.

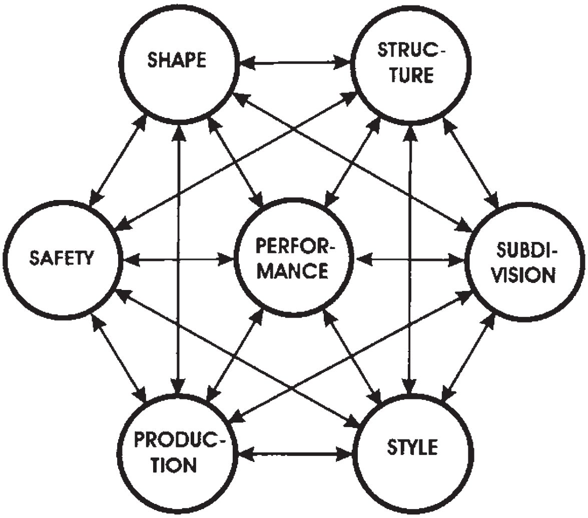


Figure 7 Structural elements of the ship design process

Structural integrity, especially in collision or grounding scenarios, is another concern for safety. The watertight subdivision of the ship, when damaged and partially flooded, is crucial for the survival of the ship and the humans aboard and for the protection of the environment. But such compartmentation is only adequate, if the ship remains floatable and stable in the damaged condition. This was a bitter lesson from the sad Titanic disaster in 1912.

Matters of style, e.g., the layout and placement of decks and spaces between keel and superstructures, may much influence the stability and damage safety performance of a ship, as some sad accidents have reminded us. Methods of production enter into the reliability of the structure (safety) but also weight distribution and centroid location (materials).

In summary, practically all essential areas of design interact with the safety parameters of the ship and must be monitored and controlled during the design process. This must be done quantitatively at the design stage by keeping track of the ship's floatability and stability in all operating conditions, especially after damaging accidents. Thus in modern times no certified ship can be designed without resorting to the principles and criteria of Archimedes.

Developments and Trends in Ship Design for Safety

Overview

Around 1800 a new era in shipbuilding and shipping was about to begin with the advent of steam-powered steel ships. It was a fortunate coincidence that by this time the scientific know-how in ship safety matters had reached sufficient maturity to be applied to this new generation of ships. In the past the technology of wooden sailing ships had been in gradual evolution for many centuries so that new designs could be based on the experience with existing and past ships. But the degree of innovation in the new ships that arrived with the industrial revolution was far too radical to base the safety concepts on inadequate experience alone. Thus science and technology together had to arrive at new methodologies to ensure safety in ship design.

The transition from wooden sailing vessels to steam-powered steel ships, which lasted more than a century, brought along many subsequent changes with new technologies and new risks. Ships grew in size and speed for greater economy but were also equipped with an increasing number of auxiliary systems to improve their safety. New specialized ship types were developed for new commodities, e.g., tankers for oil, later gas and chemical transport. The transoceanic passenger trade gained much momentum from the nineteenth into the twentieth century and created a large international fleet of fast, often luxurious passenger liners. This rapid diversification in shipping tasks and growth in seaborne trade volume generated many new technical challenges and operating risks. Simultaneously the value of the larger, faster ships and of their payload commodities grew immensely, hence also the economic risks of shipping investments.

Classification Societies

Ship owners and insurance companies consequently were interested in assessing such risks and finding technical solutions to control them during the continuing process of technical innovation. They needed advice in technical and scientific expertise from impartial, qualified maritime experts familiar with shipbuilding and shipping practice and with the scientific state of the art. Such services from the beginning of the steel ship era to this day were provided by classification societies in several leading maritime countries.

Classification societies were established, mainly during the nineteenth century, as national bodies and as legally independent, nongovernmental organizations to promote the standards of safety at sea in close cooperation with the maritime industries. They perform their functions by developing and publishing classification rules for the design, production, and operation of ships (and other offshore structures) by surveying the design process for compliance with the rules, by issuing classification certificates to approved ships, and by periodic inspection of the ships in operation. Their certificates are the basis for obtaining marine insurance contracts.

A core activity of classification has always remained the promotion of safety in the areas of ship strength, ship stability, load lines, and ship subdivision for damage control. Thereby and through their International Association of Classification Societies (IACS), they have much contributed to the national and international ship safety legislation. Their activities remain consultative since they have no executive power in the marine industry.

National and International Rules and Conventions: The Load Line

Ship safety legislation was initially based on national, sometimes even local, rules and regulations, which were slow in gaining ground. Regarding the load line of ships, it was not before 1876 in England that a law was passed requiring a mark (the Plimsoll mark) to be placed on the vessel's sides to prevent overloading the ship or to ensure a minimum freeboard. Sufficient freeboard is necessary to provide adequate reserve floatability (against sinking due to excessive heave motions) and reserve stability (against capsizing due to large heeling moments). Grave accidents initially occurred with low freeboard vessels until legislation recognized the necessity of reserve buoyancy above the load waterline. National rules set a trend, but it took until 1930 before the first International Load Line Convention was passed.

National legislation was insufficient to secure uniform safety precautions in international shipping. Conventions to prepare international agreements were long desired. It took until 1948 that a decisive step was taken. The United Nations in 1948 inaugurated an international convention, first called Inter-Governmental Maritime Consultative Organization (IMCO), later in 1982 renamed International Maritime Organization (IMO), to be concerned internationally by cooperation between governments with matters of maritime safety *to encourage and facilitate general adoption of the highest practicable standards in matters concerning maritime safety, efficiency of navigation, and prevention and control of marine pollution from ships* (cf. Lamb [58]).

The International Load Line Convention of 1930 was revised in 1966 and again in 2000 to account for the ship's seakeeping dynamics under the management of IMCO/IMO.

Damage Stability

The catastrophic accident of the Titanic in 1912 with the loss of many human lives caused international alarm and drew the attention to be focused on the safety of ships when damaged and partially flooded. An international Convention on Safety of Life at Sea (SOLAS) in 1914 presented new criteria for safety regulations for passenger vessels, but due to delays by two World Wars, the results were not passed and put into force until SOLAS 1948 under IMCO supervision. The regulations were gradually amended and stiffened, also in consequence of new grave accidents (SOLAS 60 and 74). Recent developments set safety standards for dry cargo ships in damaged condition (SOLAS 90) and moved on to probabilistic concepts of damage assessment for dry cargo and passenger ships (SOLAS 2009). The purpose of these regulations is ensuring safe design by sufficient subdivision of the hull by watertight bulkheads controlling the size, location, and number of flooded spaces so as to control sinkage, trim, and heel in the damaged floating condition not to exceed a safe margin for the ship's floatability and stability. The flooded spaces are either regarded as filled with added liquid weight or equivalently treated as lost volumes of buoyancy. In every other regard the analysis of the damaged condition is based on the same Archimedean principles as for the intact condition.

Protection of the Environment

The significant growth of ocean oil transport after WWII and sad accidents with tankers resulting in dramatic oil pollution in the ocean and on shores have caused growing concern over the threat of oil pollution in the maritime environment. This concern was addressed under the auspices of IMO at the MARPOL 73/78 conventions, which went into force in 1983. To limit the potential oil outflow in the event of tanker damages by collision or grounding, the MARPOL regulations require from all new tankers of more than 20000 tons deadweight the arrangement of segregated ballast tanks (clean tanks) in protective locations, i.e., to shield the cargo tanks. This has led to new compartment configurations in "double-hull" tankers with clean ballast tanks along the sides and in the double bottom. Such arrangements also result in more potentially empty spaces and hence more freeboard, which adds to the reserve buoyancy. How these reserves can be used to the best advantage of ship safety has been the subject of recent discussions and design optimization studies (cf. Papanikolaou [59]).

Risk-Based Ship Design

The design of complex systems operating under hazardous conditions and subject to threats of immense damages in the event of catastrophic failures has become a specialized discipline, now commonly called risk-based design. This approach has been a necessity in the nuclear industry for many decades and has also prevailed in aerospace design and in other industries with great public and economic risks. In the maritime field the offshore oil industry first introduced this approach by legislation based on risk analysis for offshore systems, e.g., in Norway in 1986, in the United Kingdom in 1992. For ships IMO is currently following a strong trend toward risk-based ship design in the development of new safety standards (cf. Sames [60], Skjong [61]).
This entails a number of methodical elements:

- Future standards and some current pilot regulations are intended to replace, at least in part, the traditional rule-based approach of classification and regulations, which describes in technical detail how a safe design is to be realized, by Goal-Based Standards, where a safety goal is set regardless of how it will be achieved. This requires quantitative risk analysis (QRA) with quantified risk assessment. Goal-Based Ship Design (GBSD) aims at an optimal solution for the overall safety of the ship. This is to be achieved in the most cost-effective manner.
- The risks will be defined for each hazardous operational scenario in probabilistic terms by the predicted probability of occurrence of the hazardous event multiplied by the economic value of the consequent damage. All damages, whether to the public, the ship owner, or to individual humans, are to be included in the analysis. The total risk is evaluated by combining the risks of all scenarios. The

total risk will be compared to the acceptable risk, chosen either relative to ships designed by existing IMO rules or in absolute terms based on forthcoming new IMO risk acceptance criteria (cf. Sames [60]),

- In optimizing designs simultaneously for their economic viability and their safety, safety is no longer regarded as a rule-based constraint but is treated as an objective in its own right. After all, the owner's and the public's interest lies in both economy and safety. Risk analysis quantifies safety targets in comparable units as the functional economic measures.

These features in the risk-based methodology set an ambitious scope for design studies. Yet, IMO discussions and pilot studies are well underway. Many details can be found in the recent book *Risk-Based Ship Design* (Papanikolaou [59]), which is in large measure a result of the EU-funded research project SAFEDOR. It contains example studies for a cruise ship (Vassalos [62]) and a double-hull tanker (Papanikolaou [63]).

Formal Safety Assessment

A systematic methodical approach that is often followed in *Risk-Based Ship Design* is the Formal Safety Assessment (FSA). It is performed in five steps:

- Identification of all relevant safety hazards
- Quantification of the risks for each hazard
- Enumeration of the design options
- Cost-benefit analysis of all design options, including the effects of all hazards
- Systematic comparison or optimization studies to recommend the chosen design

Figure 8 shows how the hazardous scenarios in ship safety are connected and depend on each other. The total risk analysis is probabilistic and must account for these interdependencies.

Interesting case studies have been performed (cf., e.g., [62, 63]). Papanikolaou and his team designed a double-hull AFRAMAX tanker, where the reference vessel was an existing rule-based design. Using a risk-based approach, it was possible to demonstrate in a multiobjective optimization study that the best goal-based designs, without changes in principal dimensions and hull form, varying double-hull tank dimensions and compartmentation, allow increases in cargo capacity and improvements in environmental safety, viz., reduction in oil outflow according to MARPOL regulations, without drawbacks to the economic performance.

Trends in Ship Design for Safety

To summarize the major developments in ship design safety, which have accompanied the rise of modern shipping, the following long-term and recent trends can be recognized:

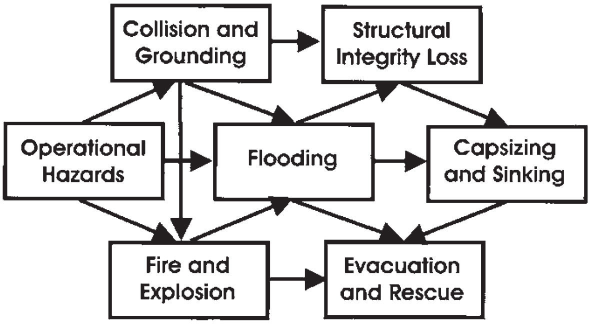


Figure 8 Structure of hazardous scenarios in ship safety (Adapted from Vassalos [62])

- From predominantly experience-based design to design on scientific-technological foundations
- From national to international rules and regulations
- Safety studies advanced from late to early design
- From deterministic to probabilistic design decision models
- From prescriptive rules to goal-oriented methods
- From feasible design to optimal design
- From modeling safety requirements as constraints to treating them as objectives
- From design against individual hazards to risk-based design for overall safety

All of these developments together have contributed to a much enhanced safety in modern ships for a greatly diversified spectrum of ship types. The ideas of the Principle of Archimedes and of his stability assessment have remained at the core of every modern ship safety assessment.

Conclusions

Archimedes laid the foundations for the hydrostatics of floating systems in his treatise OFB. He based his deductions on experiments of thought and very few physical axioms amounting to the equilibrium of forces and moments applied to an object floating in a liquid at rest. The Principle of Archimedes holds for objects of arbitrary shape, hence also for ships, and his stability criterion of positive righting arms can be extended to solids of nonhomogeneous mass distribution like ships.

However it is unlikely that Archimedes was involved in the stability analysis of contemporary ships built in Syracuse though he may have assisted in other ways, but evidently he did not have available continuous ship hull form definitions, methods of integration for areas, volumes and centroids for arbitrary shapes, and any data drawn from experience on safe margins for external heeling moments in critical scenarios.

Despite the initial lack of this further information, Archimedes' insights were recognized throughout late antiquity and the Middle Ages as physical fundamentals by those few who had access to his work, and they may have been guiding principles in qualitative assessment of design decisions. Prominent scientists, e.g., Stevin, Galileo, Huygens, Pascal, and practitioners in their treatises, made elementary contributions in order to make Archimedes' concepts applicable to ship design.

But it was only after continuous representations of ship hull forms, at least graphically by drawings, and methods of integration by calculus had become available by about 1700 that direct numerical application of Archimedes' laws could be brought to bear on practical ship design in volume and stability analysis. We owe Pierre Bouguer and Leonhard Euler the decisive scientific steps toward quantitative, scientifically founded numerical evaluation of ship hull form and stability properties in practical design. Chapman and Atwood with Vial du Clairbois in the late eighteenth century are early witnesses of the practical use of such safety relevant calculations at the ship design stage.

The new era of iron ships, propelled by steam, and further rapid innovations in shipping during the industrial revolution created many new ship types and shipping scenarios which required a new, much broader approach to ship safety. A risk-based approach, integrating all relevant hazardous scenarios in a total risk analysis, has gradually matured and is entering into ship design practice. A core element in this modern comprehensive ship safety analysis has remained the assessment of hydrostatic ship properties by Archimedean principles.

Acknowledgments I express my gratitude to Chris Rorres, the initiator and organizer of this international conference on Archimedes in the twenty-first century for stirring my interest and inviting me to contribute a section on Archimedes' role and importance for ship design. I am also grateful for the continuing support I am receiving from the Max Planck Institute for the History of Science in Berlin, where I have been a Visiting Scientist since 2001.

References

1. Jared Diamond: "Guns, Germs, and Steel, The Fates of Human Societies", W.W. Norton & Co., New York, London, 1997, 1999. ISBN 0-393-31755-2.
2. Peter Kemp: "The History of Ships", Geenwich Editions, London, 2000. First published 1978 by Macdonald & Co., Little Brown and Co., UK. ISBN 0-86288-345-8.
3. D.S. Johnson, J. Nurminen: "The History of Seafaring", John Nurminen Foundation, 2007. German edition by National Geographic Deutschland, Hamburg, 2008. ISBN 978-386690-074-5.
4. T.L. Heath: "A History of Greek Mathematics", 2 vols., republished by Dover Publ. Inc., New York, 1981. ISBN 0-486-24073-8. First published by Clarendon Press, Oxford, 1921.
5. T.L. Heath: "The Works of Archimedes", in English, translated and annotated by T.L. Heath, 1897, with a supplement "The Method of Archimedes", 1912, reissued later by Dover Publications, New York, 1981.
6. J.L. Heiberg: "Archimedis Opera Omnia cum Commentariis Eutocii", in Greek and Latin, Leipzig, 1880–1.
7. J.L. Heiberg: "Archimedis Opera Omnia cum Commentariis Eutocii", in Greek and Latin, including "De Corporibus Fluitantibus" and "De Mechanicis Propositionibus ad Eratosthenem Methodus", Leipzig, 1910, 1913, 1915.

8. E.J. Dijksterhuis: “Archimedes”, translated from Dutch by C. Dikshoorn, Princeton University Press, Princeton, N.J., 1987.
9. P. Ver Eecke: “Les Oeuvres Complètes d’ Archimède, suivies des commentaires d’ Eutocius d’Askalon”, in French, translated and annotated by P. Ver Eecke, 1st edition by Desclées De Brouwer, Bruges, 1921, 2nd edition by Librairie Scientifique et Technique Albert Blanchard, Paris, 1960.
10. A. Czwalina-Allenstein: “Abhandlungen von Archimedes”, in German, translated and annotated by Czwalina-Allenstein in 1922, Ostwald’s Klassiker der exakten Wissenschaften, Band 201, Verlag Harri Deutsch, 1996.
11. „Archimedes Werke“, in German, translated and annotated by A. Czwalina, with an appendix including “The Method”, translated by J.L. Heiberg, republished by Wissenschaftliche Buchgesellschaft, Darmstadt, 1967.
12. L. Sprague de Camp: “The Ancient Engineers”, Ballantine Books, New York, 1960.
13. J. Renn; M. Schemmel: “Balances and Knowledge in China”, in German, Preprint 136, Max Planck Institute for the History of Science, Berlin, 2000.
14. H. Nowacki: “Archimedes and Ship Stability”, Preprint 198, Max-Planck-Institute for the History of Science, Berlin, 2002.
15. J.L. Heiberg; H.G. Zeuthen: “A New Treatise by Archimedes”, in German, Bibliotheca Mathematica, B.G. Teubner Verlag, Leipzig, 1906–7.
16. Vitruvius Pollio: “De architectura libri decem”, Book IX, § 9, pp. 203–207, ca. 25 B.C., translated into English by Frank Granger, Loeb’s Classical Library, Harvard University Press, Cambridge, MA, 1934, reprinted 1995.
17. F. Costanti: “The Golden Crown: A Discussion”, chapter in S.A. Paipetis, M. Ceccarelli, eds., “The Genius of Archimedes”, Proc. of an International Conference held at Syracuse, Italy, June 8–10, 2010. Springer Publ., Dordrecht Heidelberg London New York, 2010.
18. K. Hidetaka: “What did Archimedes find at “Eureka” Moment?”, chapter in S.A. Paipetis, M. Ceccarelli, eds., “The Genius of Archimedes”, Proc. of an International Conference held at Syracuse, Italy, June 8–10, 2010. Springer Publ., Dordrecht Heidelberg London New York, 2010.
19. Athenaeus, “Deipnosophistai”, Teubner, Leipzig, 1887, 5.206d-209.d. Also Loeb’s Classical Library, vol. V, p. 433 ff.
20. M. Bonino: “Notes on the Syrakosia and on Archimedes Approach“, chapter in S.A. Paipetis, M. Ceccarelli, eds., “The Genius of Archimedes”, Proc. of an International Conference held at Syracuse, Italy, June 8–10, 2010. Springer Publ., Dordrecht Heidelberg London New York, 2010.
21. F. Zevi: “Le Invenzioni di Archimede e le Grandi Navi”, chapter in M. Giacobelli, ed., Lezioni Fabio Facenna, Conference on Underwater Archelogy, Edipuglia, Bari, 2001.
22. P. Pomey, A. Tchernia: “Les Inventions entre l’Anonymat et l’ Exploit: Le Pressoir a Vis et la Syracusia”, chapter in E. Lo Cascio, Innovatione tecnica progresso economico nel mondo romano”, Proc. Conf. on the History of Economics in Antiquity, Capri 2003, Edipuglia, Bari, 2006.
23. P. Pomey: “On the Use of Design in Ancient Mediterranean Ship Construction”, chapter in H. Nowacki, W. Lefèvre, eds.: “Creating Shapes in Civil and Naval Architecture”, Brill Publ., Leyden Boston, 2009.
24. Plutarch: “The Lives of Noble Grecians and Romans”, the Dryden translation, edited and revised by A.H. Clough, The Modern Library, New York, 1949.
25. M. Clagett: “Archimedes in the Middle Ages”, 4 vols., The University of Wisconsin Press, Madison, 1964.
26. R. Netz, W. Noel: “The Archimedes Codex. How a Medieval Prayer Book is Revealing the True Genius of Antiquity’s Greatest Scientist”. Weidenfeld & Nicolson/ Walters Art Museum, London, 2007.
27. W. Noel, R. Netz, N. Tchernetska, N. Wilson, eds.: “The Archimedes Palimpsest”. Two vols., Walters Art Museum, Cambridge University Press, Cambridge, U.K., 2011.

28. D. McGee: “The Shipbuilding Text of Michael of Rhodes”, chapter in H. Nowacki, W. Lefèvre, eds.: “Creating Shapes in Civil and Naval Architecture”, Brill Publ., Leyden Boston, 2009.
29. Z. Trombetta de Modon: “Libro de Zorzi Trombetta de Modon”, ca. 1445, British Library, London, Cotton ms., Titus A.XXVI.
30. B.Q. Drachio: “Visione del Drachio”, ca. 1599, Contarini, ms. 19, arsenal, b.1, Venetian State Archives, Venice. Published by L. Th. Lehmann, Amsterdam, 1992.
31. U. Alertz: “Naval Architecture Digitalized Introducing Arithmetic and Geometry into Late Mediaeval Shipwrightry”, chapter in H. Nowacki, W. Lefèvre, eds.: “Creating Shapes in Civil and Naval Architecture”, Brill Publ., Leyden Boston, 2009.
32. E. Rieth: “Le Maître Gabarit, la Tablette et le Trébuchet”, Editions du Cimité des Travaux Historiques et Scientifiques, Paris, 1996.
33. F. Oliveira: “Ars Nautica”, manuscript, 1579, in Barata, João da Gama Pimentel, Memórias do Centro de Estudos de Marinha, Lisbon, 1972.
34. J.B Lavanha: “Livro Primeiro da Architectura Naval”, ca. 1614–1616, reprinted by the Academia de Marinha, Lisbon, with English translation by Richard Barker, 1996.
35. M. Fernandez: “Livro de Traças de Carpenteria“, 1616, transcription and English translation, reprinted by the Academia de Marinha, Lisbon, 1995.
36. W. Bourne: “A Booke Called the Treasure for Traveylers”, Thomas Woodcocke, 1578. Facsimile, Norwood NJ: W.J. Johnson, 1979.
37. R. Barker: “Fragments from the Pepysian Library”, Revista da Universidade de Coimbra, vol. XXXII, 1986, pp. 161–178.
38. R. Dudley: “Dell’ Arcano del Mare”, Book IV: Shipbuilding, Florence, 1646.
39. E. Bushnell: “The Compleat Ship-Wright”, George Hurlock, London, 1664.
40. A. Deane: “Doctrine of Naval Architecture”, Pepys Library, Magdalene College, Cambridge University, ms. 2910, ca. 1670. Reprinted in Brian Lavery, ed.: “Deane’s Doctrine of Naval Architecture 1670”, Conway Maritime Press, London, 1981.
41. P. Hoste: “Théorie de la construction des vaisseaux”. Anisson & Posuel, Lyon, 1697.
42. J. Hudde: Mentioned in N. Witsen [43], pp. 242–247.
43. N. Witsen: “Aeloude en hedendagsche Scheeps-Bouw en Bestier”, Casparus Commelijnen & Broer en Jan Appelaer, Amsterdam, 1671.
44. C. van Yk: “De nederlandsche Scheepsbouw-Konst open gestelt” , Jan ten Hoorn, Amsterdam, 1697.
45. R. Barker: “Two architectures – a view of sources and issues”, article in H. Nowacki, W. Lefèvre, eds.: “Creating Shapes in Civil and Naval Architectures”, workshop report, Preprint 338, vol. I, pp. 42–133, Max Planck Institute for the History of Science, Berlin, 2007.
46. L.D. Ferreiro: “Ships and Science”, The MIT Press, Cambridge, MA, 2007.
47. S. Stevin: “De Beghinselen des Waterwichts” (Elements of Hydrostatics), in Dutch, Leyden, 1586, Latin translation in S. Stevin: “Hypomnemata Mathematica”, Leyden, 1608. Reprinted in E.J. Dijksterhuis, ed.: “The Principal Works of Simon Stevin”, vol. I, Mechanics, C.V. Swets & Zeitlinger, Amsterdam, 1955. Also in this volume: S.Stevin: “De Beghinselen der Weegkonst” (Elements of the Art of Weighing), in Dutch, Leyden, 1586, third part of the supplement: “On the Floating Top-Heaviness”.
48. G. Galilei: “Discorso intorno alle cose que stanno in su l’ acqua”, Appresso Cosimo Giunti, Firenze, 1612. Reprinted in Galilei Favaro, vol. IV, 1968, pp. 57–140.
49. C. Huygens: “De iis quae liquido supernatant libri 3”, 1650, published in Œuvres Complètes de Christiaan Huygens, Vol. XI, 1908, reprinted by Swets & Zeitlinger N.V., Amsterdam, 1967.
50. C.B. Boyer: “The History of Calculus and its Conceptual Development”, 1939, reprinted by Dover Publications, New York, 1949.
51. H. Nowacki, L.D. Ferreiro: “Historical Roots of the Theory of Hydrostatic Stability”, Proc. 8th International Conference, Escuela Técnica Superior de Ingenieros Navales, Madrid, 2003.
52. L.D. Ferreiro: “Measure of the Earth: The Enlightenment Expedition that Reshaped Our World”, Basic Books, New York, 2011.

53. P. Bouguer: “Traité du Navire, de sa Construction et de ses Mouvemens (sic!)”, Jombert, Paris, 1746.
54. L. Euler: “Scientia Navalis seu Tractatus de Construendis ac Dirigendis Navibus”, St. Petersburg, 1749, from Opera Omnia, vol. 18/19, Zurich, 1967.
55. D.G. Harris: “F.H. Chapman: The First Naval Architect and His Work”, Conway Maritime Press, London, 1989.
56. F.H. Chapman: “Traktat om Skepps Byggeriet” (“Treatise on Shipbuilding”), in Swedish, 1775, French translation by Vial du Clairbois, Paris, 1781, German translation from the French, by G. Timmermann, Hamburg, 1972.
57. G. Atwood and H.-S. Vial du Clairbois: “A Disquisition on the Stability of Ships”, Phil. Trans. of the Royal Soc. of London, vol. 88, 1798, pp. vi–310.
58. T. Lamb, ed.: “Ship Design and Construction”, 2 vols., The Society of Naval Architects and Marine Engineers, Jersey City, NJ, 2003.
59. A.D. Papanikolaou, ed.: “Risk-Based Ship Design. Methods, Tools and Applications”, Springer Publ., Berlin Heidelberg, ISBN 978-3-540-89041-6, 2009.
60. P.C. Sames.: “Introduction to Risk-Based Approaches in the Maritime Industry”, chapter 1 in [59].
61. R. Skjong.: “Regulatory Framework”, chapter 3 in [59]
62. D. Vassalos.: “Risk-Based Ship Design”, chapter 2 in [59].
63. A. D. Papanikolaou: “RBD Application: AFRAMAX Tanker Design”, subsection 6.2 in chapter 6 of [59].

Archimedes Screw in the Twenty-First Century

New Developments of an Old Design

Dirk M. Nuernbergk

Introduction

This article gives a short review of the Archimedes screw and its application in different areas of water pumping and water-power generation.

The Archimedes screw—a corkscrew inside a cylinder—is attributed to the great Greek mathematician, engineer, inventor, and military planner, Archimedes, who lived in the third century BC. For thousands of years, farmers have used this simple machine for irrigation and for draining mines or low-lying water. Placed at an angle, with one end submerged in a body of water, the screw is turned by hand (or sometimes by the feet) to lift water upward and out at the other end.

In the 1990s a German engineer, Karl-August Radlik, determined that running an Archimedes screw backward—that is, letting water flow in at the top—caused the screw to turn as the water fell to the bottom. He also noted that naturally flowing water in small streams could generate modest (but impressive) amounts of electricity—enough, for example, to power a village or large farm. Increasingly, this system of power generation is proving to be a robust, economical, and efficient way to generate electricity from small streams and rivers.

Also impressive is the fact that this form of power generation has a small impact on the environment. Unlike the turbine blades that spin in huge hydroelectric power plants, an Archimedes screw permits fish to swim through it and emerge at the other end almost unscathed.

In recent years, Archimedes screws have been built across Europe, particularly in Germany, Austria, and the United Kingdom, where Queen Elizabeth II commissioned two such screws to power Windsor Castle. The first unit in the United States is expected to begin operating next year.

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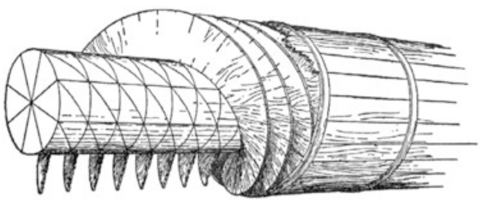
Figure 1 (*left*) Modern depiction of an Archimedes screw turned by a farmer. (*right*) Egyptian terracotta figurine (about 30 BC)

Figure 2 Reproduction of an Archimedes screw described by Vitruvius



Historical Overview

Archimedes screws as water-lifting devices have been in use for centuries (Figure 1). The invention of the screw is credited to Archimedes of Syracuse (ca. 287–212 BC). However, there is no written evidence that Archimedes invented the screw; nor is there a drawing of a screw design by Archimedes that has survived to the modern era. It may be that this type of water-lifting device is much older and that it was used by the Egyptians or the Assyrians before Archimedes' time. The discussion on this controversial matter among historians of science continues [1–4]. From the author's point of view, it makes much more sense to credit this important ancient invention to the most famous ancient mathematician, Archimedes, than to the Assyrian warlord, Sennacherib.

In the first century BC, the Roman engineer Vitruvius gave the first written evidence of a water screw in his work, *De Architectura* (Figure 2). However, Vitruvius gave no credit to Archimedes or to anyone else for its invention.

The screw described by Vitruvius consisted of a central wooden cylindrical core on which where wrapped eight helicoidal blades constructed from long willow twigs. Wooden planks were then nailed to the blades to form an enclosing outer cylinder. This type of screw was used for many centuries to drain mines as well as for irrigation purposes.

Figure 3 Roman Fresco from Pompeii (before AD 79) located in the "House of the Ephebe." The Archimedes screw was turned by foot

Figure 3 shows a fresco of a screw such as Vitruvius described. It was found in the Roman city, Pompeii, devastated AD 79 by a volcanic eruption.

Koetsier [1] has reviewed the history of the development of the theory of the Archimedes screw. He states in his work that scientists and artists like Vitruvius, Cardano, Leonardo da Vinci, Galileo, Bernoulli [5], Hachette, Eytelwein [6], and Weisbach [7] have all analyzed the theory and design of the screw. His list read like a "Who's Who" of the development of hydraulic engineering.

In the eighteenth century the famous Swiss mathematician Leonhard Euler formulated fourteen mathematical problems that he considered unsolved. One of them was this:

A theory is sought for the rising of water by the screw of Archimedes. Even if this machine is used most frequently, still its theory is desired.

Most of the theoretical work addressing Euler's problem was done in the last two centuries. A complete investigation on the discharge capacity and the efficiency was done by the Dutch engineer Muysken [8] in the year 1932. Muysken also gave the empirical equation for the proper rotational speed of a screw, which is still used by manufacturers today. The huge interest of Dutch engineers was motivated by the high water volume that could be transported by the screw and the fact that one fourth of the Netherlands is below sea level, which needed to be drained by windmills. Recent publications of Nagel [9], Kantert [10], and Nuernbergk [11] refer to Muysken's work. In the year 2000, Rorres [12] determined the optimal geometric parameters for a screw using analytical and numerical methods.

Figure 4 Flutter-Mill: Moorseer Windmill in Weser-Marsh, Germany, for the drainage of marsh land. Such wind-powered screws were also used for salt making (as in Spain)

Drainage and Irrigation

Today Archimedes screws are in widespread use throughout the world in many ways:

- Waste water treatment facilities [9, 11]
- Low-lying land pumping stations (polder pumps), as in the Netherlands [8, 13]
- Irrigation systems [4a]
- Rain detention dams
- Flood detention dams
- Industrial and fish conveyor systems [14]
- Water sports and recreational facilities

One reason for this widespread usage is the sturdiness of the screw against debris, as when the transport medium is highly polluted or when solid matter needs to be transported.

The screw is very suitable as a low-tech, high-volume pump to drain low-lying land areas using wind power to drive the screw. Two examples are given in Figures 4 and 5.

Figure 5 Windmill in the Netherlands used as a pump for polders

Archimedes screws come in various configurations, such as:

- *Casing-Tube Screws*—helicoidal blades are attached to a central inner cylinder and an outer cylinder. The entire structure is then rotated (Figures 1, 2, 3, and 4).
- *Tube Screws*—helicoidal blades are attached to a central inner cylinder and rotated in a fixed outer cylinder.
- *Trough Screw*s—helicoidal blades are attached to a central inner cylinder and rotated in a fixed U-shaped trough. The trough is open to the surrounding air (Figure 6).
- *Spiral Screws*—helical tubes are wrapped around and attached to a central inner cylinder. The entire structure is then rotated.

The trough screw is the most widely used configuration by manufacturers today. The other three configurations have insufficient ventilation and have the tendency to "swallow-up" because locked air is compressed under certain unfavorable inflow conditions.

Until the twentieth century the main application of the Archimedes screw was for drainage of low-lying land and for wastewater treatment (Figure 7). However in the twenty-first century a new application has been found, as described in the next section.

Figure 6 Installation of a hydropower screw on the River Apfelstädt in Herrenhof, Germany. The installation of a screw with an integrated steel trough takes only a few days and is very cost effective

Figure 7 Wastewater treatment plant in Memphis, Tennessee, USA

Hydropower Screw

As mentioned in the introduction, in the 1990s, Karl-August Radlik proposed the idea that Archimedes screws running backward are suitable as power-generating devices, and he submitted a German patent application [16] for this new use. There

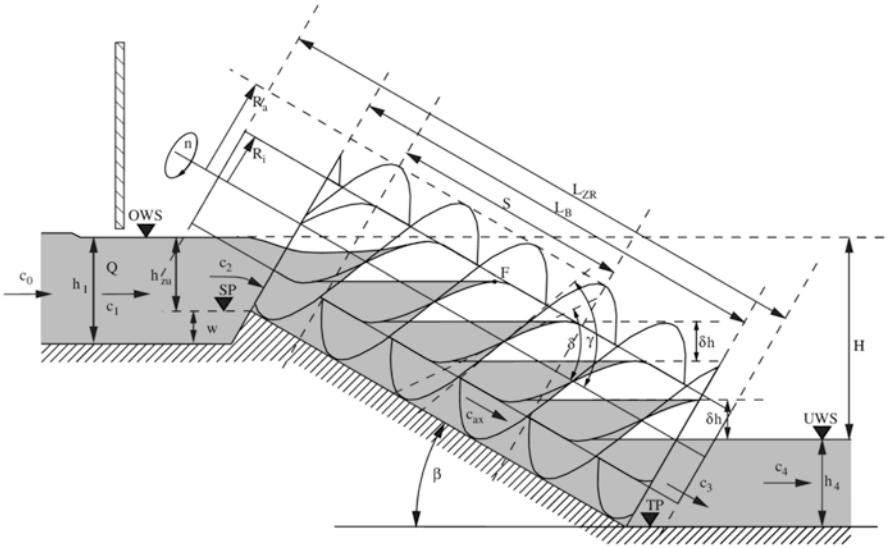


Figure 8 Geometric parameters of a hydropower screw

had been suggestions much earlier for the use of the screw as a kind of "water wheel" to generate mechanical power (cf. [17] Navier in Belidor's "Architectura Hydraulica"). It appears to the author that the screw was not used to generate mechanical power until recently because of problems with transmitting the turning of the screw to the rotation of a drive shaft. Instead, more common under-, breast-, and overshot water wheels or turbines (Francis or Kaplan turbines) were used because the shaft of the wheel can go directly into a mill house where it is then easy to transmit its rotational energy to machinery via belt drives.

Figure 8 displays the typical parameters of a modern hydropower screw. The performance of an Archimedes screw used as a power generator depends on several factors: first, the *inner* parameters, including the inner diameter (R_i), outer diameter (R_a), screw pitch (S), and number of flights (N), and, second, the *outer* parameters, including the head difference (H), inclination (β), rotational speed (n), inlet and outlet water heights (h_1 and h_4), and flow rate (Q).

Screws were first used as hydraulic motors in 1993 (Brada [18, 19]) as a means to drive electric generators. Brada put the first water-powered screw into operation and measured its efficiency. He also investigated its application range and compared the screw with turbines and water wheels. Most of his work was done in collaboration with Radlik.

Restrictions, however, apply when river or stream water flow is used for hydropower (Figure 9). Following the establishment of new European Community directives, increased instream flows are now required in most European rivers. This means a loss of available annual energy production for an owner of a hydropower site if the water is diverted from the main river. New water-powered devices have been investigated to compensate for this loss of hydraulic energy.

Figure 9 Screw supplied by residual flow from the Werra River near Meinigen, Germany

According to Meißl [20] the required residual flow in the main riverbed can be 10–30% of the discharge of the powerhouse of the hydropower site. On the other hand, at a typical water-diverting dam, a head difference of 30–70% of the head at the powerhouse is available. In addition, residual-flow water-powered devices such as the Archimedes screws can make use of a surplus of discharge at medium to high discharge conditions. In that way it is possible to increase the overall power of a particular hydropower site.

Because Archimedes screws have a robust design, are insensitive to debris, are fish-friendly, do not interfere with sediment transport, are cost-competitive, and have a higher rotational speed in comparison with water wheels [21, 22], they are often used at dams and weirs to make use of the residual flow. Water-powered screws do not interfere with downstream fish migration [23]. This is one of the most important advantages of Archimedes screws compared with water turbines.

Several investigations were carried out to check the fish-friendliness of the screw, for instance, Spaeh [24], Merkx [25], and Schmalz [23]. The effectiveness of the screw as a fish passage was proven, and improvements were suggested to avoid any harm to the fish by the blade edges of the screw, e.g., by [26].

The largest hydropower screw up to now was installed in 2012 (Figure 10). It has an outer diameter of 4.5 meters, a length of 10 meters, and a flow capacity of 8 cubic meters/second and can provide electrical power of 180 kilowatts. The rotational speed of the screw is controlled by a frequency converter so that the inlet

Figure 10 Hydropower screw manufactured by Landustrie in the Netherlands (D_a = 4.5 meters)

water level (h_1) can be kept constant to achieve optimal filling resulting in very good hydraulic efficiency.

In the near future it is expected that larger screws will be used. Water flows (Q) as high as 15 cubic meters/second and electrical power output of 800 kilowatts have been announced by manufacturers. According to Lashofer [27], 180 such generators were installed in Europe in 2011, including those commissioned by Queen Elizabeth II of England to power Windsor Castle. The first commercial installation in Canada has been established this year, and the first unit in the United States could begin operating soon.

The work of Lashofer et al. [27] also presents a site enquiry, an operator survey, extensive field measurement, as well as a literature survey on the fish tolerance of the screw. The survey found that plants on the order of 10–60 kW were most common and that fixed-speed generators are much less tolerant of large flow variations than variable speed generators. The overall efficiency (the combined hydraulic efficiency of the screw, the efficiency of the generator, the gear box, and the frequency inverter) was found to be in a range of 65–75%. The same research group published efficiency measurements of several hydropower screws during the last year showing excellent hydraulic efficiencies as high as 94% under partial-load conditions. Under full-load conditions the screw has an efficiency higher than 80%.

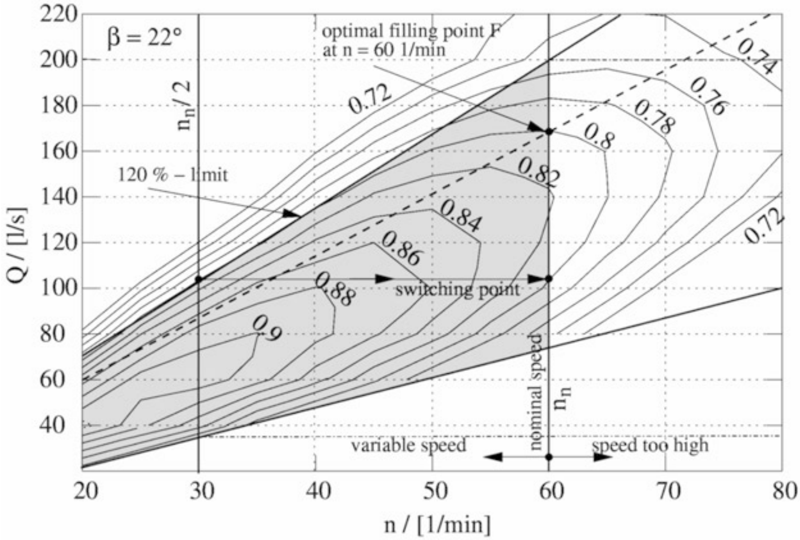


Figure 11 Efficiency measurements of a hydropower screw at the University of Natural Sciences, Vienna (R_a = 0.403 m, R_i = 0.203 m, S = 0.806 m, N = 4, β= 22°) [28]

Figure 11 illustrates the efficiency ranges under three different operation modes of a particular screw depending on the water flow rate Q:

- Constant rotational speed operation (60 rpm adjusted by a gear box and a generator)
- Constant rotational speed operation with two fixed speeds (30 rpm and 60 rpm adjusted by a gear box and a generator with Dahlander windings)
- Variable rotational speed operation (30–60 rpm adjusted by a generator controlled by a frequency inverter)

As previously mentioned, the variable-rotational-speed mode is the most effective because it can keep the screw optimally filled under changing water flow rates.

Conclusions

The Archimedes screw has been in use for at least twenty three centuries, mainly to pump water or drain low-lying land. During the past ten years, Archimedes screws have also begun to be used for power generation. This new ultra-low-head technology is still a niche application. However, Archimedes screw generators are beginning to be widely adopted at low-head hydropower sites in Europe due to their high hydraulic efficiency (greater than 80% in most installations), competitive costs, and low environmental impact. They have the greatest applicability at low-head sites—less than about 5 meters—and sites with high water flow rates.

As discussed, the performance of an Archimedes screw used as a power generator depends on several inner parameters (inner diameter, screw pitch, and number of flights) and outer parameters (outer diameter, head, inclination, and rotational speed), as well as the inlet and outlet water flow conditions. The optimization of this performance is currently an active research field.

Additional research in the Archimedes screws includes investigation of different blade shapes. Further progress is expected to be made in the near future towards the optimal design of hydropower screws.

References

1. T. Koetsier and H. Blauwendraat, *The Archimedian screwpump: A note on its invention and the development of theory*. International Symposium on History of Machines and Mechanisms: proceedings HMM2004, edited by Marco Ceccarell, Kluver Academic Publishers, first edition, 2004.
2. D.L. Simms, Adventures of an innovation over two millenia: the water-screw and its variants: Part I Origins and Designs. *Atti Della Fondazione Giorgio Ronchi*, LXV(2):197–215, 2010.
3. D.L. Simms, Adventures of an innovation over two millenia: the water-screw and its variants: Part II In and out of Use. *Atti Della Fondazione Giorgio Ronchi*, LXV(5):643–678, 2010.
4. D.L. Simms, Adventures of an innovation over two millenia: the water-screw and its variants, Part III Back in uses. *Atti Della Fondazione Giorgio Ronchi*, LXV(2):197–215, 2010.
5. Daniel Bernoulli, *Hydrodynamics by Daniel Bernoulli, Hydraulics by Johann Bernoulli*. Dover Publications, New York, 1968, first edition, 1968.
6. J.A. Eytelwein, *Handbuch der Mechanik fester Körper und der Hydraulik*. Dover Publications, Berlin, 1801, 1 edition, 1801.
7. Julius Weisbach, *Lehrbuch der Ingenieur- und Maschinen Mechanik*. Verlag von Friedrich Vieweg und Sohn, Braunscheig, first edition, 1855.
8. J. Muysken, Berekening van het nuttig effect van de vijzel. *De Ingenieur,* (21):77–91, 1932.
9. G. Nagel and K.-A. Radlik, *Wasserförderschnecken - Planung, Bau und Betrieb von Wasserhebeanlagen.* Udo PfriemerBuchverlag, Wiesbaden und Berlin, first edition, 1988
10. P.J. Kantert., *Praxishandbuch Schneckenpumpe - Ratgeber und Entscheidungshilfe für Planer, Bauherren und Betreiber.* F. Hirthammer Verlag, München, first edition, 2008.
11. D.M. Nuernbergk, *Wasserkraftschnecken – Berechnung und optimaler Entwurf von archimedischen Schnecken als Wasserkraftmaschine*. Moritz Schäfer Verlag, Detmold, 1 edition, 2012.
12. C. Rorres, The turn of the screw: Optimal design of an Archimedes screw. *Journal of Hydraulic Engineering,* 126(1):72–80, 2000.
13. J.C Horch, Proefnemingen met een watervijzel. *De Ingenieur,* (49):945–954, 1916.
14. A.C. Donaldson, Discharge tests on an Archimedean screw. *Proceedings of the Institution of Mechanical Engineers,* 157(1):93–102, 1947.
15. I.N. Bronstein, K.A. Semendjajew, G. Musiol, and H. Mühlig, *Taschenbuch der Mathematik.* Verlag Harri Deutsch, Frankfurt am Main, 5. auflage edition, 2000.
16. Karl-August Radlik, Wasserkraftschnecke zur Energieumwandlung, Patent DE 4 139 134 C2.
17. Moritz Rühlmann, *Allgemeinen Maschinenlehre*. Verlag von C.A. Schwetschke und Sohn, Braunscheig, 1 edition, 1862.
18. Editors: Sandor O. Pallfy and K. Brada*, Schneckentrogpumpe als Mikroturbine, in Wasserkraftanlagen - Klein- und Kleinstkraftwerke*. expert-Verlag, Malsheim, 1 edition, 1996.
19. K.-A. Radlik and K. Brada, Wasserkraftschnecke – Eigenschaften und Verwendung. *Proceedings of the Sixth International Symposium on Heat Exchange and Renewable Energy*, Szczecin, pages 43–52, 1996.

20. G. Meißl, Die Wasserkraftschnecke als Restwassermaschine- ein Lösungsmodell. *Kleinwasserkraftwerke, OTTI - Kolleg,* (1):160–168, 2005.
21. D.M. Nuernbergk, *Wasserräder mit Kropfgerinne - Berechnungsgrundlagen und neue Erkenntnisse.* Moritz Schäfer Verlag, Detmold, 1 edition, 2005.
22. D.M. Nuernbergk, *Wasserräder mit Freihang - Entwurfs- und Berechnungsgrundlagen*. Moritz Schäfer Verlag, Detmold, first edition, 2007.
23. W. Schmalz, Untersuchungen zum Fischabstieg und Kontrolle möglicher Fischschäden durch die Wasserkraftschnecke an der Wasserkraftanlage Walkmühle an der Werra in Meiningen - Fischökologische & Limnologische Untersuchungsstelle Südthüringen. Technical report, 2010.
24. H. Späh. Fischereibiologisches Gutachten zur Fischverträglichkeit der Patent geschützten Wasserkraftschnecke der Ritz-Atro Pumpwerksbau GmbH. Technical report, 2001.
25. J.C.A. Merkx and F.T. Vriese. Zusammenfassung – Monitoring der Schnecke und der Fischpassage der Hooidonske Watermolen - Monitoring vijzel en vispassage Hooidonske Watermolen. Technical report, 2006.
26. Fishtek Consulting, The Archimedes screw turbine – assessment of three leading edge profiles. 8mm steel edge, hard rubber, compressible bumber. Technical report, 2009.
27. A. Lashofer, F. Kaltenberger, and B. Pelikan, Wie gut bewährt sich die Wasserkraftschnecke in der Praxis. *Wasserwirtschaft*, (7-8):76–82, 2011.
28. D. Nuernbergk, A.Lashofer, W. Hawle, B. Pelikan, Betriebsarten von Wasserkraftschnecken. *Wasserwirtschaft*, 103(7-8):35–40, 2013. Buchverlag, Wiesbad en und Berlin, 1 edition, 1988.

Archimedes, Astronomy, and the Planetarium

Michael T. Wright

Editor's Note: The figures in this chapter were provided by the editor of these proceedings.

Archimedes wrote a paper that we call *The Sand Reckoner*, in which he adopts the heliocentric cosmology of Aristarchus; but he does it only so as to get the largest possible estimate of the size of the universe, all the better to make good his claim that he is able to number the grains of sand that it would take to fill it. He also cites an estimate of the relative sizes of the Moon and Sun made by his father Pheidias. This suggests that his father was an astronomer, but we have no evidence that Archimedes himself ever studied the subject. Nevertheless we link Archimedes with astronomy through the planetarium. Following his death, there arose a strong tradition that Archimedes built some sort of planetarium and that in doing so he had achieved something new and remarkable. It is this tradition that lies at the heart of my paper.

The earliest records of this instrument are three separate passages in the writings of the Roman author Cicero. It is worth giving the most informative of them, from his *De Re Publica* (Book 1), fairly fully. The following is my translation:

> *Gallus … gave orders for the [celestial] globe to be brought out which Marcellus had carried off from Syracuse when that … city was taken. Although I had heard this globe mentioned quite frequently on account of the fame of Archimedes, when I actually saw it I was not particularly impressed. Indeed, the* other *globe made by Archimedes, which Marcellus had placed in the temple of Virtue at the same time, is more beautiful and more widely known.*

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Yet, when Gallus began to give a very learned explanation of the instrument, I concluded that there was more of genius in that Sicilian than human nature seemed able to bear. Indeed, Gallus told us that the other kind of globe, solid and with no internal parts, was a very early invention. The first one of that kind had been turned by Thales of Miletus. Later, however, it was marked by Eudoxus of Cnidus (a disciple of Plato, it was said) with the stars that remain fixed in the sky. Many years later Aratus, having no knowledge of astronomy but some poetic talent, took the whole design and arrangement from Eudoxus and published it in verse.

However, this kind of globe incorporated the motions of the sun and moon and of those five wandering stars, the so-called planets, which it was impossible to include on the earlier, solid globe. There was a wonderful contrivance due to Archimedes inside; he had devised a way in which a single rotation would generate the several non-uniform motions. When Gallus set the globe in motion, the moon came into conjunction with the sun after just as many turns in that bronze instrument as it takes days for it to happen in the heavens. Therefore that very same eclipse of the sun would take place on the globe, and then the moon would pass into the cone which would be the shadow of the earth, since the sun … from the place …

… and there, to our frustration, is a break in the text.

So here were two instruments attributed to Archimedes. One was a simple celestial globe, apparently handsomely finished, which was in a temple, on public view; and Cicero tells us his version of the history of this type of instrument. Attention is, however, focused on the other instrument which belonged to the heirs of the general Marcellus. It too was called *sphaera*, globe. It was a portable instrument of metal, which contained a mechanism so that—on working some input—the user made indicators for the Sun, Moon, and planets move in a reasonably realistic way. Later ancient authors suggest two further features: a display of the phase of the Moon and the ability to show the risings and settings of stars. In addition, we have the remarks of two scholars, Pappus and Proclus: Pappus says that he believes that Archimedes wrote a book on the making of such instruments but that he has not seen it; and Proclus says that Archimedes was the master of this art.

All this is well known, but, so far as I know, until today nobody has ever offered a plausible account of what this instrument might have been like and how it could have worked.

The word *planetarium* conjures up the image of a theatre presenting its audience with a view of the night sky. In the modern planetarium, light from a central device is projected onto a domed ceiling. Visits to the London Planetarium were high points in my school holidays. When I joined the staff of the Science Museum in 1971, the regular shows in our own little planetarium were popular; but by then my interest was more closely focused on its wonderful Zeiss projector (Figure 1) than on the astronomy that it showed. Zeiss had completed the first such projector in 1923, for the *Deutsches Museum* in Munich. Our little version was originally built for teaching astral navigation in the *Luftwaffe*.

Sadly, the Science Museum's planetarium was closed decades ago, and the London Planetarium, though renamed the *Star Dome*, is now devoted to stars of another kind whose waxworks have invaded from Madame Tussaud's next door. Other planetaria of this type—such as the Hayden Planetarium in New York—

Figure 1 Zeiss planetarium projector. The Science Museum, London

continue to flourish. But the thrill that children of my generation experienced must be just a little diminished by the ease with which a fairly good display of the night sky can now be generated anywhere, thanks to the wonders of modern digital electronics. We may not see much of the night sky in our over-lit cities, but today we can see *images* of it wherever and whenever we please, on a computer screen if not projected onto a dome.

Before Zeiss developed their projector, there were more primitive attempts to achieve a similar effect. In the eighteenth century, Dr. Roger Long of Pembroke College in Cambridge built a celestial sphere 18 feet in diameter which he called the *Uranium*. Students sat on a platform inside, while the sphere was rotated by the lecturer's assistant. Dr. Long's sphere is long gone, but the Gottorf Globe, a more polished instrument of even earlier date, survives—totally rebuilt after a fire—in St. Petersburg (Figure 2).

What has any of this to do with Archimedes or with the ancient world at all? Well, to begin with, this sort of device—and the star-dome display of the night sky generally—was a logical development of the celestial globe, a globe with stars and constellations marked on it.

The globe shown in Figure 3 may be the oldest to survive. It is no bigger than a tennis ball and shows only the constellation figures.

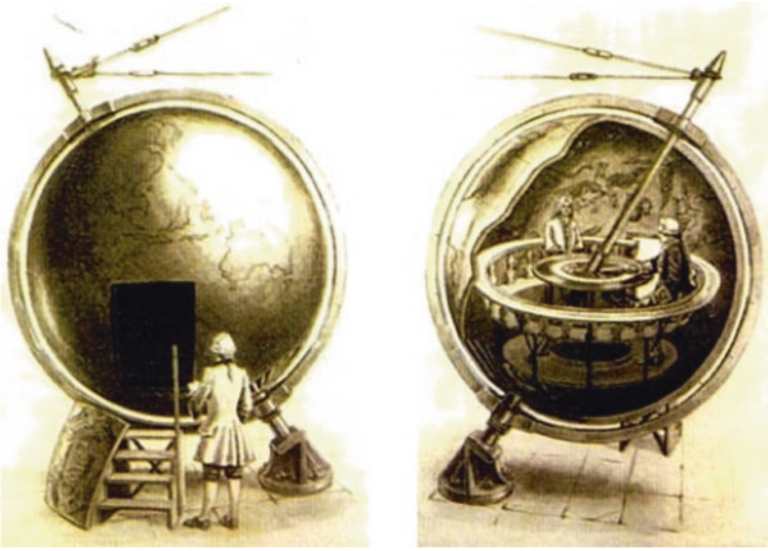

Figure 2 Gottorf Globe, seventeenth century, rebuilt

Figure 3 Celestial globe, second century BC to first century AD, Paris, Kugel Collection

The same is true of the more famous example from antiquity in Figure 4, even though Atlas's globe is over two feet across.

Perhaps the beautiful eighteenth-century example in Figure 5 makes the point more clearly. Here we see both the constellation figures and individual stars. As Cicero tells us, such an instrument, attributed to Archimedes, was taken from Syracuse and placed on public show in Rome.

The celestial globe illustrates man's developing perception of the cosmos. First, it depends on the awareness that the stars form a practically unchanging pattern; and

Figure 4 Farnese Atlas, second century Roman sculpture, the National Archaeological Museum, Naples

then, it could not have been devised until man came to think of the universe itself as spherical. So, the stars are marked on the surface of the globe just as they are imagined to be embedded in a spherical shell in the sky; when we look at the sky, we see this supposed spherical shell from the inside, but on a globe we see the stars from the outside and have to *imagine* them as viewed from inside. The later Roman author Claudian wrote about the beautiful idea of a making a celestial sphere out of a transparent globe of glass, an idea that has given rise to a number of spectacular examples; Figure 6 shows a nineteenth-century example.

However, transparent glass and the technique of blowing it to form a globe were both discovered some time after the death of Archimedes, in about the first century BC.

Cicero ascribes the invention of the celestial globe to the philosopher Thales, who lived in the sixth century BC; but we need not take his statement too seriously because the name of Thales had become a byword for everything that was clever. We now think that the idea of the cosmos as a spherical thing came only later. The Pythagoreans thought of the *Earth* as spherical, which was a good start; but it is generally agreed that the concept of the *celestial* sphere was fully developed only in the fourth century BC.

Plato, active in the fourth century BC, was not himself an astronomer but he was a strong influence on others. The mosaic in Figure 7, from Pompeii, is supposed to show an astronomy lesson at Plato's Academy; the central figure points with a rod to the celestial globe in its boxlike stand. Of course, the mosaic is later in date and

Figure 5 Celestial globe, eighteenth century

does not prove that the celestial globe actually existed in Plato's day, but we have other evidence: in the *Timaeus*, Plato described how a divine being was supposed to have made the heavens, and the only way to make sense of what Plato wrote is to accept that what he had in mind was the making of a different instrument, an armillary sphere. This instrument, made up of a set of rings fitted together, is simply an abstraction based on the celestial globe. Figure 8 shows examples of the two: a celestial globe on the left and an armillary sphere—just the reference rings of the sphere—on the right.

These ancestors of our planetarium dome date from before the time of Archimedes; but here I am using the word *planetarium* in the modern sense to mean a general display of the night sky, and this usage is fairly recent. Of course the word means, essentially, an instrument that demonstrates what is peculiar to the planets; this older usage is more directly relevant to what else I have to say.

Figure 6 Swedish "Grand Sohlberg" transparent celestial globe, late nineteenth century

The so-called fixed stars form patterns that appear practically unchanging as they roll over us night after night, except that each night the patterns are seen a little further to the West until, after a year, we see just the same part of the pattern that we did at the same time a year earlier. This slow shift is the effect of our year-long orbit round the Sun or—as ancient man would think of it—of the year-long circuit of the Sun through the fixed stars. The Moon makes the same circuit, but in a month rather than a year. Because these two drift through the pattern of stars, some ancient authors include them among the planets; but while the Sun and Moon move continually Eastward against the Westward-rolling backdrop of stars, the bodies that we call planets—of which ancient man saw five: Mercury, Venus, Mars, Jupiter, and Saturn—behave more strangely; they show what we call *retrograde motion*. Most of the time the planets also shift Eastward through the fixed stars; but occasionally they stop, go the other way, then stop again, and resume the Eastward movement, making a looped or zigzag track in the sky.

All the time, though, Sun, Moon, and planets remain on much the same path, called the Zodiac. The groups of stars along the Zodiac were imagined to represent recognizable creatures, and this gave men a handy way of recording where each planet was at any time (Figure 9).

The Zodiac is just one idea that the Greeks inherited from the older civilization in Mesopotamia. Babylon and its other great cities had a long tradition of sky-watching, and their astronomers kept written records on clay tablets. By analysis of these records, they became able to predict the movements of heavenly bodies with great success.

Figure 7 Mosaic from Pompeii, National Archaeological Museum, Naples, first century BC

Greek astronomers owed much to this earlier work, but they added something new. The Mesopotamian treatment was entirely arithmetical, but the Greeks—thinking visually —looked for ways of describing the motion of the Sun, Moon, and planets in geometrical terms. It is hard to tell how far such geometrical, kinematic systems were seen merely as a convenient way of predicting a planet's motion and how far they were thought of as the mechanism actually at work in the cosmos; but for our present purpose the point that matters is this: if the motion of a planet can be described in kinematic terms, then it becomes possible to think of a mechanical instrument that will model it. Kinematic theory leads to the idea of the planetarium.

Again, Plato provides a landmark. He seems to have expressed a prevailing view, in stating that it ought to be possible to account for the apparent behavior of each planet as a combination of uniform circular motions. After all, uniform motion in a circle was thought of as perfect and, Greek philosophers thought, only such perfection was fitting for heavenly bodies.

Figure 8 *Left*: seventeenth century celestial globe, modern facsimile. *Right*: sixteenth century armillary sphere, modern facsimile

The first kinematic planetary theory that we know about was probably a direct response to this challenge. In the system of homocentric spheres, suggested by Plato's pupil Eudoxus and later elaborated by Callippus and by Aristotle, nested spherical shells all roll about a common centre. Each shell turns on pivots planted in the one next outside it, but each axis points in a different direction so that the shells rotate obliquely inside one another. The overall result is that, although each shell rotates with a uniform motion, a point on the innermost one can be made to describe a zigzag path roughly like that of a planet. There are some lovely animations of this system on the Web; Figure 10 is a still from the website of my good friend Mogi Vicentini. It is hard to imagine, though, that this system could have been built in antiquity as an automated mechanical model: it presents difficulty both in transmitting motion to all the shells and in bringing indications of the planets' positions to the surface to be seen. Besides, this theory gives only a loose agreement with the motion of any planet; in detail the match is poor, and if we put in parameters for some of the planets, the system does not work at all.

After this came two theories that were much more successful in describing the planets' motion (Figure 11). The *epicyclic* and *eccentric* hypotheses (to give them their usual names) appear different at first sight, but it was soon recognized that they could produce identical results. In modern terms, they simply represent the addition

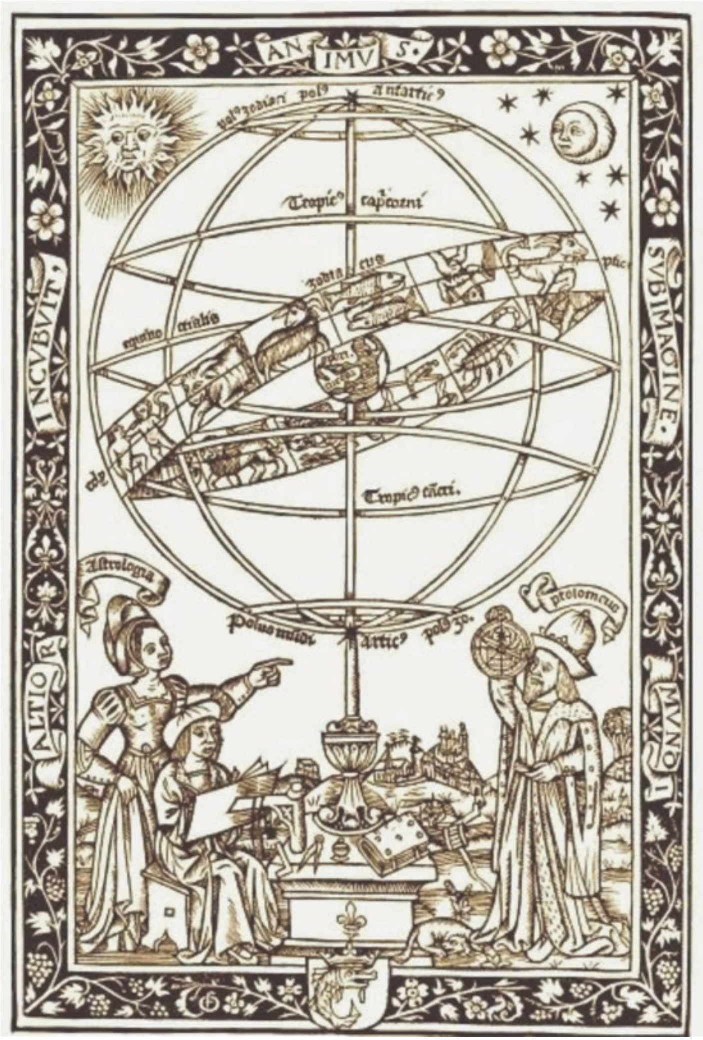

Figure 9 Frontispiece of an early edition of Ptolemy's *Tetrabiblos*

of two vectors in the alternative orders: **B+A** instead of **A+B**. An important point for us is that in their basic forms both these theories ignore the small side-to-side motion of the planets and concentrate on the main motion, forward and back along the Zodiac. As exercises in plane geometry, fully represented in flat diagrams, they are readily converted into mechanism as wheels and levers.

Unfortunately, we do not know when, or by whom, either theory was devised. The first we hear of either is that Apollonius of Perga studied the epicyclic theory, but still we do not know whether he was its originator. Apollonius was a younger contemporary of Archimedes; their lives overlapped by some 40 years. If this theory

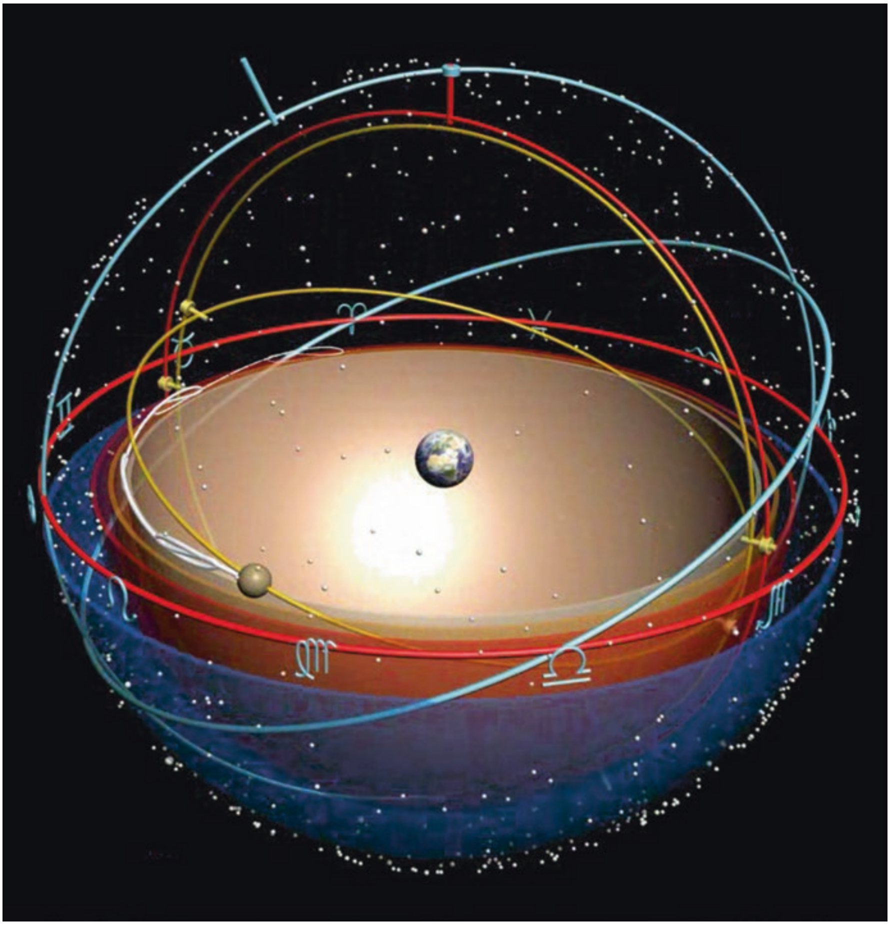

Figure 10 Still frame from an animation by Massimo Mogi Vicentini

existed during that overlap, it is likely that it was known to Archimedes as well as to Apollonius; and if Apollonius were its inventor, it is likely that he would have told the older man about it.

Knowing, as we do from his *Method*, how Archimedes was accustomed to think about mathematical problems in a mechanical way, it seems inconceivable that his genius would not have led him to translate such a planetary theory into mechanism. I suggest that it was precisely the awareness that the motion of the planets could be described in a mathematical way and that the mathematics lent itself well to being translated into mechanism that led Archimedes to devise the world's first planetarium.

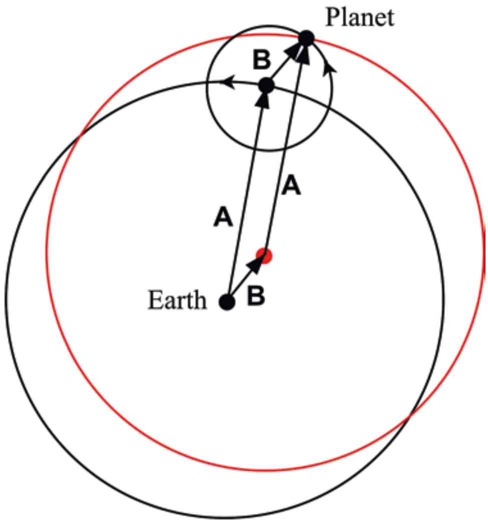

Figure 11 Diagram illustrating the equivalence of the epicyclic (**A+B**) and eccentric (**B+A**) hypotheses

There's another point. The interesting thing about the way the planets move is not just that they seem to go around the sky at different rates; it is that each has those peculiar episodes of retrograde motion at roughly regular intervals. Cautious commentators have suggested that early planetaria probably showed only the planets' mean motions; but if so, the indicators for the planets Mercury and Venus—which, because they are between us and the Sun, are always seen close to it—would simply have moved along with the Sun in a cluster, and the other planets too would have had a pretty unrealistic motion. I cannot imagine Archimedes being interested in anything so dull, and so far from reflecting reality, as mere mean motions. Nor can I imagine such a model much exciting the interest of the general Marcellus—to the extent that he claimed no other spoils—or, later, of Cicero and the rest. It is not the same as having a Copernican orrery (a heliocentric model of the solar system) with mean motions; there uniform motion is a fair approximation, but in a geocentric model the planets really do have to make those bold zigzags. Besides all this, the Latin word *dissimilis* that Cicero uses to describe the motion of the planets on the instrument is a precise translation of the Greek *anṓmalos*, the technical term for just this peculiar motion.

This, I think, was the whole point of the planetarium of Archimedes; he had found a way to mechanize the new mathematical planetary theory based on superposed circular motions. To do this, he would have needed the use of toothed gearing. But was it available?

The famous Antikythera Mechanism (Figure 12) shows that highly developed gearing existed in the first century BC at the latest, and some people argue that the instrument was built earlier, in the second century. In either case, the subtlety and accomplishment of its gearing suggest a prior history, though perhaps of simpler mechanism, reaching back at least some decades earlier still.

Figure 12 The Antikythera Mechanism, fragment A

Working forward from earlier times, a passage in the Aristotelian *Mechanical Problems*, written about 300 BC, suggests that simple toothed gearing was known that early, and—according to Vitruvius—the water-clock of Ctesibius, of about the same time, included a wheel-and-rack gear (Figure 13). And then, some authorities credit Archimedes himself with the endless screw or worm-and-wheel gear. So it does seem highly probable that Archimedes knew about toothed gearing and was able to use it in designing his planetarium.

How, though, might the gearing have been arranged? Here I think we can come right up to date and build on our growing understanding of the Antikythera Mechanism which comes, arguably, from the same tradition.

As this century dawned I had become convinced that the Antikythera Mechanism was, essentially, a planetarium, and I showed how we might restore the lost planetary part by a very simple process of modelling the astronomical theory in metal, using gear wheels to generate and combine the required circular motions, all driven from the one-rotation-per-year motion of the Sun (Figure 14). More recent work shows that my idea was certainly correct in outline. Variants on my restoration of the internal mechanism have been suggested, but so far there is no clear reason to prefer one scheme to another. All achieve the same result, and it makes no difference to the display which we choose.

In reaching for the correct restoration of the Antikythera Mechanism, we can also see something of how Archimedes could have achieved what the literary tradition claims for him. Here, in a sense that I think the organizers of this conference did not intend, is *Archimedes in the 21st Century*; it is our new understanding of the Antikythera Mechanism—the one extant specimen of Hellenistic astronomical modelling—that allows us to see more clearly the achievement of the man who, in later antiquity, was regarded as the first and greatest master of this art.

That said, what would the instrument have been like? Cicero, and all the other authors, call it *sphaera*, a globe; more than that, the way in which Cicero compares

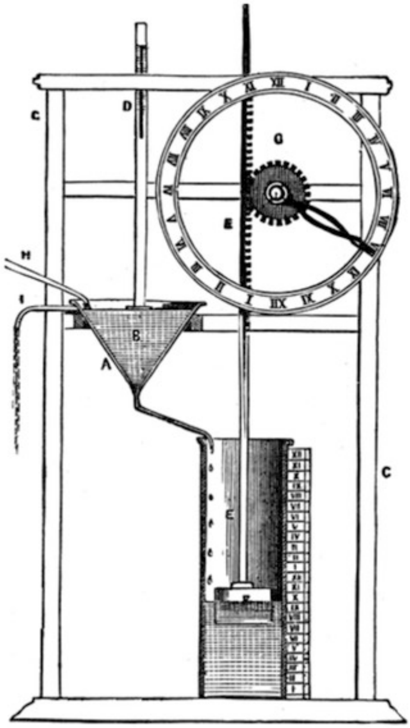

Figure 13 Water clock of Ctesibius, third century BC: a nineteenth-century reconstruction

Figure 14 The author's reconstruction of the Antikythera Mechanism

Figure 15 Detail of mosaic from Pompeii (cf. Figure 7)

it with a simple celestial globe shows that the mechanical instrument was similar in outward appearance and arrangement. It carried indicators for the positions of the Sun, Moon, and planets which were moved "with their various dissimilar motions" by a single "rotation". The mechanism inside that drove them was worked when the demonstrator "moved the sphere". Later authors refer to a display of the phase of the Moon and to the display of risings and settings. The simplest interpretation is that the instrument was, essentially, a celestial globe, supported in a stand.

Then Cicero's "rotation" and "moving the sphere" would have been rotation of the globe about its pole by hand, in imitation of the diurnal rotation, just as in using a conventional celestial sphere showing only the fixed stars. To allow the user to observe risings and settings, the stand is usually fitted with some means of marking the horizon, a horizontal component with just half the globe visible above it. For the globe in the mosaic (Figure 15), the top of the box-base probably served this function. The angle of elevation of the globe's polar axis corresponds to the observer's latitude.

To show the individual motions of the Sun, Moon, and planets through the fixed stars, there had to be a set of pointers. There are two main possibilities: either, they all radiated from the crown of the globe, reaching round so that their tips moved over the Zodiac; or, the globe was divided in two by a slot around the Zodiac—like a yo-yo—and the pointers projected through the slot. Either is feasible, but the latter solution calls for quite a wide slot which would cut all the constellations of the Zodiac in two and spoil their appearance. The first option is easier, and it provides a way of displaying the Moon's phase.

We come now to the mechanism working these pointers. If (as I argued just now) we can take the Antikythera Mechanism as our guide, this part of the scheme is straightforward. In the Mechanism we have straight pointers moving round a flat dial, but the problem is just the same. Like the dial of the Mechanism, the middle of our globe is marked out with the Zodiac. It makes no difference that the globe is

Figure 16 The author with his reconstruction of the Sphere of Archimedes

rotating, because the mechanism inside rotates with it. Whatever arrangement works for the Mechanism, a similar one can be fitted in the globe so long as the globe is large enough.

I have built an instrument according to this reconstruction (Figures 16 and 17). Notice one particular point: the Sun, Moon, and planets do not move round the equator of the globe, but around the zodiac which is tilted over to one side at an angle of about 24°. This means that the system of pointers, and the mechanism inside that drives them, have to be tilted up at this angle too, and the rotation of the globe that works them has to be transmitted through this angle as well.

The use of the diurnal motion for the input provides a pleasing completeness to the instrument that the Antikythera Mechanism lacks; but with the diurnal motion as the input, most of the output motions become very slow; the pointer for Saturn makes one full circuit of the Zodiac only after the globe has been rotated about 10,800 times. Whether such a slow output would have been seen as a difficulty

Figure 17 Details of the author's Sphere of Archimedes

depends wholly on how the instrument was to be used. In the hands of Cicero's character Gallus, demonstrating how eclipses might occur, the typical change in setting from one eclipse-possibility to the next, a shift of five synodic months, would have called for some 148 revolutions of the globe. However, in my reconstruction the drive is intermittent; if the globe is held still at a point in its rotation when it is not engaged, the pointers can all be worked about 30 times faster by moving the Moon pointer by hand.

It remains to show what significance any of this still holds today. First, despite the progress of purely electronic instruments, the mechanical planetarium is still very much with us; although the output effect of these instruments is produced by the moving of beams of light on a dome rather than pointers on a dial or globe, the motion is still generated by gearing just as in ancient ones.

Much more generally, it has been said that the whole of our modern mechanical engineering is built on the ingenuity and skill of the clockmaker. But it is becoming ever clearer that the medieval clockmaker, in turn, drew on skills first developed in antiquity, in the building of models of the cosmos. And at the head of all these stands the *Sphere of Archimedes*.

Archimedes in the Twenty-First Century Imagination

Mary Jaeger

Any discussion of Archimedes in the twenty-first century imagination must reach backward into the twentieth, because so much recent work arises from or responds to the sale of the codex containing the Archimedes Palimpsest at Christie's in New York, on October 28, 1998, and its installation in the Walters Art Gallery in Baltimore in 1999. The years since have seen the publication of the palimpsest by Reviel Netz and Nigel Wilson and the rest of their team; images are online, as is a transcription of the text [1]. In addition, Netz has published part of his ongoing project of translating Archimedes: three volumes are to come out of Cambridge University Press, the first of which, *The Two Books on the Sphere and Cylinder*, appeared in 2004. Netz has also written a three-part study of Mediterranean mathematics: *The Shaping of Deduction in GreekMathematics* (1999); *The Transformation of Mathematics in the Early Mediterranean World: from Problems to Equations* (2004); and *Ludic Proof* (2009). Finally, and firmly in the twenty-first century, an international conference on Archimedes held at Syracuse in June 2010 has resulted in a hefty volume of conference papers (*The Genius of Archimedes--23 Centuries of Influence on Mathematics, Science, and Engineering. eds.* S. A. Paipetis, Marco Ceccarelli. Springer: New York).

Clearly the specialist in ancient mathematics has plenty to work with. What about the writers who aim at a general audience, and the nonspecialist scholars and artists themselves whose projects include pedagogical material, historical fiction, and artistic representation? With some exceptions, what has captured their imaginations is not so much Archimedes' mathematics as the stories about the man, especially three: [2] first, that in which the bathing Archimedes, noting the water displaced by the submerged part of his body, realized how he might determine whether a crown made for the Syracusan king Hieron was solid gold or not, and ran naked through the streets of Syracuse crying Eureka! Eureka! ("I have found it! I

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have found it!"); second, the account of his defending Syracuse with his siege engines, including the fictitious "death ray" or burning mirrors; and third, the story of his death at the hands of a Roman soldier [3]. None of these stories was told by an eyewitness; every ancient source that reports them has its own agenda; but together they are compelling, forming as they do a neat narrative with a beginning, middle, and end.

The Eureka anecdote appears first probably because it is the most famous and possibly also because, although modern images invariably portray the bathing Archimedes as fully adult and bearded, if not old, Archimedes' emerging from the bath with a new discovery makes a nice story of origins, like that of the emergence of life from a primordial ooze. Moreover, modern artists and writers have constructed and presented this story so as to make its Archimedes appealing to children. Consider, for example, the cover art for Pamela Allen's 1994 children's book *Mr. Archimedes Takes a Bath*, or a poster from Ogilvy and Mather's 2004 ad campaign for Mentos [4]. In both cases the figure of Archimedes is that of a comical and therefore unthreatening old man, rather like the famous picture of Einstein sticking out his tongue.

The "middle" of the narrative, namely, the account of Archimedes' siege engines and other mechanical inventions, has provided material for a number of reconstructive projects and documentaries: In 2001, Chris Rorres and Harry G. Harris produced a scale model and delivered a paper titled "A Formidable War Machine: Construction and Operation of Archimedes' Iron Hand" [5]; a 2005 Discovery channel episode, from the series "Superweapons of the Ancient World," records the activity of a crew of architects and engineers reconstructing Archimedes' Claw (or 'hand') in Tunisia; and every generation, it seems, explores anew the possibility of Archimedes' using mirrors to set the Roman ships afire. After the show "Mythbusters" argued that it could not be done, in 2005 a group of students from David Wallace's Product Development class at MIT showed that it was theoretically possible to set fire to a ship by using mirrors. And they did so, first setting fire to a mock-up using 127 one-square-foot mirrors at a distance of 100 feet, then using something closer to what they thought approached the real thing. But they could achieve ignition only under ideal (i.e., windless) conditions.

I have found few recent images of Archimedes' death. The New Hampshire-based educator Marek Bennett devotes a 2014 comic strip to Valerius Maximus' version (and quotes the Latin at the end); otherwise, we have to reach farther back, to the 1950s and 1960s, when the Greek surrealist poet and painter Nikos Engonopoulos was representing mythological and historical scenes, including "The Death of Archimedes, With Syracuse in Flames"; and to 1972, when the German sculptor Gerhard Thieme installed versions of a bronze sculpture of Archimedes in various sites in Germany [6]. Thieme's bronze shows Archimedes contemplating geometrical figures drawn in the sand, just as Livy and Vitruvius say he was doing when the soldier killed him. Although this Archimedes seems absorbed in his work, his position and attitude themselves do not mean he is about to die. Unlike Archimedes in the bath, Archimedes about to die does not appear on the covers of children's books, probably because their authors would rather extol the many virtues of taking a bath than suggest that devotion to mathematics can be dangerous to one's health. As of this writing, however, it was possible to buy a "Death of Archimedes in Sack of Syracuse Twin Duvet" on Cafe Press. I do not know what psychological effect being tucked into bed under such a scene of carnage will have on the freshly bathed young mathematician.

A number of writers, some of them educators, have used the interest in Archimedes' story as a way of stimulating an interest in mathematics: the 2009 book *Eureka Man*, for example, both discusses Archimedes' biography and explains his mathematical contributions. (Its author, Alan Hirschfeld, a specialist in the history of physics and astronomy, directs the physics teacher-training program at the University of Massachusetts, Dartmouth.) The first chapter in mathematician Clifford Pickover's *Archimedes to Hawking: Laws of Science and the Great Minds Behind Them* (2008) does pretty much the same. In 2010, John Monahan, a science teacher in Baltimore, published *They Called Me Mad: Genius, Madness and the Scientists Who Pushed the Outer Limits of Knowledge.* The book's cover image, which includes the portraits of Einstein and Tesla, is just sufficiently a caricature to make both men appear less intellectually intimidating. The cover artists did not have to do this to Archimedes: Monahan's first chapter is titled "Eureka! The Mad Scientist is Born"; and, once again, the bath story is enough to make Archimedes a nonthreatening archetype of the absent-minded professor.

These authors have laudable goals: to increase science literacy and inspire young people to study STEM subjects. As a classicist, however, I hope the Archimedes anecdotes will draw readers to explore their ancient sources and in doing so learn more about other great minds of classical antiquity: the historians Polybius and Livy, for example, and Cicero, Plutarch, and Vitruvius. For those whose interest tends toward the historical and literary, several scholars provide useful guides to the ancient sources. A good starting point for the nonspecialist is Chris Rorres' Archimedes website, which offers information on all aspects of Archimedes' life, career, and times [7]. In addition to essays on Archimedes' mathematical and technical achievements, the site includes timelines, animations, videos, and a wide array of images.

Rorres' website also includes a bibliography of the many valuable papers on Archimedes by D.L. Simms. A Londoner who entered the British Scientific Civil Service and earned his PhD for work on the ignition of materials through radiation, Simms spent his early career researching the ignition and spread of fires. (This experience proved useful as he refuted again and again the argument that Archimedes used burning mirrors against the Roman fleet at Syracuse.) Simms also wrote a major report on irrigation in Great Britain and spent the later part of his career working on projects that aimed at controlling pollution. Once retired, he engaged himself full time in his research on the history of science and technology [8]. A series of articles, beginning in 1965 ("The Legend of Archimedes and the Burning Mirrors of Syracuse" in *Fire Research Notes*), and continuing until 2010 ("Adventures of an Invention Through Two Millennia: The Water-Screw and its Variants; Part III: Back in Use" in *Atti della Fondazione Giorgio Ronchi*), covers all aspects of Archimedes' life and technical achievements. Simms' work displays a thorough knowledge of the ancient sources on Archimedes and a rigorous and honest use of them in argument. Some of Simms' papers, because they appeared in regional journals or proceedings of esoteric organizations, can be difficult to find. Rorres' website has links to a number of them.

Readers of Italian will profit from Mario Geymonat's *Il Grande Archimede* [9]. Most classicists recognize the name of the late Geymonat for his magisterial edition of Vergil and his extensive work on the Vergilian commentators. However, Geymonat also had a lifelong interest in ancient science and, among his other editorial contributions, published the first edition of the fragments of the Latin translation of Euclid [10]. *Il Grande Archimede* (first ed., 2006) boasts both an introduction by the Nobel Prize-winning Zhores Alferov (2000, Physics) and a preface by the historian Luciano Canfora. Already in its third Italian edition by 2008, it has won the Corrado Alvaro award for Italian literary excellence. It has also appeared in English, translated and edited by Alden Smith (Baylor, 2010).

Il Grande Archimede first surveys Archimedes' life and times, including his experience in Alexandria and the contacts he made there with Eratosthenes, Conon, and Dositheus, his use of Doric and the style of his treatises, the intellectual range of his work, his method of argumentation, his ability to unite the theoretical and the practical, and the ancient world's social and intellection prejudices against the practical application of theory. The rest of the book covers first Archimedes' major mathematical contributions and then his mechanical inventions. The last chapter gives a history, sometimes whimsical, of legends about Archimedes, tracing them from Cicero to Walt Disney. An appendix lists references to Archimedes in Latin poetry. The book nicely interleaves the discussion of particular geometrical problems, the testimony of Archimedes himself and other ancient authors, and the ancient anecdotes.

Solving a problem by manipulating geometric features is essentially a visual activity; it is thus no surprise that a strong strand of interest in the visual runs through a number of the recent studies. Netz's work, from the commentary to the popular *Archimedes Codex*, pays particular attention to the aesthetics of Archimedes' treatises. In *The Transformation of Mathematics in the Early Mediterranean World*, Netz argues that the problems solved by Archimedes were essentially geometric problems, solved by manipulating lines, triangles, etc., as opposed to the essentially algebraic problems posed by the mathematicians of the Arab world. One of the fascinating aspects of the Archimedes palimpsest is its barely preserved diagrams; and Netz points out repeatedly that the numerical systems in modern texts of the ancient mathematicians are anachronisms: schematic diagrams, representing the topological features of a geometrical object, are what the Greeks used for clarity. When they used numbers, the result, which appears intended, was obfuscation. Netz also lays emphasis on the beauty of ancient diagrams as one of the aesthetic features of ancient Greek mathematics.

By its lavish use of historical illustrations, *Il Grande Archimede* participates in this appreciation of the visual: the diagram of Archimedes' first theorem from the *Measurement of the Circle*, taken from Jacopo of Cremona's Latin translation, which gives viewers both a grasp of the relationship between the curved and the straight and a clear impression of the beauty of early mathematical diagrams as drawings; the beginning of *Measurement of the Circle*, including text, with diagram; the end of *Measurement of the Circle* from Cardinal Bessarion's copy of the Greek manuscript of Archimedes, also including text, with diagram; some sixteenth-, seventeenth-, and eighteenth-century engravings taken from early editions of scientific and architectural works showing Archimedes solving inter alia the problem of specific gravity; and engravings of compound pulleys from Mazzucchelli's eighteenth-century biography of Archimedes. Other color plates include several of Giulio Parigi's wall paintings from the Stanzino delle Matematiche in the Uffizi, including images of catapults, the burning mirrors, the great ship being pulled to shore by pulleys, the naked Archimedes running from the bath, and the highly anthropomorphized picture of the great iron claw (the "hand" of Archimedes) that stymied the Romans at Syracuse.

Smith's English translation has kept the pictures, rearranged slightly the structure of the text, and made some additions to the bibliography. The translation has conflated the two original chapters on the ship Syracusia and the defense engines into one, and has raised the appendix on poetry to the status of a chapter. The transfer of the material on poetry to the main text is welcome, for Geymonat's knowledge of literary Latin together with his interest in the history of science has produced a dossier that will, I expect, lead to interesting results. Emma Gee's *Ovid, Aratus and Augustus: Astronomy in Ovid's Fasti* (Cambridge, 2000), has already shown how fruitful it can be to examine the interactions of science and poetry; Netz's *Ludic Proof* goes so far as to suggest that Hellenistic poetry might have had a role in the shaping of scientific texts. *II Grande Archimede* joins these works in pointing the way to an exciting subfield of literary/mathematical studies.

The most widely known popular work on Archimedes, *The Archimedes Codex*, was published by Reviel Netz and William Noel in 2007 [11]. It has been translated into at least twenty languages to date and was made into a NOVA special [12]. *The Archimedes Codex* shows how cooperative effort at the highest level of a number of discrete disciplines (Greek paleography, language, literature and textual criticism; history ancient, medieval, and modern; art history; history of the book; mathematics; computer science; and digital humanities) can achieve path breaking, truly interdisciplinary results. The work on the palimpsest has given us the only text of the treatise called *The Method*; it has helped us understand further the relationship of diagrams to text in ancient mathematical works; and it has brought about a reinterpretation of the game called the Stomachion as an exercise in combinatorics.

Part Two: The Fictional Archimedes

In relating first the perils faced by Archimedes' manuscript through the centuries, then its rediscovery and restoration, then the new discoveries in the restored text, Netz and Noel have crafted a suspenseful and exciting narrative. Indeed, Ed Rothstein's New York Times review of the "Secrets of Archimedes" exhibit at the Walters (Oct. 16, 2011) compared *The Archimedes Codex* to that other twenty-first-century publishing phenomenon, Dan Brown's 2003 *The Da Vinci Code* (the publisher of *The Archimedes Codex* includes on the cover a quote from the TLS that makes the same comparison). As is so often the case, the true story here is much, much more fascinating than fiction.

The allure of the mysterious artifact lost and then recovered and the compelling figure of the expert in esoteric knowledge who discovers ancient "secrets," the very idea that time has created secrets to be "revealed," have all proved inspiring to a number of first-time novelists [13]. Since the turn of the millennium, Archimedes has made several appearances as a fictional character in such works as: Theodore Homa's *Archimedes' Claw*, an anti-*Da Vinci Code of a sort,* in which the hero finds himself at the center of a government conspiracy having to do with time travel, with The Claw serving as a propellant helping thrust him to and fro in time; in Monte R. Anderson's *Archimedes of Syracuse: the Chest of Ideas*, which embeds the life of Archimedes within a night of Renaissance-era storytelling and shows the young mathematician enjoying the pleasures of Alexandria, including Egyptian beer and the city's lovely and compliant woman; and, finally, in Padraic Fallon's *The Circles of Archimedes*, which links the diagrams Archimedes was pondering at his death to Goddess worship and the stone circles at Avebury, England.

Together these novels show that the very scarcity of solid facts about Archimedes' life leaves plenty of room for the imagination. The comic appeal of the story of the bath, the fascinating nature of his siege engines, and his tragic death at the hands of a Roman soldier together form a matrix into which each writer inserts their interests, fantasies, prejudices, and preconceptions. Moreover, they impose their own twenty-first-century interests and experiences on the story and the figure of Archimedes.

The author of my last example imposes contemporary experience as well but, unlike the other authors, notes it explicitly. This is *The Sand-Reckoner*, which came from the pen of Gillian Bradshaw in 2000. As Bradshaw points out on her website, "real historical figures usually have too many inconvenient facts about their lives to allow for good fiction, but there aren't that many facts known about Archimedes, so I got away with it." [14] Bradshaw differs from the authors listed above in that she is a seasoned writer of historical and fantasy fiction; depending on how you count them, *The Sand-Reckoner* is her thirteenth or fifteenth novel. She is moreover, a trained classicist, who won prizes for her Greek at the University of Michigan, and then studied further at Cambridge. When she published her first historical novel and discovered that she could make a living by her writing, she said farewell to academe, except for the family connection she made by marrying the British academic physicist Robert Ball (University of Warwick). She writes of *The Sand-Reckoner*, "in a way it's a very personal book, as I drew upon the many physicists I've known to portray the man."

The Sand-Reckoner covers less than a year of Archimedes' life: his return from Alexandria, the death of his father, and his first interactions with Hieron, tyrant of Syracuse. The book does a nice job of weaving some the most tenuous strands of the biographical tradition into a compelling narrative. For example, Plutarch says that Archimedes had a connection to the royal house of Syracuse. Accordingly, Bradshaw gives Hieron a half-sister, Delia, who is an excellent musician on the aulos (the double-flute), as is Bradshaw's Archimedes. Once Hieron realizes Archimedes' importance to Syracuse's defense system, he has to find a way to keep him there. What better way than encouraging a lot of duets on the aulos and an eventual marriage alliance?

Bradshaw has a secure knowledge of ancient history and an excellent grasp of the *realia*: there are fine descriptions of neighborhoods in Syracuse, of houses and banquets, of the nature of the double flute. Her Archimedes is a charming figure, a young man who is always thinking creatively, as when, lying in bed he watches the patterns of sunlight on the wall, or sees a dead fish floating belly-up in the harbor and loses track of a conversation as he wonders why it floats that way. And he is as absent minded as Plutarch claimed: someone (usually his fictional slave Marcus) has to wake him from his trance and drag him off to a meal, a haircut, or a bath. Bradshaw also captures Archimedes' loneliness: his longing for Alexandria's lively intellectual life; his panic at his father's approaching death, which meant there would be no one at Syracuse at all capable of understanding his interests; and his desperate attempt to make himself clear to people who cannot understand what he's talking about (he knows he's not a good teacher). Or perhaps we should say instead "his research is stronger than his teaching," because this Archimedes seems very much like a modern academic. He would make an excellent professor at a research university, if he had the office staff to keep him on schedule.

Bradshaw's Archimedes is smart and creative; he is also considerate to slave and free and fond of strong women (Delia ends up running the practical side of their married life, including the considerable estate that was her dowry). He is a loyal citizen; and he speaks truth to power (Hieron), because he knows that high-tech defense jobs are always available. But he feels the emotions, shared by so many mathematicians and physicists of this past century, upon realizing that their calculations, embodied in weaponry, work as intended, which means that they wound and kill human beings.

In all these popular accounts of Archimedes, fiction and nonfiction alike, one phrase surfaces again and again: "killed by a Roman soldier." Indeed, the figure of the Brutal Roman has become essential to the twenty-first-century portrait of Archimedes, even though that figure appears in various guises, whether as the killer himself, or as another character onto whom Roman brutality is transferred. *Archimedes' Claw* begins with a brief scene of the kill; *The Chest of Ideas* saves it for nearer the end, as does *The Circles of Archimedes*. *The Sand-Reckoner* anticipates and prefigures it. Clifford Pickover (*Archimedes to Hawking* p.41) writes "Close to the time when Archimedes discovered his Principle of Buoyancy, the Septuagint Greek version of the Old Testament was being written, the La Tène Iron Age people invaded Britain, the first Roman prison Tullianum was erected, and the Carthaginian general Hannibal was born." Note the tendentious nature of Pickover's collocation of events: granted, archaeologists date the structure of Rome's prison to some point in the third century BCE. But other things happened in Rome then: the transition in temple style from more Etruscan to Hellenistic, the development of native Roman drama, the development to its highest level of the native Italian meter called Saturnians, and an act of translation that was in its own way as important as the translation of the Septuagint: Livius Andronicus' translation of the *Odyssey* into Latin verse, the first translation we know of, of a *literary* text. But a prison is a useful thing to list if you want to cast Romans as brutes.

The role of Brutal Roman extends even to the philhellenic general Marcellus: in the *Chest of Ideas*, we are told that he can barely read Latin much less Greek. After Archimedes' death, Archimedes' daughter tells Marcellus that her father wanted a diagram of the sphere and cylinder on his tomb. Marcellus answers:

> When I was in Alexandria, I saw the defenses he constructed around the city; the signal towers, the catapults, and other machines. His idea of using mirrors to signal commands was ingenious. I saw his invention of the water screw being used in silver mines in Spain. They even called it the Archimedes Screw. I saw the ship he built, the Syracusia. It was magnificent. Had he been a general, he would have conquered the world by now. And yet, he thinks a mathematics formula is his crowning achievement. I will never understand these scholars.

Likewise, at the end of *The Sand-Reckoner*, in a scene that anticipates Archimedes' death, Gaius Valerius, the brother of Archimedes' Roman-born slave Marcus, returns to Syracuse to restore to its owner the flute Archimedes had lent Marcus, who is dead. Valerius' friend Fabius comes along to translate. The interview turns sour when Fabius sees a contraption Archimedes is building, and asks what it is. Told it is a water-aulos, he infuriates Archimedes by saying what a comedown it is for the designer of great siege engines to be making musical instruments. When Fabius catches sight of Archimedes' figures of sphere and cylinder, he asks, "What use is it?" His reaction further enrages our hero, who points out that this is what is wrong with Romans: they are brutes who understand only the practical. If Bradshaw's Archimedes is the professor defending the liberal arts (especially music and mathematics), Fabius is the academic's worst kind of barbarian: a provost advocating for purely vocational education.

Both the prefigured killer, and the figures of sphere and cylinder (which would later decorate the tomb) help make this scene anticipate Archimedes' death. It continues to do so with Archimedes' last actions, because he turns away from his guests to his geometry, the circles he would later defend (p.346):

> The others [Gaius and Fabius] looked at him [Archimedes] in surprise, but he was already oblivious to them. The compass marked out its precise reckonings in the fine sand, and his face following it was rapt, intense, and joyful. For the first time in his life, Fabius felt the foundations of his own certainties tremble. The suddenly quiet room was filled with something that made the hair stand up along his arms, something that existed for no human use. Perspective altered dizzyingly, and he wondered what his own use was to a universe. Unaccountably afraid, he ducked his head and backed away.

If it feels as if some kind of conversion is imminent, that is no surprise. Bradshaw is very much a historical novelist of the old school; and *The Sand-Reckoner*, in many ways, resembles the sword-and-sandal novels of the nineteenth and early twentieth century. Consider, for example, *Ben Hur*, *The Robe*, and *Bride of Pilate*, all of which portrayed an extraordinary and mystifying character killed by Roman brutes who knew not what they did. In the imagination of the new millennium, the pure scientist dies for the sake of all scholars.

Finally, one very different literary Archimedes: Peter Hobbs' recent short story collection, *I Could Ride All Day in My Cool Blue Train* (Faber & Faber 2006), closes with "The Dead Ancients Trilogy," three vignettes portraying Archimedes, Pythagoras, and Sisyphus. Hobbs' figure of Archimedes combines the running Archimedes of the Eureka story and the idea expressed in the expression "give me a place to stand and I will move the world." In his story, Archimedes runs on the surface of the earth, like a hamster on the *outside* of his wheel, the force of his footsteps making it rotate. Like the Archimedes of the Eureka story, he too is a comic old figure, who "has lifted his robes to his waist so as not to stumble when they become tangled with his legs. His long white beard has been thrown with similar intent over his right shoulder… He's a wiry old man. His sandals flap a little as they kick back." Hobbs plays imaginatively with the idea of leverage and the idea of Archimedes' sphere. His Archimedes is on the verge of inventing the bicycle but restrains himself because the materials for such a thing are not yet available.

Archimedes continues to stimulate the imagination in a wide variety of ways: novelists rewrite his life; other writers meditate on his ideas; nor is reapplication of those ideas limited to literature. Let me close this survey with two examples of Archimedes' influence on modern design: first, the Italian firm Acquacalda has produced an Archimedean measuring bowl, a ceramic bowl with a series of Plimsoll lines engraved and painted on the outside. Place bowl in water; place the item to be measured inside; read from the water level and the lines the exact amount of displacement. Finally, according to Cicero, who said he saw it, Archimedes' tomb had on it a representation of the sphere enclosed by a cylinder and some verses expressing the ratio of their volumes. Artists have traditionally represented this as a column with a three-dimensional sculpture on top, although Netz argues that the marker held a simple diagram. I think Archimedes would have liked the idea that one early use of a 3-D printer (before its use to print a working weapon!) brings together both the "beginning" and "end" of his story. It is a three-dimensional model illustrating the relationship of the volumes of sphere and cylinder by showing the volume of water each contains [15].

Notes and References

1. *The Archimedes Palimpsest*. Reviel Netz, William Noel, Nigel Wilson and Natalie Tchernetska, eds. Cambridge University Press 2011. See also http://archimedespalimpsest.net/Supplemental/ArchimedesTranscriptions/
2. An exception is Sherman Stein's *Archimedes: What did He Do Besides Cry Eureka?* (Mathematical Association of America, 1999). Stein gives a brief biography then focuses exclusively on introducing the math.
3. The bath story appears in Vitruvius; the most important sources for the siege engines are Polybius, who could have consulted eyewitnesses, and Plutarch. The earliest references to Archimedes' death appear in Cicero, then Livy.
4. http://www.coloribus.com/adsarchive/prints/mentos-candy-archimedes-6428655/
5. The paper is available on Chris Rorres' website.
6. The most well-known is at Berlin-Treptow.

7. https://www.cs.drexel.edu/~crorres/Archimedes/contents.html
8. Information from Chris Rorres' website.
9. It was translated into English (A. Smith Baylor University Press, 2010), with additions to the original appendix on Latin poetry, but something *has* been lost in translation: *pi* has fallen out of a couple of equations (p.44); and there are some mistakes in the rendering of the Italian: Hieronymus succeeded his grandfather at the age of fifteen not "fifteen years later," (Smith, p.64). Likewise, Smith, "[a] year after Ennius' death in 168 BC, the Roman historian Polybius, who wrote in Greek, brought Archimedes to Rome as a hostage": (Smith, p.75), incorrectly renders the Italian (Geymonat, p.105), which states (correctly) that Polybius of Megalopolis came to Rome as a hostage.
10. M. Geymonat, *Euclidis Latine facti fragmenta Veronensia*, Milan, 1964.
11. *The Archimedes Codex: How a Medieval Prayer Book is Revealing the True Genius of Antiquity's Greatest Scientist.* R. Netz and William Noel (Da Capo) 2007.
12. "Infinite Secrets," PBS, originally broadcast Sept. 30, 2003.
13. They include a physician (Homa), a retired military officer (Anderson), and a former publisher and journalist (Fallon).
14. http://www.gbradshaw.net/
15. Oliver Knill and Elizabeth Slavkovsky "Thinking Like Archimedes with a 3-D Printer," Jan. 28, 2013; see also *New Scientist*, issue 2902, Jan. 30, 2013.

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