Towards a Physical Scale Decomposition of Mean Skin Friction Generation in the Turbulent Boundary Layer

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Abstract A decomposition of mean skin friction generation in zero-pressure-gradient boundary layers is presented, relying on an energy budget in an absolute reference frame. It has a direct physical interpretation and emphasizes the importance of the production of turbulent kinetic energy in the logarithmic layer in mean skin friction generation at very high Reynolds number. This leads to a new approach to the scale decomposition of mean skin friction, illustrated using a Wall-Resolved LES at $Re_\theta = 13{,}000$ obtained by the ZDES technique. The role of superstructures is especially discussed.

1 Motivation and Theoretical Decomposition of Mean Skin Friction Generation into Physical Phenomena in the Boundary Layer

Because of its relation to drag, mean skin friction is essential for applied aerodynamics. Its generation is enhanced by turbulent mixing [11], leading to the well-known excess of mean skin friction of the turbulent boundary layer compared with the laminar case at the same Reynolds number. In boundary layers at high Reynolds numbers like in aerospace applications, experimental data unveiled superstructures, i.e. coherent structures of streamwise wavelength close to 5–6δ. Evaluating their contribution to mean skin friction requires to quantify the contribution of the turbulent fluctuations as a function of their wall distance and wavelength. Moreover, interpreting such an identity in terms of physical mechanisms is needed, which is all the more complicated in the zero-pressure-gradient flat plate boundary layer case as the flow is spatially developing. The FIK decomposition [6] does identify a turbulent contribution to mean skin friction, but its relation to the excess of friction is indirect because

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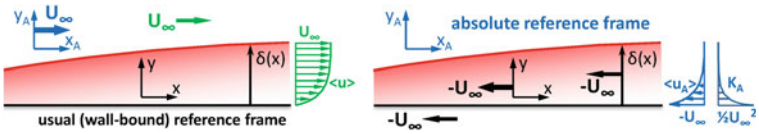

Fig. 1 Sketch of the usual and absolute frames of reference

of the spatial growth of the boundary layer [3], and the physical interpretation is not straightforward [13].

The present study is based on another decomposition of mean skin friction first introduced in [13] (where its detailed derivation is presented). This identity relies on an energy budget in a reference frame where the power of mean skin friction is non-zero (contrary to the usual wall-bound reference frame where skin friction is missing in the energy budgets because the wall is not moving). The chosen 'absolute' reference frame is associated to the outer fluid undisturbed by the wall (Fig. 1), so that the wall is moving at $-U_\infty$ along x in the absolute frame. The budget of mean streamwise kinetic energy in the absolute frame $K_A = 1/2\,\langle u_A\rangle^2$ (where $u_A = u - U_\infty$ is the streamwise velocity in the absolute frame) reads, assuming the boundary layer hypothesis in the incompressible zero-pressure-gradient flat-plate boundary layer:

$$\underbrace{\frac{\overline{D}K_A}{\overline{D}t}}_{\substack{\text{variation of } K_A\\ \text{with boundary}\\ \text{layer growth}\\ \hookrightarrow C_{f,c}(2)}} = \underbrace{\langle u_A\rangle \frac{\partial}{\partial y_A}\left(\frac{\tau}{\rho}\right)}_{\substack{\text{power of viscous}\\ \text{and turbulent efforts}\\ \hookrightarrow C_{f,c}(2)}} = \underbrace{\frac{\partial}{\partial y_A}\left(\langle u_A\rangle \frac{\tau}{\rho}\right)}_{\substack{\text{viscous and turbulent}\\ \text{diffusion of}\\ K_A\\ \hookrightarrow C_f(2)}} \underbrace{-\nu\left(\frac{\partial\langle u_A\rangle}{\partial y_A}\right)^2}_{\substack{\text{direct dissipation}\\ \text{of } K_A\\ \hookrightarrow C_{f,a}(2)}} \underbrace{+\left\langle u'_A v'_A\right\rangle \frac{\partial\langle u_A\rangle}{\partial y_A}}_{\substack{\text{'dissipation' of } K_A\\ \text{by turbulent kinetic}\\ \text{energy production}\\ \hookrightarrow C_{f,b}(2)}} \tag{1}$$

with $\frac{\overline{D}}{\overline{D}t} = \langle u\rangle \frac{\partial}{\partial x} + \langle v\rangle \frac{\partial}{\partial y} = \frac{\partial}{\partial t_A} + \langle u_A\rangle \frac{\partial}{\partial x_A} + \langle v_A\rangle \frac{\partial}{\partial y_A}$ and $\frac{\tau}{\rho} = \nu \frac{\partial\langle u\rangle}{\partial y} - \left\langle u'v'\right\rangle = \nu \frac{\partial\langle u_A\rangle}{\partial y_A} - \left\langle u'_A v'_A\right\rangle$.

Integrating the budget of K_A over the wall distance, assuming $u = 0$ ($u_A = -U_\infty$) at the smooth wall and non-dimensionalising the result leads to the following decomposition of the mean skin friction coefficient $C_f = \nu\,(\partial\langle u\rangle/\partial y)(y=0)/\left(1/2\,U_\infty^2\right)$:

$$C_f = \underbrace{\frac{2}{U_\infty^3}\int_0^\infty \nu\left(\frac{\partial\langle u\rangle}{\partial y}\right)^2 dy}_{C_{f,a}} + \underbrace{\frac{2}{U_\infty^3}\int_0^\infty -\left\langle u'v'\right\rangle \frac{\partial\langle u\rangle}{\partial y} dy}_{C_{f,b}} + \underbrace{\frac{2}{U_\infty^3}\int_0^\infty \left(\langle u\rangle - U_\infty\right)\frac{\partial}{\partial y}\left(\frac{\tau}{\rho}\right) dy}_{C_{f,c}} \tag{2}$$

This indicates that in the absolute frame, the mean energy supplied by the wall to the fluid (represented by C_f) is dissipated into heat ($C_{f,a}$), 'dissipated' by production of TKE (turbulent kinetic energy) ($C_{f,b}$) and gained as mean streamwise kinetic energy ($C_{f,c}$). The direct contribution of turbulence, $C_{f,b}$, is not associated with irreversible entropy creation since it does not involve turbulent dissipation. Focused on TKE production instead, it represents how turbulence interacts with the mean flow.

2 Role of the Turbulent Kinetic Energy Production in the Logarithmic Layer at High Reynolds Number

The decomposition of mean skin friction (2) is evaluated in Fig. 2 using DNS and WRLES datasets [5, 14–16] and RANS simulations (from the ONERA boundary layer code CLICET [1] with Jones & Launder [7] k–ε (JL) model and Michel et al. [10] model) to assess the high Reynolds number trend. This indicates that $C_{f,b}$ (2) is dominant at very high Reynolds numbers, whereas $C_{f,c}$ (2), associated with the spatial growth of the boundary layer, is negligible for $Re_\tau \rightarrow \infty$, as demonstrated in [13] (contrary to the non-negligible third term of the FIK decomposition [6] [3]). This is best understood in Fig. 3 where the integrands of each term of the decomposition (2) are non-dimensionalised and pre-multiplied so that their semi-logarithmic plot indicates the relative contribution to C_f. Because of the Reynolds invariance in the inner and outer layers, the dominant behaviour of $C_{f,b}$ is caused by the plateau of pre-multiplied TKE production in the logarithmic layer broadening with the Reynolds number (Fig. 3, detailed in [13]). Consequently, the generation of C_f at very high Reynolds number appears to be mostly driven by TKE production in the logarithmic layer (this conclusion differs from the FIK identity but is consistent with the observed importance of the logarithmic layer at high Reynolds numbers [17]). To better understand how turbulence contributes to mean skin friction, the study should focus on $C_{f,b}$, i.e. on the total TKE production, whose decomposition is attempted in the next section to identify the role of each layer and length scale.

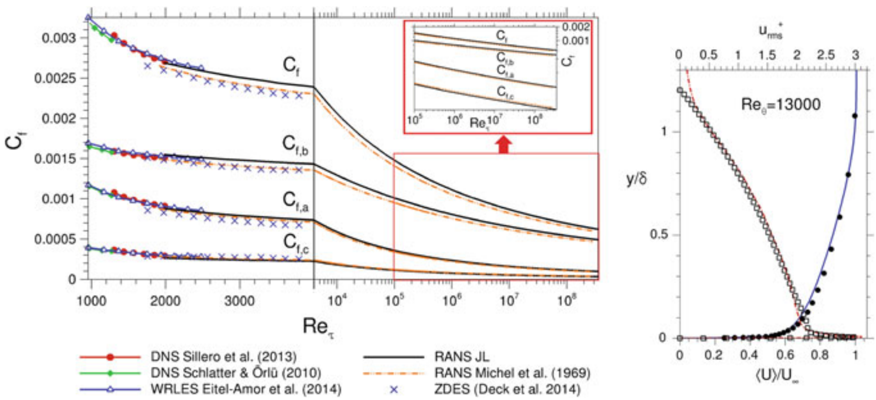

Fig. 2 Evolution of the mean skin friction decomposition (2) with the Reynolds number (*left*). ZDES profiles (see Sect. 3) at $Re_\theta = 13,000$ ($Re_\tau = 3600$): mean velocity (*solid line*) compared with experimental data by [4] (*circles*), $u^+_{\rm rms}$ (*dash–dotted line*) compared with the model by [9], [8] (*squares*) (*right*)

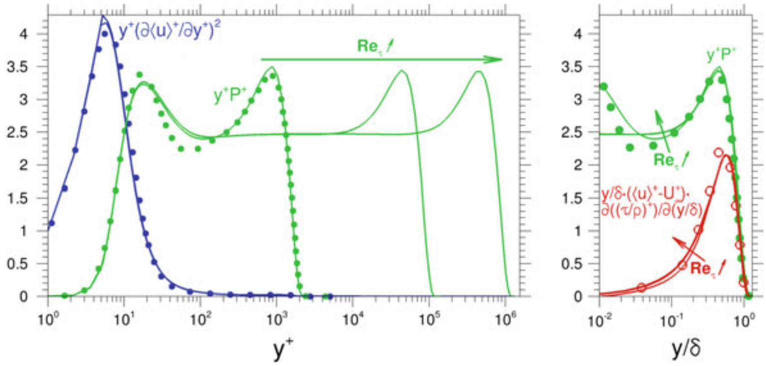

Fig. 3 Pre-multiplied integrands of the terms of the mean skin friction decomposition (2) ($y^+P^+ = -y^+\left\langle u'v'\right\rangle^+ \frac{\partial\langle u\rangle^+}{\partial y^+} = -y/\delta \left\langle u'v'\right\rangle^+ \frac{\partial\langle u\rangle^+}{\partial(y/\delta)}$). *Symbols* DNS [15] at $Re_\theta = 6500$ ($Re_\tau = 1989$). Lines: RANS simulations at $Re_\tau = 2000$, $Re_\tau = 10^5$ and $Re_\tau = 10^6$ (Michel et al. [10] model)

3 Scale Decomposition of Mean Skin Friction

According to the identity (2), the scale decomposition of mean skin friction is focused on decomposing the total TKE production [13]. However, the Reynolds number and scale separation are limited by the numerical cost in DNS. We consider here Re_θ = 13,000, for which no low-Mach-number boundary layer DNS dataset is available to our knowledge, and use a Wall-Resolved LES instead. This is acceptable because the study is focused on the outer layer contribution and because of the dominant role of the logarithmic layer at higher Reynolds numbers. The present WRLES is a Zonal Detached Eddy Simulation (mode III) described and validated in the outer layer in [3] (the ZDES hybrid RANS/LES technique is described in [2]). The profiles at Re_θ = 13,000 (Fig. 2) confirm the proper resolution of the outer layer. The evolution of C_f and of the terms of the identity (2) with the Reynolds number is correctly predicted as well (Fig. 2). The streamwise velocity spectra indicate that very large scale motions (much longer than 3δ) are resolved at Re_θ = 13,000, corresponding to superstructures which are missing at $Re_\theta = 5200$ in the same simulation (Fig. 4), suggesting that reaching at least the present Reynolds number is mandatory to evaluate the contribution of superstructures, which is affordable thanks to the WRLES approach. Because of the lack of scale separation at $Re_\theta = 5200$ where some inner-scaled fluctuations can share their wavelengths with outer-scaled ones, isolating the superstructures in Fourier space is not feasible. For this reason, the following illustration of the scale decomposition is performed at Re_θ = 13,000.

The scale decomposition of TKE production is obtained by estimating the co-spectrum of the Reynolds shear stress from time signals, with a spectral evaluation of the convection velocity [12]. Because a WRLES is used instead of a DNS, the emphasis is put on the outer layer and large scales, considering the cumulative

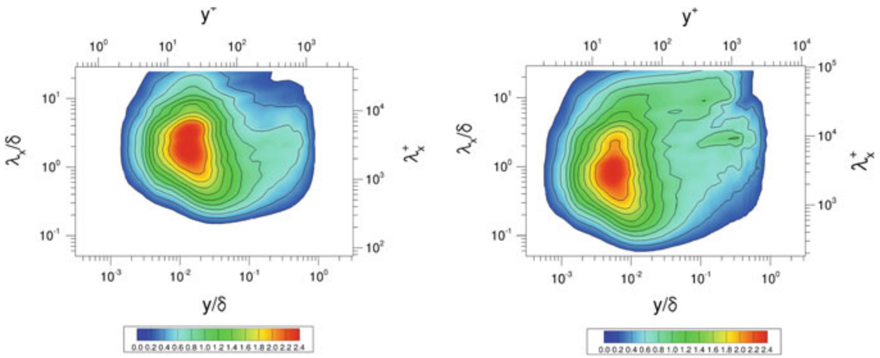

Fig. 4 Reynolds number impact on the streamwise velocity spectra $k_x G_{uu}(k_x)/u_\tau^2$ (*left* $Re_\theta =$ 5200, right: Re_θ = 13,000, reconstructed with a correlation-based convection velocity)

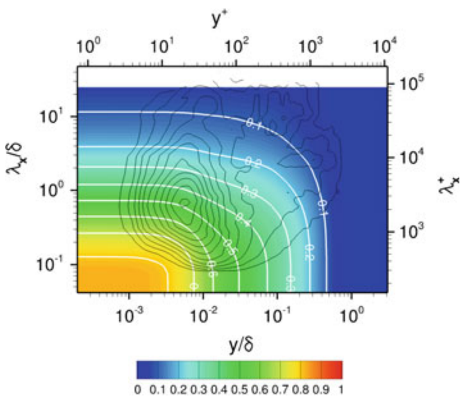

Fig. 5 Cumulative resolved contribution to TKE production $\mathscr{C}(y, \lambda_x)$ (3) at $Re_\theta = 13,000$ predicted by ZDES in WRLES mode (together with $k_x G_{uu}(k_x)/u_\tau^2$ in *black isolines*)

resolved contribution to TKE production defined as follows:

$$\mathscr{C}(y, \lambda_x) = \frac{\int_y^\infty -\left\langle u'v' \right\rangle_{\mathrm{res},[\lambda_x;+\infty[} \frac{\partial \langle u \rangle}{\partial y} dy}{\int_0^\infty -\langle u'v' \rangle \frac{\partial \langle u \rangle}{\partial y} dy} \tag{3}$$

where $-\left\langle u'v' \right\rangle_{\mathrm{res},[\lambda_x;+\infty[}$ is the Reynolds shear stress induced by the resolved fluctuations of wavelength larger than λ_x. The cumulative resolved contribution at $Re_\theta =$ 13, 000 is plotted in Fig. 5, showing for instance $\mathscr{C}(y^+ = 100, \lambda_x = 3\delta) \approx 0.19$, which means that the contribution of superstructures ($\lambda_x > 3\delta$) in the outer layer ($y^+ > 100$) at $Re_\theta = 13,000$ is approximately one fifth of the overall production of TKE, i.e. these fluctuations contribute at this Reynolds number approximately one fifth of $C_{f,b}$ (2), which is the dominant term in the decomposition of mean skin friction when $Re_\tau \to \infty$. However, the logarithmic layer still has a moderate extent at $Re_\theta = 13,000$. Because this layer plays an increasing role with Re_τ and since it contains the core of the superstructures, one may expect the contribution

$\mathscr{C}(y^+ = 100, \lambda_x = 3\delta)$ to increase with Re_τ, leading to a larger role of superstructures in mean skin friction at larger Reynolds numbers. Resolved datasets at higher Reynolds numbers are needed to confirm this, providing a larger scale separation than currently available from numerical simulations.

4 Outlook

A new approach to the scale decomposition of mean skin friction generation has been presented, relying on an energy budget in an absolute reference frame. It has a direct physical interpretation and emphasizes the importance of turbulent kinetic energy production in the logarithmic layer in mean skin friction at very high Reynolds number. The potential of the approach coupled to spectral analysis has been illustrated with a WRLES database obtained by the ZDES technique, showing a contribution of the superstructures in the outer layer close to one fifth of the turbulent mean skin friction term $C_{f,b}$ at $Re_\theta = 13,000$ and the need for higher-Reynolds-number wall-resolved databases to better understand the probably greater role of superstructures.

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