Chapter 6
Logic of Conflicts and Active Set with Uncertainty and Incoherence

An active set is a unifying space being able to act as a "bridge" for transferring information, ideas and results between distinct types of uncertainties and different types of applications. An active set is a set of agents who independently deliver true or false values for a given proposition. An active set is not a simple vector of logic values for different propositions, the results are a vector but the set is not. The difference between an ordinary set and active set is that the ordinary set has passive elements with values of the attributes defined by an external agent. In the active set, any element is an agent that internally defines the value of a given attribute for a passive element. Agents in the active set with a special criterion give the logic value for the same attribute. So agents in many cases are in a logic conflict and this generates semantic uncertainty on the logic evaluation. Criteria and agents are the two variables by which we give different logic values to the same attribute or proposition. Active set is beyond the modal logic. In fact, given a proposition in modal logic we can evaluate the proposition only when we know the worlds where the proposition is located. When we evaluate one proposition in one world we cannot evaluate the same proposition in another world. Now in epistemic logic any world is an agent that knows the proposition is true or false. The active set is a set of agents as in the epistemic logic but is different from modal logic because all the agents (worlds) are not separate but are joined in the evaluation of the given proposition. In active set for one agent and one criterion we have one logic value but for many agents and criteria the evaluation is not single true or false but is a matrix of true and false. This matrix is not only a logic evaluation as in the modal logic but gives us the conflicting structure of the active set evaluation. Matrix agent is the vector subspace of the true or false agent multidimensional space. Operations among active sets include operations in the traditional sets, with fuzzy set and rough set as special cases. The agents multi dimensional space to evaluate active set include also the Hilbert multidimensional space where it is possible to simulate quantum logic gate. New logic operations are possible as fuzzy gate operations and

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more complex operations as conflicting solving, consensus operations, syntactic inconsistency, semantic inconsistency and knowledge integration. In the space of the agents evaluations morphotronic geometric operations are the new frontier to model new types of computers, new type of model for wireless communications as cognitive radio. In conclusion, active set opens the new possibility and new models for the logic.

6.1 Agents and Logic in the Epistemic Logic

Epistemic logic is the logic which formalizes knowledge of agents. Among many applications it is used in game theories and economic behaviour in databases and in verifying cryptographic protocols shared knowledge, common knowledge. Epistemic logic is also known as the logic of knowledge, it deals with modalities, which are not part of traditional logic and which modify the meaning of a proposition. For instance such a modality is the knowledge modality: "agent Alice knows that…", written K. Alice. There is one knowledge modality Ki for each agent i, so when there are n agents, there are n knowledge modalities. From the Ki's, one can build two new modalities, namely a modality Eg of shared knowledge, which modifies a proposition p into a proposition Eg(p) which means that "everyone in the group g knows p" and a modality Cg of common knowledge. Cg(p) would say "p is known to everybody in the group g" in a very strong sense since knowledge about p is known at every level of knowledge. Slightly more precisely, if g is the group of agents and p is a proposition, Eg(p) is the conjunction over the i 2 g of the Ki(p) and Cg(p) means something like "everybody knows p and everybody knows that everybody knows p and… and everybody knows that everybody knows that everybody knows…that everybody knows p…" This infinite conjunction is handled by making Cg(p) a fix point. A typical example of common knowledge is traffic regulation. When, as a car driver, you enter an intersection you know that the person on your left will let you go, moreover you know that she knows that you have the right to go and you are sure (you know) that she will not go because she knows that you know that she knows that you have the right to go etc. Actually you pass an intersection with a car on your left, because there is a common knowledge between you as a driver and the driver of the other car on the rule of priority. But those who travel have experienced the variability of the common knowledge. Take a stop sign. In Europe it means that the person which has a stop sign will let the other to pass the intersection. In some countries, the stop sign is just a decoration of intersections. In the USA, the common knowledge is different since there are intersections of two crossing roads with four stop signs and this has puzzled more than one European. One main goal of epistemic logic is to handle properly those concepts of knowledge of an agent, shared knowledge and common knowledge. So we have the Epistemic logic evaluation 6.1.

$$K_\alpha(p) = \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & true & false & true \end{pmatrix} \tag{6.1}$$

where the proposition p is true for the agent 1, false for the agent 2 and true for the agent 3. Any agent in epistemic logic is completely separate from the others and any evaluation is given only when we know the agents as worlds. No conflict is possible because the agents are not considered together but one at the time as for the world in the modal logic.

6.2 Concepts and Definitions of Active Set

In the previous historical background we have vector evaluations but without any conflict because one evaluation is apart from the others. So p can be true in one situation and false in another but the two situations are not superposed so no conflict is possible. Only in quantum mechanics we can have superposition of different states where the proposition p can be both false and true. Now in quantum gate we use superposition and inconsistency only when we want to make a massive parallel computations. But when we want to measure the computation result, the superposition collapses and we always come back to a total separation of the states that is in agreement with the consistent classical logic by which we can make computation in the Boolean algebra. Now in a recently works on the agents appears the possibility to have inconsistent and conflict logic system where we can choose the consensus situation to come back to the classical and consistent true or false logic from inconsistency and also knowledge integration where we can know the logic value for complex propositions. Recently Cognitive radio system uses inconsistency to have a wireless efficient system. The aim of this chapter is to define a new type of set, that includes classical set theory, fuzzy set, set in evidence theory and rough set.

6.3 Properties and Definition of the Active Set

Any active set is a set of superpose agents, and any agent gives a value for the same proposition p. Active set appears similar to the Epistemic logic evaluation but the difference is that it is connected with the superposition of the world or agents whose judgment is not related to one agent but to the set of agents. We recognize active set elements in the vote process where all agents together give votes for the same person. In general the vote process is a conflicting vote because we have positive and negative votes for the same person. In epistemic logic this is impossible because we want to know where is the agent that gives a positive or negative judgment and this is possible without any conflict because we know the name of the agents. In active set, the set of agents is independent of the name that gives the judgment that must be only one for the set of agents. Now when all the agents obtain a consensus, they together give the same logic value so the conflict disappears and we have the classical non conflicting situation. The same is for knowledge integration where agents must be taken to integrate its actions to create the wanted knowledge integration. So now we begin with the formal description of the active set theory. Given three agents with all possible sets of logic values (true, false) one for any agents. So at any set of agent we have a power set of all possible evaluation for the proposition p. For example given three agents, the active set is a set of three agents with 8 sets of possible logic values for the same proposition p (as 6.2).

$$\begin{aligned}\Omega(p) = &\begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic\text{ value} & true & true & true \end{pmatrix}, \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic\text{ value} & true & true & false \end{pmatrix},\\ &\begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic\text{ value} & true & false & true \end{pmatrix}, \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic\text{ value} & false & true & true \end{pmatrix},\\ &\begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic\text{ value} & false & false & true \end{pmatrix}, \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic\text{ value} & false & true & false \end{pmatrix}\\ &\begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic\text{ value} & true & false & false \end{pmatrix}, \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic\text{ value} & false & false & false \end{pmatrix}\end{aligned} \tag{6.2}$$

In a more formal way we have 6.3.

$$SS(p) = \left\{A, \Omega(p) | A = \text{set of agents}, \Omega(p) = \text{ power set } 2^A \text{ of the evaluations } \right\} \tag{6.3}$$

Given the proposition p, we denote as Criteria C one of the possible evaluation p in the set Ω(p). For example with three agents we have eight criteria to evaluate the proposition itself so we can write 6.4.

$$
\begin{aligned}
\Omega(p, C_1) &= \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & true & true & true \end{pmatrix}, \\
\Omega(p, C_2) &= \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & true & true & false \end{pmatrix}, \\
\Omega(p, C_3) &= \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & true & false & true \end{pmatrix}, \\
\Omega(p, C_4) &= \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & false & true & true \end{pmatrix}, \\
\Omega(p, C_5) &= \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & false & false & true \end{pmatrix}, \\
\Omega(p, C_6) &= \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & false & true & false \end{pmatrix} \\
\Omega(p, C_7) &= \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & true & false & false \end{pmatrix}, \\
\Omega(p, C_8) &= \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & false & false & false \end{pmatrix}
\end{aligned}
\tag{6.4}
$$

We remark that the set of Criteria is a mathematical lattice. For the previous example we have Fig. 6.1.

Operations

The agents set A is an ordinary set with normal intersection union and complementary operator. For the logic evaluation we have two different operations.

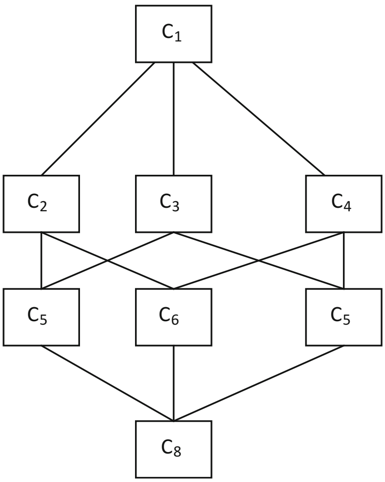


Fig. 6.1 Lattice of the uncertainty with different criteria

(1) Operation among criteria for the same proposition. Because we have the same proposition with two different criteria, we cannot compose the logic values that are heterogeneous. So we have the rule 6.5.

$$C_i \oplus C_j = \begin{pmatrix} Agent & 1 & 2 & \dots & n \\ C_i & v_{1,1} & v_{1,2} & \dots & v_{1,n} \\ C_j & v_{2,1} & v_{2,2} & \dots & v_{2,n} \end{pmatrix} \tag{6.5}$$

So we increase the dimension of the space of the evaluation. For example, given ten agents and two criteria we have 6.6.

$$\Omega(p, C_i, C_j) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 \\ p, C_i & f & f & t & t & f \\ p, C_j & t & t & f & t & f \end{pmatrix} \tag{6.6}$$

In a graphic way we have Fig. 6.2.

(2) For two different propositions p and q we have the composition rule for the active set (as 6.7).

$$\begin{aligned} \Omega(p \wedge q, C) &= \begin{pmatrix} Agents & 1 & 2 & \dots & n \\ p, C & v_{1,p} & v_{2,p} & \dots & v_{n,p} \end{pmatrix} \wedge \begin{pmatrix} Agents & 1 & 2 & \dots & n \\ q, C & v_{1,q} & v_{2,q} & \dots & v_{n,q} \end{pmatrix} \\ &= \begin{pmatrix} Agents & 1 & 2 & \dots & n \\ p, q, C & v_{1,p} \wedge v_{1,q} & v_{2,p} \wedge v_{2,q} & \dots & v_{n,p} \wedge v_{n,q} \end{pmatrix} \end{aligned} \tag{6.7}$$

$$\begin{aligned} \Omega(p \vee q, C) &= \begin{pmatrix} Agents & 1 & 2 & \dots & n \\ p, C & v_{1,p} & v_{2,p} & \dots & v_{n,p} \end{pmatrix} \vee \begin{pmatrix} Agents & 1 & 2 & \dots & n \\ q, C & v_{1,q} & v_{2,q} & \dots & v_{n,q} \end{pmatrix} \\ &= \begin{pmatrix} Agents & 1 & 2 & \dots & n \\ p, q, C & v_{1,p} \vee v_{1,q} & v_{2,p} \vee v_{2,q} & \dots & v_{n,p} \vee v_{n,q} \end{pmatrix} \end{aligned} \tag{6.8}$$

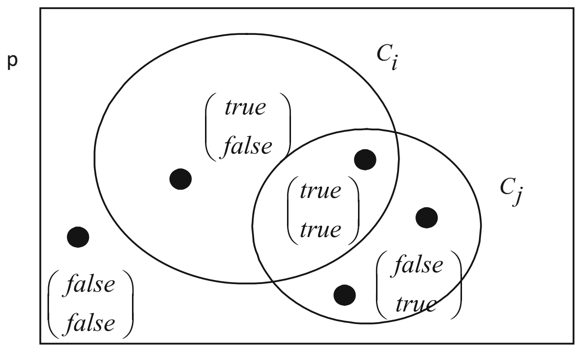


Fig. 6.2 Two dimensional evaluation for two different criteria for five agents and the same proposition p

Example 6.1

$$\Omega(\mathrm{p}) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & t & t & t & f & f & f \end{pmatrix}, \quad \Omega(\mathrm{q}) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & t & t & t & t & f & f \end{pmatrix}$$

$$\Omega(\mathrm{p} \vee \mathrm{q}) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & t & t & t & t & f & f \end{pmatrix},$$

$$\Omega(\mathrm{p} \wedge \mathrm{q}) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & t & t & t & f & f & f \end{pmatrix}$$

The two logic operators are sensible to the order of the agents as a list for the negation operator we have 6.9.

$$\Omega(\neg p) = \begin{pmatrix} Agents & 1 & 2 & \ldots & n \\ value & \alpha_1(\neg v_1) + (1-\alpha_1)(v_1) & \alpha_2(\neg v_2) + (1-\alpha_2)(v_2) & \ldots & \alpha_n(\neg v_n) + (1-\alpha_n)(v_n) \end{pmatrix} \tag{6.9}$$

Example 6.2

$$\Omega(p) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & f & f & f & t & t & t \end{pmatrix}$$

For

$$if \quad \alpha = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \text{ then } \Omega(\neg \mathrm{p}) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & t & t & t & f & f & f \end{pmatrix}$$

$$if \quad \alpha = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & 1 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \text{ then } \Omega(\neg \mathrm{p}) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & t & t & t & t & f & f \end{pmatrix}$$

$$if \quad \alpha = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \text{ then } \Omega(\neg \mathrm{p}) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 \\ values & t & t & f & f & f & f \end{pmatrix}$$

When all the values of α are equal to one, all the agents change its value in the negation operation. When one α is zero for the true values one true value agent does not change and all the others change. So in the end the number of agents with true value in the negation operation is more than in the classical negation for any agent. On the contrary, if α is zero for one agent with false value, the number of the true value in the negation is less than the classical negation for any agent.

6.4 Aggregation Rule for Active Set

Given an active set, we associate to any active set evaluation a number by an aggregation function that can be linear or non linear. For the linear case the aggregation can be simple aggregation or can be weighted aggregation. For example for simple linear aggregation rule we have the aggregation rule 6.10.

$$
\begin{aligned}
& for \;\; \Omega(p, C_1) = \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & true & true & true \end{pmatrix} \\
& \mu(p, C_1) = \frac{1}{3}|true\rangle + \frac{1}{3}|true\rangle + \frac{1}{3}|true\rangle = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \\
& for \;\; \Omega(p, C_2) = \begin{pmatrix} Agent & 1 & 2 & 3 \\ Logic \text{ value} & true & true & false \end{pmatrix} \\
& \mu(p, C_2) = \frac{1}{3}|true\rangle + \frac{1}{3}|true\rangle + \frac{1}{3}|false\rangle = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}0 = \frac{2}{3}
\end{aligned}
\tag{6.10}
$$

Where Q is the linear superposition of the logic value for the active set.

6.5 Fuzzy Set by Active Set

The probability calculus does not incorporate explicitly the concepts of irrationality or agent's state of logic conflict. It misses structural information at the level of individual objects, but preserves global information at the level of a set of objects. Given a dice the probability theory studies frequencies of the different faces E = {e} as independent (elementary) events. This set of elementary events E has no structure. It is only required that elements of E be mutually exclusive and complete, and there is no other possible alternative. The order of its elements is irrelevant to probabilities of each element of E. No irrationality or conflict is allowed in this definition relative to mutual exclusion. The classical probability calculus does not provide a mechanism for modelling uncertainty when agents communicate (collaborates or conflict). Below we present the important properties of sets of conflicting agents at one dimension Let Ω(x) the active set for the proposition x and |Ω(x)| be the numbers of agents for which proposition x is true we have

Given two propositions a and b when

$$
\begin{aligned}
&\text{If } \; |\Omega(\mathrm{a})| < |\Omega(\mathrm{b})| \; \text{ then } \; \mathrm{p} = \mathrm{a} \text{ and } \mathrm{q} = \mathrm{b} \\
&\text{If } \; |\Omega(\mathrm{b})| < |\Omega(\mathrm{a})| \; \text{ then } \mathrm{p} = \mathrm{b} \text{ and } \mathrm{q} = \mathrm{a}
\end{aligned}
$$

So we order the propositions from the proposition with less number of true value to the proposition with maximum of true values (Fig. 6.3)

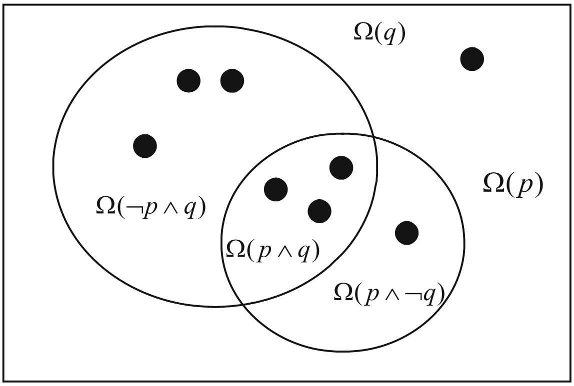


Fig. 6.3 Fuzzy rules and Active sets

$$
\begin{aligned}
&|\Omega(p)| = 4 \\
&|\Omega(q)| = 5 \\
&|\Omega(p)| < |\Omega(q)| \\
&\max(|\Omega(p)|, |\Omega(q)|) = |\Omega(q)| \\
&\min(|\Omega(p)|, |\Omega(q)|) = |\Omega(q)|
\end{aligned}
$$

$$
\begin{aligned}
&\Omega(p) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ values & f & f & t & t & t & t & f & f \end{pmatrix} \\
&\Omega(q) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ values & f & t & t & f & t & t & f & t \end{pmatrix} \\
&|\Omega(p)| = 4, \quad |\Omega(q)| = 5
\end{aligned}
$$

We have

$$
\begin{aligned}
&\Omega(p \wedge q) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ values & f & f & t & f & t & t & f & f \end{pmatrix} \\
&\Omega(p \vee q) = \begin{pmatrix} Agents & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ values & f & t & t & t & t & t & f & t, \end{pmatrix} \\
&|\Omega(p \wedge q) = 3|, \quad |\Omega(p \vee q) = 6|
\end{aligned}
$$

Now we know that

$$
\begin{aligned}
&q \vee (p \wedge \neg q) = (q \vee p) \wedge (q \vee \neg q) = q \vee p \\
&p \wedge \neg(p \wedge \neg q) = p \wedge (\neg p \vee q) = (p \wedge \neg p) \vee p \wedge q = p \wedge q
\end{aligned}
$$

But because when q is false and p is true we adjoin one logic value true at q to obtain p or q. So when we repeat this process many times for any agent we have that at the number of true values for q we must adjoin other true values for which q is false but p is true. In conclusion we have

$|\Omega(p \vee q)| = |\Omega(q)| + |\Omega(\neg q \wedge p)| = \max(|\Omega(q)|, |\Omega(p)| + |\Omega(\neg q \wedge p)|$ For any operation we have that when q is false and p is true we eliminate one element for which p is true. In conclusion when we repeat this for many times we have

$$|\Omega(p \wedge q)| = |\Omega(p)| - |\Omega(\neg q \wedge p)| = \min(|\Omega(q)|, |\Omega(p)| + |\Omega(\neg q \wedge p)|$$

In one word, in the active set we can find the Zadeh rule again when p and not q is always false.

Zadeh rule

$$\begin{aligned} |\Omega(p \wedge q)| &= \min(|\Omega(q)|, |\Omega(p)| \\ |\Omega(p \vee q)| &= \max(|\Omega(q)|, |\Omega(p)| \end{aligned}$$

So when the agents for which p is true are also the agents for which q is true. In a graphic way we have Fig. 6.4.

We can also remark that the minimum rule is the maximum possible value for AND and the maximum rule is the minimum possible value for OR. We can see that for the previous example we have

$$\begin{aligned} &\textit{for} \;\; |\Omega(p \wedge \neg q)| = 1 \\ &|\Omega(p \wedge q)| = \min(|\Omega(p)|, |\Omega(q)|) - |\Omega(p \wedge \neg q)| = 4 - 1 = 3 \\ &|\Omega(p \vee q)| = \max(|\Omega(p)|, |\Omega(q)|) + |\Omega(p \wedge \neg q)| = 5 + 1 = 6 \end{aligned}$$

For the negation we have the Zadeh rule

$$|\Omega(\neg p)| = n - |\Omega(p)|$$

When we divide agents with the number n, we have the traditional rule

$$\mu(\neg p) = \frac{|\Omega(\neg p)|}{n} = 1 - \frac{|\Omega(p)|}{n} = 1 - \mu(p)$$

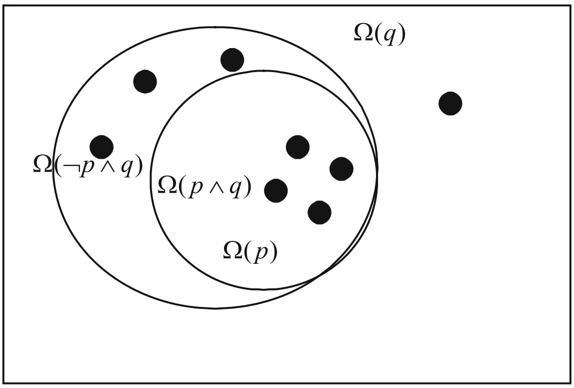
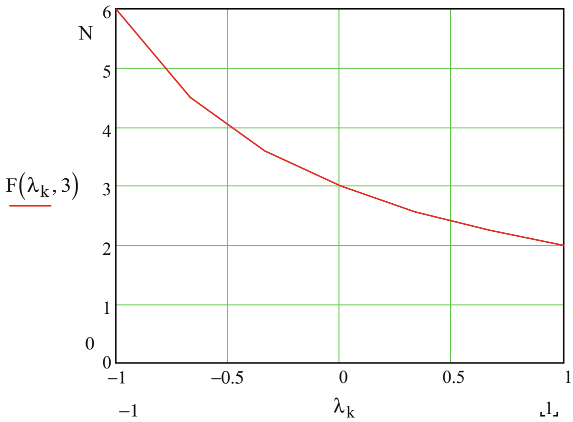


Fig. 6.4 Zadeh fuzzy rules and active sets

Fig. 6.5 Change of the Sugeno negation value for the change of lambda parameter



In this situation all the agents in the negation change all the logic values in a synchronic way. But when we have the Sugeno rule

$$|\Omega(\neg p)| = \mu(\neg p)n = \frac{1-\mu(p)}{1+\lambda\mu(p)}n = \frac{1-\frac{|\Omega(p)|}{n}}{1+\lambda\frac{|\Omega(p)|}{n}}n = n\frac{n-|\Omega(p)|}{n+\lambda|\Omega(p)|}$$

where $\lambda = [-1, \propto]$ when we change the lambda parameters for n = 6 and Ω (p) = 3 we have the negation value (Fig. 6.5).

When $\lambda = 0$ all the agents change their logic values. So before we have three true values and three false values for the negation we have the same values again but are reversed. For

$$\begin{aligned} &if\ \ \lambda = 0, \ \ |\Omega(\neg p)| = n - |\Omega(p)| = 6 - 3 = 3 \\ &if\ \ \lambda < 0, \ \ |\Omega(\neg p)| > n - |\Omega(p)| \\ &if\ \ \lambda > 0, \ \ |\Omega(\neg p)| < n - |\Omega(p)| \end{aligned}$$

When λ is negative, agents with true values do not change, when λ is positive, agents with false values do not change. In conclusion, t-norm and t-conorm and fuzzy negation can be simulates inside the active set.

6.6 Theory of Inconsistent Graph and Active Set

Given the inconsistent graph Fig. 6.6

We have the active set definition.

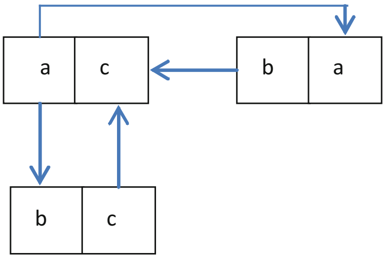


Fig. 6.6 The nodes are inconsistent

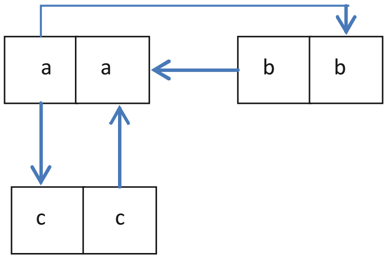
$$value(a_L) = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & T & F & F \end{bmatrix}$$
$$value(b_L) = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & T & F \end{bmatrix}$$
$$value(c_L) = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & F & T \end{bmatrix}$$
$$value(a_R) = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & T & F \end{bmatrix}$$
$$value(b_R) = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & F & T \end{bmatrix}$$
$$value(c_R) = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & T & F & F \end{bmatrix}$$

When we compose the left active sets with the right active set by logic equivalence operation we have

$$\begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & T & F & F \end{bmatrix} = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & T & F \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & F & T \end{bmatrix}$$
$$\begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & T & F \end{bmatrix} = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & F & T \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & T & F & F \end{bmatrix}$$
$$\begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & F & T \end{bmatrix} = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & T & F & F \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & T & F \end{bmatrix}$$

For consistent graph Fig. 6.7.

Fig. 6.7 The nodes are consistent



We have

$$\begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & T & F & F \end{bmatrix} = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & T & F & F \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & T & T & T \end{bmatrix}$$
$$\begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & T & F \end{bmatrix} = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & T & F \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & T & T & T \end{bmatrix}$$
$$\begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & F & T \end{bmatrix} = \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & F & F & T \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} agents & entity_1 & entity_2 & entity_3 \\ value & T & T & T \end{bmatrix}$$

For consistent graph the logic equivalence for active sets gives the value which is always true. So we have no conflicts.