Chapter 2
Crossover and Permutation

Given the permutation P

$$P = \begin{pmatrix} 1 & 2 & 3 & \ldots & n-1 & n \\ p_1 & p_2 & p_3 & \ldots & p_{n-1} & p_n \end{pmatrix}$$

Given the two crossovers

$$U = \begin{bmatrix} & A & B \\ C & a & a \\ D & b & b \end{bmatrix} \rightarrow \begin{bmatrix} DA & CB \\ b & a \end{bmatrix}$$

$$U = \begin{bmatrix} & A & B \\ C & a & a \\ D & b & b \end{bmatrix} \rightarrow \begin{bmatrix} CA & DB \\ a & b \end{bmatrix}$$

We have the elementary permutation

$$P = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

For more simple crossover we can create the matrix M

$$M = \begin{pmatrix} a & a \\ b & b \end{pmatrix}$$

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where we have two parents and two genes so we have the crossover ab, ba that are the terms of the permutation. For three parents and three genes we have the six possible crossovers from the three parents.

$$M = \begin{bmatrix} a & a & a \\ b & b & b \\ c & c & c \end{bmatrix} \rightarrow abc, acb, bac, bca, cab, cba$$

The permutation matrix is

$$a_{h,k} = \delta_{h,p_k}, \quad where \quad \begin{cases} \delta_{k,p_k} = 1 \\ \delta_{h,p_k} = 0, \quad h \neq k \end{cases}$$

For example, given the permutation P,

$$P = \begin{pmatrix} 1 & 2 & 3 \\ p_1 = 2 & p_2 = 1 & p_3 = 3 \end{pmatrix}$$

the permutation matrix is A.

$$A = \begin{bmatrix} & p_1 & p_2 & p_3 \\ k_1 & 0 & 1 & 0 \\ k_2 & 1 & 0 & 0 \\ k_3 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

With the permutation matrix we permute the columns by multiplication of R at the right. We have

$$RA = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} e_{12} & e_{11} & e_{13} \\ e_{22} & e_{21} & e_{23} \\ e_{32} & e_{31} & e_{33} \end{bmatrix}$$

So we get RA by right multiplication R with permutation matrix A. And we get AR by left multiplication R with permutation matrix A.

$$AR = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} = \begin{bmatrix} e_{21} & e_{22} & e_{23} \\ e_{11} & e_{12} & e_{13} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}$$

What is the difference between RA and AR?

In RA, Column 1 and Column 2 of R are swapped. Whereas in AR, Row 1 and Row 2 are swapped.

We write the difference between RA and AR as 2.1.

$$\begin{aligned}
&\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} - \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} - \begin{bmatrix} e_{12} & e_{11} & e_{13} \\ e_{22} & e_{21} & e_{23} \\ e_{32} & e_{31} & e_{33} \end{bmatrix} \\
&= \begin{bmatrix} e_{21}-e_{12} & e_{22}-e_{11} & e_{23}-e_{13} \\ e_{31}-e_{21} & e_{32}-e_{22} & e_{33}-e_{23} \\ e_{31}-e_{31} & e_{32}-e_{31} & 0 \end{bmatrix}
\end{aligned} \tag{2.1}$$

2.1 Right Product RA

Given Relation R and permutation matrix A,we have RA (2.2).

$$R = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RA = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{2.2}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} class \\ classroom \\ enrollment \\ teacher \\ student \end{bmatrix} \rightarrow \begin{bmatrix} class + enrollment + teacher \\ classroom \\ classroom + student \\ classroom \\ enrollment \end{bmatrix}$$

And the relation in the database of Fig. 1.2 becomes that in Fig. 2.1.

In Fig. 2.1, the changes of relations just happen on the entities directly related to entity class (represented as 1) and entity classroom (represented as 2), the reason for this is because multiplying permutation matrix A at the right of R makes R the first two columns swap. This leads to the disappearance of some relations and the generation of some new relations. For example, originally there is the relation from enrollment to class (from 3 to 1) and there is no relation from enrollment to

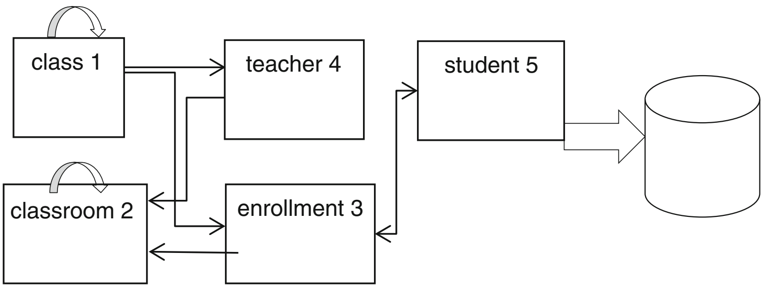


Fig. 2.1 The database scheme with right product

classroom (from 3 to 2), after permutation, the relation from enrollment to class (from 3 to 1) disappears and relation from enrollment to classroom (from 3 to 2) emerges. From the diagram, we also find the relations between 3 and 5, and those between 4 and 5 don't change on the grounds that relations between them have nothing to do with entity 1 and entity 2. Further it is not difficult to find that the number of relations doesn't change. The reason for this is that we just swap the two columns of Matrix R, not cause any change on the number of 1 in the relation RA. So, from the permutation above, what conclusion can we make? In RA we have that $e_{i,j} \to e_{i,p_j}$ or in a graphic way we have Fig. 2.2.

Here

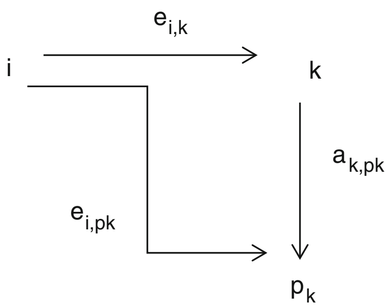
$$\begin{bmatrix} k & 1 & 2 & 3 \\ p_k & 2 & 1 & 3 \end{bmatrix}$$

In the right product RA the initial entity is the same, but the final element changes for the permutation.

We give the relations change of database.

We know that teacher has access to class but at one time he wants to have access to another entity as classroom so we permutate class with classroom and we use the permutation A in a way to change all the other entities to satisfy teacher without changing the number of relations. So we have Fig. 2.3.

Fig. 2.2 Right product RA



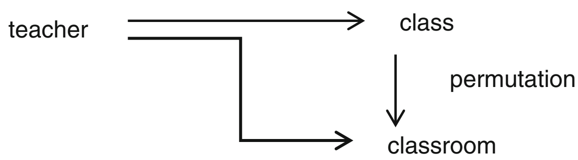


Fig. 2.3 Permutate class with classroom by right product

2.2 Left Product AR

The product AR changes the rows so we have 2.3

$$AR = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{2.3}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} class \\ classroom \\ enrollment \\ teacher \\ student \end{bmatrix} \rightarrow \begin{bmatrix} class \\ classroom + enrollment + teacher \\ class + student \\ class \\ enrollment \end{bmatrix}$$

So the relation in the database of Fig. 1.2 becomes that in Fig. 2.4.

In AR, the final entity is the same, but the initial element changes for the permutation.

We give the relations change of database.

Initially we can find the relevant teacher via class. Now on some occasions that we are urged to find the teacher, going to the relevant classroom is the only way. So by permutation, we get the relation between classroom and teacher. So we have Fig. 2.5.

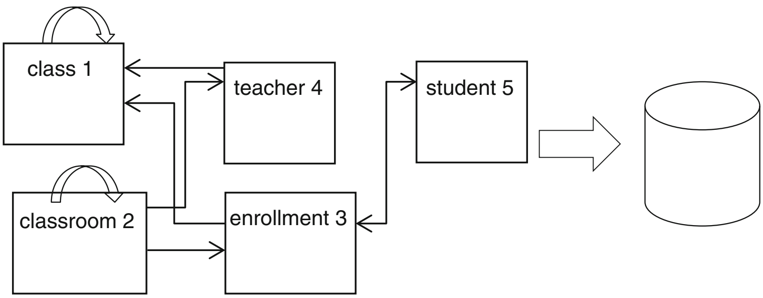


Fig. 2.4 The database scheme with left product

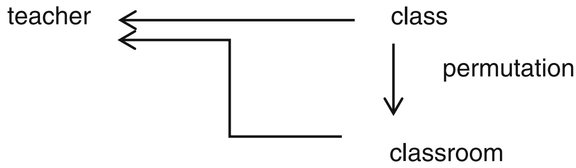


Fig. 2.5 Permutate class with classroom by left product

When we permute two entities by some way, all relations related to the two entities change correspondingly, although some relations change may not be necessary. So the graph will change.