

A *Physarum*-Inspired Ant Colony Optimization for Community Mining

Mingxin Liang¹, Chao Gao¹, Xianghua Li^{1,2,3}(✉), and Zili Zhang^{1,4}(✉)

¹ School of Computer and Information Science, Southwest University, Chongqing 400715, China

li_xianghua@163.com, zhangzli@swu.edu.cn

² Potsdam Institute for Climate Impact Research (PIK), 14473 Potsdam, Germany

³ Institute of Physics, Humboldt-University zu Berlin, 12489 Berlin, Germany

⁴ School of Information Technology, Deakin University, Geelong, VIC 3220, Australia

Abstract. Community mining is a powerful tool for discovering the knowledge of networks and has a wide application. The modularity is one of very popular measurements for evaluating the efficiency of community divisions. However, the modularity maximization is a NP-complete problem. As an effective optimization algorithm for solving NP-complete problems, ant colony based community detection algorithm has been proposed to deal with such task. However the low accuracy and premature still limit its performance. Aiming to overcome those shortcomings, this paper proposes a novel nature-inspired optimization for the community mining based on the *Physarum*, a kind of slime molds cells. In the proposed strategy, the *Physarum*-inspired model optimizes the heuristic factor of ant colony algorithm by endowing edges with weights. With the information of weights provided by the *Physarum*-inspired model, the optimized heuristic factor can improve the searching abilities of ant colony algorithms. Four real-world networks and two typical kinds of ant colony optimization algorithms are used for estimating the efficiency of proposed strategy. Experiments show that the optimized ant colony optimization algorithms can achieve a better performance in terms of robustness and accuracy with a lower computational cost.

Keywords: Community mining · Ant colony algorithm · *Physarum*

1 Introduction

Community mining is associated with the graph clustering that is a powerful tool for knowledge discovering in many real-world complex systems [1]. Identifying the structural characteristics of a network has a wide application in knowledge discovery, such as the function prediction in the protein-protein networks [2], the real-time recommendation systems construction [3], and the information diffusion analysis [4].

Many algorithms have been proposed for mining such structures in networks, such as optimization-based algorithms [5] and stochastic model based algorithms [6]. Currently, a modularity measure Q has been proposed and widely used for estimating the qualities of community divisions [7]. Specifically, the modularity maximization is a NP-complete problem, which is intractable for traditional optimization algorithms, such as mathematical programming [8]. In the field of heuristics algorithms, ant colony optimization (ACO) algorithm is popular in dealing with the NP-complete problems [9]. But low accuracy and robustness limit its performance and application.

Recently, a kind of slime molds cell, *Physarum*, has shown an intelligence of network designing and path finding in biological experiments [10,11]. Moreover, for uncovering the key mechanism of the intelligent behavior of *Physarum*, a mathematical model has been proposed by Tero et al. [12]. This *Physarum*-inspired model has been used for optimizing the heuristic algorithms [13]. Based on the characters of *Physarum*-inspired model, we wonder can the *Physarum* model optimize the ant colony optimization algorithm for community mining?

Based on the above motivation, the main contributions of this paper are as follows. Taking advantages of *Physarum*-inspired model, which could recognize the inter-community edges coarsely, a novel nature-inspired optimization algorithm has been proposed based on ant colony optimization. In the new algorithm, the heuristics factor of traditional ACO is optimized based on the recognition of *Physarum*-inspired model, which could instruct the ants to find better solutions and improve the efficiency of algorithms. Meanwhile, four real-world networks and two representative kinds of ant colony algorithms are used to demonstrate the efficiency of the proposed nature-inspired optimization algorithm, in terms of accuracy and robustness.

The remaining of this paper is organized as follows. Section 2 formulates the community mining and introduces the ant colony optimization for community mining. And then, the nature-inspired optimization is proposed based on *Physarum*-inspired model in Sect. 3. Section 4 reports the experiments on four real-world networks and two typical kinds of ant colony clustering algorithms. Finally, Sect. 5 concludes this paper.

2 Related Work

2.1 Formulation of Community Detection

Community mining is to divide the vertexes in a network into communities, where vertexes across communities are sparsely connected, and vertexes within a community are relatively densely connected. Based on inherent structural features, a modularity measure, denoted as Q , is proposed to evaluate the qualities of community divisions [1]. Therefore, the community mining problem can be formulated to an optimization problem, which is to maximize the modularity value. The formulation of community mining is shown as follows.

Considering a network $G(V, E)$, where V and E stand for the sets of vertexes and edges respectively. And a community is a subset of V , where vertexes have a common certain feature. With NC indicating the number of communities, a community division is a set of communities, $C = \{C_1, C_2, \dots, C_{NC}\}$, in which $C_i \neq \emptyset$, $C_i \neq C_j$, and $C_i \cap C_j = \emptyset$, for all i and j . After that, the mission of community mining can be represented as Eq. (1).

$$C^* = \arg \max_C Q(G, C) \tag{1}$$

The modularity Q is computed based on the topological structure of a network and its divisions, which is defined as Eq. (2). Specifically, $\delta(i, j)$ indicates the community relationship between vertexes i and j , while A and d_i stand for the adjacent matrix of a network and degree of vertex i , respectively. $A_{i,j}$ is equal to 1, if there is an edge connecting vertexes i and j . Otherwise, $A_{i,j}$ is equal to 0. Moreover, the degree of vertex i can be expressed as $d_i = \sum_j A_{i,j}$. In details, $\delta(i, j)$ is equal to 1, if and only if vertexes i and j belong to the same community. Otherwise, $\delta(i, j)$ is equal to 0.

$$Q = \frac{1}{2|E|} (A_{ij} - \frac{d_i d_j}{2|E|}) \delta(i, j) \tag{2}$$

2.2 Ant Colony Algorithms for Community Mining

Ant colony optimization is under a general category of nature-inspired algorithm, which is inspired by the collective behaviors of ants. In the ant colony optimization algorithm, each ant finds a community division based on a probability directed by the pheromone matrix and the heuristic factor. The most important parts of a ant colony algorithm are searching, mutating and updating pheromone matrix. Here, we take a typical ant colony optimization for clustering, denoted as ACOC, as an example to introduce the basic parts of an ant colony algorithm for community mining [14].

Searching Strategy: In each iteration, every ant finds a community division based on a probability matrix, which is as shown in Eq. (3). P_{i,c_j} indicates the probability of vertex i belonging to community C_j . And c_j stands for the label of community C_j .

$$P_{i,c_j} = \frac{(\eta_{i,c_j})^\beta (Tau_{i,c_j})^\alpha}{\sum_{k=1}^{NC} (\eta_{i,c_k})^\beta (Tau_{i,c_k})^\alpha} \tag{3}$$

In Eq. (3), η_{i,c_j} is the heuristic factor, which helps improve the search ability of ants based on the adjacent matrix of networks. For example, the heuristic factor in ACOC indicates the number of edges connecting the vertexes in community C_j from vertex i . Based on the character of community structure, the more edges connecting vertexes in community C_j vertex i joints, the larger probability of vertex i belonging to community C_j is. And the expression of η_{i,c_j} is shown in Eq. (4), in which n_{i,c_j} indicates the number of edges connecting vertex i and vertexes in community C_j . Here, C_j is based on the best community

division found by algorithm. And Tau stands for the pheromone matrix, which is updated by the ants in each iteration based on the qualities of solutions.

$$\eta_{i,c_j} = \frac{n_{i,c_j}}{NC \sum_{k=1}^{NC} n_{i,c_k}} \tag{4}$$

Based on the P matrix, the details in ACOC of assigning a community label to a vertex is introduced as follows. Taking the vertex i as an example, the community label c_j with maximal P value (i.e., $arg \max_{c_j} P_{i,c_j}$) is assigned to vertex i with a probability p_0 . Meanwhile, the community label of vertex i is assigned based on the roulette way, with a probability $1 - p_0$. As shown in Fig. 1, assigning community labels for all the vertexes in such way, a community division emerges.

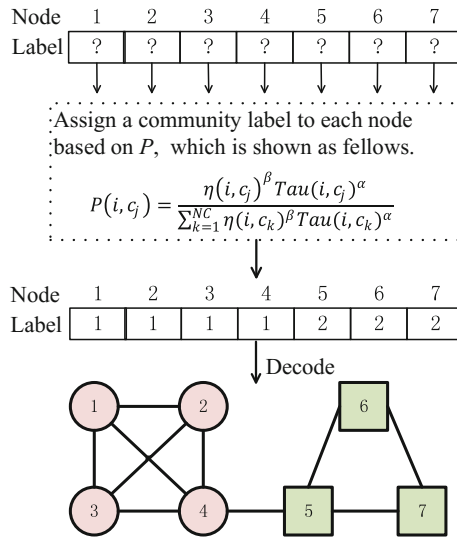


Fig. 1. The formulation of community division based on ACOC. Each community division is coded as a string of integers, which represents the community label of corresponding vertex.

Mutation Strategy: Mutation operator is a kind of random searching process, which aims to improve the diversity of solutions and protect ant colony clustering algorithms from premature. In the adopted mutation strategy of ACOC, each vertex is reassigned by a random community label with a probability P_m . And the mutation of a solution is accepted, if and only if the reassigning improves the modularity value of corresponding community division. Figure 2 shows an simple example of such mutation strategy.

Pheromone Matrix Updating Strategy: There are two phases for updating the pheromone matrix Tau . The first phase is implemented when an ant finishes

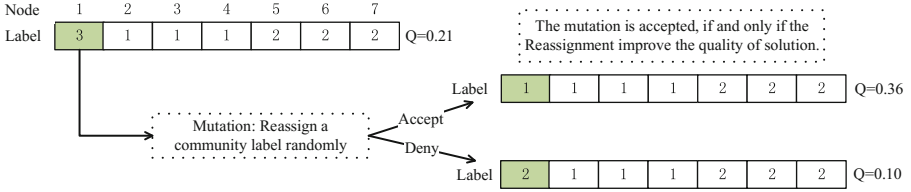


Fig. 2. An example of the mutation strategy in ACOC. Each mutated vertex is reassigned by a random community label. And the mutation is accepted, if and only if the reassigning improves the modularity value of corresponding community division.

its searching in an iteration. In this phase, every ant updates the pheromone matrix based on Eq. (5). Specifically, ρ is the volatile coefficient of pheromone, and Q indicates the quality of community division that the ant finds. The second phase executes after all the ants finish their local searching. Based on the community divisions with the top Q values in the current iteration, Eq. (6) is implemented for enhancing the effects of better community divisions. In Eq. (6), $\phi(i, c_j)$ equals to 1, if and only if vertex i within community C_j based on the corresponding community division. Otherwise, $\phi(i, c_j)$ equals to 0.

$$Tau_{i,c_j} = (1 - \rho) \cdot Tau_{i,c_j} + 2\rho \cdot Q \cdot \phi(i, c_j) \tag{5}$$

$$Tau_{i,c_j} = (1 - \rho) \cdot Tau_{i,c_j} + Q_{top} \cdot \phi(i, c_j) \tag{6}$$

With the local searching and updating for Tau , ants will aggregate to certain community divisions with higher Q values. And the solution with the highest Q value will be outputted as the optimal community division.

3 A Novel Nature-Inspired Optimization Algorithm for Community Mining

3.1 Edges Endowed with Weights Based on the *Physarum* Model

Physarum is a kind of slime with the abilities of designing networks and solving maze [10, 11]. Moreover, inspired by the bio-experiments of *Physarum*, a mathematical model is proposed and used for optimizing the heuristic algorithms [13]. In this paper, the *Physarum* model (PM) is modified to endow edges with weights, which could be used to recognize the intra-community edges in a network.

$$PQ^t_{i,j} = \frac{D^{t-1}_{i,j}}{L_{i,j}} |p^t_i - p^t_j| \tag{7}$$

The basic hypothesis of PM is a Poiseuille flow in a network. And the core mechanism of PM is the feedback system between the cytoplasmic fluxes and conductivities of tubes in the Poiseuille's flow. This feedback system has two main processes. First, $PQ^t_{i,j}$, $D^t_{i,j}$, $L_{i,j}$ and p^t_i denote the flux, the conductivity, the length of $e_{i,j}$, and the pressure of v_i at time step t , respectively. Then,

the relationship among the flux, conductivity, length, and pressure can be represented as Eq. (7). According to the Kirchhoff’s law, which is represented in Eq. (8), the pressures and fluxes can be obtained, by solving such equations at each iteration step. And then, $PQ^t_{i,j}$ feeds back to $D^t_{i,j}$ based on Eq. (9). After that, an iteration step finishes. With such feedback going on, a highly efficient network is generated.

$$\sum_i PQ^{t-1}_{i,j} = \begin{cases} I_0, & \text{if } v_j \text{ is an inlet} \\ -I_0, & \text{if } v_j \text{ is an outlet} \\ 0, & \text{others} \end{cases} \tag{8}$$

$$D^t_{i,j} = \frac{PQ^t_{i,j} + D^{t-1}_{i,j}}{k} \tag{9}$$

The major modification of PM is the scheme of choosing inlets/outlets in each iteration. In such model, when a vertex is chosen as an inlet, the others are chosen as outlets. More specifically, Eq. (8) is modified as Eq. (10), in which D and L are known. With a certain inlet and outlet, we can construct a set of equations based on Eq. (10). By solving such equations, p_i can be obtained. And, in each iteration step of PM, every vertex is chosen as the inlet once. When v_l is chosen as the inlet, a local conductivity matrix, denoted as $D^t(v_l)$, is calculated based on the feedback system (i.e., Eqs. (7), (8) and (9)). Finally, after all the local conductivity matrixes are obtained, the global conductivity matrix is updated by the average of local conductivity matrixes based on Eq. (11). A detailed description of PM is represented in Algorithm 1.

$$\sum_i \frac{D^{t-1}(v_l)_{i,j}}{L_{i,j}} |p^t_i - p^t_j| = \begin{cases} I_0, & \text{if } v_j \text{ is an inlet} \\ \frac{-I_0}{|V|-1}, & \text{others} \end{cases} \tag{10}$$

$$D^t = \frac{1}{|V|} \sum_{l=1}^{|V|} D^t(v_l) \tag{11}$$

With such modifications, the conductivities computed by *Physarum* model contain the information about inter-community edges recognition. *Physarum* model tends to endow inter-community edges with larger conductivities, vice versa. Figure 3 shows the edges with the top 20 percent conductivities in two networks based on such *Physarum* model. As it reported, the most of the edges with 20 percent conductivities connect vertexes in different communities. And there is almost no edge within the communities.

3.2 Nature-Inspired Optimization for Community Mining

Utilizing the character of conductivities computed by *Physarum* model, a novel ant colony optimization algorithm is proposed in this section, which aims to overcome the shortcomings of the low accuracy and premature. Through endowing edges with weights based on the *Physarum* model, the heuristic factor of proposed ant colony algorithm is optimized for improving the computational efficiency.

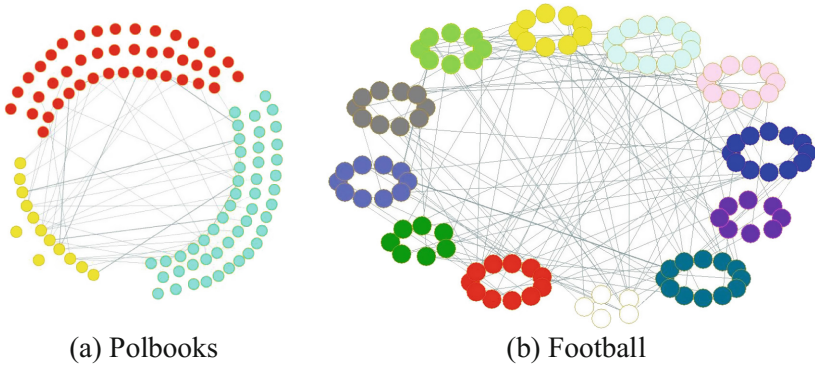


Fig. 3. Edges with top 20 percent conductivities in two networks based on the *Physarum* model. The different colors are used to label communities.

Algorithm 1. *Physarum* network mathematical model for community mining

Input: An adjacent matrix A

Output: A conductivity matrix D

1. Initializing D^0 and the maximal iteration step T
 2. For t from 1 to T
 3. For all vertexes in V
 4. Choosing v_l as the inlet
 5. Calculating $p^t_i, \forall i$, based on Eq. (10)
 6. Calculating $PQ^t_{i,j}, \forall i, j$, based on Eq. (7)
 7. Updating $D^t(v_l)$ based on Eq. (9)
 8. End for
 9. Updating D^t based on Eq. (11)
 10. End for
 11. Outputting D^T
-

$$\eta^*_{i,c_j} = \frac{n^*_{i,c_j}}{\sum_{k=1}^{NC} n^*_{i,c_k}} = \frac{\sum_{h \in C_j} 1/w_{i,h}}{\sum_{k=1}^{NC} \sum_{h \in C_k} 1/w_{i,h}} \tag{12}$$

Taking the advantages of *Physarum* model, the intra-community edges tend to have larger weights. In contrast, the inter-community edges tend to have a small ones. Based on such character, we can optimize the heuristic factor of ant colony clustering algorithms in order to improve the search ability of such algorithms during the process of community detection. Takeing ACOC as an example, we can optimize heuristic factor based on Eq. (12), in which $w_{i,k}$ indicates the conductivity of $e_{i,k}$ and $n^*_{i,c_j} = \sum_{h \in C_j} \frac{1}{w_{i,h}}$. With such expression, η^*_{i,c_j} has a larger value when there are the same intra-community edges connecting vertex i and vertexes in community C_j , compared with the original η_{i,c_j} , vice versa. Such nature-inspired optimization exaggerates the inhomogeneity of

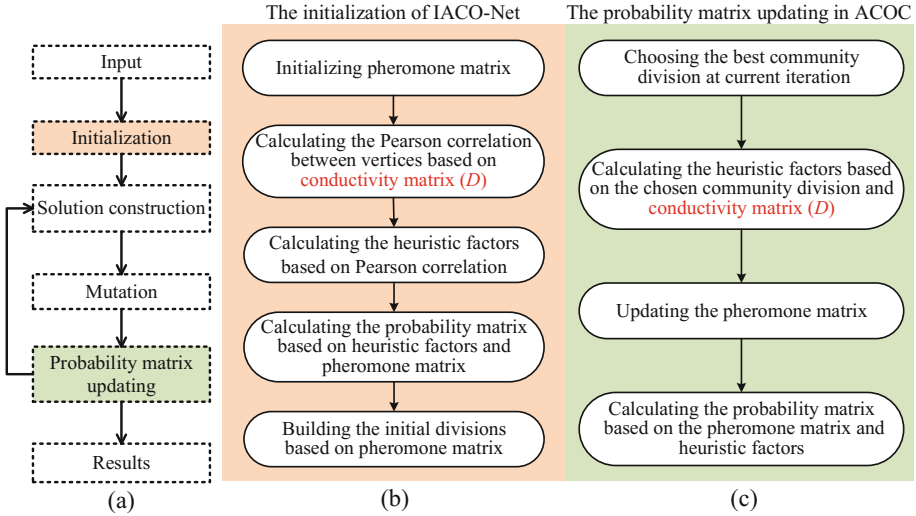


Fig. 4. Optimizing the heuristic factor in ACOC and IACO-Net based on the conductivity matrix D returned by *Physarum* model. (a) A basic framework of ant colony algorithm. (b) The optimized initialization of IACO-Net. (c) The optimized probability matrix updating process of ACOC.

original heuristic factor, and offers a more obvious information to ants, which leads to a higher accuracy and better robustness.

Although the heuristic factor is common and important in ant colony algorithms for community mining, the heuristic factor is used in different way in various ant colony algorithms. However, the *Physarum* based optimization strategy adapts various of ant colony algorithms easily. Here we employ two representative ant colony algorithms (i.e., ACOC [14] and IACO-Net [15]) to show the flexibility of our proposed method. Figure 4 illustrates the flowchart of optimizing the heuristic factors in ACOC and IACO.

4 Experiments

4.1 Datasets

Four real-world networks collected by Newman¹ and two ant colony clustering algorithms (i.e., ACOC [14] and IACO-Net [15]) are used to estimate the proposed algorithm. The basic topological features of those networks are shown in Table 1. For a clear expression, a prefix (i.e., $P-$) adds to the name of algorithm with the proposed strategy. And all the experiments are implemented in the same environment, which means that comparing algorithms have a same parameter setting and running environment. Moreover, the results are based on 20 repeated experiments to eliminate the fluctuation and evaluate the robustness.

¹ <http://www-personal.umich.edu/~mejn/netdata/>.

Table 1. The basic topological features of the real-world networks. N and E denote the number of vertexes and edges in a network. k and C stand for the average degree and clustering coefficient of networks, respectively. NC indicates the number of communities in networks based on the background.

Name	N	E	k	C	NC
Krateclub	34	78	4.588	0.588	2
Dolphins	62	160	5.129	0.303	2
Football	115	613	10.660	0.403	12
Polbooks	105	441	8.400	0.488	3

4.2 Experiment Results

Table 2 shows the box charts of modularity values returned by ACOC, IACO-Net, and their optimized algorithms on four real-world networks, which reports the distributions of results based on 20 repeated experiments. Due to the randomness of maximum and minimum, the comparison of those algorithms focuses on the first and third quartiles, and average. As is shown in such figure, the P-ACOC and P-IACO-Net have a higher average on all the four networks. And the first and third quartiles of optimized algorithms are also higher than that of original algorithms on all of four networks. Meanwhile, the distribution ranges of optimized algorithms are smaller, compared with that of original ones. It means the proposed strategy can enhance the robustness of ant colony algorithms.

Table 2. Results returned by ACOC, IACO-Net and their optimized algorithms on four networks in term of Q based on 20 repeated experiments. $Q1$ and $Q3$ indicate the first and third quartiles, respectively. And AVE stands for the average of those results on four networks.

Metrics	Algorithm	Karate	Dolphins	Football	Polbooks
$Q1$	ACOC	0.2907	0.3031	0.1613	0.4060
	P-ACOC	0.3286	0.3350	0.1943	0.4275
	ACO-Net	0.4198	0.5078	0.5769	0.5086
	P-IACO-Net	0.4198	0.5154	0.5866	0.5094
$Q3$	ACOC	0.3386	0.3751	0.2294	0.4363
	P-ACOC	0.3718	0.3778	0.2508	0.4475
	ACO-Net	0.4198	0.5170	0.5893	0.5169
	P-IACO-Net	0.4198	0.5195	0.5945	0.5168
AVE	ACOC	0.3149	0.3363	0.1947	0.4218
	P-ACOC	0.3512	0.3564	0.2261	0.4401
	ACO-Net	0.4196	0.5122	0.5824	0.5112
	P-IACO-Net	0.4197	0.5176	0.5903	0.5133

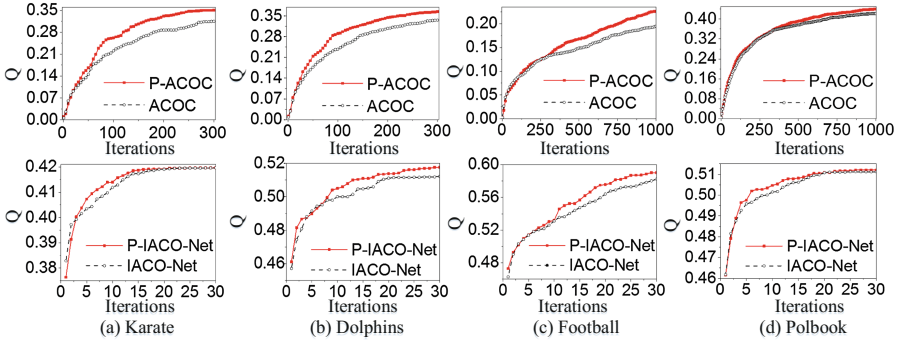


Fig. 5. The dynamic averages of Q with the increment of iteration in four networks. The results show that the proposed optimized strategy can obviously improve the search abilities of ACOCs.

For further estimating the efficiency of proposed strategy, Fig. 5 reports the dynamic changes of average modularity values with the increments of iterations. As is shown in such figure, at the initial phase, the Q values of the optimized algorithms are close to those of original algorithms. However, the algorithms with the proposed strategy have a higher growth rate, compared with the original ant colony algorithms. With the growing of iterations, the difference between original and optimized algorithms emerges. There is a distinct gap between the lines of original and optimized algorithms at the end of iterations.

Other optimization and heuristic algorithms are also used to evaluate the efficiency of proposed algorithms for community mining. The compared algorithms include the evolution algorithm (i.e., GA-Net [16]), swarm intelligence algorithm (i.e., RWACO [17]), hierarchical clustering algorithm (i.e., FN [18]), and label propagation based algorithms (i.e., LPA [19]). Table 3 reports the modularity values returned by those algorithms. As shown in such table, P-IACO-Net has the highest Q values on three of four networks. Meanwhile, the Q values of P-ACOC have significant improvements, compared with that of ACOC.

The cost of such *Physarum*-inspired optimized strategy is the computational cost. And the time complexity of *Physarum* model is analyzed as follows. For *Physarum* model, at each iterative step, every vertex should be chosen as the inlet once. When a vertex is chosen, a corresponding system of equations needs to be solved. In other words, there are N equations to solve in each iteration step. The worst computation complexity of solving a system of equations is $O(N^3)$. With an empirical setting (i.e., $T = 1$), the total computation complexity of *Physarum*-inspired optimized strategy is $O(N^4)$. For a NP-complete problem, this computation complexity is acceptable. Moreover, Table 4 shows the running time of ACOC, IACOC-Net and their optimized algorithms in seconds, which also verifies that the proposed *Physarum*-inspired optimized strategy does not increase the computational complexity noticeably.

Table 3. Comparison the proposed algorithms with other optimization and heuristic algorithms. The community divisions are evaluated by modularity value Q .

Alg.	Net.			
	KarateClub	Dolphins	Football	Polbooks
ACOC	0.314	0.336	0.194	0.421
P-ACOC	0.350	0.365	0.226	0.440
IACO-Net	0.419	0.512	0.582	0.511
P-IACO-Net	0.419	0.517	0.590	0.512
FN	0.252	0.371	0.454	0.502
LPA	0.370	0.480	0.588	0.504
GA-Net	0.406	0.467	0.598	0.490
RWACO	0.371	0.377	0.601	0.456

Table 4. The running time of optimization-based algorithms in seconds. From this table, we can conclude that our proposed computational framework does not bring more computational burden for original algorithms.

Alg.	Net.			
	Karate	Dolphins	Polbooks	Football
P-ACOC	2.2814	4.3298	8.5971	9.9121
ACOC	2.1862	4.2678	8.4494	9.8882
P-IACO-Net	1.1906	2.4156	7.5031	15.4844
IACO-Net	1.0875	2.2906	7.2562	14.8844

5 Conclusion

Inspired by the *Physarum*-inspired model, a novel nature-inspired optimization algorithm for community mining is proposed in this paper based on the optimized ant colony optimization. In the proposed novel algorithm, the heuristic factor is optimized by a *Physarum*-inspired strategy. The proposed strategy integrates the knowledge of *Physarum*-inspired model into the heuristic factor for exaggerating the inhomogeneity of original ones and offering more extra knowledge for ants. Experiments on four real-world networks and two typical kinds of ant colony algorithms show the improvements of optimized algorithm in terms of accuracy and robustness. Moreover, the time complexity analysis shows that the proposed strategy does not increase the computational complexity of ant colony clustering algorithm noticeably.

Acknowledgement. Prof. Zili Zhang and Dr. Xianghua Li are the corresponding authors. This work is supported by the National Natural Science Foundation of China (Nos. 61403315,61402379), CQ CSTC (No. cstc2015gjh40002), Fundamental Research Funds for the Central Universities (Nos. XDJK2016A008, XDJK2016B029, XDJK2016D053) and Chongqing Graduate Student Research Innovation Project (No. CYS16067).

References

1. Fortunato, S.: Community detection in graphs. *Phys. Rep.* **486**(3), 75–174 (2010)
2. Lee, J., Lee, J.: Hidden information revealed by optimal community structure from a protein-complex bipartite network improves protein function prediction. *PLoS ONE* **8**(4), 1–11 (2013)
3. Li, D., Lv, Q., Xie, X., Shang, L., Xia, H., Lu, T., Gu, N.: Interest-based real-time content recommendation in online social communities. *Knowl.-Based Syst.* **28**, 1–12 (2012)
4. Weng, L., Menczer, F., Ahn, Y.Y.: Virality prediction and community structure in social networks. *Sci. Rep.* **3**, 2522 (2013)
5. Gong, M., Cai, Q., Chen, X., Ma, L.: Complex network clustering by multiobjective discrete particle swarm optimization based on decomposition. *IEEE Trans. Evol. Comput.* **18**(1), 82–97 (2014)
6. Karrer, B., Newman, M.E.: Stochastic blockmodels and community structure in networks. *Phys. Rev. E* **83**(1), 016107 (2011)
7. Newman, M.E.: Modularity and community structure in networks. *Proc. Nat. Acad. Sci.* **103**(23), 8577–8582 (2006)
8. Mandala, S.R., Kumara, S.R., Rao, C.R., Albert, R.: Clustering social networks using ant colony optimization. *Oper. Res.* **13**(1), 47–65 (2013)
9. Mohan, B.C., Baskaran, R.: A survey: ant colony optimization based recent research and implementation on several engineering domain. *Expert Syst. Appl.* **39**(4), 4618–4627 (2012)
10. Nakagaki, T., Yamada, H., Tóth, Á.: Intelligence: maze-solving by an amoeboid organism. *Nature* **407**(6803), 470–470 (2000)
11. Tero, A., Takagi, S., Saigusa, T., Ito, K., Bebbler, D.P., Fricker, M.D., Yumiki, K., Kobayashi, R., Nakagaki, T.: Rules for biologically inspired adaptive network design. *Science* **327**(5964), 439–442 (2010)
12. Tero, A., Kobayashi, R., Nakagaki, T.: A mathematical model for adaptive transport network in path finding by true slime mold. *J. Theor. Biol.* **244**(4), 553–564 (2007)
13. Liu, Y.X., Gao, C., Zhang, Z.L., Lu, Y.X., Chen, S., Liang, M.X., Li, T.: Solving NP-hard problems with Physarum-based ant colony system. *IEEE/ACM Trans. Comput. Biol. Bioinf.* **14**(1), 108–120 (2017)
14. Shelokar, P., Jayaraman, V., Kulkarni, B.: An ant colony approach for clustering. *Anal. Chim. Acta* **509**(2), 187–195 (2004)
15. Mu, C., Zhang, J., Jiao, L.: An intelligent ant colony optimization for community detection in complex networks. In: 2014 IEEE Congress on Evolutionary Computation, pp. 700–706 (2014)
16. Pizzuti, C.: GA-Net: a genetic algorithm for community detection in social networks. In: Rudolph, G., Jansen, T., Beume, N., Lucas, S., Poloni, C. (eds.) PPSN 2008. LNCS, vol. 5199, pp. 1081–1090. Springer, Heidelberg (2008). doi:[10.1007/978-3-540-87700-4_107](https://doi.org/10.1007/978-3-540-87700-4_107)

17. Jin, D., Yang, B., Liu, J., Liu, D.Y., He, D.X.: Ant colony optimization based on random walk for community detection in complex networks. *Ruanjian Xuebao/J. Softw.* **23**(3), 451–464 (2012)
18. Newman, M.E.J.: Fast algorithm for detecting community structure in networks. *Phys. Rev. E* **69**, 066133 (2004)
19. Raghavan, U.N., Albert, R., Kumara, S.: Near linear time algorithm to detect community structures in large-scale networks. *Phys. Rev. E* **76**, 036106 (2007)