Hierarchical Multiple Stellar Systems

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Abstract. In astrophysics of hierarchical multiple stellar systems there is a contradiction between their maximum observed multiplicity (up to seven) and its theoretical limitations (about five hundred). To search for the hierarchical systems of high multiplicity we have analyzed modern catalogues of wide and close stellar pairs. We have compiled a list of objects - candidates for the stellar systems of maximum multiplicity, which includes an accurate cross-identification of their components. Presented procedure of cross-matching of multiple stellar systems is based on applying of criteria of unified form that exclude objects from sets of possible candidates to identification. Criteria are constructed using domain knowledge on astronomical objects of certain type. These criteria are not dependent on source catalogues but take into account knowledge on specific features of objects depending on conditions of observation.

Keywords: Binary stars · Visual binaries · Cross-matching · Conceptual modeling · Multi-component · Entity resolution

1 Introduction

The problem of cross-identification of celestial objects arises when working on virtually any tasks of astronomy, and is traditionally solved on a case-by-case basis. For single objects, this problem has been recognized and solved astronomical community since 80s of the last century. The problem of cross-identification of double and multiple stars is much more difficult. If a single star has, as a rule, only two celestial coordinates and magnitude, for a double star one needs to account magnitudes and coordinates of the reference and the concerned components and parameters of their orbital motion. This problem was discussed by astronomical community since the late 90s of the last century and has, in general, being solved by the authors when constructing Binary star database, BDB binaries data [11, 12]. By now BDB is the only resource of astronomical data that provides information about binary and multiple stars of all observational types. Finally, the solution for the problem of cross-identification of objects of higher multiplicity was developed for a number of special cases. In general, the difficulties arise because of a presence in the systems at the same time of the objects of different observational types: both wide, isolated (in evolutionary sense) pairs, and close pairs of stars, including

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eclipsing variables, X-ray sources and many others. Consequently, the number of parameters used for identification increases.

One of the objectives of high multiplicity stellar systems study is a search for hierarchical systems, confirming the theoretical justification for the possible existence of systems with a certain number of subordinate levels of stellar pairs. This issue was briefly described in [16] and is discussed in details in this paper.

Theoretical and observational multiplicity of hierarchical stellar systems is discussed in Sect. 2. Section 3 described catalogues of double stars used in the present study. In Sect. 4, principles of semiautomatic selection of high multiplicity systems are defined and results are presents. Efforts done to achieve automation and unification of cross-matching of systems and their components for future investigations of stellar systems with prospective catalogues are described in Sect. 5. Section 6 contain conclusions of the high multiplicity investigation.

2 Theoretical and Observational Stellar Multiplicity

Data on stellar multiplicity is important as a constraint on the problem of the formation and evolution of the Galactic stellar population. On the other hand, statistics of stellar multiplicity, i.e. the number of components, is poorly known, especially for multiplicities higher than three or four, and many questions still remain unresolved (see, e.g., the recent review in [4]).

The maximum number of components in a hierarchical multiple system depends on the number of hierarchy levels and can be estimated from theoretical considerations. A system is dynamically stable if, in a case of circular orbits, the outer orbital period exceeds the inner orbital period by a factor of five. For eccentric orbits, this factor is larger, increasing as ~ $(1-e)^3$ [21]. The mean outer/inner ratio for the semi-major axis and period is 20 and 70, respectively. On the other hand, the number of levels in hierarchical stellar systems is limited by the tidal action of regular gravitational field of the Galaxy, gravitational perturbations from passing stars, and stochastic encounters with giant molecular clouds (see, e.g., [8]). Surdin [18] demonstrated that, in these circumstances, the number of levels can reach values of 8 or 9, depending on masses and orbital parameters of the components. In the case of maximum dense “packing” of components in the system, hierarchical systems with 256 to 512 components can be produced.

On the other hand, there is no evidence to prove the existence of any hierarchical system having multiplicity of seven or higher. The most comprehensive catalogue of multiple systems [19] contains about 1350 hierarchical systems of multiplicity three to seven, and among the two catalogued septuple systems (AR Cas and ν Sco), at least the former one is a young cluster, i.e. is not necessarily hierarchical. This statistics is in a sharp contrast with the theoretical estimates given above. To eliminate this inconsistency, it is necessary to use additional sources of information, namely modern catalogues of double stars.

3 Catalogues of Double and Multiple Systems

Principal modern catalogues of visual double stars contain systems of much higher multiplicity than seven (see Table 1). Actually, WDS contains several systems of even higher multiplicity than indicated in Table 1. They represent either results of searches for sub-stellar companions to nearby stars by high-contrast and high-angular-resolution imaging, where at least some of the objects are background stars (WDS 17505−0603, 65 objects; WDS 19062−0453 = λ Aql, 107 objects) or results of speckle interferometric observations of stars in nebulae (WDS 05387−6906 = 30 Dor = Tarantula Nebula, 68 objects; WDS 05353−0523 = θ^1 Ori = Trapezium cluster in Orion nebula, 39 objects) or miniclusters/common proper motion groups (WDS 19147+1918, WDS 20315+3347, WDS 13447−6348, WDS 18354−3122, WDS 23061+6356, 38 to 44 objects per system).

Table 1. Principal catalogues of visual double and multiple systems. C, P, S are numbers of catalogued components, pairs, and systems, respectively

Catalogue, abbreviation, reference	C, P, S	M
The Washington Double Star Catalog (WDS [13])	249280, 133966, 115314	2–32
Catalogue of Components of Double and Multiple Stars (CCDM [3])	105837, 56513, 49325	1–18
Tycho Double Star Catalogue (TDSC [5])	103259, 37978, 64869	1–11

Note also in brackets that CCDM and TDSC contain some systems of multiplicity one. In the former case, this concerns astrometric binaries (with an invisible secondary component detected by its gravitational influence), while TDSC contains a fair amount of stars that the Tycho space mission failed to resolve into components.

It is instructive to plot the distribution of catalogued stellar systems on their multiplicity and compare it to the observational data. Tokovinin [20] presented statistics of catalogued multiple systems in the form $N_i/N_{i-1} = 0.11, 0.22, 0.20, 0.36$ for i = 3, 4, 5, 6, respectively, where N_i is the number of systems of the i-th multiplicity. These results are in a good accordance with conclusions made in their study [14] of multiple objects in the immediate (closer than 25 pc) solar vicinity. Later Tokovinin [22] studied hierarchical systems among F and G dwarfs in the Solar neighborhood and found the fraction of multiple systems with 1, 2, 3,… components to be 54:33:8:4:1. Our comparison shows that the most complete catalogue, WDS, satisfactory images the [14, 20] distributions, while the newer and deeper study [22] demonstrates a surplus of triple and higher multiplicity systems in comparison with the catalogued systems (or, conversely, WDS contains superfluous, obviously optical, double stars).

The listed catalogues of visual binaries contain various data for evidently overlapping sets of objects, and no one of them contains all known visual systems. Thus, to use the complete dataset, it was necessary to cross-match these catalogues, i.e. to gather all available information on visual binary stars in a single list. A comprehensive set of visual binaries using data from the current versions of the three listed catalogues was compiled in [7].

4 Selection of High Multiplicity Systems

The applied in [7] cross-matching procedure worked quite well for systems of multiplicity about five or six, but often failed to correctly cross-identify components in systems of higher multiplicity, due to high spatial density of objects.

To compile a list of candidates to hierarchical stellar systems of maximum multiplicity (and estimate the value of this maximum multiplicity), as well as to finally solve the problem of cross-identification of multiple systems, we have performed a semi-automatic identification of systems of multiplicity six and more in principal catalogues of visual double and multiple systems (see Table 1). The total number of such systems is 551. 175 of them are included in WDS only. The remaining 395 systems are included in more than one catalogue and, consequently, their components need cross-matching (the systems themselves were cross-matched in [7] and analyzed in [10].

Compiling the list of very high multiplicity systems, we were flagging optical pairs. The information about non-physical nature of a pair can be found in WDS and the textual Notes to WDS. We have also applied the criterion to select optical pairs suggested in [15], which revealed additional optical objects. Optical pair selection is described in detail in [6].

Photometrically unresolved binarity of some components can increase actual multiplicity of a system. In order to take this into account, we have cross-matched our systems with lists of closer binaries (orbital, interferometric, spectroscopic, eclipsing, X-ray systems, radio pulsars) using the Binary star database, BDB [9, 11]. Besides, indication of hidden binarity can sometimes be found in WDS Notes (34 cases). Information on sub-components of our very high multiplicity systems was found in catalogues of orbital (77 pairs), interferometric (425 pairs), spectroscopic (52), and eclipsing (16) binaries.

Table 2. Multiple system statistics. M: multiplicity of systems; N1: number of candidate systems; N2: number of prospective systems; N3: number of confirmed systems.

M	N1	N2	N3	M	N1	N2	N3	M	N1	N2	N3
6	138	54	5	15	14	1	–	24	4	–	–
7	107	34	3	16	4	3	–	25	2	1	–
8	76	20	–	17	5	2	–	26	1	–	–
9	42	16	–	18	6	2	–	27	1	–	–
10	39	13	1	19	4	–	–	28	–	1	–
11	32	9	–	20	3	–	–	29	–	1	–
12	23	3	1	21	2	1	–	30	1	1	–
13	14	4	–	22	3	–	–	31	2	1	–
14	13	1	–	23	–	–	–	32	1	–	–

Finally, we have excluded from our statistics those pairs that have no clear indication on their physical binarity, according to WDS and the textual Notes to WDS. As a result, we have compiled a list of 10 so-called “confirmed” systems of multiplicity six and higher. The list contains all systems included in the WDS, CCDM, and TDSC catalogues, and thus it is the most comprehensive list of stellar systems of multiplicity six and more. We provide extensive cross-identifications for components, pairs, and systems included in the list. We add data on photometrically unresolved binaries, taken from catalogues of closer pairs (spectroscopic, eclipsing, etc.) and flag optical pairs.

The final statistics is presented in Table 2. Column N1 contains the number of candidates to systems of multiplicity M Column N2 contains the number of candidates to systems of multiplicity M, without optical pairs Column N3 contains the number of confirmed systems of multiplicity M.

The highest-multiplicity systems are listed in Table 3. For each system, the number of components (M1), number of optical components (Opt), and number of confidently hierarchical components (M2) are given. The system WDS 17457−2900 demonstrates the highest value of possible hierarchical multiplicity (7), while possible multiplicity of several other systems (WDS 23061+6356, WDS 17378−1315, WDS 10174−5354) reaches higher values, but it should be confirmed by observations.

Table 3. Systems of highest prospective multiplicity. M1: number of components without optical ones; Opt: number of optical pairs; M2: possibly confident hierarchical multiplicity.

ID	M1	Opt	M2
WDS 23061+6356	31	8	1
WDS 17378−1315	30	2	1
WDS 10174−5354	29	3	2
WDS 10451−5941	28	6	1
WDS 15326−5221	25	0	1
WDS 17457−2900	21	0	7
WDS 01030+6914	18	1	1
WDS 05353−0522	18	0	1

It can be seen that these values are still far from those expected from theoretical predictions. Several possible ways can be considered to explain such a mismatch.

First, the theoretical possibility to construct a system with 8–9 hierarchy levels is based on purely geometrical considerations and does not necessarily mean that physical conditions in a protostellar cloud can permit to construct such a system. Consecutive fragmentation of a large contracting interstellar cloud is needed for a very high multiplicity hierarchical system to be born.

Also, very wide binaries (wider than 100 AU) are so weakly bound that they can be effectively disturbed, even disrupted, by extremely weak perturbations from inhomogeneities in the Galactic potential due to stars, molecular clouds, dark objects, or large-scale tides. Thus, the outermost components of a very high multiplicity hierarchical system will probably not survive on their orbits and leave the system.

Finally, we probably underestimate hidden multiplicity of stellar systems, and the number of photometrically unresolved components is much higher than catalogued data predict.

5 Procedure of the Multiple Systems Cross-Matching

The results of semiautomatic selection of stellar systems are useful for further advancing of cross-matching algorithms. Automation of multiple stellar systems remains an ongoing issue. The methodology of multiple entity matching proposed below is intended to be applied to systems of arbitrary multiplicity, its results are not dependent on the sequence of component identification. It may be useful for any existing and prospective catalogues since it is not configured for particular catalogues, but focused on consideration of all available domain knowledge including behavior of observational and astrophysical characteristics of given type of objects as well as characteristics of observation conditions.

Data from various astronomical catalogues of single and multiple stars of different observational types should be analyzed to identify components of stellar systems through all sources. The problem of stellar systems identification is reduced to matching of multicomponent entities from multiple data sources. Components of such entities may be typed and characterized in different data sources by different sets of attributes. A component may represent a multiple object in its turn in some data sources. For example, a close pair of stars may be discovered with spectral methods but it is catalogued as a single star in visual observations.

A multiple system is represented as a graph with components (or stellar objects unresolved yet to be multiple) as vertices, and pairs of components from catalogues denoted by the arcs from primary component to secondary one in a pair. Identification of the whole stellar systems is equivalent to construction of the connected graphs from the identified components. Every vertex, arc and graph as a whole system should be correctly identified. Any wrong identification of components or pairs obviously may cause a wrong connection of several systems into a single one, or single stars into the systems.

Matching of star systems involves a set of entity resolution approaches. Approaches to matching include various criteria based on sets of attributes, graph structures and identifiers of stars. Matching methods use not only data of observations and parameters of objects, but also identification based on objects that already have been identified [1, 2].

During analysis of data, a number of issues should be taken into account:

- different formatting data in catalogues;
- different semantics of attributes in catalogues (e.g. in different catalogues stellar coordinate may refer to the photocenter of the pair or to a brighter component in the pair);
- data input errors in catalogues (for instance, errors in the identifiers of stars);
- missing values in catalogue fields;
- variable values of attributes (for instances, changing brightness, changing coordinates in different observations as a result of proper motion or orbital movement of components);
- unstructured data (comments useful for identification of components);
- differences in representation of system structure (for example, different components may be deemed main in the pair in case of the similarity of their characteristics);

- depending on angular distance of components, components' brightness difference and resolution of a catalogue a pair may be catalogued as a single object with integral brightness or as two separate objects;

Catalogues usually contain some fields with identifiers of objects from other known catalogues. However, objects in different catalogues may have implicitly different semantics. Therefore, every identification taken from original catalogues as a cross-catalogue identifier may be wrong for some reasons and needs to be verified in a possible way, for instance, using values of observed parameters and calculated astrophysical parameters of identified entities.

5.1 Conceptualization of the Domain

Specifications of subject domain include all the concepts and knowledge used in matching of multiple systems. These include a quality of the observation instruments (angular resolution, lower and upper magnitude limits), astrometry (coordinate systems, proper motion, orbital movement, separation and position angle), photometry (photometric systems, passbands, magnitudes, variability), pair classification according to the observational technique (spectroscopic, interferometric, eclipsing pairs). Using ontological concepts, the conceptual schema is constructed for unified representation of data from heterogeneous sources [17]. Concept constraints are used for generation of sets of criteria for entity matching.

5.2 Technical Approach to Matching

General methodology for single or multiple entity matching is based on construction of a set of candidates for every entity of its component and application of sets of criteria constraining such sets of candidates. Criteria are formed from the domain knowledge limiting the interpretation of objects: expression of secondary attributes (computable) from primary ones (observed parameters), constraints of attribute values, mutual constraints of two or more attributes. Criteria have equal importance, take entities, not parameters as arguments, and are applied when all required data on entities is present. Order of application of the criteria has influence only to the effectiveness of candidate set constraining. For this purpose, the criteria may have a priority of application.

The approaches used for the entity resolution include various similarity criteria for subsets of attribute values (based on domain knowledge) and graph structures, which allow estimating identity of systems and components. Matching of entities may also include identification criteria based on the identifications of entities that already established.

It is reasonable to divide the process of entity matching into several interacting stages of matching candidate search. Firstly, components of systems from surveys of visual binaries are cross-matched as single objects. Then on the base of the first stage results, visual pairs are cross-matched (as the widest pairs in the hierarchical systems). On the next stage, consideration is complemented with the pairs of the other observational types (close pairs). Finally, existing identifiers of systems, pairs and components are cross-matched using results of the previous stages. Matching of the whole systems is a consequence of matching of all their components and pairs.

In the following subsections we describe the mentioned stages in more details.

5.3 Component Matching

For every component a set of candidates for identification in all considered catalogues (including surveys not differentiating single and multi-component objects) is constructed. Construction of candidate sets of component identity is primarily based on the proximity of the coordinate values. These sets then are used in other criteria verifying combination of parameter values such as:

- taking into account effective angular resolution, trigonometric parallax and difference due to proper motion and observational epochs;
- similar brightness in a given photometric passband taking into account sensibility to lower and higher limits of magnitude;
- limiting of brightness or color index difference in different photometric passband;
- taking into account possible variability of the star;
- similarity of evolution statuses;
- similarity of spectral classification.

In addition, we use special criteria to return from the next stages and to apply their results to choose correct candidates for component identification.

5.4 Visual Pair Matching

Work on identifying pairs of components begins with the consideration of wide visual ones. For every pair a set of candidates for identification includes possible combinations of identified components from results of the previous stage. For each candidate for pair identification, positional and photometric information is compared in criteria such as:

- position angle and separation of secondary component to primary one may differ because of the orbital movement;
- significant difference of the component proper motions in case of optical pairs (located close in the sky, but far apart in space);
- proper motions of components in a pair are similar;
- differences of component brightness are similar for identical pairs;
- sometimes it is possible to compare chemical compositions and evolution statuses of components in physical pairs;
- unique identification of components and other pairs are taken into account.

Criteria are constructed using the domain knowledge and results of statistical analysis of data from different catalogues or initial source surveys of subsets in catalogues. Limit values of parameter deviations are determined. If difference of parameter values does not exceed limiting value of the parameter, it may be a criterion of the pair identification.

Sometimes a pair should be matched not to another pair but to a component. A pair of the nearby stars depending on brightness and angular distance may be catalogued by instruments with different angular resolution as a single object (having integral brightness) or as the two distinct objects. To determine such cases an effective angular resolution of a catalogue is calculated statistically.

There are several methods to detect optical pairs. A sign of optical pair may be a difference of proper motions of components and/or their annual parallax. If there are series of observations, another sign is a linear (not orbital) relative movement of components. One more known statistical method for optical pair detection is the so-called 1% filter method [15] using field density in the direction of galactic coordinates of components, brightness of secondary components and angular distance between components. The identified probable optical pairs are marked with a special flag.

5.5 Close Pair Matching

Results of visual pair matching facilitates involvement of the close pairs. At this stage, data on binary and multiple systems of the following observational types is considered: interferometric, orbital, astrometric, spectroscopic, eclipsing, cataclysmic, X-ray binaries, as well as binaries of radio pulsars. Such pairs may be autonomous systems or components of previously considered wide visual pairs. Sets of matching criteria are mostly based on positional and photometric information but have some special criteria for specific parameters of objects. It should be noted that a pair may be listed in different catalogues as objects of different observational types.

5.6 Identifier Matching

At final stage, results of component and pair matching are complemented with identifiers from original catalogues of multiple and single stars (Bayer/Flamsteed, DM, HD, GCVS, HIP). These identifiers are commonly and widely used. However, the problem of belonging of these identifiers to a pair or to one (or the other) component in that pair often requires careful consideration. Correct identification of pairs and components using all available observed parameters on previous stages helps to solve this problem. Belonging of an identifier to a pair or to a component depends also on the difference of effective resolutions of original catalogue.

There are criteria that detect different types of identification errors. For example, an assumption of mixed up identification of components in a pair is generated if differences of brightness in different catalogues have close absolute value, but with different sign.

For each system, pair and component a specific identifier is assigned which is associated with the identifiers of all original catalogues of single and multiple stars to form a common base of identifications. Objects not resolved automatically are signed with special flags and are considered by experts for manual resolution or criterion base modification.

6 Conclusions

To explain inconsistency in stellar multiplicity between rather high values predicted by theoretical considerations and observational lack of systems with multiplicity higher than six, we have studied principal catalogues of visual double stars: WDS, CCDM and TDSC. They contain data on very high multiplicity (up to 30 components and more), though not necessarily hierarchical, systems, including moving groups and (mini-)clusters. To collect all available information on these systems, it was first necessary to make a thorough and accurate cross-matching of their components in the catalogues. Optical pairs, when known or assumed from the probability filter, were flagged and eliminated from the statistics, and information on photometrically unresolved sub-components was added.

Principal results of the current study are the following:

- a cross-identification catalogue of 551 stellar systems of multiplicity six and more;
- a list of systems, candidates to utmost multiple hierarchical systems;
- a procedure for cross-matching components of very high multiplicity systems (i.e., in crowded stellar fields), which also can be used for identification of objects in future surveys of binary/multiple stars (Gaia, LSST).

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