Development of Coordinated Ramp-Metering Based on Multi-objective Nonlinear Optimization Functions: Traffic and Safety

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Abstract. This paper is focused on the extension of the OASIS (Optimal Advanced System for Integrated Strategies) in order to take into account the multi-objective nonlinear optimization technique for coordinated ramp metering. The multi-objective function includes two costs functions: traffic and safety (Risk model) indices. OASIS is revisited and off-line simulation studies are conducted on real test site corresponding to A6W France motorway located in the south part of "Ile de France" Motorway networks. Five consecutive on-ramps are considered for the control. The obtained results are very promising.

Keywords: Risk index · Multi-objective non linear optimization · Coordinated ramp metering · Off-line evaluation

1 Introduction and Background

A number of approaches have been developed in the past for the design of control strategies that involve control measures such as route recommendation via Variable Message Signs (VMS) devices or equipped vehicles, ramp metering, motorway-to-motorway control (MTMC), automatic incident detection (AID), hard lane shoulder etc. Several approaches were investigated including expert systems, fuzzy systems, neural networks, classical feedback control and optimal control strategies based on either linear or non linear approaches.

In practice, traffic control systems within corridors or motorway networks have been developed independently for each individual control measure attempting to optimize traffic flow on the motorway or the urban road networks or on the both components. With respect to the optimal control, the problem formulation was focused on the development of the integrated strategies which simultaneously take into account several control strategies such as ramp metering, MTMC, user's guidance. This global point of view suggests that control measures within the entire network should be designed in an integrated way, or, at least, they should be suitably coordinated during operation, so as to meet

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the overall objectives [9,10]. These approaches are very promising. Nevertheless, whatever considered approach, the used cost objective functions are mainly focused on the optimization, in the considered system, of the traffic indices such as the minimization of the Total Time Spent index (TTS), the travel time, the maximization of the Total Travel Distance (TTD), the mean speed etc.

However, except the Automatic Incident Detection (AID) strategies which are focused on real time accident/incident detection aiming at minimizing the detection time and the traffic impact of the accident, the safety index is not taken into account. In general, the safety aspect is considered as an external cost function and not included in the real time control strategies. Safety indices are computed during the evaluation process. The classical approaches consist in collecting first the incident/accidents traffic data during the experimented scenarios. After the experimentation phase, the safety evaluation will commence. These evaluations are based on the statistical analysis of the number of accidents before and after the implementation of the tested strategies. Therefore, the constitution of the accident database must include a minimum number of accidents in order to guaranty the statistical significance of the evaluation process. This means that the field data collection period must have a long time periods (comprising between 5 to 10 years), which is the "price to pay" for having a significant safety evaluation result.

On the other hand, the introduction of electronic devices and computerized systems in the vehicle technologies have significantly contributed to user's safety and comfort. Nevertheless, the prediction of the crash in real time is still in investigation phase and some research efforts are dedicated in this area [4,11]. During the last decade, there is increased focus on the development of real time ("*potential crash*") prediction algorithm on urban motorway traffic [12,16,20].

This paper is focused on the development of optimal control strategy based on multi-objective index function, including traffic and safety indices. The first part of this paper is dedicated to the description of the risk model building. This risk model is validated on the ringway of Paris. The second part of this paper is focalized on the description of the generalized problem formulation based on the application of the optimal control for the development of integrated control strategies. The numerical solution of a formulated large-scale nonlinear optimal control problem is effectuated by application of a non-linear optimization technique based on the optimal control theory, which is able to deal straightforwardly with non-linear features. In this paper, focus is made on the coordinated ramp metering. The third part of this paper is dedicated to the reformulation of the cost function by including the risk index model. Summary of the developed risk model is described along with several simulation tested and evaluated scenarios. Investigation of the efficiency of this multi-objective non-linear approach is performed by using a macroscopic simulation model namely METACOR [5] which is able to simulate traffic flow phenomena in corridor networks with arbitrary topology.

2 Summary Description of the Risk Index Model

The developed risk model is based on the collection of traffic measurements synchronized with incident/accidents detection. The considered site was the ringway of Paris. Incidents/accidents database were collected from the implemented real time Automatic Incident Detection system (AID) and the operator reported incident/accident files. The characteristics of the accidents include: starting time (H:min), end-time, location, weather conditions, severity. The constituted data base covers 3 year periods (2002–2004). During this period, the total number of accidents collected is around 900. For each incident/accident, the considered measurement stations are depicted in the Fig. 1. Two upstream and two downstream measurement stations were considered. For each accident, the collected traffic data (traffic volume, occupancy rates and speed) covers two hours (one hour before and one hour after the crash). The collection time interval of the measured data is equal to one minute.

Before the statistical analysis, a selection of valid accidents is performed. The applied selection criteria correspond to the availability of all data measurements: 120 in total (60 = 1 h before the accident and 60 after the accident), the same topology of the accident location (same number of lane = 4 lanes) and sunny days. Among the total number of accidents(900), only 85 accidents are selected for the statistical analysis. Consequently, the total number of collected measurements is equal to $120 * 85 = 8200$. Each measurement includes two upstream and downstream measurement stations of each lane (4 *lanes*) of traffic volume, occupancy rates and speed for each minute time slice. In the constituted data base, each observation line includes $4 * 3 * 4 = 48$ measured traffic variables.

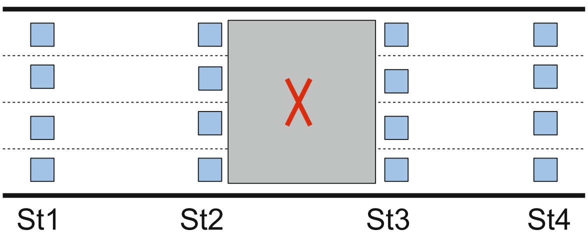


Fig. 1. Topology of the considered stretch measurements for each crash

The applied methodologies are based on statistical analysis of the traffic conditions before the accident. The main objective of this step is to analyze the traffic conditions before the occurrence of the accident and to extract the most important traffic variables to be used for the risk modeling. A series of multivariate statistical methods are used, aiming at finding the relationship between the occurrence of the accident and the traffic conditions on the stretch using the real data measurements [6,7,12].

The multivariate statistical approaches include the following steps:

1. Correlation: What are the non correlated variables to be used?
2. Principal Component Analysis (PCA): Reduction of the used variables
3. Clustering: To understand the traffic dynamics before the accidents indicated by the dynamic behavior between clusters
4. Logistic regression: Model building bases on the *logit* model application

Extensive iterative statistical computing of the four steps were performed using SAS statistical software. In particular, the following scenarios were considered:

– upstream measurements by lanes and by stations
– downstream measurements by lanes and by stations
– overall measurements by lanes and by stations

In order to alleviate the paper, reader can refer to the cited references for more details. In particular, the four steps results by scenario are well described in the reference [6] along with extended details and results analysis. In summary, the best scenario found corresponds to the use of the upstream and downstream measurement stations (means of the four lanes). In this case, the used state variables were limited to: Q1, Oc1, Oc2, Oc3 and Oc4. These 5 significant state variables to be used are provided by the statistic computation of steps (1) and (2). The clustering computations (step 3) provide 5 clusters with the following characteristics:

– **Cluster** 1**:** is the most dense one (more than 36% of all observations cases). It is characterized by quite homogeneous occupancy rates(Occ) and average flow over the 4 stations, (Occ of 11% to 12%) and a flow of 1450 to 1500 vehicles per hour and per lane) characterizing fluid traffic conditions.
– **Cluster** 2**:** presents a very high Occ on the (upstream) stations St_1 and St_2 and rather average downstream (14% to 18%). As for the flow, it is rather stable and low compared to other clusters. This cluster contains 20% of the overall observations.
– **Cluster** 3**:** presents high occupancy rates over all stations. The upstream flow is around the capacity whereas the downstream flow is less than the capacity (hight level of congestion: 40% and 37%). This cluster contains 12% of the overall observations.
– **Cluster** 4**:** has average Occ close to the usual 20% critical value, increasing from upstream to downstream (26.7%) at station St_4. The flows are higher than the other clusters, up to 1774 veh/h/lane at station St_2.
– **Cluster** 5**:** has a high average Occ (around 37%) at all stations and a lower flow (around 1230 veh/h/lane). When we consider the accidents and attribute to each time step the cluster number to which it belongs, we observe that 43 accidents out of 85 studied (51%) present cluster change during the last six minutes.

For 60 accidents (i.e. more than 70% of them), the last time step belongs to cluster 2, characterized by a rarefaction shock wave (congested upstream and fluid downstream).

The Risk model (R) is set to $R = 1$ for the observations belonging to cluster 2 and $R = 0$ elsewhere. The calibration of the Risk model is based on 55 accidents.

2.1 The Logistic Regression Model

In our case, the predictor corresponds to the occurrence of the accident ($R = 1$) or not ($R = 0$) which is a binary variable. In case of binary dependent variable, the linear regression should not be applied due to the possibility to have a probability higher to 1. Logistic regression, also called a *logit* model, is used to model dichotomous outcome variables. In the *logit* model the *logodds* of the outcome is modeled as a linear combination of the predictor variables.

The "logit" model solves these problems:

$$\ln \frac{p}{1+p} = AX + b \tag{1}$$

$$\frac{p}{(1-p)} = exp(AX + b) \tag{2}$$

where:

p is the probability that the event R occurs: $p(R = 1)$ $p/(1-p)$ is the "odds ratio" $ln[p/(1-p)]$ is the log odds ratio, or "logit"

The logistic regression model is simply a non-linear transformation of the linear regression. The "logistic" distribution is an S-shaped distribution function which is similar to the standard-normal distribution (which results in a probit regression model) but easier to work with in most applications (the probabilities are easier to calculate). The logit distribution constrains the estimated probabilities to lie between 0 and 1.

For instance, the estimated probability is:

$$p = R = \frac{1}{(1 + exp(-AX - b))} \tag{3}$$

This approach is applied for the Risk model development. The "*logit*" regression model found is the following:

$$RiskIndex = \frac{1}{1 + \exp -(\alpha + \beta Oc_1 + \gamma Oc_2 + \delta Oc_3 + \theta Oc_4 + \Phi Q_1)} \tag{4}$$

where:

Oc_i is the occupancy rates of the station (i); $i = 1..4$

Q_1 is the traffic volume of station 1.

$\alpha = -7.1677; \beta = 0.2122; \gamma = 0.1389; \delta = -0.1061;$

$\theta = -0.2052; \phi = 0.00038.$

Analyse des estimations de la vraisemblance maximum

Paramètre	DF	Estimation	Erreur std	Khi 2 de Wald	Pr > Khi 2
Intercept	1	-7.1677	0.6941	106.6361	<.0001
moytost1	1	0.2122	0.0118	320.8749	<.0001
moytost2	1	0.1383	0.00899	237.0292	<.0001
moytost3	1	-0.1061	0.00935	128.6439	<.0001
moytost4	1	-0.2052	0.0112	333.1159	<.0001
moyqst1	1	0.000385	0.000074	27.2964	<.0001

Fig. 2. Logistic regression: Table of *SAS* output statistics

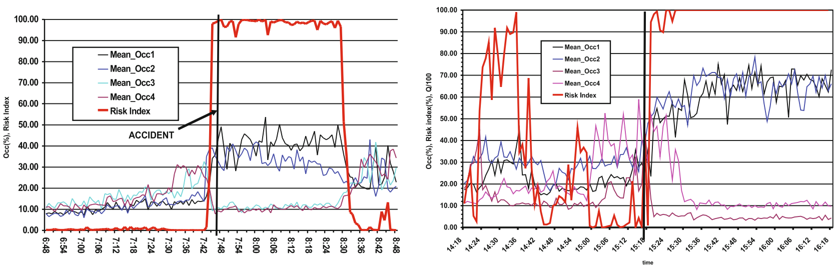
The risk index model calibration was performed on 55 accidents whereas the validation has been proceed on the rest (30 accidents) of the database (Fig. 2).

Screening the found parameters of the risk model, the following remarques can be drawn:

– the downstream parameters are negative. This means that the increase of the downstream occupancy rate measurements leads the decrease of the risk index value
– on the contrary, the upstream parameters are positive which leads, in case of increasing upstream occupancy rate measurements, the increase of the risk index.

For the risk model validation, the same parameters are used for 30 accidents which are not used for the calibration. The obtained results of the 30 accidents are very close to the two accidents selected. The risk index time evolution results are depicted in Fig. 3. The output results indicate that the risk index value is maximal just before the occurrence of the accident. We can observe that the recorded occurred accident time in both figures are not exactly synchronized when the Risk index is maximum. This remarque is related to the recording of accident times by the operator. In some cases the recorded times include some reporting error: it could be higher or less (max 6 min (+, −) are observed).

In Fig. 3 (*Acc*-1), the risk index is maximum just before the occurrence of the accident. However, in case of (*Acc*-2), we can observe that before the occurrence



"Acc-1 " "Acc-2"

Fig. 3. Time evolution of the risk index on the ringway of Paris

of the accident (14 h:18–15 h:18), the risk index is very high without occurrence of the accident. This means that the traffic flow is completely instable during this period (1 h). This instability is translated by time variation of the risk index. However, at 15 h:18, the accident is occurred. When the risk index is maximum, accident can be occurred or not.

The developed risk index can be used off-line as an evaluation index during the evaluation process. In particular the computation of the cumulative Risk index can be considered as a safety external index for the performance assessment between scenarios (e.g. assessments of without and with control). In this case, computation of the risk index using only measurements will leads to the dramatically reduction of the field test periods.

In real time, the computation of the cumulative risk index on the considered motorway axis using real time measurements can be used as safety monitoring tool (e.g. safety user warning system). In our case the risk index is integrated in a multi-criterion function to be optimized in real time (safety index combined with a traffic index).

In this paper, the developed risk model will be integrated in the cost function of OASIS Kernel for coordinated ramp metering strategy development using non linear optimization.

3 Integrated Control: Mathematical Problem Formulation

The control strategy is an essential part of any traffic-responsive motorway control system. In particular, control such urban intersection control, ramp metering, motorway to motorway control ($MTMC$) and route guidance aim at improving significantly the transport efficiency (so the safety) and reducing of environmental pollution [9]. In the past, traffic control systems within corridors or motorway networks have been developed independently for each individual control measure attempting to optimize traffic flow on the motorway or the urban road network. However, a traffic corridor constitutes an entity in terms of operational objectives, user requirements, and impact of individual control devices [10]. Therefore, the integrated control approaches have an additional potential in comparison with the individual ones. This global point of view suggests that control measures within the entire network should be designed in an integrated way, or, at least, they should be suitably coordinated during operation, so as to meet the overall objectives. The need for an integration of road and motorway traffic control measures in metropolitan areas emerges from the following considerations:

- Integration of control measures is expected to improve the use of overall network capacity.
- Particular control measures, such as driver information or route guidance, are only reasonable on the basis of the integrated network traffic state.
- Particular operational objectives, such as preference of motorway paths, can only be achieved by consideration of the overall network.

Due to the lack of a general theoretical background, the non-linear nature of the traffic process, and the constraints imposed by the control measures, the extraction of the decision rules is a highly complicated and time consuming task. The optimization techniques based on the optimal control theory allow to deals straightforwardly with important non-linear features, and have been implemented and used in many fields. Moreover, this approach seems promising because the control decisions are based on the minimization of an arbitrary control criterion rather than regulating towards a certain state of the process and because it considers explicitly the control constraints.

Consider a traffic process described by the general state equation reads:

$$x(k+1) = f[x(k), u(k), d(k)] \tag{5}$$

where $x(k) \in R^n$, $u(k) \in R^m$, $d(k) \in R^n$, denote the state, the control and the disturbance vectors respectively. The disturbance vectors include the demands at the origins, accident/incident locations, weather conditions etc.

In our case, the state vector $x(k)$ correspond to the mathematical macroscopic traffic model equations. The dynamic macroscopic traffic modeling equations are based on 3 main variables: traffic volume $q(x,t)$, density $\rho(x,t)$ and speed $v(x,t)$. The variable $d(k)$ corresponds to the traffic demand vector at each origin of the considered network whereas $u(k)$ is the vector of the control variable. The control vector includes several control strategies such as ramp-metering, motorway to motorway control, speed limits, route guidance, intersection control etc. In general, the control variables are subject to some constraints expressed as: $U_{min} \leq u(k) \leq U_{max}$.

3.1 Summary Description of the Used Macroscopic Traffic Flow Model Equations

In this section, the mathematical equations of the used traffic model are described. In this study the MAGISTER multi-model Kernel is used. MAGISTER Kernel includes several macroscopic traffic models:

– First order modeling (LWR) for the urban modeling
– 4 s order models (ACL: Acceleration Limit model [14]; METACOR [5], ARZ [17], GSOM [14])

According to the macroscopic nature of these models, the state variables of the simulated traffic process are the density $\rho(vh/km)$, the mean speed $v(km/h)$ and the traffic volume (or flow) $q(vh/h)$. Among the second order models in MAGISTER, the METACOR model is used. METACOR is able to simulate urban and motorway network for arbitrary topology including non-oriented destination (no OriginDestination (OD) matrix is needed), only the global demand at origins must be provided. The oriented destination modeling is included also. In this case, the OD matrix must be provided as an input to the model.

The motorway network is represented as a directed graph whereby the links of the graph represent motorway stretches. Each motorway stretch has uniform characteristics, i.e. no on-/off-ramps and no major changes in geometry. The nodes of the graph are placed at locations where a major change in road geometry occurs, as well as at junctions, on-ramp, and off-ramps.

The time and space arguments are discretised. The time discretisation is the same for the whole network, while the space discretisation is defined for each link separately. The discrete time step is denoted by T. A link l is divided into N_l segments of equal length. For each segment i of each link l at each time instant $t = kT, k = 0, 1, K$, where K is the time horizon, the following macroscopic variables are defined:

– Traffic density $\rho_{l,i}(k)$ (veh/km) is the number of vehicles in segment i of link l at time kT divided by the length of the segment.
– Mean speed $v_{l,i}(k)$ (km/h) is the mean speed of the vehicles included in segment i of link l at time kT.
– Traffic volume or flow $q_{l,i}(k)$ (veh/h) is the number of vehicles leaving segment i of link l during the period $[kT, (k+1)T]$, divided by T.

Let us summarize, in discrete time, the main mathematical equations (non-oriented destination modeling) for a link (l) and a segment (i): The conservation equation on each link (l) and segment (i) reads:

$$\rho_{l,i}(k+1) = \rho_{l,i}(k) + \frac{T}{\Delta_i}[u_{l,i-1}(k)q_{l,i-1}(k) - u_{l,i}q_{l,i}(k)] \tag{6}$$

Relationship between volume, speed and density:

$$q_{l,i}(k) = \rho_{l,i}(k)v_{l,i}(k) \tag{7}$$

Speed equation:

$$\begin{aligned} v_{l,i}(k+1) = v_{l,i}(k) + \tfrac{T}{\tau}[F(\rho_{l,i}(k)) - v_{l,i}(k)] + \tfrac{T}{\Delta_i}v_{l,i}(k)[v_{l,i-1}(k) - v_{l,i}(k)] \\ - \tfrac{\nu T}{\tau \Delta_i}\tfrac{\rho_{l,i+1}(k)-\rho_{l,i}(k)}{\rho_{l,i}(k)+\kappa} \\ - \tfrac{\delta T}{\Delta_i}\tfrac{q_{ramp}(k)v_{l,i}(k)}{\rho_{l,i}(k)+\kappa} \\ - \tfrac{\phi T}{\Delta_i}\tfrac{(\lambda_l-\lambda_{l+1})}{\lambda_l}\tfrac{\rho_{l,i}(k)v_{l,i}^2}{\rho_{cr}{}^l} \end{aligned} \tag{8}$$

Fundamental diagram equation:

$$v(\rho_{l,i}(k)) = V_{f,l}exp\left[-\frac{1}{a}\left[\frac{\rho_{l,i}(k)}{\rho_{cr}^l}\right]^a\right] \tag{9}$$

where
T is the simulation time step
Δ_i = length of the segment i of the link l
F: Fundamental diagram of the link l expressed by the Eq. (9).
$u_{l,i}$: denotes the control variables at the segment i (corresponds to the capacity restriction (severity) due to an accidents/incidents at the link l and segment i)

V_f, ρ_{cr}, a: denote the free-flow speed, the critical density and the parameter of the fundamental diagram of link l respectively.

$\tau, \kappa, \nu, \delta, \phi$: are the model parameters: delay term, the anticipation term and other weight terms.

The set of parameters $P = (V_f, \rho_{cr}, \mathrm{a}, \tau, \kappa, \nu, \delta, \phi)$ reflect particular characteristics of a given traffic network and depend upon street geometry, vehicle characteristics, drivers' behavior, etc.

At the origins of the simulated network a queuing modeling is applied. This modeling permits the queue length estimation and the waiting time. These indices are very useful for the evaluation of the ramp metering strategy performance.

In general, the input traffic to the network is provided at each origin where the demand is supposed to be known.
Assume that, at each origin (o), the demand during the time slice k is $d_0(k)$. The dynamic evolution of the queue length at the origin (o) can be written as:

$$Lq_o(k) = Lq_o(k-1) + T[d_0(k) - q_o(k)] \tag{10}$$

where $q_o(k)$ is the injected traffic volume in the downstream link of the considered origin. When the demand at the origin is higher than the supply of the downstream link, queue at the origin will appear. This simple queuing modeling is very useful for the application of ramp metering strategies. This variable will be considered as a state variable.

In summary, the state variable of the simulation model are:

$$x = [\rho_i \quad v_i \quad Lq_o]^T \tag{11}$$

3.2 Definition of the Traffic Cost Function

The cost criterion may be arbitrary chosen. In general, the traffic criterion used for the optimal control could be a combination of several criterion such as: the Total Travel Distance (TTD), the Total Time Spent (TTS), the travel time, the generalized mean speed $V_{moy} = TTD/TTS$, the queue length and the waiting time at all origins, the total fuel consumption etc. The general form of the traffic cost criterion is the following:

$$J_{Traf} = T \sum_K \left\{ \sum_l \sum_i \rho_{l,i}(k) \Delta_{l,i} + \sum_o Lq_o(k) \right\} \tag{12}$$

where K is the time horizon; i: the number of segment of the link l and (o): the number of origins.

In this study, the safety index reported in Eq. (4) is added to Eq. (12) in order to constitute the global cost function to be optimized, reads:

$$J_{Glob} = J_{Traf} + \alpha J_{Saf} \tag{13}$$

where the parameter (α) is a weighted factor to be calibrated during the optimization process.

3.3 Numerical Solution of the Optimal Control Approach

The optimization algorithm is one of the key ingredients in the optimal control process we attempt to resolve. In particular, since the objective is to control online, with a quick reaction to uncertainties, the convergence towards the optimal solutions has to be a low CPU time process. According to the standard methodology based on the optimal control theory [18], we can solve the optimization problem described in the next paragraph by using an optimization algorithm based on the first derivatives.

A discrete-time optimal control problem is formulated as:

$$\min J = \psi(x(K)) + \sum_{k=0}^{K-1} \phi(x(k), u(k)) \tag{14}$$

Subject to:

$$x(k+1) = f\left[x(k), u(k), d(k), P\right]; \quad x(0) = x_0 \tag{15}$$

The variable P corresponds to the set of model parameters and the constraints:

$$U_{min} \leq u(k) \leq U_{max} \tag{16}$$

This type of algorithm is well adapted but requires the explicit derivatives calculation in order to limit the computational effort. In this context, the derivatives concerns the cost criterion with respect to $u(k)$ and to $x(k)$:

$$g(k) = \frac{\partial J}{\partial u(k)} = \left[\frac{\partial f}{\partial u(k)}\right]^T \lambda(k+1), k = 0..., K-1 \tag{17}$$

The term λ denotes the constant which can be computed backwards, using the following equations:

$$\begin{aligned} \lambda(K) &= 0 \\ \lambda(k) &= \tfrac{\partial \phi}{\partial x(k)} + \left[\tfrac{\partial f}{\partial x(k)}\right]^T \lambda(k+1), k = 0..., K \end{aligned} \tag{18}$$

With all these ingredients, it is possible to optimize efficiently any cost function (13), by using some appropriate non-linear iterative optimization algorithm based on a search direction. This search direction can be found using any appropriate algorithms such as "Direction Set Methods" or "Variable Metric Methods". The optimization process steps (for the control strategy) are the following:

1. Start with random admissible trajectories of the variable (u_0), and compute the associated cost function J_0 (Eq. 13).
2. For the iteration it, compute the gradients using Eq. (18).
3. Compute the search direction using an appropriate non-linear optimization algorithm, and the new trajectory (u^{it+1}).

4. Evaluate the new trajectory by computing the cost function J^{it}.
5. if $(J^{it} - J^{it-1})/J^{it} \leq \epsilon$ then stop
   else go to step 2 with iteration $it = it + 1$

The associated algorithm, (OASIS), has been revisited by including the safety index. This tool is a generic algorithm and can be applied to any corridor network with arbitrary topology. Some investigations have been conducted on two different optimization algorithms, known to be efficient on several different optimization problem:

– A recent and very simple but fast algorithm, well-known in the field of non-linear neural networks, called RPROP [19],
– The conjugate gradient [8], comparable to the quasi-Newton method but which requires intermediate storage of order N versus $N \times N$ for the quasi-Newton method.

On the other hand, there is not, as far we know, any overwhelming advantage that the quasi-Newton-method hold over the conjugate gradient techniques. Both algorithms are reliable and provide good results in terms of optimal solutions and convergence rates. Nevertheless, the $RPROP$ algorithm seems more faster to find the *best or local minima*.

As indicated in the Eq. (13), the global cost function includes both functions: traffic and safety and both functions are antagonistically. As a matter of fact, the safety index is minimal (= 0) in both traffic condition limits: (a) No traffic (no accident) and (b) Blocked traffic (maximal density on the motorway: all users are stopped and not move). When the traffic evolves in time, the safety index will automatically increase.

On the contrary, the traffic index will be optimal when the traffic volume is around the capacity of the infrastructure. In this case, the safety index will increase. This is the antagonistic behavior of the safety and traffic cost functions. This problem is well known as $Pareto-optimal\ multiobjective\ solution$ [2,13]. Let us remind the basic properties.

Basic Properties of Pareto-Optimal Solution: Without loss of generality, let us consider the following multiobjective optimization problem:

$$\max f_1(X), f_2(X), f_3(X), ..., f_N(X) \tag{19}$$

where N = number of objective functions; $X = \{x_o, x_l, ..., x_{n-1}\}$ is a n-dimension variable vector.
Assuming $X_0, X_1, X_2 \in \psi$,

1. X_1 is said to be dominated by (or interior to) X_2 if $f(X_1)$ is partially less than $f(X_2)$, i.e. $f_N(X_1) \leq f_N(X_2)$ for $\forall i = 1, 2...N$ and for $\exists\ i = 1...N$ with $f_i(X_1) < f_i(X_2)$
2. X_0, is said to be Pareto-optimal (or non-dominated), if there does not exist any $X \in \psi$ such that X dominates X_0. This definition is based on the intuitive conviction that the point X is chosen as optimal if no criterion can be

improved without degrading at least one other criterion. If a solution is not dominated by any other solutions, then this solution is said to be a non-dominated solution.

Usually, multiobjective optimization problems have a group of trade-off, non-dominated optimal solutions that can satisfy the global cost function. In our case, the weighted parameter (α) must be calibrated and fine tuned in order to minimize the non dominated solutions in case of similar traffic conditions. The used criteria is the value of the cumulated TTS index and the cumulated risk index. The α Risk value do not dominate the optimal solution.

4 Simulation Studies and Hypothesis

4.1 Test Site Description

The test site is located in the south of the *Ile de France* (IDF) Motorway network (Fig. 4). This site is the most critical area of the overall motorway network in the (IDF) motorway networks. Morning and evening peak congestions extend over several hours and several kilometers whilst fairly dense traffic is regularly observed even during off-peak hours. The total length of the A6W motorway axis is approximately 20 km (only the direction towards Paris is considered). This motorway axis includes 5 consecutive on-ramps which are fully equipped with loop detectors and traffic signal lights. The carriageway is equipped with detector stations, one measurement stations each 500 m for traffic volume, occupancy and speed measurements respectively. This test site was the test bed of several ramp metering strategies: isolated and coordinated [3,6].

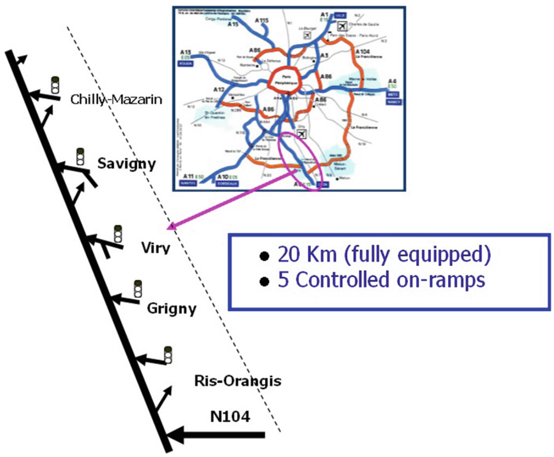


Fig. 4. A6W test site

The developed optimization algorithm (OASIS) has been implemented in the kernel of the simulation tool namely MAGISTER. The developed algorithm can be applied to any corridor network without any specific development. The first step was the METACOR model calibration and validation. This step is performed using real data measurements. The used data was split into two parts: the first part (one week from 5 h:00 to 21 h:00) is used for the model calibration and another week is used for the model validation.

The second step consists to reconstruct the demand profiles using measurements at each origin of the considered site. Several simulation runs were needed for the fine tuning of these demand profiles. They must reconstitute the congestion at the same time and the same locations than the real measurements. More description can be found in [6].

The last step is the test of the developed integrated control strategy OASIS using the multi-criteria objective function.

4.2 Risk Model Validation

Before using the safety index model in the cost function to be optimized, it is necessary to proceed to the risk model validation. A data base is constituted using the real data collection system and the existing files of the reported accidents/incidents. The used data corresponds to the collected accident synchronized with measurement traffic data. The total number of collected accidents is equal to 60. After data cleaning and accidents selection criteria (sunny day, night excluded), only 20 accidents are selected for the risk model validation. The same parameters found on the ring way of Paris are conducted on A6W motorway axis. Among the 20 accident, two accidents are chosen (see Fig. 5.). For the other accidents, the time evolutions of the risk index and traffic variables are

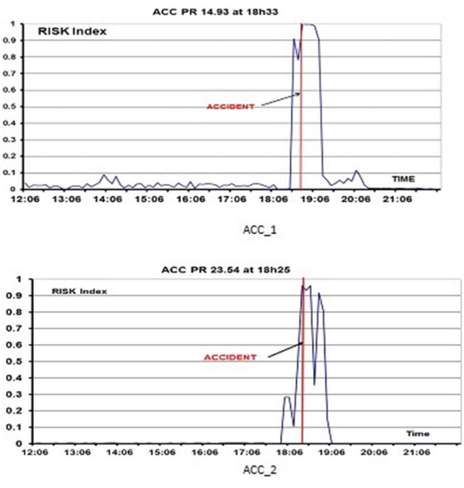


Fig. 5. Time evolution of the risk index for two accidents: Acc1 and Acc2

very similar. Figure 5 depicts the risk index time evolution of two accidents. We can underline that: using the **same parameters** found on the ring way of Paris and applying on A6W motorway axis, the obtained results are very promising. Without any parameter calibrations, the risk index value is maximal before the occurrence of the accident. Consequently, we can assume that the computation of risk index can be considered as a robust safety index and the same parameters can be used in the multi-criteria cost function optimization.

4.3 Scenario Definitions

In order to highlight the impact of the risk index function in the optimization, three main scenarios are considered:

1. Optimization of the risk index only
2. Optimization of the Traffic index only
3. Optimization of both indices: Traffic and Risk (multi-criteria)

The No control case is considered as Reference. For all considered scenarios, the same demand profiles at the origins are applied. These demand profiles are constructed from collected measurements on A6W. In particular, during the peak morning periods, the demand levels at the controlled on-ramps are very high (around 1600 to 2100 vh/h). All fixed demand profiles started and finished at fluid conditions respectively. Consequently, all vehicles are served leading to have the same value of the cumulated TTD index for each scenarios. In this case, comparative assessment will be based on the Total Time Spend (TTS) index. These demand profiles start at 5 h:00 in the morning and complete at 15 h:00. Consequently, the time Horizon of the optimization is over the duration of the simulation horizon $H = 10$ h. For each scenario, the traffic (TTS) and Risk indices are computed and reported in the output files of the simulation. The applied optimization is called Open-Loop Control because the optimization is performed on the overall time Horizon. The characteristics of the ramp metering control strategy is the following:

- The cycle duration is equal to 40 s. This value is found as the optimal cycle during the passed field trails on the same site.
- In the simulation, the control variable is the split which is defined as: $Green\ duration/cycle\ time.$
- the imposed constraints on the control variable are: $U_{min} = 0.25$ and $U_{max} = 1$. This means that the minimum of green duration is equal to 10 s whereas the maximum green = Cycle = 40 s.

Optimization Problem Dimension: The simulation time step of the simulation T is equal to 10 s. The total number of links and segments is equal to 13 and 31 respectively. The simulation time horizon (H) is equal to 10 h which equal to: $10 * 3600/10 = 3600$ time steps. The number of state variables are equal to 3 per segment (speed, density and Lq) and the number of control variables is equal to

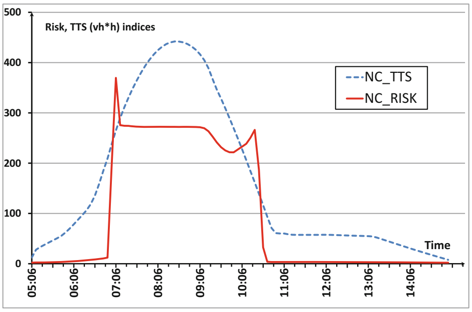


Fig. 6. No Control (NC): Time evolution of the TTS and the Risk indices

5 (5 on-ramps). Hence, the optimization problem dimension can be computed as: $(3600)(31 * 3 + 5) = 352800$ which a very-very large non linear optimization problem to be solved.

Before the results analysis for all scenarios, the prerequisite step is the calibration of the weighted parameter (α) of the global cost function reported in the Eq. (13). The used approach is the "*trial and error*" method. Several simulation runs were conducted in order to obtain a fine tuning of this parameter. The best value found is equal to ($\alpha = 2.5$). Figure 6 depicts the time evolution of no control case of the TTS and the risk index respectively.

The CPU time consuming by scenario is reported in the Table 1. The used computer is a PC, DELL precision T5610 using Windows OS with 64 GB of RAM and two CPU: 2.1 GHz. With respect to the size of the optimization problem, the CPU time consuming is largely low. The ratio between the real-time and the time consuming is roughly equal to 200. This means that the optimized control strategy is running 200 faster than the real time and could be integrated in the Traffic Management System (TMS) without any problem of computation time consuming.

Table 1. CPU Time consuming

Scenarios	Nb. Iter	CPU Time(sec)
Risk	130	60
Traffic	110	45
Risk+Traf	115	55

Figure 7 depicts the evolution of the optimized cost function J_{glob} for each iteration and during the global time horizon ($H = 10$ h) of the three optimized scenarios: Risk, Traffic and the Risk+traffic respectively. Compared to the optimization of the Risk or the Risk+traffic cost functions, we can observe that the optimization of the TTS index converges quickly to the optimal control trajectories.

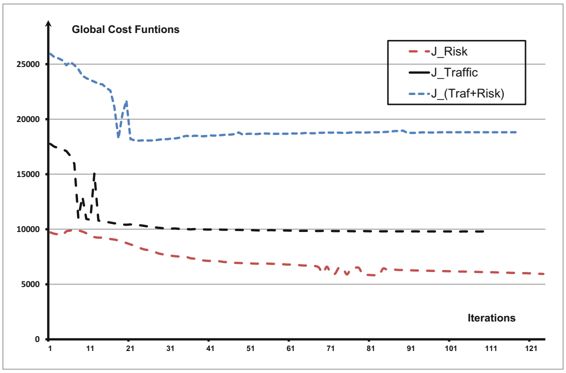


Fig. 7. CPU time consuming

5 Simulation Result Analysis

5.1 Control Output Trajectories Analysis

Before analyzing the performance in term of traffic and safety improvements by each scenarios, it is interesting to analyze first the output of the optimized control trajectories by scenarios. In our case, the optimized control variables concern 5 on-ramps. Among the 5 on-ramp, the on-ramp "*Savigny*" is selected. This on-ramp is the most critical one. During the pick morning period the observed demand is around 2200 vh/h. This level of the demand generates a high level of congestion which spill-back to the downstream on-ramps until the main origin of the A6W. Figure 8 depicts the time evolution of the ramp metering rate of the 3 scenarios.

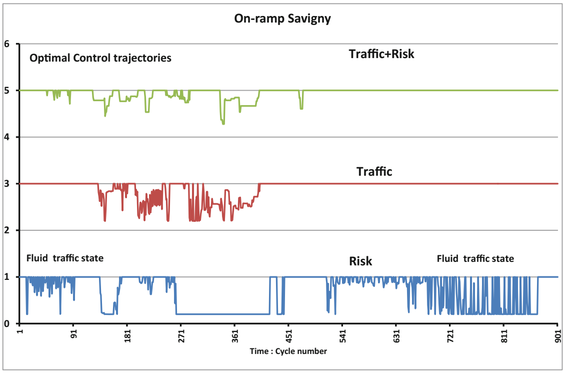


Fig. 8. Control trajectories by scenarios

Screening the control variable trajectory results, the following remarks can be drawn:

- Optimizing the Risk index only (*scenarios*1), the convergence speed of the optimization is slow compared to the other scenarios. On the other hand, the ramp metering control is always active in all traffic situations: fluid or congested. This behavior is observed also for the other on-ramps. Consequently, implementing this scenario in the Traffic Management System will not be satisfactory for the traffic managers and not acceptable by the users. In particular, controlling all the on-ramps when the traffic conditions are fluid does not have sense. Nevertheless, the safety index is dramatically improved.
- In case of scenarios (2) where the traffic index is optimized, the time evolution of control variables is correct and corresponds to real activation in the field of the ramp metering technique. When the traffic is fluid no control is applied whereas in congestion, the ramp metering is activated until the end of the congestion.
- In case of scenarios (3) where both Traffic and safety indices are optimized, the control variables are more smoothed compared to the other scenarios. However, the ramp metering is started earlier and finishing little bit late compared to the scenarios (2). In this case a trade-off is found between both indices and the *Pareto* principle is satisfied.

5.2 Global Performance Analysis

Table 2 includes all computed performance indices and the gain compared to the No Control (NC) case. In particular, the main considered performance index is the cumulated Total Time Spent (TTS) in the system over the time Horizon of the simulation. As indicated above, optimizing the Risk index only leads to a dramatic improvement of the safety index. The obtained gain is around 39% whereas the TTS index is improved by 12%. The same remark can be drawn in case of scenarios 2 where the TTS index only is optimized. The improvement of the TTS index is around 27% whereas the Risk is improved by only 10%. However, in case of multiobjective optimization scenario, the obtained optimal solutions are located in between both scenarios: (1) and (2). In this tested scenario, both indices are improved. As a matter of fact, the gain obtained are comprising between the optimization of the TTS (scenarios (2)) and the Risk (scenarios (1)). These solutions are subject to the Pareto-optimal multiobjective conditions.

Screening the time evolution of the TTS index depicted in Fig. 9, in case of the optimized multiobjective functions (traffic and safety), the time evolution of TTS is located between both scenarios: (1) and (2). At the beginning of the congestion, the optimization of the TTS index, gives better results than all scenarios. On the contrary, the optimized risk index only corresponds to worse solution with respect to the TTS index. This is due to the generation of long queue at the on-ramps leading to the increase of the waiting time and the TTS at each controlled on-ramps.

Table 2. Global performance of the 3 scenarios

	NC		OASIS		Gain(%)	
Optimized index	TTS(vh*h)	Risk	TTS(vh*h)	Risk	TTS(vh*h)	Risk
Traffic	16179	9615	11726	8688	**27**	**10**
Risk	16179	9615	14108	5910	**12**	**39**
Risk+Traf	16179	9615	12386	8191	**24**	**15**

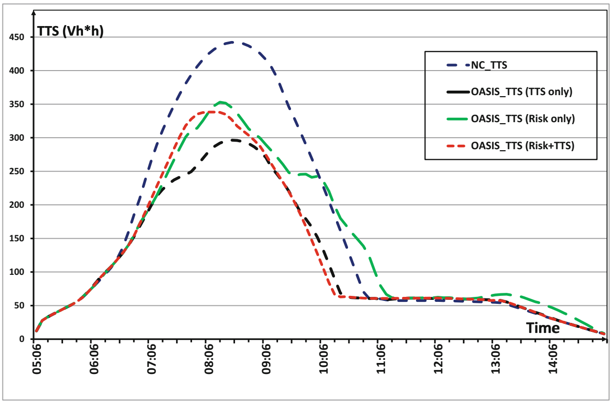


Fig. 9. TTS time evolution by scenarios

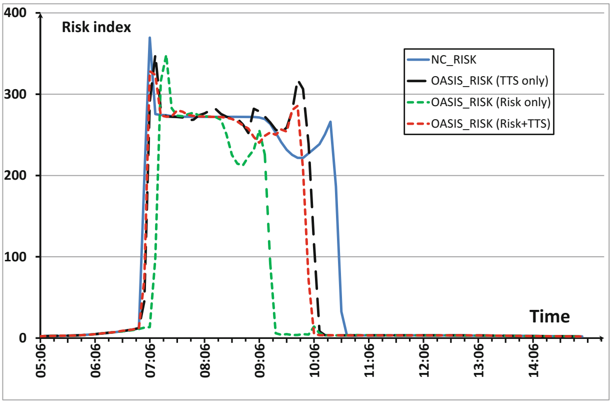


Fig. 10. Risk time evolution by scenarios

Figure 10 depicts the time evolution, on 6 min time slice, of the cumulated risk index of the considered axis for each scenario. The No-Control case is considered as a reference. Compared to the time evolution of the TTS index indicated in Fig. 9, similar time evolution of the Risk index is observed. In case of scenario (2) where the risk only is optimized, a delay is observed at the beginning of the congestion and the reduction of the level of the risk at the end of congestion. This is due to earlier starting time of the control which leads to generation of long queue at the on-ramps. In case of optimizing of Risk and TTS, the time evolution of the Risk index is in between both scenario (1) and scenario (2).

6 Conclusion and Next Steps

During the calibration and validation steps of the risk index on the test site, the obtained results are very promising. The behavior of the risk index is satisfactory with respect to the real occurrence of the accidents. Applying the risk index model on two real occurred accidents on A6W, the risk index is maximal before the occurrence of both selected accidents. In particular, the same parameters found on the ringway of Paris are used. Consequently, the risk index model can be applied, without any calibration efforts, to any motorway networks. This is a relevant result for the real time application.

The application of the non-linear multi-objective optimization approach using "OASIS" for coordinated ramp metering is very promising in terms of performance and computational effort. The obtained results indicate that the improvement of both indices TTS and $Risk$ is largely acceptable when the multi-objective cost function is optimized. Compared to the optimization of the $Risk$ or the TTS only, the optimization of multiobjective cost function results are globally better.

In this study, the considered time horizon of the optimization is equal to the duration of the simulation run. In consequence, the applied strategy is in open loop control. In order to close the loop, the hierarchical control scheme (rolling horizon) will be applied. In OASIS, the rolling Horizon technique is already developed. The next step will consist in undertaking the results analysis in case of close loop control.

On the other hand, in this paper, only coordinated ramp metering is considered. The next step consists in expanding this approach in order to make integrated control using the same multi-objective cost function. The integration control corresponds to the optimization of the global multi-objective cost function by taking into account simultaneously several types of control strategies such as ramp metering, Motorway-to-motorway control and main-lane speed limits etc.

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