Influence of Snow Cover on the Seismic Waves Propagation

Gyulnara Voskoboynikova[1(✉)], Kholmatzhon Imomnazarov[1], Aleksander Mikhailov[1], and Jian-Gang Tang[2]

[1] Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch of Russian Academy of Sciences, Akademik Lavrentjev Avenue 6, 630090 Novosibirsk, Russia
gulya@opg.sscc.ru, {imom,alex_mikh}@omzg.sscc.ru
[2] YiLi Normal University, 448 Jiefang Road, Yinning, Xinjiang, People's Republic of China
tjg@ylsy.edu.cn
http://www.sscc.ru

Abstract. In this paper the mathematical modeling of seismic waves propagation in the snow based on the theory of dynamic poroelasticity is considered. The snow cover is approximated as porous medium, saturated with liquid or air, where the three elastic parameters are expressed via three elastic wave velocities. These velocities are recalculated using the elastic wave's velocities via the Biot theory, which are expressed through elastic parameters of the snow. The obtained solutions allow the study the peculiarities of the seismic wave's propagation in the liquid or air, which saturating snow cover. In this case, the obtained equations are used for simulation the displacement velocity of the porous frame and the saturating fluid in it, as well as the pore pressure and the stress tensor components with given elastic parameters of the medium and the velocities of propagation of transverse and longitudinal waves in a porous medium.

Keywords: Snow cover · Porous medium · Seismic waves · Elastic velocities · Mathematical model · Transverse and longitudinal waves

1 Introduction

The main important geoecological problem is the problem of studying the impact from industrial such as quarry and test ground explosions to environment. It is known that different factors influence on seismic and acoustic waves propagation from explosion source. Among these factors there are weather factors, related with atmosphere state (wind, humidity, temperature), relief, state of the Earth's surface and also presence of snow cover and forest.

In the given paper, we consider the snow cover influences on the seismic and acoustic waves propagation using mathematical simulation for two-layer model. The model consist of two homogenous layers. Upper layer is snow and

I. Dimov et al. (Eds.): NAA 2016, LNCS 10187, pp. 730–736, 2017.
DOI: 10.1007/978-3-319-57099-0_84

lower layer is elastic semispace. In mathematical simulation the snow cover is presented by elastic-saturated medium which described via equations of motions and state. There is fundamental property for elastic-porous saturated medium as two longitudinal waves of the first and second kind and one rotational wave in two-layer model. The waves of the second kind are highly attenuated.

Mathematical modeling of seismic waves propagation in snow is based on the theory of dynamic poroelasticity. Three elastic parameters are expressed via three elastic wave velocities per Biot theory taking into account parameters of the snow medium. Note that obtained solutions allow studying features of the seismic waves propagation in the snow medium, saturated liquid or air. The obtained equations allow to simulate the displacement velocities of porous frame saturated fluid or air, and also porous pressure and stress tensor components by given elastic parameters of the medium and also by given velocities of the longitudinal and transverse waves propagation in the porous medium.

This problem is the subject of theoretical and experimental studies, including the using acoustic methods. Effective use of the acoustic methods involves knowledge of the processes occurring in saturated porous media, including the elastic wave's propagation as well as non-destructive methods of the snow structure classification; determination of snow physical features; developing effective methods of explosive control for snow slopes; and monitoring acoustic influence. The development of structure classification techniques and determination of the appropriate physical parameters of snow require an accurate acoustic propagation model. Pre-existing models of the seismic wave propagation made use either porous media, which has a rigid ice frame, or continuum elastic or inelastic models [1–6]. These models do not adequately explain the wave propagation phenomena observed in the snow. The air pressure waves, propagating in the interstitial pore space, and dilatational and shear stress waves, propagating in the ice frame, were detected in the snow [7–10]. However, neither the porous medium or the continuum models can explain all the three wave propagation modes. In [11], based on the Biot model the propagation of seismic waves in the snow as a porous material with an elastic frame saturated with a compressible viscous fluid (air) was discussed. The characteristics of the solutions describing acoustic waves in the snow are discussed in detail and compared with the experimental results [8,9,12,13]. In this paper, we described three elastic parameters [14–16] for the propagation of seismic waves in the snow using dynamic theory poroelasticity.

2 The Problem Statement

In poroelasticity theory [14], the stress-strain relations for a porous aggregate including the effects of fluid's pressure and dilatation are considered. As well as in [11,17] we study the dynamics of a material and the coupling between a fluid and a solid provided that the material is statistically homogeneous and isotropic in the region of interest, behaves in a linearly elastic manner and that thermoelastic effects are negligible. The macroscopic stress-strain relation for the medium was derived by assuming the isotropic medium. The coupling effect between the elastic frame and the compressible fluid was taken into account by introducing a kinetic coefficient into the dissipation function of the system [14]. Dissipation of energy by the viscous fluid was expressed in terms of a relative velocity between the fluid and the solid.

The constitutive relations describing the porous material by the tree parameters are the following [16]:

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \widetilde{\lambda}\varepsilon_{kk}\delta_{ij} - \left(1 - \frac{K}{\alpha p^2}\right) p\delta_{ij} \tag{1}$$

$$p = (K - \alpha\rho\rho_s)\varepsilon_{kk} - \alpha\rho\rho_l e_{kk} \tag{2}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right),\ e_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right),$$

where σ_{ij} is the stress in the solid framework, p is the fluid pore pressure, ε_{kk} and e_{kk} are the dilatations of the solid and the fluid, ε_{ij} is the strain in the solid, and δ_{ij} is the Kronecker delta, $u = (u_1, u_2, u_3)$ and $U = (U_1, U_2, U_3)$ are the displacement vectors of the elastic matrix and the saturating fluid with the respective partial densities ρ_s and ρ_l, $\rho = \rho_l + \rho_s$, $\widetilde{\lambda} = \lambda - (\alpha p^2)^{-1}K^2$, $K = \lambda + \frac{2}{3}\mu$, $\lambda, \mu, \alpha p^2$ are elastic parameters of the porous medium [14].

The dynamic poroelasticity theory for the porous media is derived using the method of conservation laws, which include the motion and state equations [18]

$$\frac{\partial u_i}{\partial t} + \frac{1}{\rho_s}\frac{\partial \sigma_{ik}}{\partial x_k} + \frac{\rho_l}{\rho_s\rho}\frac{\partial p}{\partial x_i} + \frac{\rho_l}{\rho_s}\chi\rho_l(u_i - v_i) = F_i, \tag{3}$$

$$\frac{\partial v_i}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x_i} - \chi\rho_i(u_i - v_i) = F_i, \tag{4}$$

$$\frac{\partial \sigma_{ik}}{\partial t} + \mu\left(\frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k}\right) + \left(\frac{\rho_s}{\rho}K - \frac{2}{3}\mu\right)\delta_{ij}\mathtt{div}\boldsymbol{u} - \frac{\rho_s}{\rho}K\delta_{ik}\mathtt{div}\boldsymbol{v} = 0, \tag{5}$$

$$\frac{\partial p}{\partial t} - (K - \alpha\rho\rho_s)\mathtt{div}\boldsymbol{u} - \alpha\rho\rho_l\mathtt{div}\boldsymbol{v} = 0. \tag{6}$$

$$u_i\mid_{t=0} = v_i\mid_{t=0} = \sigma_{ik}\mid_{t=0} = p\mid_{t=0} = 0,$$

$$\sigma_{22} + p\mid_{x_2=0} = \sigma_{12}\mid_{x_2=0} = \frac{\rho_l}{\rho}p\mid_{x_2=0} = 0,$$

where χ is the friction coefficient.

Determining the Physical Parameters for the Snow. The dynamic nature of a porous material is determined by the three elastic parameters K, μ, and α. These parameters are expressed via velocity of propagation of the shear wave c_s and the velocities of the longitudinal waves c_{p_1}, c_{p_2} [12,13].

$$\mu = \rho_s c_s^2, K = \frac{\rho}{2}\frac{\rho_s}{\rho_l}\left(c_{p_1}^2 + c_{p_2}^2 - \frac{8}{3}\frac{\rho_l}{\rho}c_s^2 - \sqrt{(c_{p_1}^2 - c_{p_2}^2)^2 - \frac{64}{9}\frac{\rho_l\rho_s}{\rho^2}c_s^4}\right),$$

$$\alpha_3 = \frac{1}{2\rho^2}\left(c_{p_1}^2 + c_{p_2}^2 - \frac{8}{3}\frac{\rho_s}{\rho}c_s^2 + \sqrt{(c_{p_1}^2 - c_{p_2}^2)^2 - \frac{64}{9}\frac{\rho_l\rho_s}{\rho^2}c_s^4}\right).$$

The velocities c_s and c_{p_1}, c_{p_2} are determined by the Biot parameters A, N, R, and Q. These parameters are calculated from the four measurable coefficients for the snow [11]. The friction coefficient is expressed by the dissipation coefficient from [11,17].

3 Algorithm of Numerical Solving

We have been used the algorithm of numerical solving of the 2D dynamic problem of seismic wave propagation in the porous medium with allowance for the energy dissipation. To solve numerically this problem, we used the method for combining the Laguerre integral transform with respect to time with a finite-difference approximation along the spatial coordinates. The proposed method of the solution can be considered as analog to the known spectral-difference method based on Fourier transform, only instead of frequency we have a parameter m, i.e. the degree of the Laguerre polynomials. However, unlike Fourier transform, application of the Laguerre integral transform with respect to time allows us to reduce the initial problem to solving a system of equations in which the parameter of division is present only in the right side of the equations and has a recurrent dependence. The algorithm used for the solution makes it possible to perform efficient calculations when simulating a complicated porous medium and studying wave effects emerging in such media.

As of result of this Laguerre transform the initial problem (3)–(6) is reduced to two-dimensional spatial differential problem in spectral domain [19]

$$\frac{h}{2}u_i^m + \frac{1}{\rho_s}\frac{\partial \sigma_{ik}^m}{\partial x_k} + \frac{1}{\rho}\frac{\partial P^m}{\partial x_i} = f_i^m + h\sum_{n=0}^{m-1} u_i^n, \tag{7}$$

$$\frac{h}{2}v_i^m + \frac{1}{\rho}\frac{\partial P^m}{\partial x_i} = f_i^m + h\sum_{n=0}^{m-1} v_i^n, \tag{8}$$

$$\frac{h}{2}\sigma_{ik}^m + \mu\left(\frac{\partial u_k^m}{\partial x_i} + \frac{\partial u_i^m}{\partial x_k}\right) + \left(\lambda - \frac{\rho_s}{\rho}K\right)\delta_{ij}\mathtt{div}\boldsymbol{u}^m - \frac{\rho_s}{\rho}K\delta_{ik}\mathtt{div}\boldsymbol{v}^m = -h\sum_{n=0}^{m-1}\sigma_{ik}^n, \tag{9}$$

$$\frac{h}{2}P^m - (K - \alpha\rho\rho_s)\mathtt{div}\boldsymbol{u}^m - \alpha\rho\rho_l\mathtt{div}\boldsymbol{v}_m = -h\sum_{n=0}^{m-1} P^n.$$

$$\sigma_{22}^m + P^m\,|_{x_2=0} = \sigma_{12}^m\,|_{x_2=0} = \frac{\rho_l}{\rho}P^m\,|_{x_2=0} = 0, \tag{10}$$

For solution of the problem (7)–(10) we used finite-difference approximation of 4th order along the spatial coordinates on the shifted grids [19]. As result of the finite-difference approximation for the problem (7)–(10) we obtain the system of linear algebraic equations in the vector form

$$\left(A_\Delta + \frac{1}{2}E\right)\boldsymbol{W}(m) = \boldsymbol{F}_\Delta(m-1) \tag{11}$$

where $\boldsymbol{W}(m) = (\boldsymbol{V}_0(m), ..., \boldsymbol{V}_{M+N}(m))$ is solution vector with the wave field components.

Finding the solution of this system of linear algebraic equations (11), we can define spectral values for all wave field components. Then we obtain the solution of the initial problem (3)–(6) using Laguerre transform.

4 Results of Numerical Simulation

In given section we present the numerical results of simulation of seismic waves fields for test model of the porous medium. The given medium is consist of two homogenous layers, namely snow (upper layer) and elastic semispace (lower layer). The physical medium's features were specified as

- snow layer with parameters $\rho_0 = 0.4\,\mathrm{g/cm^3}$ is density, $\rho_{0,l}^f = 0.01\,\mathrm{g/cm^3}$ is density, $c_{p_1} = 1.1\,\mathrm{km/s}$, $c_{p_2} = 0.25\,\mathrm{km/s}$, $c_s = 0.7\,\mathrm{km/s}$, $d = 0.5$ is porosity coefficient, $\chi = 100\,\mathrm{cm^3/(g \cdot sec)}$ is friction coefficient;
- lower elastic semispace with parameters $\rho = 1.5\,\mathrm{g/cm^3}$, $c_p = 1.2\,\mathrm{km/s}$, $c_s = 0.8\,\mathrm{km/s}$.

Wave field is modeled from point source like expansion center with coordinates $x_0 = 200\,\mathrm{m}$, $z_0 = 4\,\mathrm{m}$, situated in upper layer. Time signal in the source was given as Puzyrev's pulse with main frequency equaled 100 Hz.

$$f(t) = \exp\left(-\frac{2\pi f_0 (t - t_0)^2}{\gamma^2}\right)$$

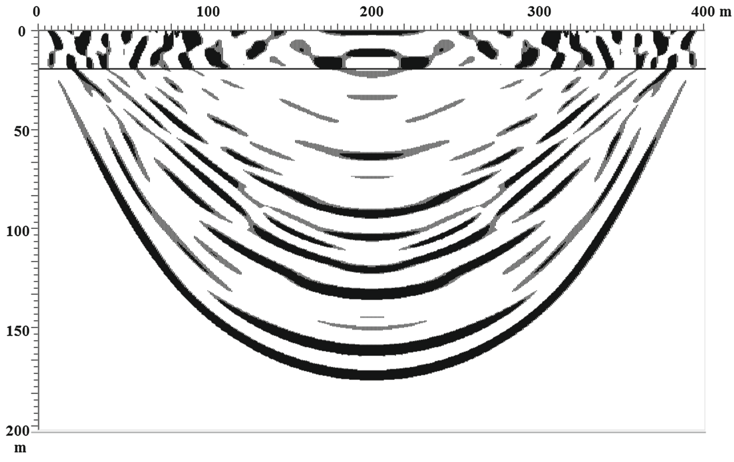

Fig. 1. Snapshot of the wave's field for vertical component of the displacement velocity $u_z(x, z)$ in the time moment t = 0.15 s. The main frequency of signal in the source is 100 Hz

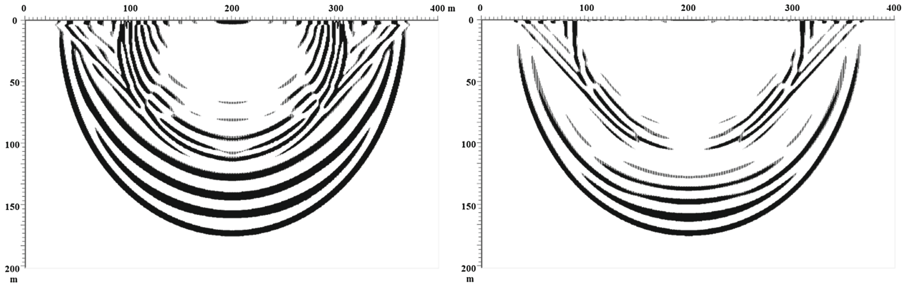

Fig. 2. Snapshots of the wave field for vertical component of the displacements velocity $u_z(x, z)$ in the moment time t = 0.15 s. On the left side without energy dissipation, on the right side with energy dissipation. The main frequency of signal in the source is 100 Hz.

The example of calculation wave's field results for different models is represented on the Figs. 1 and 2. The figure is shown that in snow's layer the multiply reflected waves arise. They generates different longitudinal and transverse waves in elastic semispace.

The Fig. 1 demonstrates snapshot of wave's field for vertical velocity component of displacement u_z in the fixed moment of time T = 0.15 s when thickness of the upper snow layer is 20 m. The Fig. 2 represents the snapshot of wave's field for vertical velocity component of displacement when thickness of the snow layer is 3 m. The right snapshot shows the wave field with energy dissipation in the snow's layer and the left snapshot presents wave field without energy dissipation. From analysis of Figs. 1 and 2. It is shown that if thickness of the snow layer less spatial length transverse and quick longitudinal wave, then presence of energy dissipation in medium significantly influences to wave field. Since the presence of energy dissipation firstly significantly influences to attenuation of the slow longitudinal waves and decreases the number of reflections in thin layers.

Acknowledgments. This work was supported by the Russian Foundation for Basic Research (grant 16-07-01052).

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