Chapter 2
An Overview of the Analysis of Jointed Structures

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The primary function of a joint in an engineering structure is to connect, usually stiffly, two separate substructures. This function is well accomplished; however, in doing so a secondary function of a joint is introduced in which it augments the dynamics of an assembled system. Joints thus introduce two features to a structure: amplitude dependent stiffness and amplitude dependent damping. With the state-of-the-art techniques, many of which are detailed in this book, the amplitude dependent stiffness can be predicted reasonably well (to within 10%); however, the amplitude dependent damping is still beyond predictive capabilities. In fact, the prediction of damping in structures (not just from joints) is the least well-characterized part of a model in structural dynamics despite it being critical to the prediction and understanding of the behavior of a structure (Akay 2015). The focus of this book is on the dynamics of jointed structures. The prediction of mechanical failure of a jointed connection is another book in and of itself.

In Part I of this book, perspectives on the need for research on jointed structures are introduced. This focuses on three areas: the current state of joints modeling and understanding discussed in this chapter, the economic ramifications of investments in joints research discussed in Chap. 3, and the challenges in developing a predictive friction model discussed in Chap. 4.

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2.1 Emergent Behavior Due to Interfaces

The amplitude dependent stiffness and amplitude dependent damping are two characteristics of the nonlinearity associated with a jointed interface. Even when these two quantities are able to be characterized, the analysis of a jointed structure is still challenging. A reasonable question to ask is, why?

The analysis of jointed structures consists of two principle parts: the deterministic/linear substructures that can be readily modeled and analyzed with existing techniques, and the often unpredictable assembly of a jointed structure that exhibits emergent behavior not observed in the deterministic substructures. To quote Wolfgang Pauli, "God created solids, surfaces were invented by the devil."

As a structure is assembled with jointed connections, emergent behavior appears—namely nonlinearities in excitation amplitude dependent stiffness and damping. This is the very notion of a complex system. In his Nobel Prize lecture, Friedrich August von Hayek said that

> While in the physical sciences it is generally assumed, probably with good reason, that any important factor which determines the observed events will itself be directly observable and measurable, in the study of such complex phenomena as the market, which depend on the actions of many individuals, all the circumstances which will determine the outcome of a process, for reasons which I shall explain later, will hardly ever be fully known or measurable.

In joint mechanics, contrary to Hayek's assertion, it is quite possible that the knowledge for fully characterizing a joint will be prohibitively difficult to obtain for modeling purposes. From experimental and numerical evidence, the nonlinearities introduced by joints seem to be dependent upon qualities introduced by the manufacturing process that cannot be measured a priori, such as the topography of the interface, the residual stress state, and the actual contact pressure field. Thus, by neglecting these poorly understood effects, epistemic uncertainty (model form error) is introduced into models. There are multiple methods, though, that can attempt to account for this type of uncertainty, which are discussed in Part V of this book and are briefly described later in this chapter.

2.2 A Brief Introduction to the Modeling of Mechanical Joints

Many individuals and institutions wrestle with the appropriate manner in which to model a jointed surface. This is a nontrivial task. A recent survey (Segalman 2013) of 20 analysts working within a single company asked "How do you model jointed connections?" The results of the survey reported more modeling approaches than analysts surveyed. As of yet, there is no consensus for the best practice of modeling a jointed connection. Some of the work detailed in this book attempts to build some consensus, but much work still needs to be done. Specific details for modeling jointed systems are presented in Parts III and IV; here, a higher level perspective is given.

A difficulty in developing a consensus for modeling bolted joints exists in the divide between theoretical and applied approaches. In academia, there are many new techniques that are discussed here within; however, these techniques are often in their infancy: significant validation and verification work is needed in addition to making them usable and efficient. It is worth noting that some companies, such as Rolls Royce, have invested a significant amount of funding to develop research codes that are now used on applied projects; however, most companies do not have similar resources and are left asking "How can a jointed connection using commercially available packages be best modeled?"

2.2.1 Analysis Levels

The first task in assessing a jointed structure is determining the strength of the nonlinearity:

1. Linear—Commercial codes can predict these responses now!
2. Quasi-Linear/Weakly Nonlinear—Research codes can do this now once calibrated.
3. Strongly Nonlinear—New tools are still needed for accurate prediction of realistic problems.

Linearity in structural dynamics is defined by the response of the structure being linearly proportional (with excitation amplitude) to the response at a low excitation amplitude. The strength of a nonlinearity is generally determined by several qualities: does the natural frequency change as the excitation amplitude is increased, does the damping ratio change as the excitation amplitude is increased, and, if the system is being driven at one natural frequency, are other modes being excited? Thus, if the excitation amplitude is doubled and the response is at the same frequencies with an amplitude that has also doubled, then the structure is linear. If the response, however, decreases in frequency by several Hz, does not double in amplitude, changes in damping characteristic, or suddenly is multi-harmonic, then that is an indication of nonlinearity. To determine if a nonlinearity is considered weak, the reader is referred to the guidelines of Eriten et al. (2013). In what follows, weak nonlinearities *do not* couple multiple modes; that is, for a weak nonlinearity, exciting at one natural frequency will result in a response at only that natural frequency. Deviations from this (i.e., multi-harmonic responses) violate the assumptions for handling weak nonlinearities in what follows.

In structures that exhibit no nonlinearity within the range of excitations considered, linear modeling approaches will suffice (so put down this book, and use a stiff spring instead). In many structures, though, there is some evidence of nonlinearity. When the nonlinearity in an assembly is able to be considered as weak, or quasi-linear, modal damping methods such as the modal Iwan model are sufficient (see Chap. 17). Two drawbacks of this modeling approach are that it assumes that there is no coupling between modes due to the nonlinearity, and that it assumes that there is no macroslip (i.e., a physical displacement or rigid body motion between two substructures, which represents a physical change in the assembled structure, as the joint is exercised) or slapping (i.e., intermittent contact between the two joint surfaces); however, it is able to capture accurately the amplitude dependent changes in stiffness and damping within the microslip regime. In practice, a jointed system in the microslip regime exhibits only a small change ($\approx 0.1\%$) in frequency as the excitation or response amplitude is varied (Allen et al. 2016). Thus, the weakly nonlinear regime is defined to end as soon as any frequency change due to increases in amplitude is observed.

The remaining case is that the structure contains a strong nonlinearity.

2.2.2 Strong Nonlinearities

Strong nonlinearities in a jointed system can entail several characteristics:

- The geometry of the system changes due to macroslip under the expected excitation levels (service conditions).
- The modes are coupled (e.g., exciting only the first mode of a structure can lead to a response of multiple other modes).
- The stiffness of the jointed interface significantly decreases.
- The damping of the jointed interface significantly increases (until macroslip, in which it decreases).

Due to the physical change in a structure from macroslip, spatial models of the joint are needed instead of a modal framework. These spatial models, by their very nature, couple the response across multiple modes. The formulation of these spatial models is elaborated upon in Part III. Less frequently observed, another nonlinear aspect in joints is an impact behavior due to slapping between the joint surfaces.

One challenge in modeling strongly nonlinear structures is that often *many* spatially discrete joint models are necessary (for instance, a convergence study might yield that a single lap joint necessitates 700 interfacial degrees of freedom for node-to-node contact, or 16 interfacial degrees of freedom for a contact patch approach). In structures with dozens of joints, this clearly is not feasible. A hypothesized modeling strategy to account for this might be:

> **The Hybrid Modal-Discrete Modeling Approach**: Account for the energy dissipation of general microslip in a modal framework. For discrete portions of the model that exhibit strong nonlinearities (including macroslip), a small number of spatially discrete joint models should be employed.

In this manner, the modal modeling approach is used to make modeling a large number of jointed connections feasible, while the small number of spatially discrete joint models accounts for the modal coupling, macroslip, and other characteristics of a strongly nonlinear system.

2.2.3 Approaches: Both Commercial and Research

For those without the resources to invest in developing modeling approaches based off the theory presented in this book, several plausible alternatives are available. In Chap. 24, results from a round-robin modeling effort are presented in which both high fidelity finite element models and reduced order models are used. The proposed modeling approach is the same for both high fidelity and reduced order modeling methods. The consensus arrived at in that chapter is that analysis is divided into three stages:

1. Nonlinear Static Solve
2. High Fidelity Contact Interface Modeling
3. Dynamic Analysis

The first step, the nonlinear static solve, is used to determine which nodes in the interface are stuck together, and which nodes are not expected to be in contact. Additionally, this step yields the strength of the contact forces N and the contact pressure distribution, both of which are used as inputs to the high fidelity contact interface modeling.

In the second step, the problem is reduced slightly by assuming that nodes that are out of contact will remain out of contact during the analysis. The remaining nodes are considered to be potentially slipping (though under certain circumstances these could be divided into stuck and slipping, in which case the stuck nodes just need to be attached with linear springs). The contact pairs identified as slipping need a joint model of some form. The simplest might be Coulomb friction; however, this neglects the tangential stiffness observed in microslip (see Part III). Commercial codes, though, should be able to specify a friction slider (also known as a Jenkins element or a Masing element for a two-dimensional representation). The friction slider, as shown in Fig. 2.1, consists of an elastic spring of strength K_T and a frictional interaction of strength $\mu N = F_S$, where F_S is the force needed to initiate sliding. This is thus a two-parameter model (K_T and F_S or μ). The normal force N can be prescribed as the contact force determined during the nonlinear static solve.

The third step is the dynamic analysis. Once the contact interface has been prescribed, the next challenge is determining the quantity of interest. For frequency responses, harmonic balance-based simulations are preferable as the frequency response will exhibit nonlinearities dependent upon the excitation amplitude. For response to shock events or random vibrations, transient simulations (also referred

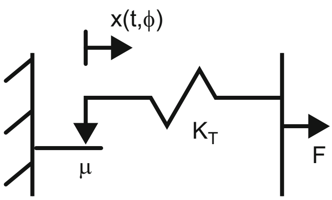

Fig. 2.1 Illustration of a Jenkins element, also referred to as a frictional slider

to as direct time integrations) are useful. A fundamental issue raised by the analysis, though, is that the quantities typically taken from a response—damping ratio, natural frequency, etc.—are all quantities formulated for linear systems. In nonlinear systems, the meaning of these quantities is less clear and can even be misleading. Alternative approaches such as amplitude dependent measures of damping and stiffness (e.g., see Chap. 21) or the nonlinear normal modes (e.g., see Chap. 30) are more appropriate.

2.2.3.1 Commercial Codes

One immediate challenge that becomes evident is that for convergence, large numbers of node-to-node contacts will need to be specified for high fidelity finite element commercial codes. Recent advances by commercial finite element packages exhibit capabilities to minimize the burden placed on the user for this modeling. Several methods have been successfully implemented on commercial codes, including:

- The use of ADAMS, a multibody dynamics code, with flexible bodies modeled in NASTRAN (Hopkins and Heitmann 2016). This approach is very efficient (simulation times on the order of minutes) for macroslip, but not amenable for microslip applications
- The use of ANSYS to introduce bolts to geometrically flat surfaces
- The use of ABAQUS, Hyperworks, or other codes with node-to-node contact and preloaded bolts (Chap. 24)

2.3 Joint Sensitivities: Reducible Uncertainty in Interfacial Mechanics

The challenges inherent in modeling jointed assemblies are due to the interfaces. Some of these challenges are due to the difficulty of directly measuring the properties of an interface (such as contact pressure, portions of the interface sliding versus sticking, etc.) during an experiment. Further complicating these challenges is the sensitivity of an assembled system to localized interfacial properties (see, for instance, Chap. 31). Aspects including surface curvature, surface roughness, residual stress, and prestress all contribute to the manner in which energy is dissipated across the interface. Many of these properties, though, are introduced by the manufacturing process. This means that in order to predict a likely response of a jointed structure, a stochastic approach will be needed as it is nearly impossible, a priori, to quantify accurately the topography and stress state of the interface. Because many of these quantities are unlikely to be directly modeled in the near-term, statistical methods are necessary to account for the physics neglected by the models.

2.3.1 Contact Pressure

Contact pressure is one of the most important derived quantities for determining the local kinematics and energy dissipation within a joint. In areas of high contact pressure, the contact force N is high, resulting in a high frictional force $f_f = \mu N$. High frictional forces lead to the interface sticking, resulting in the local kinematics being dominated by the tangential stiffness of the interface K_T. In areas where N is low, there is a low frictional force, which results in the potential for localized slipping, known as microslip. In practice, these areas of low contact pressure are found away from the bolt locations (such as detailed in Chap. 11) and dominate the energy dissipation characteristics of a joint.

The contact pressure found in an interface is sensitive to many factors, including surface roughness, surface waviness, bolt force, bolt tightening order, geometrical design of the interface, and even residual stresses. While some of these quantities might be known in advance, many (such as the machined features of a surface including its roughness and waviness) are not, even when tight tolerances are specified for the manufacturing of a system. Consequently, dramatically different pressure distributions can be achieved for surfaces that are nominally identical, as shown in Fig. 2.2.

In situations where the contact pressure can be measured and directly used in a simulation, relatively good agreement is found between the experimental and numerical results for high fidelity modeling of the interface. Given the dramatic variations observed in Fig. 2.2, it is very important to be able to measure the contact pressure of a real interface. Without this measurement, it is often prohibitively difficult for models and experiments to agree.

In Fig. 2.2, the variations in contact pressure are due to surface features (predominantly roughness, but also machining features from the fabrication process such as waviness), bolt tightening order, and bolt force. These, however, are not the only sources of influence on the distribution of contact pressure.

2.3.2 Residual Stress

Residual stresses within two components that are joined at an interface can develop as a result of multiple factors: the machining process, the original casting of the material, heat treatment, and loading history amongst other sources. Even when the surface features are well controlled for, small changes in residual stress can lead to very large changes in the contact pressures (Fig. 2.3).

These changes in contact pressure distributions are indicative of the variability observed in experiments of nominally identical specimens, as highlighted in Chaps. 9 and 10, and are also related to the sensitivity of an interface to initial conditions (see, for instance, Chap. 31). As the residual stress at an interface is

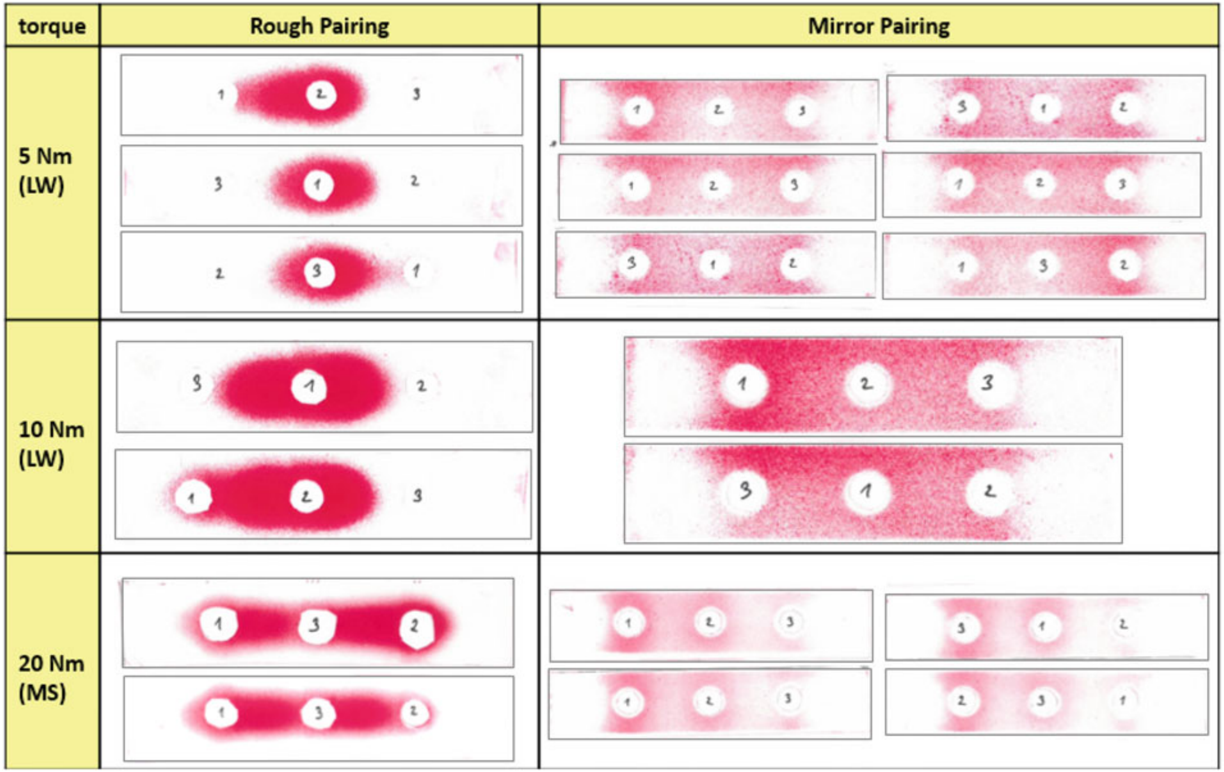

Fig. 2.2 Measured pressure distributions of the specimens with bolt tightening order indicated on each pressure distribution, which is shown for both a rough surface and a surface with a mirror-like finish

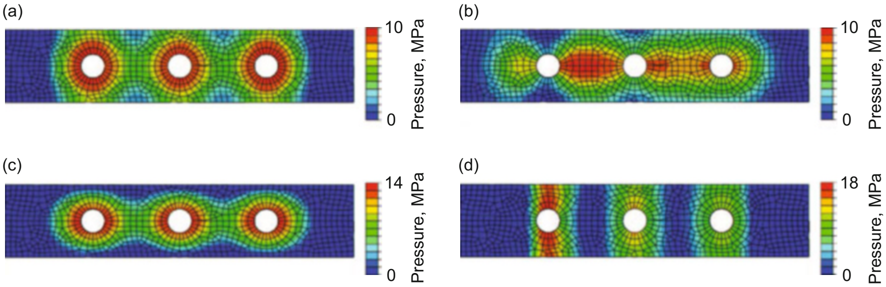

Fig. 2.3 Contact pressure in the interface for different residual stress states: (**a**) no residual stress, (**b**) axial tension, (**c**) transverse compression, and (**d**) axial compression. Image courtesy of R. Flicek and K. Moore

varied, the pattern of nodes that are out of contact, in contact and slipping, and in contact and stuck varies dramatically. This variation is directly related to the amount of energy dissipated by an interface during excitations.

2.3.3 Accounting for the Unknown in Modeling

Knowing or characterizing the sources of uncertainty in a jointed structure is not always sufficient or necessary. To develop a modeling program in which every grain near an interface is well-characterized is prohibitively difficult and should not be attempted. Reflecting back on Friedrich August von Hayek's quote in which he stipulates that some sources of the observed variations may never be characterized, it is necessary to develop modeling techniques that account for the unknown quantities (the epistemic uncertainty) within a jointed system. To that end, there are two approaches for a way forward: complexity theory and uncertainty modeling.

2.3.3.1 Complexity Theory

In complex systems theory, there is a spectrum of approaches for analyzing systems that exhibit emergent behavior. At one extreme is the use of a black box model (e.g., see Fig. 2.4), which is based on a phenomenological description of the system and often includes differential equations or stochastic elements. At the other extreme is a white box model, which is derived from logic-based rules describing the interactions between two subsystems, such as cellular automata. The current state of the art for joint modeling utilizes a gray box model, which is a hybrid between these two approaches: for systems with strong nonlinearities, the contact interface is divided into a number of nodes or contact patches (similar to the white box modeling approach of cellular automata approach of describing an interaction via a series of cells governed by logic-based rules). Each node or contact patch on

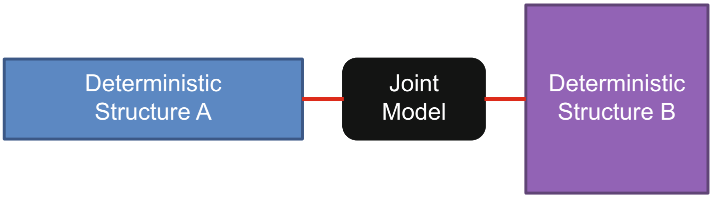

Fig. 2.4 Conceptualization of a model of a jointed system in which the interface is represented as a *black box* connecting two deterministic substructures

one subsystem is connected to a node or contact patch on the other subsystem, and a constitutive model is introduced to describe contact versus no contact and sticking versus slipping. These constitutive models, whether they are Jenkins or Iwan elements, are the black box modeling element of the system.

2.3.3.2 Uncertainty Modeling

As discussed in detail in Part V of this book, there are two types of uncertainty that are typically considered in bolted structures: epistemic and aleatoric. Epistemic uncertainty, also referred to as model form error, is the uncertainty introduced to a model by neglecting physics. This is sometimes intentional—to simplify things when the neglected physics are deemed to have a small effect, such as in the formulation of a reduced order model that neglects the Giga-Hertz response of a structure, or due to a lack of knowledge regarding the actual physics, such as in modeling interfaces as nominally flat. More often, this is done unknowingly when an important aspect of the model is neglected (e.g., the actual interface is curved, not flat as modeled, or Coulomb friction is assumed to be correct). By its nature, epistemic uncertainty is reducible: the more knowledge about this type of uncertainty that a modeler has, the more accurate that a model can be made, resulting in lower amounts of uncertainty. Aleatoric uncertainty, on the other hand, is the uncertainty associated with variations of known quantities, such as the width of a flange, the roughness of a surface, or the modulus of a material. This type of uncertainty, also referred to as parametric uncertainty, is irreducible: as more knowledge describing the aleatoric uncertainties in a system is gained, the number of sources of uncertainty in the system is unaffected, so that the uncertainty is only better characterized. Taking advantage of this key distinction between epistemic and aleatoric uncertainty—that one type is reducible while the other is irreducible—Soize developed a method to decouple these two types of uncertainty (Soize 2010), as detailed in Chap. 34.

2.4 Perspective for a Way Forward

There are many open questions facing the jointed structures community. While many challenges focus on developing predictive models of interfaces, a second set of questions and challenges deal with redesigning jointed connections to facilitate their analysis. Not listed in the following set of challenges is the need to motivate the importance of joints research, which is highlighted in the following chapters.

2.4.1 Designing a Better Joint

What if a joint was designed to have repeatable properties? Then, the surrounding structure of the joint could be optimized with those properties as a constant in order to have the desired structural dynamic properties. As of the writing of this book, though, there are no known methods for making an arbitrary joint repeatable. Several recent studies within the solid mechanics community highlight that the geometry of the interface plays a significant role and that several geometries not typically used within structural dynamics applications may be more amenable to the design of a repeatable joint [as detailed by Chap. 31 and the references included therein as well as Andersson et al. (2014)]. A significant research effort is needed, both experimentally and numerically, to investigate the effect of the interfacial geometry on the sensitivity of the joint's properties.

2.4.2 New Definitions for Better Context

As previously mentioned, many techniques used in analyzing jointed systems were developed for linear systems (natural frequency, log decrement, half power points, damping ratio, etc.). A fundamental need, going forward, is for a new set of definitions specifically for discussing stiffness changes and energy dissipation in the context of nonlinear systems. Proposed starting points include the definition of amplitude dependent curves to describe the evolution of stiffness and damping with excitation amplitude (Roettgen and Allen 2017; Kerschen et al. 2006 and Chap. 21) and nonlinear normal modes (Chap. 30 and Kerschen et al. 2009; Vakakis 1997), which are not-necessarily synchronous periodic solutions to the equations of motion. Analysis of the efficacy of these concepts for describing jointed systems and consensus on terminology to use is necessary for introducing a more sophisticated method of analyzing jointed systems.

2.4.3 Advancements in Physics for Predictive Capabilities

The ultimate goal of the fundamental research on mechanical joints is to develop a predictive model of energy dissipation within a jointed interface. Currently, though, there are no predictive models. Fundamental research is needed to determine the precise mechanisms that contribute to the behavior of a jointed interface, to determine the relative contribution of each mechanism, and to develop a theoretical connection between the mechanisms and the structural dynamics properties of a jointed interface. Part of the difficulty associated with this proposed research is that many of the mechanisms that contribute to frictional energy dissipation are coupled (e.g., see Chap. 4); that is, many mechanisms occur simultaneously and cannot be decoupled. The concept of friction as a whole encompasses many competing processes: the fracture and dislocation of grains, the interaction and fracture of asperities, elastic and plastic processes, acoustic emission, and thermal generation, amongst other mechanisms. Collaboration with tribologists and skilled experimentalists is needed in order to design well crafted experiments that isolate as many mechanisms as possible.

2.4.4 Advancements in Experimental Techniques

Advances in the understanding of the physics related to interfacial mechanics cannot come in isolation. These advancements will be dependent upon new techniques to characterize and investigate, in situ, the behavior and properties of an interface. In particular, methods to image the evolution of the contact patch and slip field with load will be instrumental in advancing our understanding of the dissipation mechanisms within a jointed interface. Some of these methods are beginning to become commercially available today, such as time varying measurements of contact pressures. Other methods, such as X-ray based digital image correlation, are in their infancy, and many methods have yet to even be formulated.

2.4.5 Advancements in Numerical and Stochastic Techniques

As discussed in Part IV, it is not enough to develop a predictive model. Fundamental work on developing more efficient computational methods and ensuring that models are *usable* is needed. In order to make a model usable, a direct connection between the parameters that populate a model and simple/common experiments is needed so that the task of describing a model is not impractical for an analyst. Further, as high fidelity computing allows for larger and more complex systems to be modeled, and as the need to consider uncertainty increases, computational techniques (such as improved reduced order modeling theories and multiscale methods) are needed to ensure that numerical simulations are useful in the design phase of a system. Further, improved descriptions of uncertainty for jointed systems are needed, and advanced sampling techniques, such as stochastic reduced order models (Field et al. 2015; Mignolet and Soize 2008), promise to reduce significantly the computational burden of assessing realistic ranges of uncertainty for high-dimensional systems.

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