Chapter 7
Thinking, Analogy, and Reasoning

Thinking is a natural phenomenon generated in the human brain thanks to its neural networks, and whose experimental and theoretic study corresponds to the neurosciences. Speaking and reasoning are but external manifestations of thinking in which both the configuration of the throat and the brain's functioning play decisive roles; if thinking is not always directed by the person, speaking and reasoning are often so. Thinking could be unconscious, but reasoning is almost always conscious and directed to a goal, and for many years attempts have been made to study and mathematically model it; the history of logic contains such an evolution from old philosophy to modern mathematical logic. Reasoning is deeply related to rationality, and at the end, the old Greek term *Logos* referred to both word and reason; we refer to conscious reasoning.

Usually, the natural interlinking of speaking and reasoning is seen as the ground on which rationality is anchored; and it was in the field of psychology where the disturbances, and illnesses, of speaking and reasoning began to be studied. The term "Logos" referred to both people's capability of conducting articulate reasoning, and the use of "the word" to speak for communicating the reasoning and its conclusions. Now, once a new theoretic presentation on how to build up complex statements by capturing their meanings is done, a moment for advancing a view on reasoning seems to have arrived.

7.1. It should be first pointed out that by "reasoning" it is here understood the kind of reasoning laypeople consciously conduct, a reasoning that scarcely shows all the characteristics of the formal and deductive reasoning mathematicians manage for proving their conclusions, the theorems constituting the corpus of mathematics. Ordinary, plain, everyday, or commonsense reasoning (for short, sometimes and afterwards, reasoning), is often not formally deductive, even if it can sometimes present some features approaching formal deduction. Nevertheless, a first characteristic of ordinary reasoning is the "unsafe" character of its conclusions, because (contrary to formal deduction among whose conclusions, or consequences, contradictory pairs of them can never be found) this is not at all rare in the ordinary

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reasoning done by people. Formal deduction is reasoning with a safety net, but commonsense reasoning is without; it is always like walking in a loose robe.

In part, it was the importance Euclid's *Elements* acquired during the Middle Ages as a model for performing correct reasoning that identified correct reasoning with the safe, and beautiful, deductive reasoning of geometry. Nevertheless, what should essentially be preserved from such a view is just the impossibility of conducting any reasoning without previously accounting for some actual information on the corresponding subject; without such information, for instance, the axioms in geometry, no geometrically interesting conclusions of reasoning are possible. Reasoning cannot be based on an empty set of informative items; to start from some premises reflecting the information, or evidence, is strictly necessary for concluding something actually significant.

The specification of the premises can come from some previous, and perhaps disorganized, thinking on a related question that can involve images, not only words; nevertheless, and in the end, the premises should be specified by words. Without specifying them by words it is difficult, if not impossible, to know if there are, or are not, contradictions among them, that is, if the starting information is liable. Reasoning is done in an environment of knowledge and inside thinking.

The set of premises is just constrained to not contain contradictions, that is, neither pairs of contradictory premises, nor self-contradictory ones, something accepted as a minimal caution for conducting any reasoning. Nobody admits that contradictions among the evidence can be accepted; before starting to reason, it should have been secured that no contradiction exists among its premises. Were a pair of contradictory premises found, at least one of them should be excluded; a previous analysis of the premises' lack of contradictions is compulsory.

The category of liable information is never assigned to a set of contradictory premises on a given subject, and in general to contradictory statements whatsoever. For instance, in a conversation on the state of today's weather, which information could actually be furnished by two statements such as "Right now there are very dense clouds," and "Right now it is totally sunny"? Who will accept these statements as an actual piece of information on the current weather's state? Contradiction is the worst sin of reasoning; it should be prohibited in, at least, the premises, and, provided two contradictory conclusions were obtained at the end of the reasoning, at least one of them should finally be rejected, and the reasoning reviewed.

Two statements p and q are said to be contradictory if the conditional statement holds, "If p, then not q", and q and p are contradictory provided "If q, then not p" holds; p is self-contradictory provided "If p, then not p" holds. Contradiction is not always a symmetric relation even if it can happen under some conditions such as those in Boolean calculus, in which $p \leq q' \Rightarrow (q')' \leq p' \Leftrightarrow q \leq p'$ holds, and reciprocally; nevertheless, it should be pointed out that this proof is made by presuming that stating "If p, then q" is represented by the lattice's order $\leq$ of the Boolean algebra in the form $p \leq q$, and that negation is strong, $(p')' = p$, for all p. Note that it will also hold provided the negation were to be weak, that is, it verifies $q \leq (q')'$, for all q. In a path towards analyzing ordinary reasoning, it seems essential to have a previous reflection on how people can obtain relationships among the meanings of the intervening words for arriving at conceptualizing them and for managing such meanings to reason. In principle, it can be said that reasoning needs a linkage of meanings with something in common, allowing a view of a real or virtual situation; when the "overlapping of meanings" is large enough, the idea of synonymy appears and shows that a reasoning by synonymy will stop. In some way, all reasoning is but a kind of chain of statements whose meanings are related in some form. Only in the deductive/proving reasoning of mathematics are the steps of such chains perfectly linked each one with the following in such a form that not a single hole can exist between them. In common reasoning, the existence of holes, or jumps, is not at all rare, and hence the relationship between its conclusions and its premises is not always clear; only in the case of formal deduction, can the reasoning be perfectly repeated step by step by another person.

If common reasoning could be compared with a house that is badly constructed without the help of an architect, and without taking too much care of its sustaining structure, formal deduction is comparable to a house perfectly well constructed by a competent architect. In the first, removing a single brick can cause the failing of the house, but in the second and for its failing, removing much more than a brick is necessary. Formal deduction requires a perfect enslaving of statements, but in the ordinary forms of reasoning such a kind of chain is often far from being perfect.

People's reasoning is, very rarely, of a totally deductive type, even if people try, in what it is possible, to approach formal deduction; at the end, formal deduction is but a way to control the effectiveness of a reasoning. The uncontestable fact is that formal deduction is the safest type of reasoning, but it requires symbols working in an as "natural" as possible mathematical framework. Hence to attempt a symbolic study of ordinary reasoning allowing its classification in its diverse types and, at the end, clarifying what reasoning actually consists in, it is necessary to know previously how reasoning is initially fueled, and which are its main natural engines.

7.2. Analogy seems to be the first natural engine facilitating reasoning. Analogy is based on the brain's human capability for capturing partially overlapping resemblances among the real, or virtual, objects that are taken into account; it is the linkage of such overlapping meanings that permits reasoning.

Note that thinking is larger than reasoning, because it generates and comprises, for instance, wishing, imagining, memory or storage of information, emotions, dreaming, mixing of old images, and so on, all of them useful for doing reasoning. If thinking is possible without articulate language, conscious reasoning is not, and analogy moves between thinking and ordinary language, inasmuch as without previous conceptualization and naming concepts, it seems very difficult for analogy to support reasoning extensively.

Speaking a language of which reasoning is the most important feature is shared by all people on Earth. From childhood, people simultaneously learn to speak well one, two, or even more languages, to appreciate metaphors, to establish analogies and express them with words, to manage conditionals, and so on. With all this, people learn how to reason, and learn to feel perplexed by an absurd conclusion;

they informally learn how dangerous a contradiction is, and that one should flee from it. The simultaneous learning of both language and reasoning is, in good part, a social phenomenon whose physical possibility lies in that of thinking which all sane children naturally have. All that becomes as it actually appears, due to the brain's physical configuration and functioning that controls the body, allows speaking and thinking, as well as their manifestations such as singing and reasoning, and even enjoying or hating something.

It should be pointed out that one of the main goals of reasoning is satisfactory support for the taking of decisions, and particularly to decide "intelligent" actions; reasoning is basically an important and natural tool with which people are endowed for their survival, not only of each person but, and essentially, of the species *Homo*. This is the reason for considering that the essential characteristic of *Homo*, differentiating its members from those in other animal species, is rationality, the *Logos* externally expressed by telling stories, posing questions, guessing answers, and by computing. Our only way of knowing how to do things, how things are, and how situations will evolve, is reasoning; its possibility is of the greatest help for satisfying human curiosity towards knowing and understanding what is still unknown, or not yet understood, for foreseeing and for answering the questions people pose continuously. Paraphrasing Albert Einstein, not to stop questioning is the human characteristic on which all progress is based. Note that in this book, "intelligence" only appears sometimes as the adjective "intelligent," and just for signaling decisions leading to actions beneficial for both the person and society. Intelligence is a concept referring to apply rationality in the best form possible and, for instance, if the capability for computing were considered an important attribute of intelligence, today it is by and large surpassed by computing machines. The great technological problem is how to endow computers with rationality, with *Logos*.

7.3. Referring to human rationality requires jointly considering conceptualization, or categorization, and analogy, because both permeate language and reasoning. If concepts are abstract products of thought, analogy is what allows their linking, the establishment of relations between them that is the essence of reasoning. But neither is reasoning just analogy, even if analogy seems to play in it the character of an unavoidable natural mechanism, nor can reasoning be identified with "thinking" that comprises aspects only indirectly related to reasoning, for instance, imagining and remembering, to say nothing of the sometimes relevant influence emotions and previous beliefs have on reasoning. In any case, memory or storage is basic for both conceptualization and analogy, that is, for organizing in the form of concepts or categories what is being known, and for establishing some relationships with what was known earlier and with the external reality.

Without some relationship to reality, knowledge risks being insufficient for both acting and taking decisions; additionally, it is important to be conscious that any current knowledge, at least of the world, is not forever, but that it has a caducity date that is not previously known as it is with medicines. Knowledge is not preserved like medicines in a closed container; knowledge is not fixed and isolated; it is always in flux; it varies with time.

The often-repeated linguistic expressions "of the same kind of", or "like that", already announce in language the relevance of analogy in creating "categories" or "collectives", naming concepts, the basis of, at least, not spontaneous reasoning. It seems that comprehending something new lies in the possibility of comparing it with what is already comprehended, or believed to be so; namely, in making analogies with past experience or with already acquired knowledge. Denoting or labeling concepts, or categories, by words, that is, by "naming" them, is essential for remembering, managing, and relating concepts; naming nonempty categories is an efficient way to help people's reasoning.

For comprehending a new concept, or to find a new one, or to pose a question, or to capture some unobvious relationship, the analogy with something previously known or firmly believed seems to be a step forwards either good enough, or maybe necessary; it is, perhaps, for this reason that analogy plays so an important role in questioning. Posing questions, and guessing their answers through analogous former situations, are typical expressions of rationality. At the end, in a rough summary, science consists in a specialized and ordered guessing in particular domains, aiming at solving previously identified problems whose solution could be stated in the conditional form "*provided* this, *then* that". Foreseeing, essential for surviving, is almost always done in conditional terms; for instance, weather forecasting is always done under the antecedent of what is currently known about the atmospheric situation, and it is due to the antecedent's variation that weather forecast failures lie.

Thinking is a wide natural phenomenon whose functioning, produced by neural connections in the brain, still remains partially unknown, and neuroscientists are today searching for the role language plays in reasoning. Actually, and from what has recently been discovered, it seems that relational thinking is what supports natural reasoning jointly with how emotions, intentions, and desires can direct it, and even if forms of spontaneous thought are being explored. For many years, thinking has been recognized thanks to some of its signs such as, in a first instance, the capability of organizing knowledge in categories, of comprehending and managing conditional statements well, and the linking of categories to reach conclusions, reasoning, in sum.

Hence analogy is an important subject to reflect on how concepts are generated, on how they can be scientifically domesticated, and on how analogy initially helps thinking with the kind of organization in which reasoning consists. In this respect the following imaginary example can be in order.

7.4. In the beginning, when primitive people started to manage stones for building walls preserving them from predators, they, at least in some way after several trials, empirically realized that stones were of different forms, sizes, and weights (even without having these concepts/words), and that it was better to place those that were bigger in size and height in the wall's first level for grounding the wall properly in the land. In this experiential way, and after trial and error, the concept of "big" was born, and it soon was applied to materials other than stones, such as pieces of wood obtained from trees, as well as to the same trees, walls, animals, and the like. The category *big*, referring to stones, walls, trees, animals, and the like, could then be managed to facilitate communication. That is, the meaning of “big” appeared for the first time allowing the people to say, for instance, that some lake was big by comparing it with others that were able to be skirted around in fewer days; possibly, this also led to taking into account the concept of the “size” of lakes, trees, animals, hills, and so on, and the new and bigger category *size* was born. Possibly, also the qualitative concept of the “weight” of stones, animals, pieces of wood, and so on, jointly appeared; it only turned to be quantitative when, fueled by trade, the first mold for grains, or the first balance, appeared.

It seems that concepts don’t appear isolated from each other, but linked in the form of chains of concepts keeping between them some empirically perceived relationships. Maybe some concepts, appearing from the direct management of physical objects, are either followed, or intermingled, with other concepts coming from the management of the former ones; this can be, for instance, the cases with *length* and *weight* if derived from the pair *big* and *small*, and allow the introduction of “large” and “middle” size, for instance. In ways like those, the use of the concept *big* spread all over language thanks to the analogies established by comparing between them several different families of objects, and, perhaps, also originating other concepts such as *size*. Furthermore, and perhaps by comparing with the naked eye the height of trees, hills, and mountains, the concepts of *high* and *top* appeared, in short, playing some Wittgensteinian sort of language game with words.

It seems difficult to believe that a concept could appear isolated from others, and it is more credible that concepts are generated inside a more or less organized conglomerate of old and new concepts that migrate between several universes of discourse. If it is very difficult, if not impossible, to establish the genealogy of a given particular concept, what seems without doubt is that its genesis is grounded on experience and analogy; once experience is presupposed, rationality is not even imaginable without the capability of making analogies, of appreciating some family resemblances. It is like in the movie *2001: A Space Odyssey*, in the beginning of which the monkeys imagine that some bones can be used for fighting against their enemies by beating their heads the way they beat a stone, and showing how the first weapon appeared.

Very often, something is considered to be understood when both the similarities and dissimilarities it shows with something else previously known are captured; the roots of understanding and creativity are immersed in analogy. The perhaps false story of Newton’s apple, and also that of the Einstein lift, are but examples of it.

Those simple and imagined cases with primitive people just try to illustrate how categorizing could empirically have been started and, once connected by analogy, fuel reasoning. That is, helping the acquisition of reasons/arguments enough for acting or deciding to do or not to do something, such as to place a settlement at the border of a big lake and at the top of a hill, for counting with both water and fish, and the possibility of observing, with enough time to react, if dangerous animals, potential enemies, or just unknown people, approach the settlement, as well as for doing the necessary constructive actions for building such a settlement, in sum, constituted conjecturing or refuting the adequacy of placing the settlement there. Note that, in the first place, it implies managing conditional statements well such as, "If the settlement is placed here, then it is possible to foresee the danger that could come from an unexpected approach," not implying, of course, the impossibility of placing the settlement elsewhere. Indeed, conditional statements play a central role in reasoning.

For mentally establishing a commonly shared concept, a previous group's experience with elements of some first family of objects in a universe of discourse interesting for the group is required; later on, the newly acquired concept migrates to be applied in other universes by analogy. Hence, it is important trying, more or less formally, to consider as an initial point what concerns the meaning's representation of imprecise concepts (such as they are "big", "high", etc.); they actually permeate plain language and ordinary reasoning. The goal of its scientific domestication as done before, is actually relevant. Note that concepts are singularized, even isolated among them, and also remembered, thanks to their names; they are commonly recognized, jointly with their opposites and negations, thanks to the names, or linguistic labels, or predicates, that once stated, help to avoid the confusion their mixing could produce. Predicates P, through the elemental statements "x is P", allow capturing the meaning of concepts; it is in this sense that each predicate can be seen as the "mother" of a corresponding concept, and that overlapping of meaning and synonymy are seen as important linguistic phenomena.

7.5. Either a kind of Platonic precedence of concepts over previous experience with physical or virtual samples for their application, or the possibility of a strictly private reasoning on them, before their naming or their inclusion in language, is something illusory and always dangerous. Concepts can only be communicated after counting with an initial meaning. The before-mentioned elemental statements "x is P", once intercommunicated among a group of people, are what allow the common comprehension of the concept, and its use in complex statements, after the group's members share some common meaning of them. Only after some concepts can be managed by recognizing "this is that" can abstract reasoning start. There cannot be human rationality without meaning; semantics and rationality are strongly intermingled. Semantics, the essence of understanding, comes from observing and analyzing what is external to people; from, for instance, distinguishing between a lake and a river by describing, after being acquainted with them, what is a lake and what is a river, which characteristics distinguish one from the other, passing, for instance, from "these lakes" to "lake".

Human beings need intercommunication, sharing concepts and reasoning, accepting or denying them; they are social beings who are not only directed by genetics but, also, by a common culture of which language is a manifestation helping communication by designating concepts by words that allow us to share, modify, and improve them. Each concept charges with a cultural back for the members of a more or less culturally homogeneous group of people, and it is often modified through the typically social experience provided by talking; for instance, as neuroscientists observed, it is easier to reason with familiar than with unfamiliar concepts. In the case of an unfamiliar concept, analogy permits a first approach to what it means.

It should be pointed out that almost all those concepts, if not all of them, are of an *imprecise* character; that is, they cannot be expressed/captured by *if and only if* definitions, as they are the *precise* concepts of mathematics, for example, "prime number", "continuous function", and so on.

Nevertheless, concepts such as the mathematical ones only can appear once the capability of *abstracting* is acquired from experience, and from something stored in the memory with the aim, for instance, of clarifying a subject through its representation in a formal setting, and perhaps for being able to compute some of its aspects with the help of an artificial language based on some new but related and stringently defined precise concepts. This is possibly how the surveyors at the Nile in ancient Egypt went from the cord with 3, 4, and 5 knots to mark rectangular triangles in the land to Pythagoras' famous theorem. Note that theorems are, essentially, conditional statements the validity of whose consequent depends on that of its antecedent; for instance, Pythagoras' theorem holds on a Euclidean plane but not on a spherical surface. Indeed, in some way it "defines" the Euclidean plane.

In any case, the way leading from ordinary to abstract-formal reasoning is a long one, even if it were noticed that the abstraction capability is manifested from the very beginning, because without it neither imagining nor understanding a conditional statement nor that of making any analogy is actually possible. Abstraction and imagining seem to be the brain's capabilities developed, for instance, through mental processes such as that of passing from "this stone" to (the category) *stone*, and that in no way is limited to formal abstraction. For instance, just arriving at the category *bird*, and stating, "Almost all birds fly," already requires some abstraction; without a minimum capability of abstraction, reasoning does not seem actually possible.

7.6. All that jointly proceeds with the human capability for questioning; posing and trying to answer questions seems to be the reasoning task. In this sense, reasoning can even be seen as a "mechanism" directed to answer questions. It is for this that *Homo Sapiens* could be better known as *Homo Quaerens* (the seeker and the sought), the current *Homo* species able to pose questions expressed by words. There are other animals that also "know", but they seem unable to pose questions to themselves, and especially complex questions articulating chains of simpler questions; questions coming from a deep curiosity seem to be a strong intellectual engine of reasoning. Without categorization, analogy, abstraction, and good management of conditional statements, complex questioning coming from dynamical situations does not seem actually possible, at least in forms intellectually fruitful, whose answers can allow foreseeing and surviving in a partially unknown environment, or concerning difficult problems for whose solutions good guessing, and some suitable kind of proving conclusions, is necessary.

Language, abstraction, and questioning are what facilitated our ancestors in arriving at fiction and especially at "telling fictions", and at "talking on fictions", or at "writing fiction", through imagining virtual, not always properly physical, entities and situations. These imaginations, sometimes transformed in myths, are what allowed Homo Quaerens, over the last 70,000 years, to cooperate effectively by associating a number of them surpassing the small number of those that can be directly acquainted with some concepts, questions, guessing, and answers. That does not seem to be much greater than around 100 people, a more or less big herd. In such a way, commerce, religion, law, social and political organizations, science, and especially the capabilities of transmitting information and foreseeing, were born. This seems to be the cultural evolution giving to Homo Quaerens its singular and marvelous capability for questioning and answering with fruitful conjectures and, in consequence, innovating and even showing creativity. Without such capability, the species surely would either disappear, or remain bound into regions where predators were kept under sufficient control, with food and water enough to survive, more or less what currently happens with other mammals, gorillas and chimpanzees that, for instance, neither spread around the planet, nor show but a minimum innovative capability, nor are able to write; they seem to lack the word, although they can learn some words once a person teaches them.

Along the intellectual processes leading to know deeply and to innovate, it cannot be forgotten that recognizing contradictions also plays an important role; if it can be said that a person grows intellectually through successive infections of contradiction, it is clear which benefits recognizing contradiction facilitate mankind, and which advantages of escaping from contradiction add to Homo Quaerens' life; there is no doubt that the appearance of a contradiction makes mentally sane Homo Quaerens stop any reasoning and that it seems to be a central part of an abstract formal study of reasoning. When the Pythagorean philosophers believed that the length of the diagonal of the square contradicted their belief that it should be a fraction of its side's length and that, consequently, such length cannot be a number, they just hid the result, and it took many centuries to recognize that "irrational" entities such as $\sqrt{2}$ are but numbers.

The history of mathematics, and also that of science, contains many cases in which the final task was to surpass (apparent) contradictions by constructively creating new formal entities, new symbols endowed with a new meaning, permitting the guessing of new questions, and opening new frontiers to research. Because without real and complex numbers, differential and integral calculus would not exist, current science is indebted to the passing over of rational numbers done by introducing the irrational ones. It can be said that if the positive integers are "found" in the world and were abstracted by the necessity of counting, the other numbers are a human creation that needed former knowledge and abstraction, some previous mathematical operations, and representation frameworks for it, for instance, quotients for the rational and square roots for the first irrational numbers, to say nothing of transcendent numbers such as π and e. Without formal abstraction numbers would not be known. It is nevertheless obvious that people don't like to continue when actual, or believed, contradictions appear; everyone considers that contradiction is not acceptable.

What marks the basic jump in the way to arrive at formal abstraction is the *representation,* in a suitable framework, of concepts by symbols endowed with meaning. Representation is a concept older and wider than in a mathematical setting alone, as shown, for instance, by the old representations of animals and people in the pictures on the walls of caves or, in a conversation, drawing in sand a shape by helping to graphically show some aspects of something. But a representation's frame, be it a wall or the sand, is always necessary.

Representation is inherent to reasoning, and the formal case itself requires several previous analogies, abstractions, managing of conditionals, and formal constructions; for instance, when representing a sinusoidal curve in a Euclidean plane, even if it is a smudged one, the mathematical plane should have been previously constructed by abstraction of an analogy with a flat surface, and the name of the function by an abstraction by analogy with a physical figure, and so on. It does not seem that a so-relevant concept for "reasoning on reasoning" as is that of *representation,* could be deeply analyzed without, at least, taking into account *meaning, conditionality,* and *analogy.* The possibility of "representing" is crucial for articulating ordered reasoning, and representing something by symbols abstracting some features of the real things is still more important.

Additionally, it should be pointed out that even based on analogy, reasoning is not a uniform mechanism but one that specializes itself by attending each wished goal. For instance, the deductive reasoning of the mathematical proof is a specialization, done in a previously constructed formal framework and with an invented artificial language, directed to check that either a conjecture, or a refutation, can be actually accepted in the corpus of mathematics, and defining the inferential relation through a special interpretation of the meaning of conditional statements. Reasoning continuously refines itself by more reasoning, leading to the adoption in each case and through several abstractions of some special organization that allows the advance of knowledge on the territory to which it is applied, such as with reasoning on reasoning.

This view is a good reason for, directly or indirectly, trying to learn from a first-class mentor when, for instance, either one decides to prepare a PhD dissertation, or to become a sculptor, a painter, a writer, a musician, a singer, and so on. In all of these cases, and apart from learning the necessary technicalities, there is the strictly important aspect of learning how to recognize "good" questions, the form of actually facing them, looking for antecedents further than technicalities but in the context in which questions are posed, seeing interesting analogies, figuring out their answers, capturing when an answer could be seen as possibly fertile, and the like, in sum, by acquiring what is to count with a clinical eye on the corresponding topic.

All that requires big doses of analogy, which is only possible to learn in close contact with the best real praxis possible. A "good office" is always acquired by intensively working as an apprentice in the workshop's team of a great master artisan, seeing many times the difficulties appearing in both the design and along the process for arriving at the end, and, especially, keeping permanently excited, worried, and critical for all them. After graduation, specialized forms of reasoning require an apprenticeship period; good researchers fight with, and against, the work of the greatest scientists. Parodying the words of the great geometer Luis A. Santaló, a scientist only can be considered so if, from time to time, he or she publishes original, new, and interesting results; at least those that are neither original, nor new, are useless; they don't contribute to advance knowledge or practice.

7.7. With all that has been said, there lacks the addition of a short comment on formal deduction, that is, on the deduction done in a formal framework, the reasoning whose conclusions, or consequences, are attainable by means of algorithms reproducing, step by step and without any jump, the passing from a statement to the next one by using the rule of modus ponens.

As said above, this type of reasoning is the safest one, but it does not mean that everything that can be posed in a formal framework can be attained by an algorithm, as shown by those mathematical problems having been formally proven to be undecidable, that is, impossible to neither prove them nor their negation. In those problems and in each case, some solution can be envisaged by guessing at it, but it cannot be reached deductively. Not all that is thinkable has, notwithstanding, the possibility of being deductively found and hence for the advancement of knowledge, guessing is necessary; without conjecturing and refuting, without guessing, no knowledge seems to be possible. The human mind's capability of reaching solutions by deduction is, sometimes, blocked; something that forces us to be humble, and paraphrasing Albert Einstein, accepting that what is actually marvelous is not only formally to deduce some conclusions, but also formally to prove deductively that there are cases in which this is not possible.

For all that, some kind of black over white symbolic representation can be necessary, but it should be taken into account that, as with visual monochromatism, intellectual monochromatism is dangerous for interpreting realities inasmuch as things are seen in color and, at least, greys are important. Not only is gradation basic, but considering the particular situation in which something is going on is essential when attempting to construct a model for it.

The kind of intellectual achromatopsis consisting in reducing situations to only a model in black over white, can lead to simplifications that do not allow us to appreciate deeply all that is actually there, and can limit the model's usefulness; this is what Lotfi A. Zadeh tried to avoid in 1965 with the introduction of fuzzy sets.

Nevertheless, starting from a simplistic black over white description can sometimes help to pose some problems with an initial and sufficient clarity for a subsequent advance of thought.

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