Chapter 1
Introduction

It can be said that reasoning introduces some kind of organization in thinking for directing it to a goal. What is required for a theoretical study of ordinary reasoning is to establish as simple and natural a framework as possible, in which symbols represent what corresponds to the setting into which the subject under study is inscribed. Without "representation" nothing of a scientific type seems to be possible. Science deserves representation and measuring, and from the very beginning of the scientific revolution (and even before), symbolic representation seemed to be a good instrument for acquiring scientific knowledge through measuring.

But symbolic representation and measuring need a mathematical framework, and not all mathematical structure is adequate to a given subject; nobody will try to study thermodynamics in, for instance, only the formal framework of a finite geometry, without mathematical analysis or experimentation.

1.1. Hence finding a suitable, or natural, framework for a subject is of paramount importance for, at least, not adding laws that cannot be immediately presumed for it. The relatively natural character of a particular mathematical framework depends on the kind of problems that are posed, and whose solutions are not only analyzed but processed. For instance, the framework of a Boolean algebra is a natural one for the reasoning dealing with precise statements, in which it is supposed that all the needed information on them is, in principle, available, and all the laws of such algebras are valid because each precise predicative word specifies a subset (is represented by it) in the corresponding universe of discourse. Inasmuch as its complement represents the negation of the word, each precise predicative word produces a perfect classification of the universe of discourse, and composed precise statements also produce partitions of such a universe. Boolean algebra is the undisputed natural framework of representation for computing with precise words; it facilitates a suitable calculus for mimicking reasoning with them.

Notwithstanding, and because neither the distributive laws, nor the law of perfect repartition, nor the equivalence between contradiction and incompatibility, are valid for reasoning on quantum physics phenomena, their statements cannot be

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represented by sets but by functions in a Hilbert space whose structure is weaker than that of sets. Hence Boolean algebra ceases to be the natural framework for such a kind of reasoning, and weaker algebraic structures, such as orthomodular lattices, were taken for it instead of Boolean algebras. Analogously, Boolean algebras are not a natural framework for the analysis of the reasoning involving both precise and imprecise statements, in which the last cannot be specified by sets but by the membership functions of fuzzy sets. With them, for instance, the negation is not a "Boolean complement", and neither the principle of contradiction, nor that of the excluded-middle, can always be supposed to hold as they do with sets. No Boolean algebra and no orthomodular lattice can be taken as a natural framework for it even if, in some special cases, the weaker framework of a De Morgan algebra can be suitable.

Hence, for the analysis of ordinary reasoning that comprises all particular types of reasoning, a natural mathematical framework necessarily should be extremely weak and, in particular, weaker than Boolean and De Morgan algebras, than orthomodular lattices, and than the weakest standard algebras of fuzzy sets.

It should be pointed out that in the methods for "logically proving", such as the first one established in 1934 by Gerhard Gentzen, and called Gentzen's natural system, Boolean laws are not explicitly formulated, but implicitly used; these systems are for ruled deduction, that is, for formal deduction without jumps and whose conclusions are but "logical consequences". That is, its set (following from that of the given premises) satisfies the properties of a compact Tarski operator of consequences characterizing formal deduction. In ordinary reasoning it cannot be presumed that statements perfectly classify the universe of discourse; things are more complex, imprecision is pervasive, and degrees should be considered. Nevertheless, and for all this, arriving at a "computing" with both precise and imprecise words requires a calculus rooted in a very general and weak mathematical framework, able to be compacted in a stronger mathematical structure in some particular cases, for instance, to a Boolean algebra when the subject is computing with precise words, or to a De Morgan algebra in some particular cases with imprecise words.

The mentioned Boolean algebra, De Morgan algebra, orthomodular lattice, and standard fuzzy algebra frameworks are but models for the corresponding types of reasoning, and, as such, are just simplifications of the actual reasoning by trying to take into account those of its characteristics considered to be the basic ones. Of the four models, only the first can be considered definitive, because it reflects formal reasoning with precise words well; this formal reasoning is exactly what is done when doing such kinds of proving reasoning. For a reasoning that can be conducted with pencil and paper, to speak metaphorically, there is nothing else; the model is the reasoning and hence the frame is a fully natural one. Nevertheless, in the cases of De Morgan, orthomodular, and standard fuzzy algebras, the model is still provisory in the sense that there is no general agreement on its full suitability for the corresponding subjects and, for instance, some logicians prefer to use weaker structures than orthomodular lattices for the analysis of reasoning on quantum physics. Note that one thing proves that "this" follows from "that" (the so-called context of proof), but that a very different one is to find an unknown “that” to be proven after knowing the “this” (context of search). In any case, the “that” and the “this” are, respectively, the reasoning’s conclusion and premise.

A model can be seen as the mockup an architect makes to visualize in three dimensions how the true building finally will be, or as a map that does not contain all that is in the mapped land, yet allows finding its places. But, neither the mockup nor the map is exactly the building or the land; they are just simplified representations of them. A basic theoretical task consists in finding a model allowing us to, at least, present the essential characteristics of what is being modeled, and allowing us to foresee more things; a model should arise from some knowledge of the corresponding subject, and a good one also allows finding novelties.

In the case of ordinary reasoning, models should arise from the currently scarce knowledge of it. Which is such knowledge, and how can it be improved and extended? This question could be seen similarly to how visual knowledge of the heavens in Copernicus’ time passed to that after the telescope and Galileo, to Newton–Leibniz’s invention of differential calculus and Newton’s mathematical theory of gravitation, and, perhaps finally, to Einstein’s mathematical model of general relativity, which today nobody can predict whether it actually models the full macrophysical reality, even if experimentation seems continuously to confirm its validity.

Each time, frontiers for new theories and for new experiments (usually also helped by new technology) either confirming or falsifying the former knowledge with their measurements were open. This is the form in which science typically advances through a continuous interlinking of mathematical models, technology, and experimentation, but without stating that things can be, definitively and exactly, identified with the model. Models are engines driving the only safe type of knowledge of the reality, the always uncertain scientific one; the rest is but metaphysics just based on pure abstract thinking in which the meaning of the employed words is taken as something universal and without considering their possible measurability. It is a kind of elemental reasoning coming from the old times where primitive metaphors were taken as reality, but not as ways for just reflecting on them, and that should be embodied and clarified in a theory of ordinary reasoning; it is a type of reasoning by analogy that, fueled by ordinary deduction, is able to create the brain’s images and is very useful for producing emotion, exciting sentiments, and provoking ordered speculations. If it does not properly serve to describe reality deeply, it has, nevertheless, the power of directing the intellect towards creativity, the “last mystery” in the words of the writer, Stephan Zweig. Many of our current concepts have their roots in metaphor, and the metaphysic mode of reasoning is not at all contemptible.

1.2. To construct a theory on the meaning words show in plain language, let’s say their linguistic meaning, it should be taken into account that meaning is not universally associated with a word in itself, but that words are usually context-dependent and purpose-driven. For instance, in the context of the positive integer numbers, “prime” has a precise meaning given by an “if and only if” definition, with a purpose restricted to the field of arithmetic, and related to expressing all natural numbers by the product of their prime divisors; but in a context of people the meaning of “odd” cannot be defined in such a precise form, it can only be described as a synonym of rare-person and perhaps used with some denigrating, or hilarious, purpose. It is not with the same meaning that it is said, “Seven is odd”, that “He is odd”. Nevertheless, in the setting of the positive integer numbers, “odd” has a precise meaning given by an if and only if definition, and, without a previous definition of what a “very odd number” can consist in, it is not possible to state and understand, “This number is very odd”, although it is not necessary to add anything to the use of odd with either people or houses, and so on, to recognize immediately what the statement, “Such person/house/… is a very odd one”, describes in a given context.

A look at a dictionary shows that it contains words mainly belonging to three categories: those whose use is imprecise in a given context and precise in another, those whose use is always precise, and those whose use is always imprecise, the second category of which contains a relatively small number of words. The meaning of words is always contextual or situational; precise words abruptly change their meaning when the modifier “very” affects them, and the imprecise ones are those that, once their meaning is captured and, if affected by “very”, the meaning of the new expression is immediately captured. Without taking into account the purpose for its use, precise words have a rigid meaning, but that of the imprecise is flexible and how they are recognized after being affected by the linguistic modifier “very” seems to indicate that, at least for imprecise words, there are some qualitative variations of their use, and going from less to more, that the meaning of imprecise words is not static but flows in the universe of discourse.

As Ludwig Wittgenstein wrote, “The meaning of a word is its use in language”, and, for instance, “odd” is used in the language of arithmetic by using it according to the rule, “n is odd” $\Leftrightarrow$ “the rest of dividing n by 2 is 1”, but “big” is used in the interval [0, 10] according to the rules, (1) x is “less big than” $y \Leftrightarrow x \leq y$; (2) 10 is maximal for the relation “less big than”; and (3) 0 is minimal for such a relation. These rules allow several interpretations, such as the rigid one “x is big” $y \Leftrightarrow 7 \leq x$, and its flexibility is additionally shown by the fact that if x can be qualified as big, there is $\varepsilon > 0$ such that all numbers between x and $x + \varepsilon$ can also be qualified as big; if “7 is big”, then also “7 + 0.0001 is big”; with it, any rigid use of “big” should be avoided. Often, the use of words can be described by instances of their application, or by rules (not always precise ones) describing how they can be used. Note that 5 is an odd number, but that adding unity to it (the smaller positive integer) what is obtained, 5 + 1 = 6, is not an odd number; odd is not flexible among the positive integers.

Without capturing the linguistic relationship “less than”, or its inverse “more than”, it is unknown how the word’s application flows along the universe of discourse, how its application to the elements in the universe varies, or how it is imprecisely used. Without knowing the prototypes (or maximal elements) the universe contains, and the antiprototypes (or minimal elements), provided they were to exist, it seems difficult to have instances of the total verification, or total nonverification, of the property to the elements the word names, and there is a lack of information on the word's use. Maximal refers to the nonexistence of elements showing more what the word carries, and minimal to the nonexistence of elements showing it less; without capturing the variations from less to more, prototypes, and antiprototypes, the meaning of a word is not well captured. For instance, understanding the use of "big" in [0, 10] as given by the set [7, 10], those elements in [0, 7) are antiprototypes of big, and those in [7, 10] are prototypes; the word's use is precise, and there are no more than prototypes and antiprototypes of it. Nevertheless, as shown, the word *big* has many imprecise uses in [0, 10].

Meaning is always related to a given universe of discourse X; hence to study the linguistic meaning of a word P, it is necessary to consider both the pair (X, P), and that P names a "property" that is recognizable, at least empirically, for the elements x of X; P is a predicative word in X. The word P says something about the elements in X that is translated by the elemental statements "x is P", resuming the information carried by P on X, with which the meaning of P flows along X, and eventually showing some vortices of concentration (the prototypes) and some of dissolution (the antiprototypes).

Once this can be formally established, the pair constituted by X and the relationship "less P than", allows us to define what a measure for the meaning is, as shown in what follows. When the use of P in X is precise, or rigid, there are only prototypes and antiprototypes, and the relationship "less P than" collapses with that "equally P than".

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