Optimization and Simulation of Fuel Distribution. Case Study: Mexico City

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Abstract In this chapter, the combined use of optimization and simulation in the design of a distribution network for hazardous materials in the northern region of Mexico City is assessed. A mathematical programming model was developed to allow for fuel dispatch truck allocation, minimizing the total distribution cost. Heuristics were used to solve the model and different simulation scenarios were applied to do what-if analysis to be able to decide on different managerial situations. Reviewing simulation and optimization results, an appropriate estimate of the fuel quantity to order (EOQ), the best type of truck to carry out the supply, as well as the ordering schedule that minimizes the associated costs of distribution and inventory, is provided. This real-life Mexican case study shows how a combined optimization-simulation approach, specifically taking advantage of heuristic methods to diminish computing time, can provide a practical, efficient and flexible tool for optimization assessment in operational research.

1 Introduction

Fuel supply has been studied since 1959 when Dantzig and Ramser publish *The Truck Dispatching Problem*, assessing the optimization of the routing of vehicles transporting gasoline from a terminal to different service stations. Since then, a

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variety of bibliographical material on optimization and simulation of fuel supply has been published, most of them on optimization of the distribution from production sites to refineries, as well as from refineries to mayor storage terminals, mostly by pipelines.

For fuel distribution from a minor deposit or distribution terminal, as Dantzig and Ramser, most of the authors consider trucks that can dispatch part of its load at different service stations. However, due to ruling standards, in México only trucks without compartments and with only one valve are allowed, changing the nature of the fuel distribution problem.

A Mexican company that distributes gasoline in the north of Mexico City using C3 type trucks $[63]$ $[63]$ having a 20 (exactly 20.108) $m³$ capacity, wants to know if the inclusion of T3-S3 and T3-S2-R4 type trucks $[63]$ $[63]$ with capacities of 45 (46.149) m³ and 60 (61.504) $m³$ respectively, will minimize the distribution costs given a constant demand. The previous problem corresponds to a Designing Distribution Networks (DDN) problem, where the main goal is to distribute the fuel in the cheapest possible way. Figure 1 represents the problem graphically.

This chapter is organized in the following way: Sect. [2](#page-2-0) addresses the theoretical background on designing distribution networks for hazardous materials and inventory optimization and management, Sect. [3](#page-8-0) presents the used methodology and Sect. [4](#page-10-0) shows the observed results, including data collection, determination of the distribution costs and model formulation and results.

The goal of this study is to optimize the distribution network of a hazardous material for a company that operates in the north of Mexico City, allowing the use

Fig. 1 Representation of the study problem

of a heterogeneous fleet, through a mathematical programming model and simulation.

As specific objectives, the following can be mentioned:

- Collection of information on and statistical analysis of the present state of the fuel supply system in the Azcapotzalco territorial delegation.
- Estimation of fuel demands and distribution costs for the service stations in the Azcapotzalco region.
- Construction of a mathematical programming model to be able to obtain a good solution for fuel ordering quantity, periodicity and the type of truck to be used, considering the collected and estimated information in the previous steps.
- Selection and simulation of different scenarios representing possible critical situations.
- Evaluation of the proposed scenarios by determining the corresponding performance measures, to be able to define possible improvements that can be implemented in the fuel supply system.

2 Theoretical Background

The optimization of vehicle routing and scheduling problems has been studied extensively in specialized literature. This kind of problems aim at establishing the best possible way to distribute products and goods from an origin node to a destiny node, considering changes in the network structure, satisfying the customer demands and minimizing the total costs. This cost is usually expressed in terms of transport costs, inventory costs, opportunity costs, investment, and locationallocation costs.

2.1 Designing Distribution Networks

The models for designing distribution networks are composed of several sub-problems to be optimized. The main ones are: location, allocation, routing, and inventory; different models result when variables are static or dynamic, deterministic or stochastic, discrete or continuous, among others.

The design of the distribution network considers different types of decisions, as for example the location of the elements of the network, fleet dispatching, client and provider assignment, inventory and routing management [[8\]](#page-29-0). Each of these decisions can be optimized independently or jointly. For example, the *vehicle routing problem* (VRP) combines the decisions of selecting the best route and client assignment with homogeneous or heterogeneous fleet [[15,](#page-29-0) [31](#page-30-0), [49\]](#page-31-0). The *location routing problem* (LRP) combines the decision of locating and assigning clients to distribution terminals [[35,](#page-30-0) [45,](#page-31-0) [90](#page-33-0)].

Min et al. [[57\]](#page-31-0) present the origin and evolution of LRP problems, including different mathematical formulations; they present different LRP classifications based on the number of deposits to locate, demand variations, vehicle number and capacities, distance between nodes, time restrictions or the form of the objective function. Routing and locating models for real-life problems are reported by several authors; see for example [[1,](#page-29-0) [5,](#page-29-0) [6](#page-29-0), [9](#page-29-0), [10](#page-29-0), [12,](#page-29-0) [14,](#page-29-0) [24,](#page-30-0) [29](#page-30-0), [30](#page-30-0), [33,](#page-30-0) [38,](#page-30-0) [44,](#page-31-0) [50](#page-31-0), [52](#page-31-0)–[54](#page-31-0), [56](#page-31-0), [60,](#page-31-0) [62,](#page-31-0) [67](#page-32-0), [69](#page-32-0), [72](#page-32-0), [73,](#page-32-0) [75,](#page-32-0) [87](#page-32-0)–[89](#page-33-0)].

Problems studying inventory control and vehicle routing jointly are known as *inventory routing problems* or IRP [[23,](#page-30-0) [35,](#page-30-0) [42,](#page-31-0) [45](#page-31-0), [90](#page-33-0)]. IRP problems are closely related to *vendor managed inventory* (VMI) problems, having the following characteristics: inventory levels are monitored by the vendor, which decides order quantity and moment, and if shortage of stock is allowed.

At present, models have been developed that consider at the same time decisions on localization, routing and inventories [[1,](#page-29-0) [13\]](#page-29-0). However, the high complexity involved in solving the complete problem with a sole algorithm originated the formulation of models that solve the problem in stages, in order to find a good solution in the smallest possible computer time. These methodologies involve exact or heuristic algorithms to solve the required decisions. For example, this is the case in the study presented by Flisberg et al. [\[27](#page-30-0)] where an exact solution algorithm is proposed to obtain vehicle flows whereas the TABU search method is used in a second step to find optimal routes towards the clients. The use of matheuristics for solving different types of vehicle routing problems, making use of mathematical programming models in a heuristic framework, is assessed in the interesting review presented by Archetti and Speranza [\[3](#page-29-0)].

Different kinds of transportation networks include direct shipping, milk runs, crossdocking and tailored networks [[18\]](#page-30-0). The direct shipping network delivers products from suppliers to their customers and is the one used in this study.

2.2 Models for Designing Distribution Networks for Hazardous Materials (DDNHM)

Since the publication of *The Truck Dispatching Problem* [\[22](#page-30-0)], several studies have been published on the optimization and simulation of fuel supply; see for example [\[77](#page-32-0)] or [\[58](#page-31-0)].

According to Winkler [\[91](#page-33-0)], the fuel distribution process consists of three steps. The first step includes the distribution from the extraction and/or production plant to the storage terminal, the second step corresponds to the transport of the fuel from the storage terminal to the retail customers (in this case the service stations), while the third and last step corresponds to the distribution to the final client, being cars and/or trucks in this case.

The project presented in this study focuses on fuel distribution in the second stage, that is, from the storage terminals to the retail customers, service stations or petrol stations. In this stage, distribution is carried out by fixed-capacity trucks, as specified in corresponding regulations.

The work of Çetinkaya et al. [[16\]](#page-29-0) shows that fuel truck dispatch policies for stock replacement can be carried out regarding two metrics, based on quantity or time. Results showed that truck dispatch based on required quantity provides higher savings in transport costs. In this study, truck dispatching is thus planned based on quantity and using the EOQ inventory model.

Chopra [[17\]](#page-30-0) considers the parameters associated with the designing of the distribution network to be directly related to the customer's necessities and the costs needed to implement the network. The first of them include the response time, the variety and availability of offered products, post-sales services, etc. The latter involve the holding costs, transport costs, costs of physical installations and the associated cost of the information system used.

The study by Flisberg et al. [\[27](#page-30-0)], mentioned before, presents a truck dispatching problem where daily routes of woodworking trucks deliver to a combination of clients using heterogeneous fleet and taking multiple planning horizons trough mathematical programming and TABU search.

An analysis of literature in the field shows that one of the heuristic algorithms more frequently used to solve the optimization of distribution networks is GRASP (see for example [[26,](#page-30-0) [70\]](#page-32-0)) in combination with mathematical programming. Table 1

Author	Model
Sousa et al. [83]	GRASP
Corberán et al. $\lceil 20 \rceil$	GRASP
Sombat $[81]$	EOO, GRASP. IRP
Boudia et al. [11]	GRASP
Martí et al. [55]	GRASP
Shen et al. [78]	GRASP, IRP
Hamdi-Dhaoui et al. $[36]$	GRASP. VRPC, ILS. ELS
Crainic et al. $\left\lceil 21 \right\rceil$	VRP. GRASP
Dubedout et al. $\lceil 25 \rceil$	GRASP, IRP
Lacomme et al. [48]	VRP. GRASP
Sombat $[82]$	GRASP, IRP, VLSN, EOQ

Table 1 Most relevant studies for the optimization of distribution networks

shows the most relevant studies that optimize the DDM for different products, specifically hazardous materials.

In this study, the GRASP heuristic was initially explored as the solution method for the optimization model; however, due to the specific nature of the problem where ordering quantity is limited by the used storage tank sizes, feasible solutions are only very small proportion of all possible solutions. As unfeasible solutions increase drastically when the size of the problem increases, the GRASP heuristic would not be time efficient in this study. Still, it was considered the basis of a problem tailored heuristic.

2.3 DDNHM Model Construction

As presented by Chopra [\[17](#page-30-0)], the basic components of a DDN model are:

- Localization of the network elements
- Inventory management
- Fleet design
- Vehicle routing

Reyes et al. [\[71](#page-32-0)] propose the development of a distribution network in three phases: diagnostics of the distribution system, design of the logistic network and implementation of the network. Each of these phases includes a series of steps, as shown in Table 2.

Phases	Steps		
PHASE I: Diagnosis of the	Step 1: Inventory of the existing equipment		
distribution system	Step 2: Obtaining information on the current organization of the distribution system		
	Step 3: Graphical description and map analysis of the territory of the study object		
	Step 4: Description of the existing route		
	Step 5: Feasibility study		
	Step 6: Temporal analysis of the distribution system		
	Step 7: Analysis of the demand by segment and customers		
	Step 8: Cost analysis		
PHASE II: Design of the logistic	Step 9: Description of the proposed route		
network	Step 10: Analysis of the feasibility of the design		
	Step 11: Development of the information system		
PHASE III: Implementation of	Step 12: Implementation of the new logistic network		
the network	Step 13: Measurement and analysis		

Table 2 Procedure to construct a distribution network

2.4 Inventory Management and Optimization

Inventory management is defined as the inventory planning and control carried out to meet competitive priorities of the organization $[47]$ $[47]$. Taha $[85]$ $[85]$ states that the inventory problem consists of keeping in stock just enough articles to satisfy fluctuations of the demand, based on an inventory policy that answers the question of how much and when to order.

Different models have been presented for the optimization of inventories, including models based on dynamic programming [\[4](#page-29-0)], linear programming models [\[41](#page-31-0)], non-linear programming models [[76\]](#page-32-0) and geometric models [[46\]](#page-31-0). Dynamic programming of inventories is based on the minimization of production, retention or holding costs [[28\]](#page-30-0). The Wagner-Whitin algorithm is a classical dynamic programming model that minimizes the fixed ordering and linear procurement and holding costs, over a finite horizon, providing good results [\[37](#page-30-0)]. Non-linear programming models to mathematically optimize inventories are proposed by [[2,](#page-29-0) [43](#page-31-0), [46](#page-31-0), [76\]](#page-32-0). These models look for the optimal ordering quantity by optimizing the EOQ model.

To know the behaviour of the demand it is necessary to analyse it statistically and know if it is deterministic or stochastic $[92]$ $[92]$; a suitable criterion is the variation coefficient (VC) introduced by Silver and Peterson [\[80](#page-32-0)]. The VC is determined by Eq. (1), where σ is the standard deviation and μ the mean value of the demand.

$$
CV = \frac{\sigma}{\mu} \tag{1}
$$

If its value is below or equal to 0.2, the data has a low dispersion with respect to the mean value, indicating that the demand can be considered to be deterministic. In the opposite case, it is stochastic. For stochastic demands, a goodness-of-fit test must be carried out to determine the corresponding type of distribution [[2\]](#page-29-0).

Taha [\[85](#page-32-0)] distinguishes four types of cost related to inventory problems, being the acquisition cost, preparation or ordering cost, retention or holding cost and the stockout cost (see Table 3).

Cost type	Definition
Acquisition cost	Unitary price of an inventory product
Ordering cost	Fixed charge due to placing an order, regardless of the ordered quantity
Holding cost	Costs due to having a certain level of existence during a specific time-period; these include the opportunity cost of the inverted money, the storing cost (rental fees, heating, illumination, refrigeration, security etc.), depreciations, taxes, insurance fees, deterioration and obsolescence [59]
Stockout cost	Penalty incurred when the company runs out of a product of the inventory. It includes the loss of income, production disruptions, transaction costs to replace inventory and loss of customer's goodwill

Table 3 Types of inventory costs

Different methods of inventory replenishment exist; their application depends on the used inventory model, as the demand is the leading factor for replenishment. Examples of procurement systems are given by the Wilson equation (EOQ), the Wagner-Within dynamic programming procedure and the Silver-Meal heuristic, being the EOQ model one of the most extendedly used, as it can be adapted for both deterministic and stochastic demands [\[4,](#page-29-0) [79](#page-32-0)]. In this study, the EOQ model will be applied.

2.5 Simulation

An optimization model is useful to establish the best possible way to distribute products and goods from an origin node to a destiny node; however, in real-life situations observed parameters and or variables change constantly, in which case the proposed schedule must be adjusted. A simulation analysis that compares two different possible scenarios is a cost-efficient and cheap way to decide for one of two future options before these changes take place.

Simulation is the process of reproducing the features, appearance and behaviour of a real system. It is based on three ideas: (1) Imitate, with a mathematical model, a real situation, (2) Study the model's operative characteristics and the system's expected properties making analogies within the simulation model; and (3) Make conclusions and take actions in the system based on the results obtained in the model [\[37](#page-30-0)].

Simulation studies are developed in three research levels: descriptive, exploratory and explanatory. Different authors apply the methodology proposed by Law [\[51](#page-31-0)], consisting of the steps shown in Fig. 2; see for example [\[19](#page-30-0)]. After formulating and planning the study, the data is collected and the model can be constructed. If the model is shown to be valid for the study system, it is implemented in a computer program and should be verified. Experimentation is done for different scenarios of interest; finally, output data is analysed and interpreted.

Fig. 2 Simulation methodology, adapted from Law [\[51\]](#page-31-0)

3 Methodology

The problem addressed in this study is a Design Distribution Network problem where the optimized variables are inventory level, optimal ordering quantity, fleet size and vehicle quantity, as well as order scheduling, corresponding the latter to fleet dispatching or management; a one year planning horizon is used.

Specialized literature describes problems that optimize the previous concepts as inventory routing problems with fleet dispatching. This study presents a special case where each vehicle coming from the storage terminal visits only one client (service station), as the distributed fuel cannot be discharged into fragmented batches due to legal regulations. Reducing the fuel discharge to only one service station by vehicle, the problems seems to be simplified considerably; however, even this special case remains to have an important combinatorial of solutions and therefore stays highly complex.

In the presented case study, both service stations and distribution terminal pertain to the same company, so no out of stocks are considered. Supplied quantities are governed by the vehicle size and required filling level, causing small residuals to exist for technical reasons at the end of the year. These residuals will always be smaller than the truck capacity and are assumed to be transferred without any problem to the next planning horizon. In consequence, demand and supply are assumed constant and always satisfied. According to the information above, this study designs a distribution network with the lowest possible operational costs considering constant fuel demand without stock disruption, heterogeneous fleet, continuous inventory review policy, and fixed capacity of vehicles and storage tanks.

To solve the problem, a methodology of nine steps was used, including the tailored heuristic solution algorithm and mathematical programming.

- Step 1: At first, a *data collection* was carried out to obtain existing sales information for the three fuel types considered in the study. Information was found for two service stations; consistency of data was analysed. Storage tank size was obtained for all service stations. Missing information was estimated.
- Step 2: The *behaviour of the demand* was analysed for the existing information, including variability, normality and distribution parameters.
- Step 3: Based on the previous information, *demand estimation* was done for the rest of the service stations in the study region, considering demographic information and service station characteristics.
- Step 4: *Distribution costs* were determined, including holding costs, variable and fixed transport costs and ordering costs.
- Step 5: A *mathematical programming model* was developed to describe the truck assignment problem in the study problem.
- Step 6: The state space was downsized before solving the model heuristically, so that only *feasible solutions* would be analysed. Truck combinations were restricted to the existing storage tank sizes in each service station. This,

in fact, is an optimization step in the solving process, as solution are found in a more efficient and quick way.

- Step 7: *Programming* of the linear programming model and its heuristic solution algorithm in R.
- Step 8: Determination of specific *simulation scenarios* representing possible critical situations, used to compare different management policies for truck assignment in the fuel distribution.
- Step 9: Determination of cost performance measures to *evaluate* the proposed scenarios and definition of possible improvements to be implemented in the fuel supply system.

Figure 3 represents the study methodology graphically.

Fig. 3 Study methodology

4 Case Study

4.1 Description of the Problem

In the México City greater area four fuel Distribution Terminals (DT) can be found, being three of them located in Mexico City and the fourth one in the State of Mexico (EDOMEX), from where three types of fuel (gasoline A, gasoline B and Diesel) are distributed towards 371 service stations (SS) [\[64](#page-31-0)] in one of the 16 political delegations and some municipalities in the EDOMEX state.

In a pilot phase, this study was carried out in the Azcapotzalco political delegation, located in the northern part of Mexico City (Fig. 4). This delegation has approximately 400 000 inhabitants and one of the DT is located in this area, supplying 18 service stations. Each of the service stations has implemented an inventory review and control system that provides a forecasting method and a weekly ordering schedule into maintain an appropriate service level for fuel consumers, both people and industries.

At present, the Azcapotzalco distribution terminal is using a homogeneous fleet with a 20 m^3 capacity to provision periodically fuel to each of the service stations. The company wants to know if total costs can be minimized when using a heterogeneous fleet and optimizing the supply frequency for the different service stations.

Fig. 4 Study area. Adapted from: [http://www.mapa-mexico.com/Mapa_Ubicacion_](http://www.mapa-mexico.com/Mapa_Ubicacion_Azcapotzalco_Mexico_DF.htm) [Azcapotzalco_Mexico_DF.htm](http://www.mapa-mexico.com/Mapa_Ubicacion_Azcapotzalco_Mexico_DF.htm)

4.2 Data Collection

4.2.1 Monthly Sales Information

Monthly sales information was available from January 2014 until October 2015 for service stations SS1 and SS6; this information is presented in Table 4.

As can be seen in Table 4, SS6 sells about double the quantity of fuel for each of the three fuel types. In both service stations, the most sold fuel is gasoline A, which determines about 53% of the total sales. This information is consistent with information reported by INEGI [[39\]](#page-30-0) from where it can be determined that average sales per service station for the same period in the Mexican republic were respectively 271.26, 61.27 and 332.54 for gasoline A, gasoline B and diesel. The specific quantity sold in each service station depends on its size and correspondingly on the number of hoses installed for each type of fuel.

In service station SS6 two values were missing for the diesel sales; a simple average of the two closest values was used to estimate these missing values.

	SS ₁			SS6		
	Gasoline A $(m^3/month)$	Gasoline B (m ³ /month)	Diesel $(m^3/month)$	Gasoline A $(m^3/month)$	Gasoline B $(m^3/month)$	Diesel $(m^3/month)$
Jan 2014	333.16	50.86	210.46	663.25	102.92	427.79
Feb 2014	314.15	52.85	203.07	618.09	96.85	430.44
Mar 2014	339.81	55.73	242.77	679.91	110.69	516.37
Apr 2014	319.84	51.15	213.92	622.61	104.36	454.36
May 2014	344.65	57.20	232.58	687.75	104.36	490.70
Jun 2014	324.78	53.33	228.97	648.80	106.42	NA
Jul 2014	322.36	54.64	240.94	635.28	105.56	502.76
Aug 2014	332.63	59.46	212.36	662.18	112.38	469.20
Sep 2014	325.78	56.52	216.90	652.32	107.25	456.31
Oct 2014	339.96	61.07	232.38	679.04	113.79	504.48
Nov 2014	318.11	57.96	213.37	632.71	104.01	450.75
Dec 2014	318.61	63.26	208.06	666.96	119.38	418.25
Jan 2015	306.74	56.88	190.22	636.68	117.50	434.75
Feb 2015	299.33	54.56	181.12	597.82	106.95	429.72
Mar 2015	329.73	62.70	201.74	650.40	123.67	471.84
Apr 2015	305.77	60.82	181.87	586.13	118.42	495.90
May 2015	325.38	65.44	199.77	621.87	125.79	501.20
Jun 2015	325.12	65.03	191.08	614.57	121.96	488.31
Jul 2015	332.40	65.26	198.83	629.44	129.38	506.51
Aug 2015	322.23	69.93	177.31	625.63	137.74	479.99
Sep 2015	331.59	67.29	147.94	637.38	147.12	494.85
Oct 2015	357.04	74.91	192.00	678.59	151.80	NA
Average	325.87	59.86	205.35	642.15	116.74	471.22

Table 4 Monthly sales for SS1 and SS6, January 2014–October 2015

4.2.2 Behaviour of the Demand

The service stations are franchises of the same company that owns the fuel supplier (DT), so they are supposed never to run out of stock. In consequence, the monthly demand for both stations SS1 and SS6 are matched to the monthly sales presented in Table [4.](#page-11-0)

Similar behaviour is observed for fuel demand at different service stations, despite differences in quantities sold (Fig. 5). Of the three fuel types, only gasoline

B is presenting a marked trend in both service stations (Fig. [5b](#page-12-0)). This was corroborated determining the corresponding linear regression models and hypothesis tests to verify if the slope is statistically significant. Calculations in R show slope and p-values as presented in Table 5.

As can be expected from graphical results presented in Fig. [5b](#page-12-0), gasoline B demand has a statistically significant positive slope in both service stations, while diesel has a statistically significant negative slope or slight level change in service station SS6. To carry out normality tests, the corresponding tendency was removed, presenting the following averages and standard deviations (Table 6).

Analysis of the demand behaviour was carried out through the determination of the variation coefficient (VC), as defined in Eq. ([1\)](#page-6-0). Variation coefficients between 0.041 or 4.1% and 0.073 or 7.3% (0.12 or 12% for non-corrected values) were observed for the three types of fuel in SS1 and SS6; all values were below 0.2 so, although slight level changes and/or trends were observed, the demand can be considered deterministic.

Finally, normality was tested with the Jaque-Bera statistic adjusted to small samples; corresponding p-values and conclusions for each of the demand series are presented in Table 7.

	Series	Slope	p-value
SS ₁	Gasoline A	0.041	0.928
	Gasoline B	0.863	4.85e-08
	Diesel	-2.570	0.0001
SS ₆	Gasoline A	-1.281	0.169
	Gasoline B	1.964	$9.63e-08$
	Diesel	1.265	0.224

Table 5 Slope and p-value for the hypothesis tests on slope significance

Table 6 Corrected averages and standard deviations for fuel demand, SS1 and SS6

	Gasoline A		Gasoline B			Diesel			
	Average	SD	CV	Average	SD	CV	Average	SD	CV
SS ₁	325.87	13.26	0.041	50.86	$^{\circ}2.96$	0.058	226.09	16.11	0.071
SS ₆	642.16	27.37	0.043	95.92	7.04	0.073	473.12	30.37	0.064

Table 7 Jaque-Bera normality test results for fuel demand in SS1 and SS6

All series can be considered to have a normal distribution, unless gasoline B in SS6, for which the hypothesis test was rejected at a 5% significance level. However, even for this type of gasoline, normality is accepted at a 3% level.

4.2.3 Demand Estimation

As the first step in the determination of the demand for the other 16 SS, Voronoi diagrams [\[34](#page-30-0), [40](#page-30-0)] were implemented to delimit the areas to be supplied [\[65](#page-31-0)] for each of the service stations; they capture information on the proximity of a set of points P decomposing the plane in convex polygonal regions. AutoCAD tools were used to define these areas, whereas INEGI [[39\]](#page-30-0) information was used to obtain the corresponding number of inhabitants. Land use classification of the service stations was obtained from SEDUVI [\[74](#page-32-0)].

Figure [6](#page-15-0) represents respectively the political divisions in Azcapotzalco (a) and the corresponding Voronoi polygons (b). The red dots indicate the location of the service stations.

Table [8](#page-16-0) shows the resulting Voronoi area for each service station, as well as the corresponding number of inhabitants and land use type.

The Voronoi diagram method supposes that customers will get their fuel supplies in the establishment closest to their domicile. However, as an important difference exists in inhabitants registered in residential and industrial areas, the number of inhabitants resulted not to be a suitable measure to determine the demand; using it as a proportionality coefficient to estimate the demand in each SS, industrial areas would have an artificially low demand as a low number of inhabitants can be expected. On the other hand, land use analysis indicates that both SS where information exists are located in areas classified as mixed residential.

An additional proportionality coefficient was needed, so SS1 and SS6 demands were compared regarding the number of hoses installed for each type of fuel; the results are presented in Table [9.](#page-16-0) The standard deviations for gasolines A and B can be considered statistically equivalent in both service stations, being mean demand per hose slightly lower for gasoline A in SS1 with respect to SS6. The demand of gasoline B per hose can be considered statistically equivalent in both stations. The standard deviation for the diesel demand is almost two times higher in SS1 with regard to ES6, while the mean diesel demand per hose is also higher in SS1. This higher variability in diesel demand can be explained by the proximity of ES6 to the industrial areas, where a more constant diesel demand is expected. The previous analysis shows differences in the demand per hose in both service stations; however, values are of the same order of magnitude, so the number of hoses for each type of gasoline installed in the service station in combination with the information presented in Table [9](#page-16-0) will be used to estimate demands in the other 16 service stations.

The number of serving hoses for each type of gasoline and service station was obtained from information provided by PROFECO [[68\]](#page-32-0), visual inspection in a field visit and/or photographical analysis in Google Street View [[32\]](#page-30-0); the results are

Fig. 6 Azcapotzalco political delegation. **a** Political divisions and **b** corresponding Voronoi polygons

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Service station	Voronoi area	Nr of inhabitants	Land use
SS ₁	1.88	11534	Mixed residential
SS ₂	1.18	1598	Industrial
SS ₃	1.399	10501	Residential
SS ₄	1.435	14856	Mixed residential
SS ₅	2.068	29838	Residential
SS ₆	1.469	348	Mixed residential
SS ₇	0.536	6302	Mixed residential
SS 8	1.975	2650	Residential/commercial
SS ₉	2.262	429	Industrial
SS 10	1.322	18603	Industrial
SS 11	1.987	16248	Residential
SS 12	2.1	36381	Residential
SS 13	1.017	25008	Mixed residential
SS 14	2.12	14190	Residential
SS 15	1.42	30845	Residential
SS 16	1.145	16257	Residential
SS 17	1.164	10796	Industrial
SS 18	0.881	3800	Industrial

Table 8 Voronoi area, number of inhabitants and land use type for each service station

Table 9 Number of hoses and corresponding demands (l/hose) for SS1 and SS6

		Gasoline A	Gasoline B	Diesel
SS1	Number of hoses	h	6	\overline{c}
	Average demand (l/hose)	54.06	9.86	102.99
	Standard deviation (l/hose)	2.21	1.06	11.43
SS ₆	Number of hoses	11	11	
	Demand (l/hose)	58.22	10.46	94.24
	Standard deviation (l/hose)	2.49	1.32	6.25
	Average estimated demand (1/hose)	56.14	10.16	98.62

presented in Table [10](#page-17-0). Values equal to 0 correspond to service stations that do not sell the corresponding gasoline.

Multiplying the average demand per hose (Table 9) by the number of hoses (Table [10\)](#page-17-0) for each type of gasoline and service station, the estimated demand can be found (Table [11](#page-17-0)). The demand of SS2 calls the attention; it is considerably higher than the demand in the other service stations due to its location in the biggest industrial area of Azcapotzalco.

4.2.4 Storage Tank Size

As the ordering quantity for each gasoline type and service station depends on the size of the storage deposit, deposit sizes were obtained for the 18 service stations in study. Table 12 presents the corresponding information. Values indicating $(*)$ correspond to estimated values, considering the estimated demand. Values of 0 indicate the station does not sell diesel.

Note that the presented values correspond to the tank size, but not to the maximum tank capacity. Due to security regulations, fuel volume in the storage tank should be maximum 90% of its nominal capacity. All storage tanks have an overfill valve installed [[61,](#page-31-0) [66](#page-32-0)].

4.3 Determination of the Distribution Costs

The costs that must be considered in the distribution network include holding costs, transport costs and ordering costs.

Service station	Gasoline A (m^3)	Gasoline B (m^3)	Diesel $(m3)$
SS ₁	50	40	40
SS ₂	80	50	$120^{(*)}$
SS ₃	40	40	$\mathbf{0}$
SS ₄	120	60	$\mathbf{0}$
SS ₅	50	50	$\mathbf{0}$
SS ₆	80	40	40
SS ₇	80	60	$\overline{0}$
SS 8	100	100	$60^{(*)}$
SS ₉	100	100	$\mathbf{0}$
SS 10	50	40	$\mathbf{0}$
SS 11	60	60	60
SS 12	50	40	40
SS 13	100	100	$60^{(*)}$
SS 14	100	100	$\mathbf{0}$
SS 15	160	80	$\mathbf{0}$
SS 16	200	80	60
SS 17	100	50	$60^{(*)}$
SS 18	100	50	$60^{(*)}$

Table 12 Storage tank size for each type of gasoline and service station

4.3.1 Holding Costs

Benitez [\[7](#page-29-0)] considers among the holding costs rates for physical storage, return on capital detained in stock, insurance, transport, manipulation and distribution of material and finally obsolescence of the material in stock. Tawfik and Chauvel [\[86](#page-32-0)] consider holding costs are generally between 14 and 36% of the mean valuation of the stocked products. In the case of fuel service stations, the material is discharged directly in the storage tanks, so no intern transport costs must be considered. On the other hand, the obsolescence concept is not applicable in fuel supply. Accordingly, in this study the holding cost, C_{hik} , is considered as a 20% rate of the cost required to acquire the average monthly demand, being the latter half the ordered demand. Considering a purchase cost, C_{p_i} , of 10 MXN/l, the holding cost for fuel *i* in service station *k* is:

$$
C_{h_{ik}} = 20\% \cdot C_{p_i} \cdot \frac{Q_{ik}}{2} \quad \forall i, k \tag{2}
$$

where

 $C_{h_{ik}}$ Holding cost for fuel *i* in SS *k*, MXN/month C_{p_i} Purchase cost for fuel *i*, MXN/m³

Purchase cost for fuel *i*, MXN/m³

 Q_{ik} Demanded quantity of fuel *i* in SS *k* in each order, m³

4.3.2 Transport Costs

Transport costs include fixed and variable components. Variable components are directly proportional to the distance between origin and destiny, in addition to taking into account the type of merchandise transported and its weight and volume. These costs change depending on the road type, and if the transport is long range or short range. Fixed costs include purchase costs of the fleet, salaries, driving licenses, insurance, installations for maintenance workshops and parking lots, taxes and recovery of financial capital. Information provided in a report presented to the Ministerio de Transportes y Telecomunicaciones in Chile [\[84](#page-32-0)] suggest that fixed costs are about 125% of the fuel cost. No specific information was found for Mexico; as an approximation, the average obtained in the above transport report will be used in this analysis.

Considering that each fuel truck is supplying only one service station in a round-trip, variable fuel costs per trip were determined for three types of trucks *j*, having capacities of respectively 20, 45 and 60 $m³$, as follows:

$$
C_{vijk} = \frac{2d_k}{R_j \cdot \eta_j} \cdot c_D \quad \forall \ i, j, k \tag{3}
$$

where

Cvijk Variable transport cost for fuel *i*, truck type *j* and SS *k*, MXN/order

- *dk* Distance between the DT and SS *k*, km
- *Rj* Fuel consumption rate of truck type *j*, km/l
- *η^j* Performance efficiency of truck type *j*, %
- c_D Required fuel cost for the trip, MXN/l

Table 13 shows the specifications considered for each of the transporting units. The trucks run on diesel; for the diesel cost, a value of 13.77 MXN/l was used (diesel cost in México in August 2016).

For the fixed transport costs, average values of distance, fuel consumption, efficiency and trip number were considered for each service station, in accordance with the demand obtained in Table [11](#page-17-0) and the tank capacity of the service station (Table [12\)](#page-18-0), resulting in:

$$
C_{f_k} = 1.25 \frac{2\overline{d}_k}{\overline{R}_j \cdot \overline{\eta}_j} \overline{n} \cdot c_D = 1.25 \frac{2(5)}{(2.42)(0.7)} 623 \cdot c_D = 63300 \text{ MXN/year} \tag{4}
$$

The average number of trips per year, \overline{n} , was determined including information on required trips for the three fuel types sold in each of the service stations. The amount determined by Eq. (5) is for the whole service station and has to be divided by the number of trips carried out per year in each service station, n_k , to obtain the fixed cost per trip; n_k depends on the obtained supply schedule.

Considering both fixed and variable transport costs, the total transport cost can be determined thus by:

$$
c_T = C_{f_k} + \sum_{j=1}^{3} n_{ijk} \cdot C_{v_{ijk}} = \frac{63300}{n_k} + \sum_{j=1}^{3} n_{ijk} \cdot \frac{2d_k}{R_j \cdot \eta_j} \cdot C_D \tag{5}
$$

where n_{ijk} is the number of trips carried out per order for fuel *i*, truck type *j* and SS_k .

Table 13 Data sheet for the 20, 45 and 60 m³ capacity used by the transporting company

	Truck type		
Specifications	20 m^3	45 m^3	60 m^3
Truck type	3 ^C	$T3-R2$	$T3-S2-R4$
Minimum fuel consumption rate (km/l)	3.66	ND.	ND.
Real fuel consumption rate (km/l)	2.95	2.48	1.83
Performance efficiency $(\%)$	0.8	0.65	0.65
Model	Freighliner M2 35k	Freighliner Columbia	Freighliner Columbia
Motor type	MBE4000 de 12.8L EPA 04	Cummins ISX	Cummins ISX
Size of the fuel deposit (1)	189	270	271

4.3.3 Ordering Costs

Concepts used for the determination of the ordering cost include the salary of the personnel that intervene in the ordering process and fixed costs as electricity, telephone, computer use, security clothing and equipment, wheel shims for the tank truck, fire extinguishers and measuring equipment to check the fuel quality.

Regarding the personnel cost, two people are considered to have to be present at the time of discharging, including the person in charge of the service station during the first part of the fuel discharge. A salary of 10 000 MXN/month is considered for the employee, while the station manager has a higher salary but must only be present part of the time. The charge and/or discharge of a 20 $m³$ tank truck takes between 30 and 45 min, but time must be added for operations like connection and disconnection of the discharge hoses, security revisions of equipment, quality measuring of the material to discharge, leading to an estimate of 1 h for the complete operation [\[63](#page-31-0)]. Considering a finite truck fleet, transport times to and from the DT and recharging times, a maximum of 4 trips per day is considered. In addition to the discharging personnel, a secretary with a monthly salary of 10 000 MXN is considered to dedicate 1 h of her time to each order. Considering 6 weekly working days per week or 25 per month, and 8 daily working hours, a salary of 50 MXN per hour and a total salary cost of 150 MNX per emitted and supplied order.

Fixed ordering costs apportioned per order are assumed to ascend to the same amount, giving a total of 300 MNX per order emitted and per service station.

4.4 Mathematical Optimization Model

4.4.1 Model Formulation

Defining the indexes, decision variables and employed parameters, a mathematical model can be developed which can be used for the determination of a good solution for the ordering quantity in each service station, in addition to the type of truck that minimizes the objective function. The model is based on mixed integer programming, with linear restrictions but a non-linear objective function.

In addition to the previously defined variables (see Sect. 4.3), the following indices, variables and parameters are used in the model (Table 14):

Indexes		Model parameters and decision variables	
	Fuel type $(i = 1, 2, 3)$	C_i	Truck capacity for truck $j \text{ [m}^3]$
	Truck type $(i = 1, 2, 3)$	O_{ik}	Number of orders for fuel i in SS k
k	Service station (SS) number $(k = 1, 2,)$, 18)	D_{ik}	Yearly demand for fuel type <i>i</i> in SS $k \, \mathrm{[m^3]}$
		S_{ik}	Storage tank size for fuel type i in SS $k \, \mathrm{[m^3]}$

Table 14 Indexes and additional variables and parameters used in the model

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The objective function minimizes total distribution costs. It is a function of the ordered quantity Q_{ik} for fuel *i* and service station *k*, which corresponds in each case to the sum of the number of ordered trucks of each type *j* multiplied by the capacity of the truck, C_i (Eq. 6). Note the factor of 0.9, which indicates that the fuel vessel should be filled to approximately 90% of its rated capacity; this restriction is imposed by corresponding safety regulations (see for example [[61\]](#page-31-0)) to avoid accidents due to overload and/or fuel leaks.

$$
Q_{ik} = \sum_{j=1}^{3} n_{ijk} \cdot 0.9 \cdot C_j \quad \forall i, k
$$
 (6)

The distribution costs C_{ik} for fuel *i* and service station *k* (Eq. 7) are calculated as the sum of holding costs and transport costs as defined by Eqs. (2) (2) and (5) (5) . The transport cost in Eq. ([5\)](#page-20-0) was determined considering the number of fuel trucks *nijk* in one order, so it must be multiplied by the number or orders for that fuel and service station.

$$
C_{ik} = 20\% \cdot C_{p_i} \cdot \frac{Q_{ik}}{2} + O_{ik} \cdot \left(C_{f_k} \sum_{j=1}^3 n_{ijk} \cdot C_{v_{ijk}} \right) \quad \forall \ i, k \tag{7}
$$

To obtain the total cost (Eq. 8), the above costs are summarized for the three fuel types in each service station *k* and this quantity is increased with the ordering cost. If the total cost is to be minimized, orders for the different fuel types in a specific service station should be concurrent. Assuming concurrency, the number of orders for service station *k* in an annual planning horizon equals the order number of the most frequently ordered fuel. The latter depends on both the demand and storage capacities of the fuels.

$$
C_T = \sum_{k=1}^{18} \left(C_{o_k} \cdot \max_{\forall i} [O_{ik}] + \sum_{j=1}^{3} C_{ijk} \right) \quad \forall \ i, k \tag{8}
$$

The constraints of the model are the following:

- Each truck only supplies one service station in each trip.
- Security constraint: fuel trucks must not be overloaded; they are assumed to be charged at 90%. This restriction is considered in the formulation of the ordered quantity (Eq. 6).
- A single order is considered for any combination of truck and fuel types arriving at a service station on a specific day.
- The demand is always satisfied; only a remnant smaller that the ordering quantity can exist and will be transferred to the next planning horizon. As mentioned before, this is a direct consequence of the specific restrictions in loading capacity in fuel transport and containers vessels. For each iteration, the

number of orders corresponds to the demand divided by the ordered quantity and the following restrictions must be fulfilled:

$$
O_{ik} = \frac{D_{ik}}{Q_{ik}} \quad \forall \ i, k \tag{9}
$$

- An unlimited fleet is considered.
- Capacity constraint: the total ordered quantity for fuel *i* and service station *k* for all types of truck in each order cannot exceed the capacity of the corresponding storage tank:

$$
Q_{ik} \leq S_{ik} \quad \forall \ i, k \tag{10}
$$

The tank is assumed to be at its minimum level at the time of ordering.

• No negativity constraint: all physical quantities should be positive.

Finally, the optimization model was implemented in the R programming language, being the leading open-source tool in data analysis. Its main advantages are that it is platform independent, open-source, free and very flexible and straightforward to use. It can handle big amounts of data due to its power and efficient calculations. In addition, R allows integration with other languages as C/C++, Java or Phyton and has packages allowing to integrate the optimization model within a user-friendly interface.

4.4.2 Proposed Solution

Greedy randomized adaptive search procedure (GRASP) is a metaheuristic technique for combinatory optimization. The technique consists of an iterative process, where each iteration is composed of two stages: in the first stage, a feasible solution is constructed, while the second stage consists of a local search in the neighbourhood of the previous solution [[70\]](#page-32-0).

The proposed model is highly combinatorial; considering the maximum storage tank capacity of the fuel involved in the problem (200 m^3) , a maximum of 66 different truck combinations can be found for each fuel type and service station, giving a total of 1.8×10^{98} combinations. Only a very small amount of them correspond to feasible truck combinations, as a certain combination cannot deliver more fuel than is possible to receive in the storage tank.

Instead of determining the objective function for all possible combinations and discarding those that violate the storage capacity restriction, the number of feasible truck delivery combinations is determined for each storage tank size. As, due to security reasons, fuel vessels should be loaded at a maximum of 90% and only fixed storage tanks exist (see Table [16](#page-24-0)), in this study the 45 $m³$ truck is assumed to be loaded with a maximum of 40 $m³$ of fuel to simplify calculations. As an example of how feasible combinations were determined, Table [15](#page-24-0) presents all possible Optimization and Simulation of Fuel Distribution … 273

Combination	Number of 20 m^3 trucks	Number of 45 m^3 trucks	Number of 60 m^3 trucks	Total quantity delivered (m^3)
		Ω		20
2	Ω			40
	2	0	0	40
	Ω	0		60
				60
	3			60
	0	2		80
8		θ		80
9	\overline{c}			80
10	4			80

Table 15 Feasible combinations for a 80 m³ storage tank

combinations to deliver a maximum of 80 $m³$. This amount can be supplied with two 45 m³ trucks, one 20 m³ and one 60 m³ truck, two 20 m³ trucks and one 45 m³ truck or, finally, four 20 m^3 trucks. However, since the storage tank must not necessarily be filled completely, there exist other combinations where the total quantity supplied is less than 80 $m³$. For this example, there are a total of 10 feasible combinations.

The number of feasible combinations in this problem for commercial storage tanks in Mexico are given in Table 16.

Selecting only feasible delivery combinations, the solution space was reduced to 5.75×10^{38} , which is only a fraction of the original problem search space.

To obtain a solution, at each iteration a feasible delivery combination was randomly selected for each type of fuel and service station. For this combination, total distribution costs are determined. If the solution obtained is better than the best one from previous iterations, it is stored as the best solution. If not, it is discarded. The algorithm stops at a fixed number of iterations, or when the solutions are not improving at a given number of iterations.

4.5 Simulation

To show how simulation can be used as a tool to assess managerial decisions, we analysed if it is convenient to consider a heterogeneous fleet to deliver the fuel orders, instead of the current homogeneous fleet. A planning horizon of one year was considered.

4.5.1 Scenario Definition

To be able to analyse if the inclusion of tank trucks with a higher capacity (45 and 60 m3 respectively), will minimize total distribution costs, the model was set up for two situations:

- If the present scheme of homogeneous fleet is considered (only 20 $m³$ trucks), the 54 storage tanks in the 18 service stations will be supplied only with these trucks. As the maximum storage size is 200 m^3 , maximum ten feasible truck combinations exist. For instance, the 80 $m³$ storage tanks can be supplied with one, two, three or four 20 $m³$ trucks (combinations 1, 3, 6 or 10 in Table [15\)](#page-24-0). If the supply in all fifty four storage tanks is considered at the same time, a total of 2.2×10^{22} feasible combinations exist.
- When 45 and 60 m³ trucks are included, 5.75×10^{38} feasible combinations exist, as explained in Sect. [4.4.2.](#page-23-0)

Simulation conditions:

- Considering the parameters included in the model (for example, unlimited number of trucks and drivers) for the present study, the distribution cost at a certain service station does not depend on information at other service stations. For this reason, the optimization of the above scenarios can be carried out at each service station independently to increase the algorithm's efficiency. The independency between service stations can be lost, of course, if more information becomes available in a later stage and for example resources are shared between them.
- A total of 100 000 iterations per simulation and 10 repetitions were carried out for each scenario. Running time was about 25 s in a MacBookPro 2 GHz for each run. Analysis of the repetitions suggested that 100 000 iterations was enough to come to a good solution.

4.5.2 Experiments and Discussion

Solving the model for the first scenario where only 20 $m³$ trucks are programmed, a minimum total distribution cost of 1 980 076 MXN was obtained.

SS	20 m^3 trucks			45 m ³ trucks			60 m^3 trucks		
	Gasoline A	Gasoline B	Diesel	Gasoline A	Gasoline B	Diesel	Gasoline A	Gasoline B	Diesel
-1	$\mathbf{0}$	θ	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	Ω	Ω	$\mathbf{0}$
\overline{c}	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\overline{0}$	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\overline{0}$	\overline{c}
$\overline{\mathbf{3}}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\overline{4}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{0}$	$\overline{3}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
5	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
6	Ω	Ω	$\mathbf{0}$	$\overline{2}$	1	$\mathbf{1}$	θ	Ω	Ω
7	θ	$\mathbf{0}$	$\mathbf{0}$	$\overline{2}$	Ω	$\overline{0}$	$\overline{0}$	$\mathbf{1}$	$\mathbf{0}$
8	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
9	$\mathbf{0}$	θ	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{0}$
10	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
11	$\mathbf{0}$	θ	$\mathbf{0}$	$\overline{0}$	Ω	$\boldsymbol{0}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
12	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{0}$	$\overline{0}$	$\mathbf{0}$
13	Ω	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{1}$	1	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
14	Ω	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{1}$	\overline{c}	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{0}$
15	\overline{c}	Ω	$\mathbf{0}$	$\mathbf{0}$	\overline{c}	$\boldsymbol{0}$	$\overline{2}$	$\mathbf{0}$	$\mathbf{0}$
16	Ω	Ω	$\mathbf{0}$	Ω	\overline{c}	$\overline{0}$	3	θ	$\mathbf{1}$
17	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{1}$
18	$\mathbf{0}$	$\overline{0}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{1}$

Table 17 Truck allocation scheme proposed by the model

The model was rerun for the scenario that considers a heterogeneous fleet (20, 45 and 60 m³). For this case, a minimum distribution cost of 1 733 585 MXN was found, showing an improvement of minimum 12.4% with respect to the current costs. Since up to now no optimization rules have been applied, the current distribution costs can still be higher than the 1 980 076 MXN obtained in the simulation scenario. In other words, the inclusion of tank trucks with more capacity seems to decrease distribution costs considerably.

The best truck allocation scheme found by the model is presented in Table 17.

The proposed allocation scheme shows preference towards trucks with a higher capacity, which is consistent with the conclusion that the use of a heterogeneous fleet can cut distribution costs.

The corresponding ordered quantity, the number of orders in the yearly planning horizon and the order frequency (in days) can be found in the Table [18.](#page-27-0)

If the company is not willing to buy trucks with other capacities on a short term (vehicles can be substituted for example only when their useful life is over), the proposed model can still be used to optimize the fuel distribution with the current homogeneous fleet, as this study showed the following:

• Transport costs seem to be an important portion of the total distribution cost. This is suggested by the fact that bigger trucks are preferred.

SS	Ordered quantity (m^3)			Annual orders			Order frequency (days)		
	Gasoline A	Gasoline B	Diesel	Gasoline A	Gasoline B	Diesel	Gasoline A	Gasoline B	Diesel
1	36	36	36	110	20	70	3.3	18.3	5.2
\overline{c}	54	36	108	203	55	200	1.8	6.6	1.8
$\overline{3}$	36	36	$\mathbf{0}$	152	28	$\mathbf{0}$	2.4	13.0	$\overline{}$
$\overline{4}$	108	54	$\mathbf{0}$	83	30	$\mathbf{0}$	4.4	12.2	$\overline{}$
5	36	36	$\mathbf{0}$	152	28	$\mathbf{0}$	2.4	13.0	$\overline{}$
6	72	36	36	109	39	160	3.3	9.4	2.3
τ	72	54	Ω	95	23	Ω	3.8	15.9	$\overline{}$
8	90	90	54	92	17	89	4.0	21.5	4.1
9	90	90	$\mathbf{0}$	122	22	$\overline{0}$	3.0	16.6	\equiv
10	36	36	18	152	28	200	2.4	13.0	1.8
11	54	54	54	102	14	45	3.6	26.1	8.1
12	36	36	36	304	42	134	1.2	8.7	2.7
13	90	90	54	92	17	67	4.0	21.5	5.4
14	90	72	Ω	61	14	$\mathbf{0}$	6.0	26.1	$\overline{}$
15	144	72	Ω	76	28	$\mathbf{0}$	4.8	13.0	$\qquad \qquad -$
16	162	72	54	51	21	156	7.2	17.4	2.3
17	90	36	54	92	28	89	4.0	13.0	4.1
18	90	36	54	61	28	178	6.0	13.0	2.1

Table 18 Ordered quantity, number of orders and order frequency proposed by the model

- For the same reason, it can be cheaper to supply bigger and less frequent orders; obviously, considering the maximum capacity of the storage tank for each type of fuel. As mentioned before, in this study an unlimited existing homogeneous fleet is considered. Restrictions in the number of available trucks can change the outcome of the model.
- The fuel with the highest demand (in most cases gasoline A) seems to govern the ordering scheme, suggesting that it is possible to make the planning in stages and adjust the reordering schedule of the less requested gasolines based on the optimal ordering schedule for gasoline A. More simulation runs should be carried out to revise this assumption.

In conclusion, the proposed scenario of including trucks of different capacity showed to be less costly than the current situation in which a homogeneous fleet is used; the tool presented in this study can be used for the optimization of the allocation and delivery scheme with the current fleet or when other vehicles with different capacity are included, as well as for different "what-if?" questions raised by the management of the company.

5 Conclusions

The methodology applied in this chapter corresponds to a mathematical programming model with a tailored heuristic solution, originally based on the GRASP algorithm, to optimize total distribution costs in a fuel distribution network. The model provides estimates of the fuel quantity to order, the best type of truck to carry out the supply, as well as the ordering schedule that minimizes the associated costs of distribution and inventory. Subsequent simulation of several scenarios related to critical situations provides a cheap, flexible and quick way to assess different managerial decisions.

Scenarios that were analysed include the selection of a homogeneous versus a heterogeneous fleet. The current homogeneous fleet was not proven to be the most cost-effective option. In addition, the model is an interesting tool to learn more about the posed supply problem, as for example the preference of supplying bigger quantities on a less frequent basis.

With the present model, what-if analysis can easily be carried out on questions as for example:

- What if the fuel company decides to construct a new service station?
- What if the fleet is limited? In which case should new trucks be purchased?
- What if in the future more companies (and thus different fuel terminals in the same region) start to operate?
- Is a bigger storage tank needed for some fuels?
- In the last months of 2016, the price of diesel increased by approximately 25% due to political and economic instability in Mexico. Does this affect the optimal selection of the fleet? If this raise in diesel cost would persist, would the previous conclusions remain valid?

The flexibility of R to program this kind of optimization model makes it very easy to include more advanced features or extend the problem to a larger spatial scale. Programming in R gives very fast answers, so it should not be a problem to consider, among other, more service stations, political divisions, truck types or cost concepts. Even unexpected situations such as traffic problems due to major maintenance or construction roadworks in a heavily congested city such as Mexico can be evaluated, for example by considering an "equivalent distance" for detours in the determination of variable transport costs. With these relatively simple adaptations, the nature of "what if?" questions which can be posed is very extensive.

As several variables, such as demand or fuel price, are stochastic in nature, future investigations may include determining the corresponding behaviour with a probability density function; in this case, the quantity to be ordered will be determined based on these probability functions and it may be necessary to change the solution strategy. More efficient solution strategies can be considered in the future to find a solution more quickly in these complex situations.

Finally, it should be noted that, due to the specific nature of the problem studied in this chapter, an existing heuristic was not necessarily the best option to find a

quick and good solution. Although today there are very powerful mathematical tools and computers to solve an operational research problem without worrying too much about the required computing resources, common sense indicates that the very nature of optimization and engineering prefers to apply simpler strategies, based on previous knowledge of the problem, if they achieve a less intensive use of resources.

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