Classification Methods in the Research on the Financial Standing of Construction Enterprises After Bankruptcy in Poland

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Abstract In the literature devoted to applications of multivariate statistical analysis to finance, the issue of bankruptcy forecasting is dealt with at length, but few papers concern the statistical evaluation of financial standing of companies after they have been declared bankrupt. The examination of their way out from the insolvency problem may be a source of valuable information, useful for the assessment of the probability that other bankrupt enterprises achieve success as a result of the execution of restructuring proposals. The purpose of this article is to present a proposal to use selected classification methods when studying the financial standing of companies after the declaration of bankruptcy in comparison with the situation of financially sound companies. The logit model and the classification tree were used to classify companies. The evaluation of the classification efficiency was based on the following measures: sensitivity, specificity and AUC. In the study, both univariate (Tukey's criterion) and multivariate (projection depth function) methods for detecting outliers were considered. The study covered construction companies in Poland in the years 2005–2009.

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1 Introduction

The problem of corporate bankruptcy is an important issue in economic sciences. The establishment of new companies and the closure of business by some of the existing companies are natural phenomena occurring in a free-market economy. The cessation of business as a consequence of declared corporate bankruptcy attracts the particular interest of academics, business practitioners and financial institutions. Such interest can be justified by, among other matters, the serious socio-economic consequences of corporate bankruptcy, and it promotes the development of methods for predicting the risk of corporate bankruptcy.

In the literature concerning applications of multivariate statistical analysis to finance, the issue of bankruptcy forecasting is dealt with at length, but few papers concern the forecasting of repeated corporate bankruptcy (Fig. 1).

The purpose of this article is to present a proposal to use selected classification methods when studying the financial standing of companies after the declaration of bankruptcy in comparison with the situation of financially sound companies. The research hypothesis is that classification methods used in forecasting corporate bankruptcy are effective tools for assessing the financial standing of companies after the declaration of bankruptcy. The novelty in the article is the use of methods applied in bankruptcy forecasting to assess the financial standing of companies trying to get out of the problem of insolvency after they have been declared bankrupt by the courts.

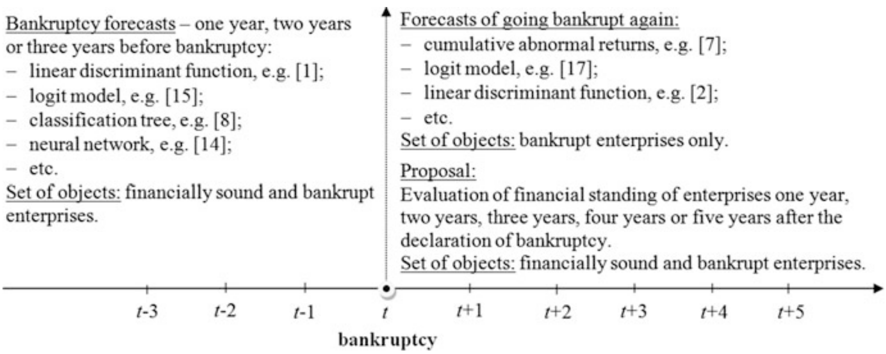

Fig. 1 Brief review of enterprise bankruptcy examinations

2 State of Research

Literature on the subject contains review papers concerning the issue of forecasting bankruptcy (e.g. [9, 20]). In the paper [9], the following criteria of the comparative analysis of papers related to the forecasting of enterprise bankruptcy based on an unbalanced sample have been adopted: type of database, approach applied to evaluate the classification effectiveness, evaluation measures of the classification effectiveness and application of statistical tests. The typical features of this type of research have been indicated after an analysis of more than 140 papers from the years 2000–2013. The basis for considerations is primarily the actual data concerning the economies of certain states; databases contain mainly up to 1000 objects, and then from 1000 to 10,000 objects; a division into training and testing datasets or a cross-validation test is often applied; such measures as the overall effectiveness, type I and type II errors, as well as, increasingly, the AUC measure are primarily used to evaluate the classification effectiveness; no statistical tests are applied.

On the other hand, in the paper [20] about 140 publications from the years 1966–2014 were reviewed. They concerned the subject under discussion in terms of definitions of unfavourable financial standing and corporate bankruptcy, considered methods for forecasting corporate bankruptcy, approaches adopted to construct a research sample, as well as procedures applied to select exogenous variables for the model. The authors highlighted, among other matters, the variety of definitions of poor financial standing of a company. They found that in theoretical considerations, different levels of adverse financial situation are distinguished, whereas empirical studies are generally limited to an analysis of two conditions: a financially sound company and a company declared bankrupt. The applied methods of bankruptcy forecasting can be divided into, for example, univariate and multivariate methods, traditional statistical models and methods based on artificial intelligence, static and dynamic models, etc. In empirical studies, the sector to which companies belong is taken into account when selecting a sample. Sets underlying empirical analyses differ in structure (balanced and unbalanced samples). The selection of an initial set of exogenous variables of the model (financial ratios and non-financial variables) is carried out based on various criteria, including expert opinions, incidence in other studies, availability of financial data, etc. A reduction in the set of potential variables is based on both qualitative and quantitative criteria.

A separate trend in forecasting corporate bankruptcy is the predicting of repeated corporate bankruptcy (e.g. [2, 7, 17]). To justify the usefulness of conducting this type of research, arguments presented by the authors of the paper [2] can be cited. According to them, about 18.25% of companies in the USA land in court once again (or more than once) after a court has declared their bankruptcy open for arrangement. They claim that the Altman Z"-score model may be useful for a court to predict the success or failure of further activity of bankrupt companies. They believe that both courts and persons responsible for preparing and conducting the restructuring of companies should use statistical methods applied to predicting corporate bankruptcy as a supplement to traditional analysis. Those methods can be used to evaluate the restructuring plan and monitor the situation after the bankruptcy in order to introduce adjustments to the plan. In their study, the authors analysed only companies with a court verdict divided into those which had landed in court only once and those which are ‘recidivists’. A return to court means that the restructuring has failed in terms of both the concept and socio-economic costs. They emphasised the importance of the early warning system in order to reduce the likelihood of repeated bankruptcy, often preceded by long and costly restructuring.

An alternative approach to the one proposed in the paper [2] is a statistical evaluation of the financial standing of companies in the years following the declaration of bankruptcy compared with the situation of financially sound companies. The examination of companies’ paths to get out of the insolvency problem may be a source of valuable information, useful for the assessment of the likelihood that other bankrupt enterprises achieve success as a result of the execution of restructuring proposals.

The results of such an assessment may be helpful in selecting appropriate remedial programmes for companies with a solvency issue. However, caution is advised when interpreting the received results. Tax law regulations, the policies of financial institutions, etc. may affect the assessment of the economic situation of a company and its future on the market. It is also worth remembering that differences in accounting, particularly the degree of implementation of international accounting standards, make it difficult or even impossible to compare data between countries. This, in turn, makes it necessary to conduct research on the financial standing of companies after the declaration of bankruptcy separately for each country.

3 Data and Research Procedure

Bankruptcy can be equated with bankruptcy declared by the courts. In Poland to 31 December 2015, there were two types of bankruptcy: arrangement bankruptcy and liquidation bankruptcy. The courts are guided by the principle according to which bankruptcy proceedings must be conducted in such a way that the creditors’ claims can be satisfied as far as possible, and the debtor’s enterprise can be preserved. One should be aware that within a particular country there may be no comparability or very limited comparability of data among companies that are subject to the obligation to have their financial statements audited and companies exempt from this obligation. In Poland, some companies with unfavourable financial ratios do not go bankrupt and even ‘bloom’ due to tax law regulations allowing them to utilise tax losses in the case of mergers of companies based on a company with losses instead of the one with profits. On the other hand, financial institutions are sceptical about companies with unfavourable values of financial ratios and they strive to quickly recover their funds by bringing such companies to liquidation bankruptcy. Thus, a company in a worse financial situation can survive, and the one in a better financial situation may go bankrupt, if the latter has been more

Table 1 Financial ratios

Symbol	Description	Symbol	Description
R_{01}	Current liquidity ratio	R_{08}	Net profitability
R_{02}	Quick liquidity ratio	R_{09}	ROE
R_{03}	Cash ratio	R_{10}	ROA
R_{04}	Total debts to assets	R_{11}	Accounts receivable turnover
R_{05}	Debt to equity	R_{12}	Fixed asset turnover
R_{06}	Long-term debt to equity	R_{13}	Total asset turnover
R_{07}	Gross profitability	R_{14}	Operation cost to sales revenues

indebted to financial institutions. The above observations indicate the difficulties occurring when trying to evaluate companies' financial standing after the declaration of bankruptcy.

The data used in this analysis have been downloaded from the website of the Emerging Markets Information Service (http://www.emis.com). The research objects were 369 construction companies in Poland, five of them were bankrupts (B_1–B_5). Court verdicts were passed between November 17, 2003 and August 30, 2004. The study used 14 financial ratios broken down into the groups of the following indicators: liquidity (R_{01}–R_{03}), liability (R_{04}–R_{06}), profitability (R_{07}–R_{10}) and productivity (R_{11}–R_{14}) (Table 1). The financial data were taken from the period 2005–2009. The following designations were adopted: *NB*—financially sound company (i.e. the one which had not been declared bankrupt before 2009) and *B*—bankrupt company. Due to a small number of bankrupt companies, it was not possible to create a test sample.

An unbalanced set was the basis of empirical studies. In the case of this type of set, the problem of low classification efficiency of bankrupt enterprises occurs more often than in the case of analysing balanced sets within the framework of the considered methods of predicting corporate bankruptcy. Apart from a small share of bankrupt enterprises in the examined sets, this may be caused, among other factors, by the occurrence of atypical objects among financially sound companies [16].

Papers devoted to the forecasting of enterprise bankruptcy present considerations related to the occurrence of outliers among the data. Proposed solutions to this problem oscillate from ignoring [19], through substitution or removal of outliers (e.g. [6, 16, 18, 22]), to the use of robust methods.

An atypical financially sound company is understood here as an object with outlying financial ratios. It was assumed that an outlier is a value that seems to differ significantly from other elements of the group in which it occurs (e.g. [3, 10]). Companies so defined may be characterised by both a very good financial situation and a poor financial situation, similar in terms of many indicators to the situation of companies declared bankrupt. The detecting and removing of atypical financially sound companies from a set of objects also has a substantive justification. Economic practice shows that companies that have a poor financial standing (i.e. those characterised by unfavourable financial ratios) may not fulfil the obligation to file a petition for bankruptcy. In the absence of such a petition also on the part of creditors, such companies may exist on the market and influence the situation of the entire industry. Therefore, the set of financially sound companies was cleaned of atypical financially sound objects. The following were used to detect outliers:

- A univariate method based on Tukey's criterion [21]
- A multivariate method based on a projection depth function [23]

Financial ratios of typical financially sound companies reflect the sector's financial standing, which depends, for example, on the economic situation in Poland.

For each year, the procedure based on Tukey's criterion [21] had the following stages:

- For each financial ratio, the first (Q_1) and third (Q_3) quartiles and the quartile deviation (Q) were calculated. The analysis used financial ratios of financially sound companies.
- Values outside the range: $< Q_1 - 1.5Q, Q_3 + 1.5Q >$ were regarded as outliers.
- A financially sound company was considered atypical if at least one of the values of the financial ratios had been regarded as an outlier.

The concept of data depth is an issue of non-parametric resistant multivariate statistical analysis, developed within the framework of an exploratory data analysis [11]. It enables one to determine a linear order of multivariate observations with the use of a multivariate median, defined as a multivariate centre of the observation set [24]. There are many proposals of functions called depth functions (e.g. Euclidean depth function, Mahalanobis' depth, Tukey's depth, projection depth and Student's depth) assigning a positive number to each observation originating from a certain distribution, which number is a measure of the observation divergence from the centre, due to this distribution.

In the case of applying a projection depth function [11], 10% of all financially sound companies which were furthest from the multivariate centre designated for financially sound companies were considered atypical financially sound companies in a given year.

Selected methods of classification applied to predicting corporate bankruptcy (e.g. [5, 12]) were used in the statistical assessment of the financial standing of companies:

- A logit model
- A classification tree based on CART algorithm

It is worth stressing that the first mentioned method belongs to the group of statistical techniques, whereas the second is the data mining method. Selecting variables is one of the crucial issues while constructing models. The backward stepwise method was used for the logit model taking into account the analysis of the correlation between the explanatory variables. The following technique was applied: in the case the algorithm is interrupted, the correlation matrix is analysed, the most correlated variable is omitted and the backward stepwise method is applied again. In order to create classification trees, the CART algorithm, which simultaneously triggers variables reduction, was used. In classification trees, the Gini Index was employed to assess the quality of the obtained splits of objects in the nodes. The tree pruning was based on the cost-complexity criterion.

To evaluate the classification effectiveness of the considered methods, the following was applied (e.g. [4]):

- Sensitivity (percentage of bankrupt enterprises which had been correctly classified)
- Specificity (percentage of financially sound companies which had been correctly classified)
- AUC measure (area of the figure under the ROC curve, where the ROC curve demonstrates sensitivity as the 1-specificity function)

The calculations were made in the R, Statistica and Excel programs.

4 Results of Empirical Research

The results of the classification of bankrupt companies with the use of the logit model are shown in Table 2. The identification of bankrupt companies characterised by a financial standing significantly different from the standing of the majority of financially sound companies was based on the result of both point estimation and interval estimation of the probability of belonging to the class of financially sound companies. Point probabilities for each enterprise have been calculated by substituting observed values of financial ratios for individual enterprise into the logit model. Confidence intervals for theoretical (predicted) probabilities have been obtained according to [13, p. 244] taking transformation of the dependent variable in the logit model on board.

On the basis of the obtained results, it can be concluded that after removing atypical objects from the set of financially sound companies, the logit model was characterised by generally higher values of the AUC measure. This proves the higher classification effectiveness of this model in comparison with the model estimated based on data not cleaned of atypical financially sound companies. Also, the specificity measure of this model was at a very high level, which means that few financially sound companies were classified as companies with a financial standing similar to the financial situation of companies that had a problem with solvency in the past. Therefore, in the following part of the paper, the interpretation is limited to the results obtained from the logit model estimated on the basis of sets cleaned of atypical financially sound companies.

Figures 2 and 3 present point estimates and 95% confidence intervals estimated for the probability of belonging to the class of financially sound companies, calculated for the considered bankrupt enterprises. The charts show a low precision of the obtained probability estimates in the studied years, mainly in the case of the bankrupt enterprises B_1, B_2, B_4 and B_5 in the years 2007–2009 (with Tukey's

Table 2 Results for the logit model

Outliers	Year	Sample size	Ratios	$B_{\_}^{a}$	$B_{\_}^{b}$	Sensitivity[a]	Specificity[a]	AUC[a]
Not removed	2005	369	$R_{04}R_{13}$	B_3	$B_1B_3B_4$	0.2	1.000	0.959
	2006	369	$R_{06}R_{07}R_{08}$	–	B_2B_3	0.0	1.000	0.833
	2007	369	$R_{05}R_{06}$	B_1	B_1	0.2	0.997	0.767
	2008	369	R_{09}	B_4	B_4	0.2	1.000	0.599
	2009	369	R_{14}	–	–	0.0	1.000	0.826
Removed —Tukey's criterion	2005	188	R_{04}	$B_1B_2B_3B_4$	$B_1B_2B_3B_4$	0.8	1.000	0.976
	2006	205	R_{05}	B_1B_2	$B_1B_2B_4$	0.4	1.000	0.733
	2007	197	$R_{05}R_{06}R_{07}R_{08}$	B_1B_2	$B_1B_2B_4B_5$	0.4	0.995	0.981
	2008	176	$R_{05}R_{06}R_{10}R_{14}$	$B_1B_2B_4B_5$	$B_1B_2B_4B_5$	0.8	1.000	0.961
	2009	188	$R_{06}R_{14}$	B_1B_5	$B_1B_2B_4B_5$	0.4	0.995	0.940
Removed —depth function	2005	333	R_{04}	$B_1B_2B_3B_4$	$B_1B_2B_3B_4$	0.8	1.000	0.955
	2006	333	$R_{04}R_{05}R_{08}$	$B_1B_2B_4$	$B_1B_2B_3B_4$	0.6	1.000	0.912
	2007	333	$R_{05}R_{06}R_{08}R_{14}$	B_1B_4	$B_1B_2B_4$	0.4	1.000	0.920
	2008	333	$R_{05}R_{06}R_{11}R_{14}$	B_4	$B_1B_2B_4$	0.2	1.000	0.976
	2009	333	$R_{08}R_{09}$	B_1	B_1B_4	0.2	1.000	0.794

$B_{\_}$—Bankrupt enterprises, which have not been classified as financially sound ones

[a]According to the point probability of belonging to the class of financially sound companies

[b]According to the lower limit of the 95% confidence interval for the probability of belonging to the class of financially sound companies

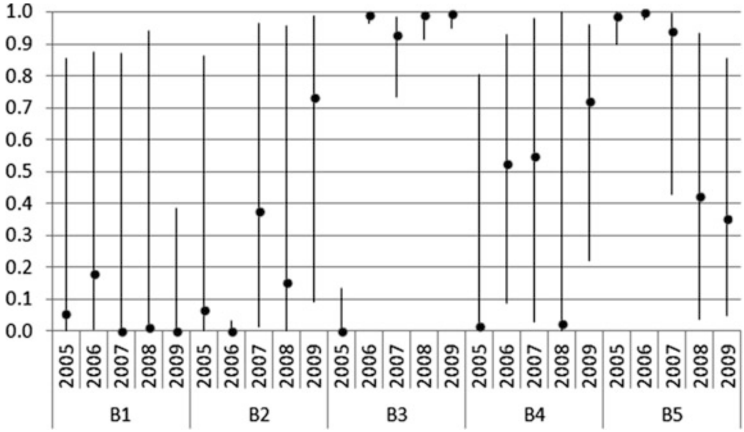

Fig. 2 Point and interval estimates ($1 - \alpha = 0.95$) of the probability of belonging to the class of financially sound companies on the basis of the logit model based on a set cleared of atypical financially sound objects according to Tukey's criterion

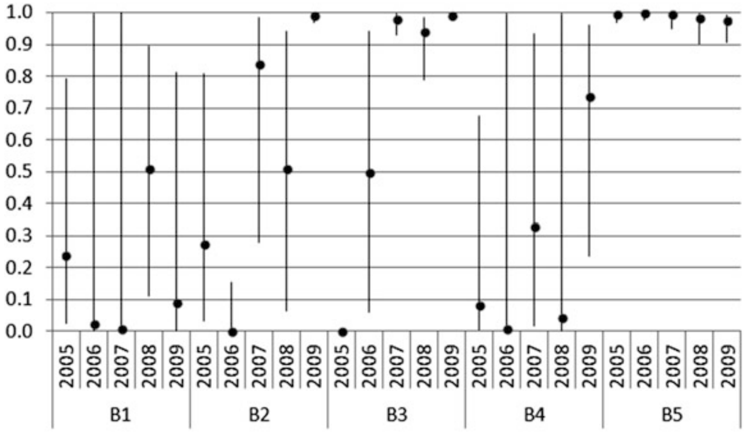

Fig. 3 Point and interval estimates ($1 - \alpha = 0.95$) of the probability of belonging to the class of financially sound companies on the basis of the logit model based on a set cleared of atypical financially sound objects according to the projection depth function

criterion). Only results obtained for bankrupt B_3 can be generally regarded as accurate.

Due to the considerable uncertainty associated with the obtained results, it was decided to apply a very strong criterion for the assessment of the financial standing of bankrupt companies. It was assumed that the lower limit of the 95% confidence interval for the probability of belonging to the class of financially sound companies would be decisive as regards the recognition of a given bankrupt company as a company with a financial standing similar to that typical of financially sound companies.

The adoption of Tukey's criterion enabled the authors to indicate bankrupt enterprise B_3 as a company with a financial standing similar to that observed in the group of financially sound companies in the years 2006–2009. In the case of applying the projection depth function, the financial standing of this bankrupt company was assessed as good in the years 2007–2009. Therefore, it can be concluded that it was a company that had coped best with the problem of insolvency among the considered bankrupt enterprises after having been declared bankrupt by a court. Another bankrupt company whose financial standing was assessed as generally good was company B_5. With the use of the projection depth function, this concerned the entire period of study, but in the case of Tukey's criterion, it applied only to the period from 2006 to 2007. B_1, B_2 and B_4 were categorised as bankrupt enterprises whose financial standing diverged from the standing characteristic of financially sound companies.

The classification results of bankrupt companies with the use of the classification tree based on CART algorithm are shown in Table 3.

The constructed classification trees are characterised by excellent classification efficiency of bankrupt enterprises (Table 3). However, financially sound companies are classified as bankrupt enterprises (i.e. companies which had a solvency issue in the past) to a greater extent than in the case of the logit model. This problem is relevant to both sets not cleaned of atypical financially sound companies and sets that have been cleaned of outliers. Also, the AUC measure shows high classification efficiency in the case of this method. The achieved results also confirm the robustness of classification trees to the presence of atypical observations, since

Table 3 Results for the classification tree

Outliers	Year	Sample size	Ratios	Sensitivity	Specificity	AUC
Not removed	2005	369	$R_{04}R_{14}$	1.0	0.995	0.998
	2006	369	$R_{04}R_{05}R_{12}$	1.0	0.967	0.984
	2007	369	$R_{03}R_{11}R_{12}$	1.0	0.926	0.963
	2008	369	$R_{02}R_{07}R_{11}R_{12}$	1.0	0.956	0.978
	2009	369	$R_{01}R_{03}R_{07}R_{12}$	1.0	0.967	0.984
Removed —Tukey's criterion	2005	188	$R_{04}R_{12}$	1.0	1.000	1.000
	2006	205	$R_{02}R_{04}R_{12}$	1.0	0.985	0.993
	2007	197	$R_{03}R_{11}R_{12}R_{14}$	1.0	0.906	0.985
	2008	176	$R_{02}R_{07}R_{11}R_{12}$	1.0	0.912	0.956
	2009	188	$R_{01}R_{03}R_{12}$	1.0	0.934	0.967
Removed —depth function	2005	333	$R_{01}R_{13}$	1.0	0.976	0.988
	2006	333	$R_{04}R_{06}R_{12}$	1.0	0.976	0.988
	2007	333	$R_{03}R_{11}R_{12}$	1.0	0.902	0.951
	2008	333	$R_{02}R_{07}R_{11}R_{12}$	1.0	0.936	0.968
	2009	333	$R_{01}R_{03}R_{12}$	1.0	0.945	0.973

Table 4 Decision rules in classification trees constructed on sets not cleaned of atypical financially sound objects (sample size = 369)

Year	Decision rules	Probability
2005	$R_{04} > 1.045 \rightarrow B$	0.667
	$R_{04} \leq 1.045 \wedge R_{14} \leq 51.345 \rightarrow B$	1.000
	$R_{04} \leq 1.045 \wedge R_{14} > 51.345 \rightarrow NB$	1.000
2006	$R_{12} > 5.005 \rightarrow NB$	1.000
	$R_{12} \leq 5.005 \wedge R_{04} \leq 0.545 \rightarrow NB$	1.000
	$R_{12} \leq 5.005 \wedge R_{04} > 0.545 \wedge R_{05} > 2.045 \rightarrow NB$	1.000
	$R_{12} \leq 5.005 \wedge R_{04} > 0.545 \wedge R_{05} \leq 2.045 \rightarrow B$	0.294
2007	$R_{11} > 5.265 \rightarrow NB$	1.000
	$R_{11} \leq 5.265 \wedge R_{12} > 7.745 \rightarrow NB$	1.000
	$R_{11} \leq 5.265 \wedge R_{12} \leq 7.745 \wedge R_{03} > 0.615 \rightarrow NB$	1.000
	$R_{11} \leq 5.265 \wedge R_{12} \leq 7.745 \wedge R_{03} \leq 0.615 \wedge R_{11} \leq 2.650 \rightarrow NB$	1.000
	$R_{11} \leq 5.265 \wedge R_{12} \leq 7.745 \wedge R_{03} \leq 0.615 \wedge R_{11} > 2.650 \rightarrow B$	0.156
2008	$R_{11} > 4.930 \rightarrow NB$	1.000
	$R_{11} \leq 4.930 \wedge R_{02} \leq 1.325 \rightarrow NB$	1.000
	$R_{11} \leq 4.930 \wedge R_{02} > 1.325 \wedge R_{07} > 10.165 \rightarrow NB$	1.000
	$R_{11} \leq 4.930 \wedge R_{02} > 1.325 \wedge R_{07} \leq 10.165 \wedge R_{12} > 10.700 \rightarrow NB$	1.000
	$R_{11} \leq 4.930 \wedge R_{02} > 1.325 \wedge R_{07} \leq 10.165 \wedge R_{12} \leq 10.700 \wedge R_{11} \leq 2.740 \rightarrow NB$	1.000
	$R_{11} \leq 4.930 \wedge R_{02} > 1.325 \wedge R_{07} \leq 10.165 \wedge R_{12} \leq 10.700 \wedge R_{11} > 2.740 \rightarrow B$	0.238
2009	$R_{12} > 4.125 \rightarrow NB$	1.000
	$R_{12} \leq 4.125 \wedge R_{03} > 0.455 \rightarrow NB$	1.000
	$R_{12} \leq 4.125 \wedge R_{03} \leq 0.455 \wedge R_{01} \leq 1.160 \rightarrow NB$	1.000
	$R_{12} \leq 4.125 \wedge R_{03} \leq 0.455 \wedge R_{01} > 1.160 \wedge R_{07} > 4.920 \rightarrow NB$	1.000
	$R_{12} \leq 4.125 \wedge R_{03} \leq 0.455 \wedge R_{01} > 1.160 \wedge R_{07} \leq 4.920 \rightarrow B$	0.294

Probability—the estimated probability for a terminal node

there are no significant differences between the trees constructed for a given year on the basis of a set cleaned and not cleaned of atypical financially sound companies. Due to the above-mentioned similarity of results, it was decided to discuss at length classification trees constructed on sets not cleaned of atypical financially sound companies.

Table 4 presents decision rules resulting from the construction of classification trees for subsequent years on the basis of sets not cleaned of atypical financially sound companies.

Based on the results presented in Table 4, it can be concluded that the estimated probability for terminal node, which allows to extract the class of companies that had problems with solvency before 2005, decreases from 0.667 in 2005 to 0.156 in 2007 and then slightly increases to 0.294 in 2009. It may indicate positive effects of the remedial process conducted in the companies concerned. The smallest estimated probability for a terminal node was recorded in 2007. This may be associated with the slowdown of the Polish economy in this period, resulting from the global economic crisis, and—consequently—difficult conditions for the operation of companies, including financially sound ones.

5 Summary

The results of the conducted empirical studies confirm the usefulness of classification methods applied to predicting bankruptcy of companies for the evaluation of the financial standing of companies after the declaration of bankruptcy in comparison with the situation of financially sound companies. The logit model enabled the authors to single out, from among companies declared bankrupt, those bankrupt enterprises whose financial standing in the five consecutive years after the declaration of bankruptcy had improved to such a degree that it could be regarded as similar to the situation typical of financially sound companies. The classification trees, in turn, provided decision rules indicating those areas of operations of companies which require particular attention in the remedial process of bankrupt companies.

The logit models estimated on the basis of sets cleaned of atypical financially sound companies indicated the importance in the evaluation of the financial standing of companies after the declaration of bankruptcy:

- 1 year—only of the liability ratios
- 2 years—of the liability and profitability ratios
- 3–5 years—of the liability, profitability, and productivity ratios

The classification trees provided decision rules based on:

- 1–2 years after the bankruptcy—the liquidity, liability and productivity ratios
- 3 years after the bankruptcy—the liquidity and productivity ratios
- 4 years after the bankruptcy—the liquidity, profitability and productivity ratios
- 5 years after the bankruptcy—the liquidity and productivity ratios

In light of the results obtained, it can be concluded that the development paths of companies B_3 and B_5 can be a source of valuable guidance on selecting appropriate remedial programmes for companies having a problem with solvency. On the other hand, the analysis of decisions taken in companies B_1, B_2 and B_4 after the declaration of bankruptcy can provide information useful for avoiding the deterioration of their financial standing. In the authors' opinion, it is worth continuing the undertaken studies and attempting to test the results achieved and confront them with reality, depending on the availability of data.

In further research, authors are planning to incorporate other classification methods. They also intend to consider other methods of detecting outliers, and other approaches (e.g. V-fold cross-validation) in the verification of the obtained results.

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