Integrated Location-Inventory Optimization in Spare Parts Networks

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Abstract This research work is concerned with integrated location-inventory optimization in spare parts networks. A semi-Markov decision process (SMDP) is developed, formulated as linear program (LP) and finally, embedded into a set-covering problem framework. The resulting model is a mixed integer linear program (MILP) which integrates (1) strategic facility choice, (2) tactical base-stock level setting and (3) operational sourcing decisions. Due to the integration of these decision stages, physical and virtual inventory pooling opportunities can be evaluated at the same time. Experimental results emphasize the value of the integrated model compared to the sequential 'location first, inventory and sourcing second' approach. The cost savings are particularly high in networks with low fixed facility location cost, high shipment cost and high demand rates as virtual inventory sharing opportunities increase in these cases.

1 Introduction

After-sales service becomes increasingly important in today's marketplace as competition is strong and companies are looking for ways to distinguish themselves from their competitors. At the heart of after-sales service is providing the customer with spare parts in case of breakdowns that happen during regular operation. This work focuses on expensive and critical spare parts which are characterized by low demand rates and fast delivery requirements. The inventory holding cost of such parts are typically high which sets incentives to keep inventories low. Traditionally, low inventory levels have been achieved by consolidating multiple stocking points into one physical location and thereby, reducing the amount of system-wide safety stock [3]. However, the downside of this approach is that delivery times and outbound shipment cost typically increase since the centralized inventory is stored relatively far away from the markets. Instead of pooling inventory *physically*, there is also the possibility of

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sharing inventory *virtually* among warehouses [5]. With this approach, the system-wide inventory level can be reduced as well while the distance between warehouses and markets tends to be shorter.

In this research work, we consider a spare parts manufacturer that outsources supply chain management to a third-party logistics service provider (3PL) and that needs to decide at which of the (already existing) warehouse locations to stock spare parts. This decision problem has the notion of the classical strategic facility *location* decision but, in fact, it is rather a facility *choice* or an assignment problem. For solving this decision problem, we propose a mixed integer linear program (MILP) that simultaneously evaluates physical and virtual sharing opportunities. Current research mostly focuses on physical pooling opportunities with the notable exception of Mak [4] who considers virtual inventory sharing in a location-inventory framework. To the best of our knowledge, there is no study yet that integrates both pooling variants in one model. The proposed MILP contains the following decision stages.

1. Strategic supply network design, i.e. at which warehouses to store spare parts.
2. Tactical inventory level optimization, i.e. which base-stock level to choose at each warehouse.
3. Operational sourcing, i.e. from which warehouse to satisfy spare part orders.

To evaluate virtual inventory sharing opportunities, it is necessary to include the inventory and sourcing decisions into the framework. The idea is that sourcing warehouses may vary dynamically depending e.g. on the current inventory level at each of the warehouses. Thus, demand can be allocated to multiple warehouses which then exhibit virtual inventory sharing. The MILP consists of a semi-Markov decision process (SMDP) that is formulated as linear program (LP) and embedded into a set-covering framework. The model is briefly presented in this article and our findings from an experimental study are provided. For further details on the model or the solution algorithm deployed, the reader is referred to [8].

2 Model Formulation

We consider a three-tiered supply chain consisting of one supplier, multiple warehouses $r \in R$ and markets $m \in M$. Each warehouse r replenishes items from an external supplier with infinite supply according to an $(S-1, S)$ review policy, i.e. the delivery of a part to a market immediately triggers a replenishment order at the respective sourcing warehouse r. Furthermore, the replenishment lead-time of warehouse r is exponentially distributed with mean $1/\mu_r$, where μ_r constitutes the replenishment rate of warehouse r per time unit. Assuming an exponential lead-time distribution appears rather restrictive at first glance—however, Alfredsson and Verrijdt [1] have shown that the overall system performance is rather insensitive with regard to the chosen lead-time distribution which makes our assumption robust. Each market m faces a Poisson demand process with an expected number of demand arrivals λ_m

per time unit. Furthermore, every market m can only be served by a subset of warehouses R_m because of service time constraints related to the geographical distance between warehouses and markets.

Cost of $t_{r,m}$ are incurred for shipping one item from warehouse r to market m. If no item is available at a warehouse within a market's service region, the part is express-shipped from an external supplier at cost of l_m. The unit replenishment cost of warehouse r are v_r and the unit inventory holding cost at warehouse r are h_r per time unit. Moreover, fixed cost of f_r are incurred if warehouse r is used to store spares.

For solving the outlined three-stage decision problem, we propose a MILP which integrates an SMDP with a classical set-covering model. The latter is concerned with the strategic network design decision and selects a subset out of a set of candidate warehouses. Inventory and sourcing decisions are modeled with an SMDP which is a reformulated version of the one in Seidscher and Minner [6]. The SMDP essentially models an inventory system that contains the candidate warehouses $r \in R$ as stocking points. By minimizing replenishment cost, inventory holding cost, shipment cost and express-shipment cost, the SMDP specifies in each state of the system from which warehouse to source an incoming part order. Thus, it determines the optimal sourcing policy for a given set of stocking points and base-stock levels.

The states $i \in I$ of the SMDP represent the allocation of inventory to the stocking points. We distinguish between auxiliary states $i \in I^A$ and decision states $i \in I^D$. The former is used to determine whether the next event will happen at a warehouse (arrival of an outstanding replenishment order) or at a market (arrival of a new spare part order). In those states, the system is not allowed to take a sourcing decision, i.e. to specify from which warehouse to source the demand of a market. In contrast, decision states $i \in I^D$ are concerned with taking these sourcing decisions $q \in R_{c_i}$ for a particular market $c_i \in M$.

Let us introduce the following sets and parameters. First, the sets $V^A(r, u)$ and $V^D(r, u)$ contain those states $i \in I^A$ and $i \in I^D$ that have an inventory level larger than u at warehouse $r \in R$, respectively. Second, $O(r)$ comprises those decision states $i \in I^D$ where warehouse $r \in R$ is out of stock. Furthermore, let U_r^{max} denote the (preprocessed) maximum possible base-stock level at warehouse r. U_r^{max} is not to be confused with a maximum storage capacity and is determined by optimizing an $M|M|S|S$ queue [8]. Moreover, the following decision variables are introduced.

- y_r Binary decision variable that indicates whether warehouse $r \in R$ is used for inventory placement.
- $S_{r,u}$ Binary decision variable that indicates whether the base-stock level u at warehouse $r \in R$ is active.
- $x_{i,q}$ Decision variable that denotes the long-run fraction of decision epochs where the system is in decision state $i \in I^D$ and decision $q \in R_{c_i}$ is taken.
- $x_{i,0}$ Decision variable that denotes the long-run fraction of decision epochs where the system is in auxiliary state $i \in I^A$.
- $z_{i,0,r,u}$ Decision variable that replaces the product $S_{r,u} \cdot x_{i,0}$, $\forall i \in I^A, \forall r \in R, u = 0, 1, \ldots, U_r^{max}$.

$$\min \quad \sum_{r \in R} f_r \cdot y_r + C(SMDP) \tag{1}$$

$$s.t. \quad \sum_{r \in R_m} y_r \geq 1 \qquad \forall m \in M \tag{2}$$

$$S_{r,u+1} \leq S_{r,u} \qquad \forall r \in R, u = 0, \ldots, U_r^{max} - 1 \tag{3}$$

$$S_{r,0} = 1 \qquad \forall r \in R \tag{4}$$

$$\sum_{u=1}^{U_r^{max}} S_{r,u} \leq U_r^{max} \cdot y_r \qquad \forall r \in R \tag{5}$$

$$\sum_{i \in V^A(r,u)} x_{i,0} + \sum_{i \in V^D(r,u)} \sum_{q \in R_{c_i}} x_{i,q} \leq S_{r,u+1} \qquad \forall r \in R, \forall u = 0, \ldots, U_r^{max} - 1 \tag{6}$$

$$\sum_{i \in O(r)} \sum_{q \in R_{c_i | q=r}} x_{i,q} \leq y_r \qquad \forall r \in R \tag{7}$$

$$z_{i,0,r,u} \leq S_{r,u} \qquad \forall i \in I^A, \forall r \in R, \forall u = 1, 2, \ldots, U_r^{max} \tag{8}$$

$$z_{i,0,r,u} \leq x_{i,0} \qquad \forall i \in I^A, \forall r \in R, \forall u = 1, 2, \ldots, U_r^{max} \tag{9}$$

$$z_{i,0,r,u} \geq x_{i,0} - \left(1 - S_{r,u}\right) \qquad \forall i \in I^A, \forall r \in R, \forall u = 1, 2, \ldots, U_r^{max} \tag{10}$$

$$y_r \in \{0, 1\} \qquad \forall r \in R \tag{11}$$

$$S_{r,u} \in \{0, 1\} \qquad \forall r \in R, \forall u = 0, 1, 2, \ldots, U_r^{max} \tag{12}$$

$$x_{i,0} \geq 0 \qquad \forall i \in I^A \tag{13}$$

$$x_{i,q} \geq 0 \qquad \forall i \in I^D, \forall q \in R_{c_i} \tag{14}$$

$$z_{i,0,r,u} \geq 0 \qquad \forall i \in I^A, \forall r \in R, \forall u = 1, 2, \ldots, U_r^{max} \tag{15}$$

$$+ \boldsymbol{SMDP\ constraints} \tag{16}$$

The objective function is given by (1) which consists of two cost terms. The first part refers to the costs associated with the strategic facility choice decision. The second term denotes the total SMDP costs which is the sum of inventory holding cost in auxiliary states as well as shipment, express-shipment and replenishment cost in decision states associated with sourcing decisions.

Constraint (2) ensures that at least one warehouse location that can serve market m within the required service time window is used to stock spares. Constraint (3) represents the incremental definition of the $S_{r,u}$ variables and ensures that base-stock level $u + 1$ can only be active if the predecessor base-stock level u is also active. Moreover, constraint (4) requires that base-stock level $u = 0$ is active at each warehouse $r \in R$. Furthermore, constraint (5) connects the inventory and location decision, i.e. only at the selected warehouses inventory can be placed.

Constraints (6) and (7) connect the set-covering problem framework with the SMDP model. Constraint (6) applies the following logic: Those states that would involve inventory levels higher than the base-stock levels need to be forbidden, i.e. the relative fraction of being in that state (taking any decision) have to be equal to zero. Additionally, constraint (7) ensures that demand can only be assigned to out-of-stock warehouses that are also open (incurring unfavorable express-shipment cost).

When integrating the SMDP with the set-covering framework, the model (at first) becomes non-linear as binary ($S_{r,u}$) and continuous decision variables ($x_{i,0}$) are multiplied with each other. We resolve the non-linearity by introducing a new set of continuous decision variables $z_{i,0,r,u}$ that replace the product term. Furthermore, we add constraints (8)–(10) to the model. This approach is consistent with the literature, see e.g. [2]. Moreover, constraints (11)–(15) define the variable domains.

For the sake of clarity, the SMDP constraints as well as the SMDP objective function are not formulated explicitly in this article. The interested reader is referred to [8] for a full exposition of the MILP, in particular the SMDP. Nevertheless, in order to give a notion of the SMDP model, we provide the general LP formulation that can be used to solve SMDPs [7]. τ_i is the average time of being in state $i \in I$ and $p_{i,j,q}$ denotes the transition probability from state $i \in I$ into state $j \in I$ under decision $q \in Q(i)$. $C_{i,q}$ denotes the cost in state $i \in I$ associated with decision $q \in Q(i)$.

$$\min \sum_{i \in I} \sum_{q \in Q(i)} C_{i,q} \cdot \frac{x_{i,q}}{\tau_i} \tag{17}$$

$$s.t. \qquad \sum_{q \in Q(j)} \frac{x_{j,q}}{\tau_j} - \sum_{i \in I} \sum_{q \in Q(i)} p_{i,j,q} \cdot \frac{x_{i,q}}{\tau_i} = 0 \qquad \forall j \in I \tag{18}$$

$$\sum_{i \in I} \sum_{q \in Q(i)} x_{i,q} = 1 \tag{19}$$

$$x_{i,q} \geq 0 \qquad \forall i \in I, \forall q \in Q(i) \tag{20}$$

The objective function (17) minimizes the sum of the expected long-run average cost per time unit. Constraint (18) refers to a set of balance equations which ensure that for any state $j \in I$ the long-run average number of transitions *from* state j per time unit are equal to the long-run average number of transitions *into* state j per time unit. Moreover, the convexity constraint (19) forces the sum of all $x_{i,q}$ variables (over all states and decisions) to be equal to 1. Furthermore, (20) requires $x_{i,q}$ to be non-negative.

3 Findings and Conclusion

The integrated model is compared to the sequential 'location first, inventory and sourcing second' approach which essentially maximizes physical pooling opportunities. In a network with 3 warehouses and 6 markets (3×6), three model input

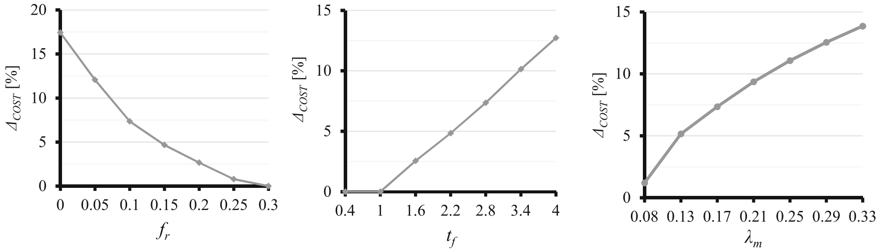

Fig. 1 Cost comparison between integrated and sequential approach in a 3×6 network

parameters are varied and the cost differences between integrated and sequential approach are measured. The experiments reveal that the cost savings (Δ_{COST}) are particularly high in networks with low fixed facility location cost (f_r), high shipment cost (t_f) and high demand rates (λ_m) as virtual inventory sharing opportunities increase in these cases, see Fig. 1. Note that t_f is a linear scaling factor for the shipment cost $t_{r,m}$.

Our results clearly show that there is a huge cost saving potential in evaluating both physical and virtual inventory sharing opportunities simultaneously rather than focusing on only one of the extremes.

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