

# Optimal Dynamic Assignment of Internal Vehicle Fleet at a Maritime Rail Terminal with Uncertain Processing Times

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**Abstract** This study aims to improve the efficiency of container loading process at a seaport by optimizing the dynamic assignment of internal vehicle fleet in the process of moving containers from storage yards at maritime terminals to the train at the rail terminal. We formulate the problem into a stochastic dynamic programming model taking into account uncertain processing times. Numerical experiments based on a case study are performed to illustrate the effectiveness and the sensitivity of the model.

## 1 Introduction

The growing traffic volume puts a huge pressure on container port as an interface between seaborne transport and hinterland transport. Rail transport is regarded as an effective way to tackle the above challenges due to its high capability and low emission. Therefore, improving the efficiency of rail terminal operations at seaports is essential to ensure the sustainability of global container transport chains. This study aims to improve the efficiency of container loading process at a seaport by optimizing the dynamic assignment of internal vehicle fleet in the process of moving containers from storage yards at maritime terminals to the train at the rail terminal.

A number of survey papers have reviewed operations management at container ports and terminals, e.g. Stahlbock and Voss [6]; Carlo et al. [4]. However, the operations management issues directly associated with rail terminals at seaports

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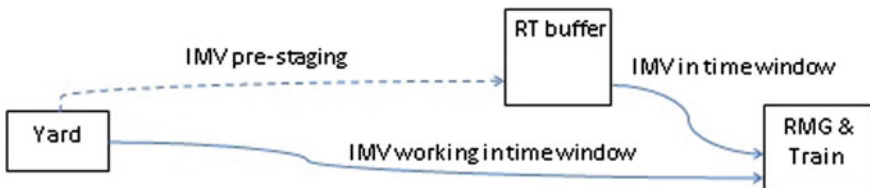
have been understudied. A few papers focused on the container loading problem, which aims to assign containers to the wagon slots of the train by minimizing the unproductive operations at the rail terminal and/or in the storage areas [1, 2]. Caballini et al. [3] developed a mixed integer linear mathematical programming model to optimize the timings of the trains and the use of the handling resources devoted to rail port operations. The authors further extended the deterministic model to deal with unexpected situations or uncertainty by adopting an event-triggered receding-horizon planning approach. Their model does not consider the regular uncertainty in the container processing times.

There is a high level of uncertainty/variability in the process of moving containers from storage yards to the rail terminal. A need has emerged for tools that have the capability of appropriately determining the dynamic internal vehicle assignment in order to load containers onto the train within the time window. In this paper, we focus on the container loading process at a seaport from storage yards to trains. We will formulate the problem into a stochastic dynamic programming model, with the aim to minimize the total logistics costs associated with moving containers from storage yards to the train plus the penalty cost of underutilizing the train capacity.

## 2 Model and Solution

The process of transporting containers from storage yards to the train includes the following main activities (see Fig. 1): Internal Moving Vehicle (IMV) receives a message to collect a container; the container is landed on the IMV; IMV transports the container to the rail terminal (either to the Rail Terminal (RT) buffer area before the working time window, which is called pre-staging, or to the Rail Mounted Gantry crane (RMG) directly during working time window); the pre-staged containers are moved from the RT buffers to the RMG; RMG loads the container to a wagon slot on the train.

Consider the loading process of a single train under periodic-review scheme with a working time window  $(0, T)$ . The decision variables include:  $q$ : the number of containers to be pre-staged from storage yards to the rail terminal (RT) buffer before the working time window;  $u^V(t)$ : the planned flow rate (i.e., the number of assigned



**Fig. 1** The process of transporting containers from yards to train

**Table 1** Notation of static parameters

$T$ :	The planning horizon, assuming the working time window is $(0, T)$
$Q^B$ :	The capacity of the RT buffer space
$Q^C$ :	The maximum handling capacity of the RMG within one period
$Q^T$ :	The capacity of the train
$c^P$ :	The unit cost of pre-staging containers (including transport and storage)
$c^V$ :	The unit cost of vehicle deployed to transport a container from yard to RMG
$c^B$ :	The unit cost of vehicle deployed to transport a container from RT buffer to RMG
$c^C$ :	The unit cost of the RMG loading a container to train
$c^S$ :	The storage cost at RT buffer per container per period
$c^U$ :	The unit penalty cost of underutilizing the train capacity

**Table 2** Notation of dynamic parameters and variables

$U^V(t)$ :	The maximum number of assigned IMV from yard to RMG in period $t$
$U^B(t)$ :	The maximum number of assigned IMV from RT buffer to RMG in period $t$
$\xi^V(t)$ :	The random flow rate from yard to RMG in period $t$
$\xi^B(t)$ :	The random flow rate from RT buffer to the RMG in period $t$
$x^B(t)$ :	The number of containers in the RT buffer at the end of time period $t$
$x^T(t)$ :	The number of containers on the train at the end of time period $t$

IMV) to move containers from yards to the RMG over the working time window; and  $u^B(t)$ : the planned flow rate (i.e., the assigned number of IMV) to move containers from RT buffer to the RMG over the working time window. We assume that one IMV carries one container. Other parameters are introduced and shown in Tables 1 and 2. The objective is to minimize the total cost incurred during pre-staging containers, transporting containers from yards to RMG, transporting containers from RM buffer to RMG, RMG crane handling containers, container storage at buffers, and penalty for underutilizing the train capacity.

### 2.1 Model

The discrete-time dynamics of the transportation system can be described by

$$x^B(t) = x^B(t - 1) - \xi^B(t), \text{ for } t = 1, 2, \dots, T; \tag{1}$$

$$x^T(t) = x^T(t - 1) + \xi^V(t) + \xi^B(t), \text{ for } t = 1, 2, \dots, T. \tag{2}$$

$$x^B(0) = q; x^T(0) = 0; 0 \leq q \leq Q^B; \tag{3}$$

$$0 \leq \xi^V(t) \leq u^V(t); 0 \leq \xi^B(t) \leq u^B(t); \tag{4}$$

$$0 \leq u^V(t) \leq U^V(t); \quad 0 \leq u^B(t) \leq \min(x^B(t-1), U^B(t)); \tag{5}$$

$$u^V(t) + u^B(t) \leq \min(Q^T - x^T(t-1), Q^C); \tag{6}$$

The initial number of containers in the RT buffer is  $q$ , and the initial number of containers on the train is 0. It should be noted that due to the uncertainty in container processing time, the actual number of containers that reach the RMG in one period, represented by  $\xi(t)$ , is often lower than the planned flow rate  $u(t)$ . Thus we have constraints in (4). The planned flow rate  $u(t)$  is also constrained by the maximum number of IMVs available, by the capacity of the train and by the capacity of the RMG, as shown in (5, 6).

The objective function is given by:

$$J_0(q, 0, 0) = E[q \cdot c^P + \sum_{t=0}^T c^S x^B(t) + \sum_{t=1}^T (c^V u^V(t) + c^B u^B(t) + c^C (x^B(t) + x^V(t))) + c^U \cdot (Q^T - \xi^T(T))] \tag{7}$$

On the right-hand-side of the above equation, the first term is the pre-staging cost; the second term is the storage costs at RT buffer; the third term represents the container movement costs from yard to RMG, from RT buffer to RMG, from RMG to train; the fourth term represents the penalty cost for underutilizing the train capacity. Following the stochastic dynamic programming theory [5], the backwards optimality equation is given by (for  $t = 0, 1, \dots, T$ ):

$$J_t(x^B(t), x^T(t)) = \min\{q \cdot c^P \cdot \mathbf{I}\{t=0\} + c^S x^B(t) + c^V u^V(t+1) + c^B u^B(t+1) c^U \cdot (Q^T - x^T(t)) \cdot \mathbf{I}\{t=T\} + E[c^C (\xi^B(t+1) + \xi^V(t+1)) + J_{t+1}(x^B(t+1), x^T(t+1))]\} \tag{8}$$

where  $J_{T+1}(x^B(T+1), x^T(T+1)) = 0$ , and  $\mathbf{I}\{\text{condition}\}$  is an indicator function. It takes 1 if the condition in  $\{\}$  is true, 0 otherwise.

## 2.2 Solution

The stochastic dynamic programming problem in (1)–(8) can be solved using the backwards value iteration algorithm (c.f. [5]).

Step 1: Let  $J_{T+1}(x^B, x^T) = 0$  for any  $(x^B, x^T)$ . Let  $t = T$ .

Step 2: Use (8) to calculate the optimal value function  $J_t(x^B(t), x^T(t))$  subject to (1)–(6), and the optimal control  $u_t^V(x^B(t), x^T(t))$  and  $u_t^B(x^B(t), x^T(t))$ .

- Step 3: Let  $t = t - 1$ . If  $t \geq 0$ , go to Step 2.
- Step 4: Identify the optimal  $q^*$ . Return the optimal cost  $J_0(q^*, 0)$ ; the optimal decision variables  $q^*, u_t^V(x^B(t), x^T(t)), u_t^B(x^B(t), x^T(t))$ .

### 3 Numerical Examples

In this section, we first provide an empirical case to demonstrate the container loading process at a seaport rail terminal and calibrate the input data. Secondly, we perform a range of experiments to illustrate the application of the proposed models.

Figure 2 shows the empirical data of container loading rates at a real rail terminal within a day (from a real case study in the UK). In total six trains are handled within a day, and each time period is 30 min. The number of containers handled per period ranges from 0 to 15. The working time window for each train ranges from 4 periods (i.e. 2 h) to 8 periods (i.e. 4 h). We calibrate the input data of the reference scenario as follows: the time period is 30 min;  $Q^T = 40$ ;  $Q^B = 30$ ;  $Q^C = 15$ ;  $U^V(t) = 15$ ;  $U^B(t) = 15$ . We assume  $x^B(t) \equiv u^B(t)$  and  $\xi x^V(t) = u^V(t) \cdot z$ , where  $z$  follows a uniform distribution. Here we want to focus on the uncertainty in the process from yards to rail terminal by assuming deterministic operations from RT buffer to RMG. Moreover, let  $c^P = 4$ ;  $c^V = 5$ ;  $c^B = 2$ ;  $c^C = 1$ ;  $c^S = 1$ ;  $c^U = 100$ . It should be noted that the above cost coefficients are hypothetical and only the relative values of these cost elements are meaningful.

Now we apply the model to optimize the pre-staging decision and the dynamic IMV assignment. As the length of working time window is an important factor, we experiment with three levels of working window, i.e.  $T = 4, 6, 8$ , which correspond to 2, 3, and 4 h working windows respectively. The results are given in Table 3.

From Table 3, it can be seen that: (i) in the deterministic situation, we have  $q^* = 0$ , which means zero pre-staging is optimal. This is intuitively true due to the facts: (a) pre-staging plus moving containers from RT buffer to RMG costs more than directly moving containers from yard to RMG; (b) the working time window is

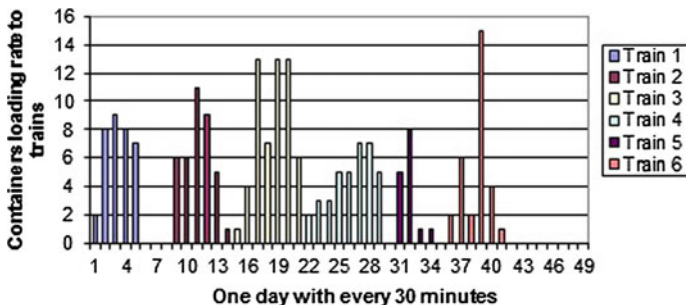


Fig. 2 Empirical data of container loading rate at a rail terminal within a day

**Table 3** Optimizing IMV assignment and pre-staging

	T = 4		T = 6		T = 8	
$z$	$q^*$	$J_0(q^*, 0)$	$q^*$	$J_0(q^*, 0)$	$q^*$	$J_0(q^*, 0)$
U(1, 1)	0	240.00	0	240.00	0	240.00
U(0.8, 1)	3	283.05	1	270.80	0	267.95
U(0.6, 1)	12	334.78	1	300.70	0	297.44
U(0.4, 1)	30	363.63	17	340.45	16	334.25
U(0.2, 1)	30	394.35	30	372.42	30	361.29

sufficiently large to move containers directly from yards to RMG to fully load the train; (ii)  $J_0(q^*, 0)$  is increasing in the degree of uncertainty; and  $q^*$  is increasing in the degree of uncertainty; (iii) by comparing the results with that of zero pre-staging cases (not included in this paper due to page limit), the cost saving of the best pre-staging decision from zero pre-staging is increasing as the degree of uncertainty increases. This indicates the importance of determine appropriate pre-staging. (iv) At the same degree of uncertainty,  $J_0(q^*, 0)$  is decreasing as the time window increases; and  $q^*$  is decreasing as the time window increases. When the time window is adequately large, zero pre-staging tends to be optimal.

## 4 Conclusions

This study considers the optimal assignment of IMV fleet and container pre-staging at a seaport rail terminal in the presence of uncertainty. The mathematical model developed using stochastic dynamic programming can plan the container flow at aggregate level, without the need to address the detailed discrete events, therefore can avoid the NP hard combinational optimization problem. Another innovation of the developed model is the ability of yielding optimal plans under dynamic mode and accommodating stochastic factors. However, when the dimension of state and decision variables increases, the computation complexity of the model also increases significantly. Numerical examples based on a real case are provided to illustrate the effectiveness of the model. Further research includes combining both discharge and load trains into a single optimization model.

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