Analysis of Operating Modes of Complex Compressor Stations

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Abstract We consider the modeling of operation modes for complex compressor stations (i.e., ones with several in- or outlets) in gas networks. In particular, we propose a refined model that allows to precompute tighter relaxations for each operation mode. These relaxations may be used to strengthen the compressor station submodels in gas network optimization problems. We provide a procedure to obtain the refined model from the input data for the original model.

1 Introduction

Gas transmission networks are a crucial part of the European energy supply infrastructure. The gas flow is driven by pressure potentials. To maintain the necessary pressure levels and control the routing of the gas in the network, compressor stations are used. In the German network compressor stations usually interconnect two or more pipeline systems. They often have a complex internal structure, allowing them to realize different routing patterns between the boundary nodes, which may serve as inlet or outlet depending on the requirements of the surrounding network [2]. An example of such a complex compressor station is shown in Fig. 1.

In this paper, we consider the compressor station modeling introduced in [2]. This model combines a network containing compressors and valves and a set of switching states for these elements to describe all feasible operation modes of a compressor

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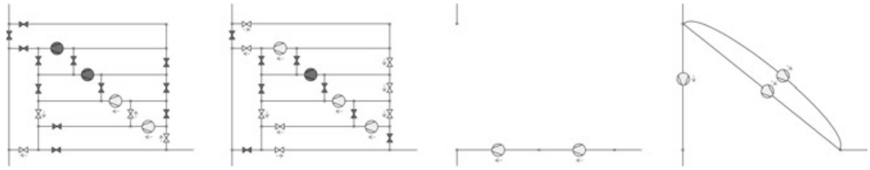

(a) Flow from east to south, double serial compression. Elements colored dark are "closed".

(b) Flow from north to east with double and to south with single compression.

(c) Reduced representation of operation mode in (a).

(d) Reduced representation of operation mode in (b).

Fig. 1 Two operation modes of a large compressor station (**a, b**) and their reduced representations (**c, d**) obtained with the methods of Sect. 3

station. The constraints describing the technical capability of a compressor may be nonlinear and nonconvex, leading to hard-to-solve MINLP models for a compressor station. To improve the model, a natural idea is to precompute, for each operation mode, bounds on the minimum and maximum flow and pressure that can be handled and to include this information in the model. This should help the solution process to detect unsuitable operation modes early. However, the modeling of an operation mode from [2] does not specify whether a compressor is actively compressing or bypassed. Thus, no nontrivial flow bounds may be obtained for an operation mode.

Contribution We develop techniques for analyzing the original representation of operation modes to obtain a more detailed representation prescribing for each compressor whether it is compressing or in bypass. This allows to compute tight bounds (or even convex hulls) for the pressure/flow combinations that can be handled by each operation mode. The crucial ingredient is a method to obtain a reduced representation of an operation mode to cope with redundancies due to the original representation. Examples of such reduced representations are also shown in Fig. 1.

Related work We briefly mention some related papers and refer to [5] for a comprehensive overview. Most work on optimization of compressor stations has focused on simple compressor stations compressing from a single inlet to a single outlet. The fact that a compressor station usually features several (often distinct) compressor units has been dealt with by using an aggregated model, like a range for the power required by the compression process [3], box constraints for flows and pressures [1], or a polyhedral model [6]. Papers using a detailed model for the operation of a single compressor unit usually assume that a compressor station consists of several parallel identical units [7], the only discrete decision being the number of units switched on. A recent exception is the work of [4], considering configurations consisting of serial stages of units used in parallel. Complex multi-way compressor stations with multiple operating modes are only considered in [2] and related work.

The remaining paper ist structured as follows. Section 2 recalls the model introduced in [2]. In Sect. 3, we propose a method that reduces the description of a single operation mode to a kind of "normal form". This is used in order to detect redundancy and generate a set of redundancy-free operation modes that, as a set, are equivalent to the original operation modes. Finally, we report on some computational results in Sect. 4.

2 Model for Complex Compressor Stations

Our model for compressor stations closely follows that proposed in [2]. We represent a compressor station as a directed graph $(V, A_{\mathrm{va}} \cup A_{\mathrm{cg}} \cup A_{\mathrm{sc}})$, where the arc set consists of the set of valves A_{va}, the set of compressors A_{cg}, and the set of shortcuts A_{sc}. Moreover, we partition the set of nodes into boundary nodes $V_{\pm}$ and inner nodes V_0. For each node $u \in V$ we introduce a variable for the pressure p_u with non-negative lower and upper bounds $\underline{p}_u$ and $\overline{p}_u$. For each arc $a \in A$ there is a variable for the mass flow q_a with lower and upper bounds $\underline{q}_a$ and $\overline{q}_a$. Positive mass flow values indicate flow in the direction of the arc, whereas negative values represent flow in the opposite direction. The precise values of the bounds depend on the type and state of an element. We define the excess of mass flow at nodes by

$$b_u := \sum_{a \in \delta^-(u)} q_a - \sum_{a \in \delta^+(u)} q_a \quad \text{for all } u \in V. \tag{1}$$

where $\delta^-(u)$ and $\delta^+(u)$ denote the sets of ingoing and outgoing arcs for node u. At inner nodes, the mass flow is conserved, i.e., we have $b_u = 0$ for $u \in V_0$.

Valves can be *open* or *closed* and are used to control the route of gas through the compressor station. A binary variable s_a distinguishes between these states ($s_a = 1$: *open*, $s_a = 0$: *closed*). A closed valve is like a missing connection, i.e., there is no flow and the pressures are decoupled. An open valve admits arbitrary flow and the pressures at its nodes are identical.

Compressors may operate in one of the states *closed* (no gas flow), *active* (compressing), and *bypass* (gas flow without compression). Binary variables $s_a, s_a^{\mathrm{ac}}, s_a^{\mathrm{bp}}$ distinguish between the states *active*, *bypass* and *closed* where $s_a = 1, s_a^{\mathrm{ac}} = 1$ corresponds to *active*, $s_a = 1, s_a^{\mathrm{bp}} = 1$ corresponds to *bypass* and $s_a = 0$ corresponds to *closed*. A closed compressor again corresponds to a missing connection and one in bypass to an open valve. We model the capabilities of an active compressor $a \in A_{\mathrm{cg}}$ by an abstract set $P_a \subseteq \mathbb{R}^3_{\geq 0}$ of feasible inlet pressure, outlet pressure, and mass flow. Thus our methods apply to a large range of compressor models. The constraints describing P_a, the capability set of a compressor, may be nonlinear and nonconvex, leading to hard-to-solve MINLPs for the entire compressor station.

Shortcuts are convenient modeling elements that allow arbitrary gas flow between two nodes without pressure drop.

An *operation mode* specifies the switching state of each active element (valves, compressors) and thus the route of the gas flow through the compressor station. Operation modes are modeled in [2, Section 6.1.8] by a triple $(A_{\mathrm{active}}, \mathscr{M}, d)$, where $A_{\mathrm{active}} = A_{\mathrm{va}} \cup A_{\mathrm{cg}}$ is the set of active elements. The set $\mathscr{M} \subseteq \{0,1\}^{A_{\mathrm{active}}}$ describes each operation mode $m \in \mathscr{M}$ by stating whether an active element a is *open* ($m_a = 1$) or *closed* ($m_a = 0$). In the case of an *open* compressor it is not yet specified whether this compressor is in *bypass* or is *active*. Finally, the function $d : A_{\mathrm{active}} \times \mathscr{M} \to \{-1, 0, 1\}$ describes whether the flow direction for an active arc $a = (u, v)$ is restricted or not (-1: flow in opposite direction of arc, 0: direction unspecified, -1: flow in arc direction).

As mentioned in the introduction, the fact that this representation does not specify whether an *open* compressor is *active* or running in *bypass* precludes us from obtaining tight bounds for flows and pressures obtainable by an operation mode. We thus propose a more detailed representation where each operation mode is *fully specified* by prescribing for each compressor whether it is *active* or in *bypass*. To obtain this representation from the original one we enumerate all *active*/*bypass* combinations for each operation mode. Since this leads to many and redundant operation modes, we apply the methods from Sect. 3 to obtain an equivalent smaller set of fully specified operation modes. These are described by a tuple $(A_{\text{active}}, \mathscr{M}^{\text{va}}, \mathscr{M}^{\text{cg}}, d')$, where $\mathscr{M}^{\text{va}} \subseteq \{0,1\}^{A_{\text{va}}}$ prescribes the state of each valve and $\mathscr{M}^{\text{cg}} \subseteq \{0,1\}^{A^2_{\text{cg}}}$ prescribes the state of each compressor. For each of these operation modes, we can now compute tight pressure and inflow bounds by solving the optimization problem given by (4)–(8) together with respective objective functions. Then, with $\underline{p}_u(m), \overline{p}_u(m)$ and $\underline{b}_u(m), \overline{b}_u(m)$ denoting the pressure and mass flow excess bounds for node u in operation mode m, the following inequalities are valid:

$$\sum_{m \in \mathscr{M}} \underline{p}_u(m)\, s_m \le p_u \le \sum_{m \in \mathscr{M}} \overline{p}_u(m)\, s_m \qquad \text{for all } u \in V, \tag{2}$$

$$\sum_{m \in \mathscr{M}} \underline{b}_u(m)\, s_m \le b_u \le \sum_{m \in \mathscr{M}} \overline{b}_u(m)\, s_m \qquad \text{for all } u \in V. \tag{3}$$

We call the model using the original operation modes the *compact model*, the one using fully specified operation modes the *extended model* and the extended model together with (2)–(3) the *bounded extended model*.

3 Topology Simplification for a Single Operation Mode

Our goal is to simplify the topology of a single operation mode of a compressor station to obtain a small "canonical" representation suitable for comparing operation modes via graph isomorphism detection (see Fig. 1).

We consider the network $N^m = (V, A^m, \underline{q}, \overline{q}, \underline{p}, \overline{p})$ corresponding to a fully specified operation mode m derived from the station network as follows. First, all closed elements are removed. Second, every shortcut, open valve and compressor in bypass is replaced by two opposing shortcuts with lower flow bound equal to zero. This is an equivalent transformation since the constraints for open valves or bypassed compressors are equivalent to those of shortcuts. Hence, the arc set A^m consists only of shortcuts and active compressors. Thus the model for a single operation mode becomes

$$0 \le \underline{p}_u \le p_u \le \overline{p}_u \qquad \text{for all } u \in V, \tag{4}$$

$$b_u = 0 \qquad \text{for all } u \in V_0, \tag{5}$$
$$\underline{q}_a = 0, \overline{q}_a = \infty \qquad \text{for all } a \in A_{\text{sc}}, \tag{6}$$
$$p_u = p_v \qquad \text{for all } (u,v) \in A_{\text{sc}}, \tag{7}$$
$$(p_u, p_v, q_a) \in P_a \subseteq \mathbb{R}^3_{\geq 0} \qquad \text{for all } (u,v) \in A_{\text{cg}}. \tag{8}$$

However, the network may be highly redundant, as a shortcut usually indicates that the incident nodes are identical. Thus we can reduce the size of the network by contracting a shortcut as follows. We identify the incident nodes of the shortcut and update the pressure bounds of the remaining node to be the intersection of the pressure intervals for the original nodes. If there are any other arcs between the two nodes, we do keep them as self-loops. But we need to be careful when applying this contraction since shortcuts sometimes do carry important information on the topology of feasible flows. We now devise a criterion for safely removing shortcuts. For this, we consider the shortcut subgraph of N^m, G^{sc}, its set of entries V_+^{sc}, its set of exits V_-^{sc} and for all entries $w \in V_+^{sc}$ the set $\vec{R}_N(w) \subseteq V_-^{sc}$ of exits reachable using only shortcuts:

$$G^{sc} := (V, A_{\text{sc}}) \tag{9}$$
$$V_+^{sc} := V_\pm \cup \{w \in V \mid \exists u \in V : (u,w) \in A_{\text{cg}}\} \tag{10}$$
$$V_-^{sc} := V_\pm \cup \{w \in V \mid \exists u \in V : (w,u) \in A_{\text{cg}}\} \tag{11}$$
$$\vec{R}_{N^m}(w) := \{u \in V_-^{sc} : \exists\ w - u - \text{path in } G^{sc}\} \qquad \text{for all } w \in V_+^{sc} \tag{12}$$

Proposition 1 *Consider a shortcut $\tilde{a} = (u,v)$ with $u \in V \setminus \{V_+^{sc}\}$ and the network N' arising from N when contracting $\tilde{a}$ to v. If*

$$\vec{R}_N(w) = \vec{R}_{N'}(w) \quad \textit{for all } w \in V_+^{sc} \tag{13}$$

then for every admissible flow-pressure combination (p', q') for N' there exists an admissible flow-pressure combination (p, q) for N such that

$$q'_a = q_a \quad \textit{for all } a \in A_{\text{cg}}, \tag{14}$$
$$b'_w = b_w \quad \textit{for all } w \in V_\pm, \tag{15}$$
$$p'_w = p_w \quad \textit{for all } w \in V_\pm, \tag{16}$$

and vice-versa.

4 Computational Results

To investigate the effect of our method, we consider the compressor station network with three boundary nodes and four compressors shown in Fig. 1. We model the operating range $P_a = \{(p_u, p_v, q_a)\} \subseteq \mathbb{R}^3_{\geq 0}$ of each compressor $a = (u,v) \in A_{\text{cg}}$ by a

Table 1 Computational results on sample compressor station network. The first number is for feasible, the second number for infeasible instances

	Compact model	Extended model	Bounded extended model
Number of binary variables after presolve	29.6/33.5	34.1/33.2	27.6/33.2
Number of solving nodes	9.2/16.3	10.4 / 24.4	11.1/20.5
Presolving detected infeasibility	–/80.4%	–/80.1%	–/84.2%

simplified polyhedral model since we are only interested in the combinatorics of the compressor station model. In the original data there are 53 operation modes; these are used in the compact model. Enumerating all combinations of *active* and *bypass* for compressors leads to 655 fully specified operation modes. Removing infeasible operation modes and eliminating redundant modes using graph isomorphism detection after applying topology simplifications presented in Sect. 3 leaves 109 operation modes. These are used in our extended and bounded extended models.

We generated a large set of 58463 instances with varying flow amounts from one boundary node to one or both of the others at multiple different pressure levels, and checked whether each instance is feasible. We have used SCIP to solve our problems and the results showed that ca. 55% of the instances were feasible. To compare the performance of our extended models to the original compact one we consider the mean number of binary variables that have not been fixed by SCIP presolving and the mean number of branch-and-bound nodes required for solving. The solving times were negligible in all cases due to the absence of nonlinear constraints (see Table 1).

The results show that our preprocessing methods only have limited impact on the solver performance. We conjecture this to be due to the fact that we are considering the compressor station in isolation where combinatorics are simple enough for SCIP to perform well without further support. The next step is thus to apply our methods to optimizing large-scale gas networks.

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