Chapter 12
Probing Faster than Light Travel and Chronology Protection with Superluminal Warp Drives

Carlos Barceló and Stefano Liberati

12.1 Introduction

The possibility of "faster than light" (FTL) travel and time machines (TM) have been a fascinating theme for human beings since the dawn of storytelling and they are at the centre of many science fiction novels and movies. What is less known to the broad pubic is that these tantalising phenomena are definitely an open possibility in Einstein's general relativity (GR) theory. The very same "plasticity" of spacetime that is at the core of the Einstein's grand construction allows at the same time for extremely warped spacetimes where the causal structure can be subverted and realise these extreme features. In this chapter, we are going to see how FTL and TM are related and why they are so relevant in assessing the viability of GR as a fundamental theory of reality. We shall consider explicit examples allowing for these features, explain why the latter are so problematic for our understanding of reality, and focus on those spacetimes that seems so far not forbidden or evidently unphysical. In particular, we shall discuss a class of spacetimes called "warp drives", presented in the previous chapter—which allow for FTL travel and can be used to built a TM—and show that they will generically present a semiclassical instability just for the fact of allowing faster than light propagation.

The structure of this contribution will be the following. We shall first provide some technical definitions about what we mean by time travel. We shall then review some spacetimes which allow for TM and FTL travel. We shall then discuss in the next

C. Barceló (✉)
Instituto de Astrofísica de Andalucía (CSIC), Glorieta de la Astronomía,
18008 Granada, Spain
e-mail: carlos@iaa.es

S. Liberati
SISSA, Via Bonomea 265, 34136 Trieste, Italy

S. Liberati
INFN, sezione di Trieste, Italy
e-mail: liberati@sissa.it

F.S.N. Lobo (ed.), *Wormholes, Warp Drives and Energy Conditions*,
Fundamental Theories of Physics 189, DOI 10.1007/978-3-319-55182-1_12

section why FTL travel can be easily manipulated for building TMs. In Sect. 12.5, we shall consider why TM are paradoxical and which solutions have been devised to come out of these paradoxes and explain why so far it seems that for reaching a definitive conclusion one should go beyond the realm of semiclassical gravity. Finally, in Sect. 12.6 we shall consider the specific case of warp drives and show that a generic prediction about their instability can be reached by a quantum field theory calculation in curved spacetime. We shall argue that this result seems to strongly hint that structures allowing for FTL travel and hence TM building are intrinsically unstable even within the well-known framework of semiclassical gravity.

12.2 Time Machines: Basic Technical Definitions

Worldlines of observers/particles in spacetime are typically timelike curves if they have mass, and null curves if the particles are massless (respectively inside and on the border of light cones). A causal curve is defined to be a curve which is nowhere spacelike (i.e. lying outside of a light cone). A chronological curve is a curve which is everywhere timelike. So in layman language, the first kind of curves characterises the propagation of signals via massive or massless mediators while the second class pertains to massive objects/observers.

Time travelling notoriously requires a time machine (exactly like in the H.G. Well novel). A time machine is defined as some device able to provide closed causal or chronological curves, loops in propagation of signals or massive objects. Albeit closed timelike/chronological curves (CTC) imply automatically closed causal curves (CCC) the reverse is of course not true, i.e. one might have a spacetime where only massless particles can form loops but massive objects cannot. Of course this last case is also not seriously dangerous as it does not lead to the aforementioned paradoxes. So, the culprit of our search, the smoking gun of time travel, is the presence of a CTC in a given spacetime.

However we can imagine, and indeed we have many examples of spacetime which are almost everywhere "sane" except for regions where time travel is allowed, or alternatively spacetimes which in the past look globally hyperbolic but later develop a region with CTC and hence do not admit a well-posed Cauchy evolution. In order to describe precisely these situations, a first step is to characterise points within a chronology-violating region (a region with CTC). There are events p belonging to a chronology-violating region I^0_p if

$$I^0_p \equiv I^+(p) \bigcap I^-(p) \neq \emptyset,$$

in words, if the intersection of the chronological past and future for this set of events (i.e. the interiors of the past and future light cone stemming from these events) is not an empty set. As usual, we shall then define the total chronology-violating region in a given manifold $\mathscr{M}$ as the union of all these I^0_p, i.e.

$$I^0(\mathscr{M}) = \bigcup_{p \in \mathscr{M}} I^0_p.$$

Of course in the same way we can define causality violating regions by just replacing in the above formulas chronological future and past $I^{\pm}$ with causal future and past $J^{\pm}$.

We can now define the onset of a time machine region, its border, as the *future chronological horizon* which is technically characterised as

$$H^+(I) \equiv \partial \left[I^+ \left(I^0(\mathscr{M}) \right) \right].$$

Of course we can also straightforwardly define a *past chronological horizon* (the border in the past of a time machine region) by the same definition replacing the chronological future with the chronological past. Similarly, the definition for a past/future causal horizon can be obtained replacing the chronological past/future with their causal counterparts.

It is worth stressing that chronological horizons are always generated by null geodesics [33] and that while they do not need to coincide mathematically with causal horizons, they do so in many cases. Finally, it is easy to see that chronological horizons are just a special case of Cauchy horizons as they also characterise a breakdown of the Cauchy evolution of data and a lack of global hyperbolicity of the spacetime (the other typical case of Cauchy horizons being those associated to missing points in spacetime e.g. due to timelike singularities).

12.3 Causality Challenging Spacetimes

The list of spacetimes in GR which are problematic from the point of view of the causal structure is quite long and basically splits in two big families. The first family is made by what we might call "time machine rotating solutions". For all these solutions rotation is the key ingredient as the swirling of spacetime and frame dragging tilts the light cones till they eventually include closed orbits around the rotation axis. These are spacetimes which self-evidently include regions with CTC and hence TMs. The second family of spacetimes is in a sense milder. These spacetimes do not per se entail CTCs but, as we shall see in the following sections, can be easily manipulated so to form them.

12.3.1 Time Machine Rotating Solutions

A succinct (incomplete) list of this kind of solutions includes (see [33] for a more complete treatment)

- **van Stockum spacetimes**: Infinitely long cylinders of spinning dust with density increasing with radii. Tipler cylinders are included in this family (with a massive cylinder replacing the cylindrical distribution of dust).
- **Gott's spacetimes**: these are spacetimes, e.g. with two infinitely long, parallel, cosmic strings longitudinally spinning around each other.
- **Gödel universe**: this is the famous solution found by Kurt Gödel in 1949 which describes a globally rotating spacetime filled with (homogeneously distributed) dust and a (positive) cosmological constant fine-tuned to match the energy density of the dust. The solution is not isotropic due to the rotation, however the homogeneity implied by the matter distribution requires that the direction but not the position of the rotation "axis" is determined. Indeed, any event in the spacetime can be considered at the origin of the rotating frame and lies on some CTCs.
- **Kerr–Newman black holes**: Rotating (neutral or charged) black holes are endowed with a region, inside the Cauchy horizon, where the Killing vector associated to invariance under rotations about the spin axis becomes timelike. Given that the orbits of this vector are necessarily closed curves this implies the presence of CTCs.

The first three items of this list are often (somewhat irreverently) characterised as GIGO, i.e. as solutions "garbage in-garbage out", which means that these are solutions characterised by somewhat unphysical matter/energy distributions (e.g. infinitely long cylinders or strings) and/or global rotation (e.g. Gödel universe). In the case of the rotating black hole solutions, the common objection is instead that the chronology-violating region is located well within the inner/Cauchy horizon. The well-known instability associated to this global structure (infinite blueshift) is per se a warning bell that this (classical) instability might lead to an internal geometry very different from the one predicted by the Kerr family of solutions. There is however a second class of dangerous spacetimes that are not as easily dismissed because they "per se" do not posses CTC and can be grouped under the name of "superluminal travel allowing" spacetimes.

12.3.2 Faster than Light Travel Spacetimes

There are two well-known kinds of spacetimes in general relativity which allow for faster than light travel, these are warp drives and traversable wormholes. Let us summarise their main features here.

Warp drives: Warp drives (where the most known declination is the Alcubierre warp drive (see the previous chapter) discovered in 1994 [1]) are a different sort of spacetimes where a spherical region with flat (or almost flat) geometry, a bubble, can move at arbitrary speed thanks to the simultaneous contraction and expansion of spacetime respectively at its front and rear. Of course such a geometry does not violate local Lorentz invariance as this still holds locally around each point of spacetime. However, geometrically speaking general relativity does not pose any bound on the relative motion of different regions of spacetime. Noticeably, these geometries need exotic matter for being realised, i.e. matter which violates some energy condition (typically the null one).

Traversable wormholes: The concept of a "spacetime shortcut", a sort of tunnel connecting points of spacetime which might be otherwise separated by long distances arose several times in the literature. The first to be discovered was the so-called Schwarzschild wormhole often called Einstein–Rosen bridge (after the name of the authors of the 1935 paper [12] that brought it to the general attention albeit it was already discovered in 1916 by Flamm [14]). The latter connects two asymptotically flat regions in an eternal black hole spacetime. Unfortunately this class of wormholes is not traversable. Indeed if they connect two parts of the same universe, it was shown [16] that they will pinch off too quickly for light (or any massive particle) to travel from one exterior region to the other. However, traversable wormhole solutions were found in 1988 by Morris and Thorne [26, 27] (and later developed by several authors, see e.g. [33, 34] and references therein). These are wormhole structures whose throat is held open by variably large amounts of exotic matter.

Let us consider more in detail one solution for each of these classes.

12.3.2.1 The Alcubierre Warp Drive in a Nutshell

The Alcubierre warp drive geometry was introduced by Miguel Alcubierre in 1994 (see Ref. [1]) and represents a bubble containing an almost flat region, moving at arbitrary speed within an asymptotically flat spacetime. Mathematically its metric can be written as

$$ds^2 = -c^2 dt^2 + [dx - v(r)dt]^2 + dy^2 + dz^2 \, , \tag{12.1}$$

where $r \equiv \sqrt{[x - x_c(t)]^2 + y^2 + z^2}$ is the distance from the centre of the bubble, $\{x_c(t), 0, 0\}$, which is moving in the x direction with arbitrary speed $v_c = dx_c/dt$. Here $v(r) = v_c f(r)$ and f is a suitable smooth function satisfying $f(0) = 1$ and $f(r) \to 0$ for $r \to \infty$. To make the warp drive travel at the speed $v_c(t)$, the spacetime has to contract in front of the warp drive bubble and expand behind it. It is easy to see that the worldline $\{x_c(t), 0, 0\}$ is a geodesic for the above metric. Roughly speaking, if one places a spaceship at $\{x_c(t), 0, 0\}$, it is not subject to any acceleration, while moving faster than light with respect to someone living outside of the bubble (here the spaceship is basically treated as a test particle, see Ref. [23] for a more general treatment). The spaceship inside the warp drive bubble is, as a matter of fact, isolated from the spacetime surroundings and cannot interact with them, however one could in principle conceive to build a sort of "interstellar railway" running from Earth to a distant planet which by a coordinated generation of energy violating matter could locally produce and move a warp drive, with a spaceship inside, at superluminal speeds.

A curious characteristic of the Alcubierre warp drive geometry, which is seldom mentioned, is that in a superluminal travel between two events in spacetime A and B, the proper time as measured by the traveller inside the bubble is not subject to the standard relativistic time slowdown. This is due to the fact that the observer inside the warp drive finds itself at rest in a basically flat portion of spacetime and (classically) does not perceive the bubble motion. Indeed, observers at rest outside, as well as internal travellers, will measure essentially the same amount of travel duration in terms of their proper times (see Eq. (12.1)). This contrasts with what normally happens in standard special relativity. From the traveller's point of view if one approaches the speed of light, the proper time duration to go from A to B becomes arbitrarily short. Instead, using a warp drive to shorten this time duration one would need to increase the warp drive velocity to larger and larger speeds which will not affect the relation between the proper time of the warp drive traveller and that of some observer outside at rest.

12.3.2.2 Traversable Morris–Thorne Wormholes in a Nutshell

Traversable Morris–Thorne wormholes [26, 27] are time independent, non-rotating and spherically symmetric solutions of general relativity describing a bridge/passage between two asymptotically flat regions (not necessarily in the same universe albeit this is the case we are interested in here). They are described by a simple metric

$$ds^2 = -e^{2\Phi(\ell)}dt^2 + d\ell^2 + r^2(\ell)\left[d\theta^2 + \sin^2\theta d\phi^2\right], \tag{12.2}$$

where one requires $\ell \in (-\infty, +\infty)$, absence of event horizons and metric components at least C^2 in ℓ. Furthermore asymptotic flatness for $\ell \to \pm\infty$ is imposed by requiring

$$\lim_{\ell\to\pm\infty}\left\{\frac{r(\ell)}{|\ell|}\right\} = 1 \qquad \text{i.e.} \quad r(\ell) = |\ell| + O(1) \quad \text{for} \quad \ell \to \pm\infty \tag{12.3}$$

for space asymptotic flatness and

$$\lim_{\ell\to\pm\infty}\Phi(\ell) = \Phi_{\pm} = \text{constant and finite} \tag{12.4}$$

for spacetime asymptotic flatness. The radius at the wormhole throat is $r_0 \equiv \min\{r(\ell)\}$ which can always be chosen to be at $\ell = 0$.

This metric is a solution of the Einstein equations but requires a stress–energy tensor (SET) $T^{\mu}{}_{\nu} = \text{diag}(-\rho(r), \tau(r), p_\theta(r), p_\phi(r))$ with radial tension $\tau(r) = -p_r(r)$ which violates the null energy condition (NEC) which states the positivity of the product $T_{\mu\nu}k^\mu k^\nu$ for any null-like vector k^μ. More precisely there is always some r_* such that for any $r \in (r_0, r_*)$ one has $\rho(r) - \tau(r) < 0$ which is tantamount to NEC violation (which also implies the violation of the weak, strong and dominant energy conditions, see e.g. [33]). Of course this implies that also at the throat the NEC is violated and specifically one finds

$$\tau(r_0) = \frac{1}{8\pi G r_0^2} \approx 5 \cdot 10^{36} \left(\frac{10 \text{ m}}{r_0}\right)^2 \frac{\text{N}}{\text{cm}^2}, \qquad (12.5)$$

so that as anticipated a huge tension is required for keeping a macroscopic wormhole open.

12.3.2.3 Objections and Answers

The common objections against these spacetimes are twofold. One consists in noticing that this “superluminal travel allowing” spacetimes generically require large amounts of exotic (energy violating) matter (e.g. for a traversable wormhole with a throat of about one metre one would need about one Jupiter of matter with negative energy density), at least within GR (in alternative theories of gravity this does not need to be the case). The other objection has to do with the way such structures could be formed in the first place. For example, forming from nowhere a wormhole would imply a change in topology which is forbidden in classical GR and anyway is known to lead to unacceptable large particle production from the vacuum [2, 25]. However, such objections could be seen as engineering challenges, in the sense that no no-go theorem tells us that we cannot generate exotic matter in a given region (indeed we can do so, in small quantities, e.g. with the Casimir effect) or that in principle one cannot “grow” macroscopic wormholes from those expected to be spontaneously produced at the Planck scale in the so-called Wheeler spacetime foam.

So in conclusion, within GR nothing seems to prevent, at least in principle, the possibility of superluminal travel at the moderate cost to solve few (daunting) engineering problems and to produce and control sufficiently large amounts of exotic matter. Of course this sounds pretty exciting but on second thoughts also very worrisome. Indeed, it is quite easy to transform any superluminal travel capable structure into a time machine. This is what we shall analyse in the following section.

12.4 Time Machines from Faster than Light Travel

Wormholes and warp drives are two ways in which one could travel faster than light (FTL). As we shall see, within the GR framework the existence of CTCs is strongly linked with the seemingly milder concept of FTL travel.

Probably the question that lay public asked more frequently to specialists in relativity is “why is it not possible to travel faster than light”. We do not know why but, leaving aside for the moment the cosmological realm, we have not observed any counterexample. Moreover, the theory of special relativity was born out of requiring this as a postulate and after one hundred years of development we can say with confidence that the number of predictions and verifications of this theory are quite robust (at least at currently explored energies, see e.g. [21] for a recent review).

In special relativity, the proper time of an observer freezes out when the observer approaches the speed of light. Beyond that, travelling in a spacelike trajectory amounts to having your proper time running backwards in time, the seed of time travel. But these trajectories are precisely the ones not allowed for real massive particles. By allowing the deformation of the light-cone structure, general relativity opens new venues into the connection between CTC and FTL travel.

As we already mentioned general relativity is constructed so that locally nothing can travel outside the light cone, so locally nothing can travel faster than light. However, somewhat surprisingly general relativity can indeed accommodate FTL travel. The cleanest situation one can think of is the following. Consider an asymptotically flat spacetime. In the asymptotic limit a light ray will traverse this spacetime in a straight line and at the speed of light. However, geometrically the light-cone structure deep in the spacetime can be such that a light ray could traverse this region faster than it would be done by his homologous at infinity.

This is possible because there is a tight connection between this sort of formulation of FTL travel and the existence of violations of the energy conditions. While the presence of positive distributions of energy results in Shapiro time delays, when light rays traverse negative energy distributions they are advanced. For instance, K. Olum [28] proved that to produce a FTL configuration one needs to violate the Weak Energy Condition. Related investigations were carried over in [35] dealing with weak fields. This association between ray advancement and negative energy has also been used to produce a slightly different formulation and proof of a positive mass theorem [29].

From this perspective, the problem of FTL travel is transmuted in general relativity into the problem of understanding the nature of the possible sources of gravity. In fact, without restricting the possible matter content general relativity allows in principle all imaginable Lorentzian geometries.

Leaving aside the energy-condition-violations issue, how can one manipulate FTL configurations such as warp drives or wormholes to build a time machine?

12.4.1 The Warp Drive Case

Before any warp drive is operating, consider an inertial reference frame in which two locations A and B are at rest. At some point, we arrange a warp drive so that someone in the bubble travels from A and B in a time interval $0 < t_B - t_A < |x_B - x_A|$ as measured from this inertial frame. From the perspective external to the warp drive the trajectory of the bubble seems to be spacelike (although it is not). Now, make a change of inertial frame to describe the same configuration, one travelling with a velocity close to the speed of light in the same direction of the warp drive. In this inertial frame the trajectory of the warp drive will be such that $t'_B - t'_A < 0$, it is as if the warp drive was running backwards in time. Then, in this frame one just needs to arrange another warp drive to travel back from B to A so that $0 < t'_{AB} - t'_B < |t'_B - t'_A|$. In this way the time t'_{Ab} can be made smaller than t'_A, that is, one can come back to the initial position before the very forth and back journey has started: CTCs and a chronological horizon have been formed. Of course, the discussed warp drive is just a special case, as the above reasoning would apply to any device allowing faster than light propagation at arbitrary speeds while special relativity holds (see e.g. [22] for an extensive discussion).

12.4.2 The Wormhole Case

The same type of time machine configuration can be produced using a wormhole. Consider one mouth of the wormhole to be always at rest in one inertial frame. Then, in principle one can make time to run slower in the other mouth, for instance by making this mouth to travel towards the other at a certain velocity (a special relativistic effect) or by placing it close to a very compact object (a general relativistic effect). In the first case, if we wait long enough since the second mouth starts moving, a light ray produced in the first mouth and sent through the throat will come up again in the inertial frame in the past. This travel back in time can be strong enough so that the ray now travelling from the second mouth to the first through the standard path in spacetime arrives earlier than it was sent in the first place. Again, CTCs and a chronological horizon have been formed.

12.4.3 Some Analogue Gravity Lessons

One can build warp drive geometries in analogue gravity systems, such as a flowing fluid [5]. Is it then possible to build a time machine using a fluid? No, definitely it is not. From the perspective of the lab one of the two warp drives needs to be running backwards in time. From the same perspective one can perfectly tilt the sound cones (by producing background flows) so that the sound travels quicker or slower than in a fluid at rest. However, to tilt a sound cone so that it runs backwards in time needs to pass through a configuration in which the sound velocity becomes infinite (and beyond). This is clearly not possible as the background fluid flow cannot be arranged to reach an infinite velocity. So although an internal observer could follow the same line of reasoning leading from warp drives to time machines the existence of an external and more fundamental causality forbids to perform this step. Any proper manipulation of the system in terms of initial data developments will show that no causal paradoxes can occur. The fundamental causality of the Minkowskian lab is the one that controls the evolution of the system and that is always running forward in time.

If the general relativity causality were not the only one in reality, a similar argument could be in place. The fact that there is a hand waving argument to build a time machine does not imply that this can be realised as a well-defined initial data problem. There might be deeper layers of reality, with a perfectly well-defined causality but possessing FTL characteristics, fooling us (a nice discussion of these possibilities can be found in [18]). Nature shows that indeed there are many different causalities operating for different degrees of freedom. To this day it is a well-supported hypothesis that the general relativity causality encompassed all of the rest. It is this fundamentality that allows to think about FTL travel and time machines. However, it is clear that whether the general relativity causality is the ultimate causality and whether one could build this type of configurations can only be ultimately distinguished by experiments.

An example of the previous discussion is provided by dispersive modifications of the general relativity behaviour. A superluminal dispersion can produce FTL travel without the need *per se* of allowing the construction of time machines. It will all depend on the specific causal characteristics of the final system of differential equations. Superluminal modifications of the dispersion relations could be obtained as a semiclassical result of an underlying theory of quantum gravity (see e.g [17]). Also, it was shown by Scharnhorst [31] that in a Casimir vacuum photons travelling perpendicular to the plate boundaries could achieve FTL (although the suppossed effect is too small to be measured in current experimental setups). In this case, it has been shown that even if this effect existed it will not break the tenets of special relativity [22] maintaining the existence of a constant c which is invariant under Lorentz transformations. On the other hand, they also show that the Scharnhorst effects cannot be used to build a time machine in the form described above. In this sense, the Scharnhorst effect would exhibit a mild violation of the speed of light limit.

But even before reflecting onto more or less speculative possibilities based on beyond general relativity effects, the very construction of a time machine has to be examined at the light of a well-defined initial value problem within the very framework of general relativity. As is conjectured to happen with Cauchy horizons, it might be that the formation of a chronological horizon is non-generic and unstable to perturbations. The analysis of possible classical instabilities has to be complemented by one of semiclassical instabilities in a hierarchy of possibilities. One of these semiclassical instabilities will be discussed later on in this chapter.

12.5 Time Travel Paradoxes and Possible Solutions

We have seen in the previous sections that GR seems to entail the possibility of time travel and that at the classical level nothing seems to forbid a priori the production of CTCs in otherwise healthy spacetimes, especially by making a careful use of the superluminal travel allowed by some solution with exotic matter.

The reason why the possibility of time travel is per se considered a deeply worrying weakness of the theory is that time travel to the past is per se logically paradoxical (apart from being a limit to the predictive power of the theory as e.g. singularities are as well). Indeed, the possibility to travel back in time (there is no logical paradox with travelling into the future, which is anyway already allowed by special relativity, think about the *apparent* twin paradox often quoted in text books) is *logically* paradoxical in two ways:

Grandfather paradox: The first way is related to the fact that when travelling back into the past our actions can affect what was supposed to stem from that past. So, for example, we could end up killing our grandfather and in this way prevent our own birth. Then how can we exist and change the past in the first place?

Bootstrap paradox: If we could travel back in time we could generate information from nothing. For example, someone could copy a mathematical theorem proof from a textbook, then travel back in time to find the mathematician who first published the proof before he published it, and simply pass the proof to the mathematician. In this case, the information in the proof has no origin.

So it is easy to see that time travel is a quite dangerous feature to have built in within your theory of spacetime dynamics. In what comes next we shall analyse why GR allows for this feature. But before doing this, we need to have the mathematical tools for being precise about what we mean by time travel.

In order to avoid such paradoxes, several approaches have been proposed in the past. We shall here summarise them briefly.

- **Bifurcating reality**: The most radical approach consists in accepting the possibility of modifying the past so creating a bifurcation in the future. This obviously requires also a radical rewriting in physics and possibly stepping from pseudo-Riemannian manifolds to non-Hausdorff ones.[1]
- **Multiverse/many-worlds reality**: This alternative is much in line with the one above but is based on quantum rather than classical reasoning. Reality would be made of several (possibly infinite) copies of our universe corresponding to all the possible outcomes of the many choices/measurements continuously performed. In this sense, changing the past does not produce any paradox as it would just correspond to take a pre-existing parallel reality while all the realities in which we never changed the past continue to exist.
- **Novikov's consistency conjecture**: In this case, one postulates CTCs never lead to paradoxes because only periodic solutions are really allowed. More rigorously, one conjectures a principle of self-consistency, which states that the only solutions to the laws of physics that can occur locally in the real Universe are those which are globally self-consistent [15]. That means that if you go back in time and attempt any action to change the past you will not only be unable to succeed but any of these actions will have to be there in first place for things to unfold the

[1] A Hausdorff manifold is a manifold for which for any two different points x_1 and x_2 belonging to the manifold admit open sets O_1 and O_2, with $x_1 \in O_1$ and $x_2 \in O_2$, such that $O_1 \cap O_2 = \emptyset$.

way they did in your past. You had to travel back in time and do what you had to do. A strict consistency without escape. The conjecture has been subject of debate with attempts to prove it in simple systems such as billiard balls moving in spacetimes with traversable wormholes [11]. The upshot from this and other studies is that there seems to exist self-consistent solutions for every possible billiard ball initial trajectory. However, this only applies to initial conditions outside of the chronology-violating region of spacetime, which is, as said, bounded by a Chronological/Cauchy horizon and moreover it is very difficult to extend to more complex, realistic, systems.

- **Hawking Chronology protection conjecture**: Finally a more orthodox solution to the paradoxes entailed by time machines is that the latter are always unstable due to quantum effects. More precisely, the chronology protection conjecture says that the laws of physics will always prevent the formation of CTCs [19]. Much work was put in the last 25 years for proving this conjecture on the base of calculations using QFT in curved spacetime (see e.g. [32, 34] and references therein). Generically, all these calculations confirm that the renormalized stress–energy tensor (SET) tends to blow up in the proximity of a chronological horizon. Nonetheless, no conclusive proof up to date can be provided due to the fact that generically the Green functions fail to be Hadamard on a Chronological/Cauchy horizon [20]. Given that the Hadamard behaviour of two points Green functions is a crucial requirement for deriving a renormalised SET and hence the semiclassical Einstein equations that would predict the evolution of spacetime, we are hence obliged to accept that no definitive proof is derivable from this framework. Actually, the behaviour of the Green functions close to a chronological horizon seems to indicate that contributions for Planck scale physics will always be non-negligible in its proximity and that consequently a full quantum gravity framework will be the only hope to prove the conjecture.

So it looks like no conclusive answer is available for how to cope with the widespread possibility of spacetimes with CTCs in GR missing a deeper understanding of spacetime as it should be provided by full-fledged quantum gravity. Nonetheless, a more humble approach can sometimes be taken. As we have seen most of the "rotation-based" time machines can be discarded on physical grounds (including the interior of a Kerr black hole). So, much more dangerous is the appearance of spacetimes such as wormholes and warp drives that would "only" require sufficiently large amounts of exotic matter. In particular, we have also seen that it is very debatable if a macroscopic wormhole can be created (problems with topology change) from nothing or growth from a hypothetic spacetime foam. No such objection though are present for a superluminal warp drive (the type that could be used to build a time machine). So is there anything forbidding the formation of such structures? Can semiclassical gravity provide at least in this case a definitive prediction? In what follows, we shall show that the answer to this question is in fact affirmative.

12.6 Superluminal Warp Drive Instabilities: A Pre-emptive Chronology Protection at Work?

We are going to discuss now the instability associated with a superluminal warp drive. In the actual computation, we shall restrict our attention to the $1 + 1$ dimensions case (since in this case one can carry out a complete analytic treatment). Changing coordinates to those associated with an observer at the centre of the bubble, the warp drive metric (12.1) becomes

$$ds^2 = -c^2 dt^2 + [dr - \bar{v}(r)dt]^2 , \qquad \bar{v} = v - v_c , \tag{12.6}$$

where $r \equiv x - x_c(t)$ is now the signed distance from the centre. Let us consider a dynamical situation in which the warp drive geometry interpolates between an initial Minkowski spacetime $[\hat{v}(t, r) \to 0$, for $t \to -\infty]$ and a final stationary superluminal ($v_c > c$) bubble $[\hat{v}(t, r) \to \bar{v}(r)$, for $t \to +\infty]$. To an observer living inside the bubble this geometry has two horizons, a *black horizon* $\mathscr{H}^+$ located at some $r = r_1$ and a *white horizon* $\mathscr{H}^-$ located at $r = r_2$. Here, let us just add that from the point of view of the Cauchy development of $\mathscr{I}^-$ these spacetimes possess Cauchy horizons.

12.6.1 Light Ray Propagation

Let us now consider light ray propagation in the above-described geometry. Only the behaviour of right-going rays determines the universal features of the renormalised stress–energy tensor (RSET), just like outgoing modes do in the case of a black hole collapse (see Refs. [3, 4, 13]). Therefore, we need essentially the relation between the past and future null coordinates U and u, labelling right-going light rays. There are two special right-going rays defining, respectively, the asymptotic location of the black and white horizons. In terms of the right-going past null coordinate U, let us denote these two rays by $U_{\rm BH}$ and $U_{\rm WH}$, respectively. The finite interval $U \in (U_{\rm WH}, U_{\rm BH})$ is mapped to the infinite interval $u \in (-\infty, +\infty)$ covering all the rays travelling inside the bubble. For rays which are close to the black horizon, the relation between U and u can be approximated as a series of the form [13]

$$U(u \to +\infty) \simeq U_{\rm BH} + A_1 e^{-\kappa_1 u} + \frac{A_2}{2} e^{-2\kappa_1 u} + \ldots . \tag{12.7}$$

Here A_n are constants (with $A_1 < 0$) and $\kappa_1 > 0$ represents the surface gravity of the black horizon. This relation is the standard result for the formation of a black hole through gravitational collapse. As a consequence, the quantum state which is vacuum on $\mathscr{I}^-$ will show, for an observer inside the warp drive bubble, Hawking radiation with temperature $T_H = \kappa_1/2\pi$.

Equivalently, we find that the corresponding expansion in the proximity of the white horizon is [13]

$$U(u \to -\infty) \simeq U_{\rm WH} + D_1 e^{\kappa_2 u} + \frac{D_2}{2} e^{2\kappa_2 u} + \dots , \tag{12.8}$$

where $D_1 > 0$ and κ_2 is the white hole surface gravity and is also defined to be positive. The interpretation of this relation in terms of particle production is not as clear as in the black horizon case and a full study of the RSET is required.

12.6.2 Renormalized Stress–Energy Tensor

In past null coordinates U and W the metric can be written as

$$ds^2 = -C(U, W) dU dW . \tag{12.9}$$

In the stationary region at late times, we can use the previous future null coordinate u and a new coordinate $\tilde{w}$, defined as

$$\tilde{w}(t, r) = t + \int_0^r \frac{dr}{c - \bar{v}(r)} . \tag{12.10}$$

In these coordinates the metric is expressed as

$$ds^2 = -\bar{C}(u, \tilde{w}) du d\tilde{w} , \quad C(U, W) = \frac{\bar{C}(u, \tilde{w})}{\dot{p}(u)\dot{q}(\tilde{w})} , \tag{12.11}$$

where $U = p(u)$ and $W = q(\tilde{w})$. In this way, $\bar{C}$ depends only on r through $u, \tilde{w}$.

For concreteness, we refer to the RSET associated with a quantum massless scalar field living on the spacetime. The RSET components for this case acquire the well-known expressions [6, 10]

$$T_{UU} = -\frac{1}{12\pi} C^{1/2} \partial_U^2 C^{-1/2} , \tag{12.12}$$

$$T_{WW} = -\frac{1}{12\pi} C^{1/2} \partial_W^2 C^{-1/2} , \tag{12.13}$$

$$T_{UW} = T_{WU} = \frac{1}{96\pi} C \, R . \tag{12.14}$$

Using the relationships $U = p(u)$, $W = q(\tilde{w})$ and the time independence of u and $\tilde{w}$, one can calculate the RSET components in the stationary (late times) region (see [13]).

Let us here focus on the energy density inside the bubble, in particular at the energy density ρ as measured by a set of free-falling observers, whose four velocity is $u_c^\mu = (1, \bar{v})$ in (t, r) components. For these observers neglecting transient terms one obtains [13] $\rho = T_{\mu\nu} u_c^\mu u_c^\nu = \rho_{\rm st} + \rho_{\rm dyn}$, where we define a static term $\rho_{\rm st}$, depending only on the r coordinate through $\bar{v}(r)$,

$$\rho_{\rm st} \equiv -\frac{1}{24\pi}\left[\frac{\left(\bar{v}^4 - \bar{v}^2 + 2\right)}{\left(1-\bar{v}^2\right)^2}\bar{v}'^{\,2} + \frac{2\bar{v}}{1-\bar{v}^2}\bar{v}''\right], \tag{12.15}$$

and a, time-dependent, dynamic term

$$\rho_{\rm dyn} \equiv \frac{1}{48\pi}\frac{\mathscr{F}(u)}{(1+\bar{v})^2}, \quad \text{where} \quad \mathscr{F}(u) \equiv \frac{3\ddot{p}^2(u) - 2\dot{p}(u)\,\dddot{p}(u)}{\dot{p}^2(u)}. \tag{12.16}$$

12.6.3 Physical Interpretation

Let us start by looking at the behaviour of the RSET in the centre of the bubble at late times. Here $\rho_{\rm st} = 0$, because $\bar{v}(r=0) = \bar{v}'(r=0) = 0$. One can evaluate $\rho_{\rm dyn}$ from Eq. (12.16) by using a late-time expansion for $\mathscr{F}(u)$, which gives $\mathscr{F}(u) \approx \kappa_1^2$, so that $\rho(r=0) \approx \kappa_1^2/(48\pi) = \pi T_H^2/12$, where $T_H \equiv \kappa_1/(2\pi)$ is the usual Hawking temperature. This result confirms that an observer inside the bubble measures a thermal flux of radiation at temperature T_H. Thus, apart from the energy-condition-violating mass distribution engineered to create the warp drive, the semiclassical calculation shows that one would need to add to the configuration an energy supply to maintain the unavoidable Hawking flux.

Let us now study ρ on the horizons $\mathscr{H}^+$ and $\mathscr{H}^-$. Here, both $\rho_{\rm st}$ and $\rho_{\rm dyn}$ are divergent because of the $(1+\bar{v})$ factors in the denominators. Using the late time expansion of $\mathscr{F}(u)$ in the proximity of the black horizon (see Ref. [13]) one gets

$$\lim_{r\to r_1} \mathscr{F}(u) = \kappa_1^2 \left\{1 + \left[3\left(\frac{A_2}{A_1}\right)^2 - 2\frac{A_3}{A_1}\right] e^{-2\kappa_1 t}\,(r-r_1)^2 + \mathscr{O}\left((r-r_1)^3\right)\right\}, \tag{12.17}$$

and expanding both the static and the dynamic terms up to order $\mathscr{O}(r-r_1)$, one obtains that the diverging terms ($\propto (r-r_1)^{-2}$ and $\propto (r-r_1)^{-1}$) in $\rho_{\rm st}$ and $\rho_{\rm dyn}$ exactly cancel each other [13]. It is now clear that the total ρ is $\mathscr{O}(1)$ on the horizon and does not diverge at any finite time. By looking at the subleading terms,

$$\rho = \frac{e^{-2\kappa_1 t}}{48\pi}\left[3\left(\frac{A_2}{A_1}\right)^2 - 2\frac{A_3}{A_1}\right] + A + \mathscr{O}(r-r_1), \tag{12.18}$$

where A is a constant, we see that on the black horizon the contribution of the transient radiation (different from Hawking radiation) dies off exponentially with time, on a time scale $\sim 1/\kappa_1$.

Close to the white horizon, the divergences in the static and dynamical contributions cancel each other, as in the black horizon case. However, something distinctive occurs with the subleading contributions. In fact, they now become

$$\rho = \frac{e^{2\kappa_2 t}}{48\pi}\left[3\left(\frac{D_2}{D_1}\right)^2 - 2\frac{D_3}{D_1}\right] + D + \mathscr{O}\left(r - r_1\right) \ . \qquad (12.19)$$

This expression shows an exponential increase of the energy density with time. This means that ρ grows exponentially and eventually diverges along $\mathscr{H}^-$. In a completely analogous way, one can study ρ close to the Cauchy horizon. Performing an expansion at late times ($t \to +\infty$) one finds that the RSET diverges also there [13].

Note that the above-mentioned divergences are very different in nature. The divergence at late times on $\mathscr{H}^-$ stems from the untamed growth of the transient disturbances produced by the white horizon formation. The Hawking radiation produced in the black hole will be accumulating at the white horizon. The RSET divergence on the Cauchy horizon is due instead to the well-known infinite blueshift suffered by light rays while approaching this kind of horizon. While the second can be deemed inconclusive because of the Kay–Radikowski–Wald theorem, the first one is inescapable. Apart from the energy supply necessary to maintain the Hawking fluxes, one would have to compensate an ever-increasing accumulation of energy at the white horizon.

The appearance of event horizons can of course be avoided if the superluminal travel does not last forever. However, these two exponentially fast accumulations of energy will still occur. The exponential increase will be controlled by $1/\kappa = \Delta/c$, where Δ represents the thickness of the warp drive walls. Note that, in order to get a time scale of even 1s, one would need $\Delta \sim 3 \times 10^8$ m.

Another way of taming the exponential accumulations of energy could be to travel in non-straight trajectories.[2] But also this solution seems impractical. First of all, to reach B starting from A in a superluminal manner with a wiggling trajectory would require faster velocities than in the straight trajectory case. Furthermore, one might avoid the problem of energy accumulation at the cost of introducing additional devices to produce the wiggling. At the semiclassical level, it is to be expected that the fast wiggling would also produce additional energy fluxes that one would also have to sustain (a sort of analogue of superradiant particle production).

Summarising, although not logically impossible, finite duration superluminal warp drive configurations will be very costly to produce and probably technically extremely hard to realise in practice.

[2] A possibility brought to us by E. Martín-Martínez and L.J. Garay in a personal communication.

12.7 Warp Drive Instabilities Under Dispersion

The semiclassical instability described above stems from standard, relativistic, QFT in curved spacetimes. One might wonder if the story could be different in scenarios, where a UV completion of the theory is provided by some quantum gravity (QG) scenario. This is the case of analogue gravity inspired Lorentz breaking scenarios (see e.g. Ref. [5]), where generically one expects the standard relativistic dispersion relation for the matter field to be replaced by $E^2 = c^2(p^2 + p^n/M_{\rm LIV}^{n-2})$, where $M_{\rm LIV}$ is normally assumed to be of the order of the Planck mass and n is some integer greater than two.

Indeed, this is a modification that could potentially stabilise the warp drive, as it is by now understood that modified LIV dispersion relations are able to remove Cauchy horizons instabilities and tame the divergence of fluxes at white hole horizons. The reason for this is simple, UV rays in the above dispersion relations are faster or slower than light, in both cases light rays will not accumulate at the horizons (past or forward in time depending on the black or white nature of the horizon) as they normally do. Hence no built-up of divergences can take place.

Can this be a scenario where a quantum gravity inspired UV completion/ regularisation could appear? This problem was dealt with in Ref. [9] and surprisingly it leads to a negative answer, i.e. not even the breakdown of Lorentz invariance can stabilise superluminal warp drives. Let us see how this works.

For the sake of simplicity, we work in 1 + 1 dimensions and consider a stationary situation. We can define a new spatial coordinate $X = x - v_c t$ (we use a different notation to avoid confusion between the two calculations) so the warp drive metric becomes

$$ds^2 = -c^2 dt^2 + [dX - V(X)dt]^2\,, \tag{12.20}$$

where $V(X) = v_c(f(X) - 1)$ is negative. In this spacetime, ∂_t is a globally defined Killing vector field whose norm is given by $c^2 - V^2$: it is timelike within the bubble, its norm vanishes on the two horizons, and it is spacelike outside. In a fluid flow analogy, this would correspond to two superluminal asymptotic regions separated by a black and a white horizon from a compact internal subluminal region [9].

We can now consider a massless scalar field with a quartic dispersion relation. In covariant terms, its action reads

$$S_\pm = \frac{1}{2}\int d^2x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \pm \frac{(h^{\mu\nu}\partial_\mu\partial_\nu\phi)^2}{M_{\rm LIV}^2}\right], \tag{12.21}$$

where $h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ is the spatial metric in the direction orthogonal to the unit timelike vector field u^μ which specifies the preferred frame used to implement the dispersion relation. The sign $\pm$ in Eq. (12.21) holds for superluminal and subluminal dispersion, respectively.

Using Eq. (12.20) and taking $u^\mu = (1, V)$ in the t, X frame, the wave equation is

$$\left[(\partial_t + \partial_X V)(\partial_t + V\partial_X) - \partial_X^2 \pm \frac{1}{M_{\rm LIV}^2}\partial_X^4\right]\phi = 0. \tag{12.22}$$

and $V(x)$ can be shaped so to mimic the warp drive geometry. Because of stationarity, the field can be decomposed in stationary modes $\phi = \int d\omega e^{-i\omega t}\phi_\omega$, where ω is the conserved (Killing) frequency. Correspondingly, at fixed ω the dispersion relation reads

$$(\omega - Vk_\omega)^2 = k_\omega^2 \pm \frac{k_\omega^4}{M_{\rm LIV}^2} \equiv \Omega_\pm^2, \tag{12.23}$$

where $k_\omega(X)$ is the spatial wave vector, and Ω the *comoving* frequency, *i.e.* the frequency in the aether frame. The quartic nature of the dispersion relations allows up to four solutions/modes and the problem in the end reduces to solve a Bogoliubov matrix of coefficients relating the mode in the asymptotic regions (assuming $M_{\rm LIV}$ to be larger than any other scale in the problem) [9].

The upshot is that in the case of subluminal dispersion relation there is an instability related to the well-known "laser effect" [8]. In the case of superluminal dispersion relations there is an infrared divergence that leads to a linear growth in time of the energy density proportional to $M_{\rm LIV}$ and the square of the warp drive wall surface gravity κ (we are assuming $\kappa_1 = \kappa_2 = \kappa$) [9]. Using quantum inequalities, Ref. [30], one can argue that κ must be of the order of the Planck scale, which implies that the growth rate is also of that order (unless $M_{\rm LIV}$ is very different from that scale). So, even in the presence of superluminal dispersion, warp drives would be unstable on short time scales. This instability, although not logically impossible to compensate for, would be extremely costly making it technologically implausible.

In conclusion with or without local Lorentz invariance FTL travel via warp drives seems inherently extremely problematic. This might seem a very depressing piece of news to science fiction fans, however let us stress that still any subluminal propagation (even just at 99.999% of the speed of light) seems to be free of most of the superluminal problems, of course, if the daunting engineering problems related to the very formation of the warp drive will be solved in a (possibly distant) future. Not too bad for novels and movies...

12.8 Conclusions

In summary, the above-discussed warp drive case shows that the very formation of the spacetime structure, allowing for a TM built up via FTL travel, can be very problematic—to say the least—well within the realm of semiclassical gravity. Remarkably, this conclusion still holds even when one allows for high energy deviations from local Lorentz invariance as inspired by analogue models of gravity. So, this "pre-emptive" chronology protection seems to strongly suggest that FTL travel will be always forbidden by the underlying structure of spacetime and gravity.

Interestingly enough, there are hints how this might work in some QG models. For examples in Causal Set Theory [7], the discrete structure replacing spacetime is made of sets of points which must be acyclic, meaning that no element in a causal set can causally precede itself. This in turn implies that CTCs are ruled out a priori. Actually, there is concrete evidence that generically Planck scale approaches based only on the causal structure of a spacetime cannot permit CTCs in the continuous classical limit (neither a corresponding phenomenon in their quantum counterparts) [24].

Looking forward it would be interesting to see if the warp drive analysis presented here could be extended to the formation of a traversable wormhole. As we have said in this chapter there are already hints that the very formation, e.g. with an otherwise flat spacetime, might be forbidden due to a divergent particle production generated by the topology change [2, 25] but it is not so clear if quantum gravity (via Wheeler spacetime foam "harvesting" or by allowing relic wormholes from the big bang era) could not get around this apparent obstruction.

In any case, whatever the final answer on the viability of FTL travels and TM might be, let us stress that it will be very relevant to our understanding of the fabric of reality as it might very well tell us crucial features that will have to be embedded in our models of the spacetime at the Planck scale and beyond. So this research is not just a fun field for sci-fi fans, but should be considered a very crucial aspect of general relativity which might teach us the way forward. We hence hope that this humble contribution would stimulate further research in this field.

Acknowledgements The authors wish to acknowledge that the research reported here was done also in collaboration with A. Coutant, S. Finazzi and R. Parentani. S.L. also wish to acknowledge the John Templeton Foundation for the supporting grant #51876. C.B. acknowledge support provided by the Spanish MINECO through the project FIS2014-54800-C2-1 (with FEDER contribution), and by the Junta de Andalucìa through the project FQM219.

References

1. Alcubierre M. The warp drive: hyperfast travel within general relativity. Class Quant Grav. 1994;11:L73–7.
2. Anderson A, DeWitt B. Does the topology of space fluctuate? Found Phys. 1986;16(2):91–105.
3. Barcelo C, Liberati S, Sonego S, Visser M. Quasi-particle creation by analogue black holes. Class Quant Grav. 2006;23:5341–66.
4. Barcelo C, Liberati S, Sonego S, Visser M. Fate of gravitational collapse in semiclassical gravity. Phys. Rev. 2008;D77:044,032.
5. Barcelo C, Liberati S, Visser M. Analogue gravity. Living Rev. Rel. 2005;8:12 [Living Rev. Rel.14,3(2011)].
6. Birrell ND, Davies PCW. Quantum fields in curved space., Cambridge monographs on mathematical physicsCambridge: Cambridge Univ. Press; 1984. http://www.cambridge.org/mw/academic/subjects/physics/theoretical-physics-and-mathematical-physics/quantum-fields-curved-space?format=PB.
7. Bombelli L, Lee J, Meyer D, Sorkin R. Space-time as a causal set. Phys Rev Lett. 1987;59:521–4.
8. Corley S, Jacobson T. Black hole lasers. Phys Rev. 1999;D59:124,011.

9. Coutant A, Finazzi S, Liberati S, Parentani R. Impossibility of superluminal travel in Lorentz violating theories. Phys Rev. 2012;D85:064,020.
10. Davies PCW, Fulling SA, Unruh WG. Energy momentum tensor near an evaporating black hole. Phys Rev D. 1976;13:2720–3.
11. Echeverria F, Klinkhammer G, Thorne KS. Billiard balls in wormhole spacetimes with closed timelike curves: classical theory. Phys Rev D. 1991;44:1077–99.
12. Einstein A, Rosen N. The particle problem in the general theory of relativity. Phys Rev. 1935;48:73–7. http://link.aps.org/doi/10.1103/PhysRev.48.73.
13. Finazzi S, Liberati S, Barcelo C. Semiclassical instability of dynamical warp drives. Phys Rev. 2009;D79:124,017.
14. Flamm L. Beiträge zur Einsteinschen Gravitationstheorie. Hirzel (1916)
15. Friedman J, Morris MS, Novikov ID, Echeverria F, Klinkhammer G, Thorne KS, Yurtsever U. Cauchy problem in spacetimes with closed timelike curves. Phys Rev D. 1990;42:1915–30.
16. Fuller RW, Wheeler JA. Causality and multiply connected space-time. Phys Rev. 1962;128:919–29.
17. Gambini R, Pullin J. Nonstandard optics from quantum space-time. Phys Rev. 1999;D59:124,021.
18. Geroch R. Faster Than Light? AMS/IP Stud Adv Math. 2001;49:59–70.
19. Hawking SW. Chronology protection conjecture. Phys Rev D. 1992;46:603–11.
20. Kay BS, Radzikowski MJ, Wald RM. Quantum field theory on space-times with a compactly generated Cauchy horizon. Commun Math Phys. 1997;183:533–56.
21. Liberati S. Tests of Lorentz invariance: a 2013 update. Class Quant Grav. 2013;30:133,001.
22. Liberati S, Sonego S, Visser M. Faster than c signals, special relativity, and causality. Ann Phys. 2002;298:167–85.
23. Lobo FSN, Visser M. Fundamental limitations on 'warp drive' spacetimes. Class Quant Grav. 2004;21:5871–92.
24. Malament D. Causal theories of time and the conventionality of simultaneity. Noûs. 1977;11(3):293–300.
25. Manogue CA, Copeland E, Dray T. The trousers problem revisited. Pramana. 1988;30:279–92.
26. Morris MS, Thorne KS. Wormholes in spacetime and their use for interstellar travel: a tool for teaching general relativity. Am J Phys. 1988;56(5):395–412.
27. Morris MS, Thorne KS, Yurtsever U. Wormholes, time machines, and the weak energy condition. Phys Rev Lett. 1988;61(13):1446.
28. Olum KD. Superluminal travel requires negative energies. Phys Rev Lett. 1998;81:3567–70.
29. Penrose R, Sorkin RD, Woolgar E. A Positive mass theorem based on the focusing and retardation of null geodesics. 1993.
30. Roman TA. Some thoughts on energy conditions and wormholes. In: Proceedings, 10th Marcel Grossmann Meeting On recent developments in theoretical and experimental general relativity, gravitation, and relativistic field theories. MG10, Rio de Janeiro: Brazil; 2004. 20-26 July 2003. pp. 1909–1922
31. Scharnhorst K. On Propagation of light in the vacuum between plates. Phys. Lett. B. 1990;236:354–9.
32. Visser M. Hawking's chronology protection conjecture: Singularity structure of the quantum stress energy tensor. Nucl. Phys. B. 1994;416:895–906.
33. Visser, M.: Lorentzian wormholes: From Einstein to Hawking. Springer (1995)
34. Visser, M.: The Quantum physics of chronology protection. In: Workshop on Conference on the Future of Theoretical Physics and Cosmology in Honor of Steven Hawking's 60th Birthday Cambridge, England, January 7-10, 2002, pp. 161–176 (2002). URL http://alice.cern.ch/format/showfull?sysnb=2303505
35. Visser M, Bassett B, Liberati S. Superluminal censorship. Nucl. Phys. Proc. Suppl. 2000;88:267–70.