From Information Systems to Interactive Information Systems

Andrzej Skowron and Soma Dutta

Abstract In this chapter we propose a departure from classical notion of information systems. We propose to bring in the background of agent's interaction with physical reality in arriving at a specific information system. The proposals for generalizing the notion of information systems are made from two aspects. In the first aspect, we talk about incorporating relational structures over the value sets from where objects assumes values with respect to a set of attributes. In the second aspect, we introduce interaction with physical reality within the formal definition of information systems, and call them as interactive information systems.

1 Introduction

Professor Zdzisław Pawlak published several papers [8–14, 16, 18–24, 26, 27] as well as a book (in Polish) [25] on information systems (see Figs. 1, 2, 3 and 4). The first definition of information systems, as proposed by him, appeared in [18, 19].

An information system was defined as a tuple consisting of a finite set of objects and a set of attributes defined over the set of objects with values in attribute value sets. More formally, an information system is a tuple

$$IS = (U, \mathscr{A}, \{f_a : U \to V_a\}_{a \in \mathscr{A}}), \tag{1}$$

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G. Wang et al. (eds.), *Thriving Rough Sets*, Studies in Computational
Intelligence 708, DOI 10.1007/978-3-319-54966-8_10

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Zdzisław Pawlak

Mathematical Foundations
of Information Retrieval

101
1973

CENTRUM OBLICZENIOWE POLSKIEJ AKADEMII NAUK
COMPUTATION CENTRE POLISH ACADEMY OF SCIENCES
WARSAW, PKiN, PO Box 22, Poland

This note contains a simple mathematical formulation of basic ideas concerning information retrieval and its computer implementation. The presented theory is based on the results in [1], [2], and [3].

1. Descriptive systems

By a descriptive system we mean a triplet

$$D = < A_D, \chi_D, \delta_D >$$

(or briefly $D = < A, \chi, \delta >$), where

A – is a (finite or infinite) set; elements of *A* are called objects of *D*,

X – is a finite set of symbols; elements of *X* are referred to as elementary descriptors of *D*,

$\delta \subseteq A \times X$ – is a binary relation, called description relation (or description) in *D*.

Relation δ may be replaced by the function:

$$\psi : X \to 2^A$$

such that:

$$\psi(x) = \{a \in A : \delta(a, x)\}.$$

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COMPUTING RESEARCH CENTRE UNITED NATIONS D.P. BRATISLAVA

mathematical foundations of computer science

proceedings of symposium and summer school

HIGH TATRAS SEPTEMBER 3-8, 1973

Fig. 1 The first papers on information systems by Zdzisław Pawlak [18, 19]

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Wiktor Marek, Zdzisław Pawlak

Mathematical Foundations
of Information Storage and
Retrieval
Part 1-3

135-137
1973

CENTRUM OBLICZENIOWE POLSKIEJ AKADEMII NAUK
COMPUTATION CENTRE POLISH ACADEMY OF SCIENCES
WARSAW, PKiN, PO Box 22, Poland

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Information Storage
and Retrieval System --
Mathematical Foundations

149
1974

CENTRUM OBLICZENIOWE POLSKIEJ AKADEMII NAUK
COMPUTATION CENTRE POLISH ACADEMY OF SCIENCES
WARSAW, PKiN, PO Box 22, Poland

Annales Societatis Mathematicae Polonae
Series IV: Fundamenmta Informaticae VII. 1 (1984)

ROUGH SETS AND INFORMATION SYSTEMS
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Theoretical Computer Science 1 (1976) 331-354.
© North-Holland Publishing Company

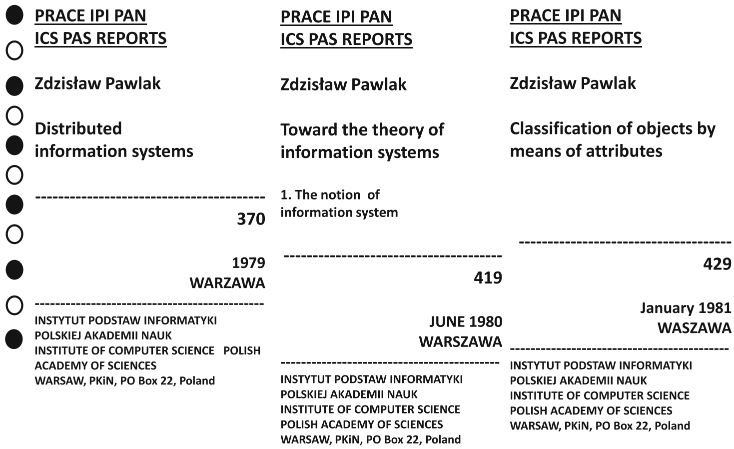
INFORMATION STORAGE AND RETRIEVAL SYSTEMS:
MATHEMATICAL FOUNDATIONS
Wiktor MAREK and Zdzisław PAWLAK
Communicated by E. Engeler
Received April 1974
Revised January 1975

BULLETIN DE L'ACADÉMIE POLONAISE DE SCIENCES
Série des sciences math., astr. et phys. – Vol. XXII, No. 4, 1974

MATHEMATICS
(COMPUTER SCIENCE)

On the Foundations of Information Retrieval
by
W. MAREK and Z. PAWLAK
Presented by A. Mostowski on October 5, 1973

Fig. 2 Further papers on information systems by Zdzisław Pawlak et al. [8–14]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ICS PAS REPORTS
Zdzisław Pawlak
Distributed information systems
370
1979
WARZAWA
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WARSAW, PKiN, PO Box 22, Poland

PRACE IPI PAN
ICS PAS REPORTS
Zdzisław Pawlak
Toward the theory of information systems
1. The notion of information system
419
JUNE 1980
WARSZAWA
INSTYTUT PODSTAW INFORMATYKI POLSKIEJ AKADEMII NAUK
INSTITUTE OF COMPUTER SCIENCE POLISH ACADEMY OF SCIENCES
WARSAW, PKiN, PO Box 22, Poland

PRACE IPI PAN
ICS PAS REPORTS
Zdzisław Pawlak
Classification of objects by means of attributes
429
January 1981
WASZAWA
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INSTITUTE OF COMPUTER SCIENCE POLISH ACADEMY OF SCIENCES
WARSAW, PKiN, PO Box 22, Poland

Fig. 3 Further papers on information systems by Zdzisław Pawlak [21–23] (CONT)

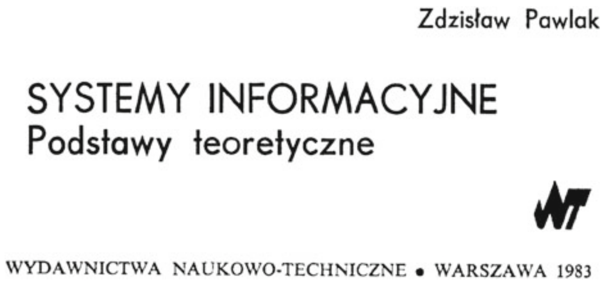
Zdzisław Pawlak
SYSTEMY INFORMACYJNE
Podstawy teoretyczne
WYDAWNICTWA NAUKOWO-TECHNICZNE • WARSZAWA 1983

Fig. 4 Book (in Polish) dedicated to information systems by Zdzisław Pawlak [25]

where

- U is a finite set of objects,
- $\mathscr{A}$ is a finite set of attributes,
- any attribute a from $\mathscr{A}$ can be characterized as a function f_a from U to a value set V_a corresponding to a.

In the mentioned papers and in the book, Pawlak investigated different kinds of information systems such as deterministic, nondeterministic, information systems with missing values, probabilistic, stochastic as well as distributed. From the point of view of rough sets, information systems are used for constructive definition of indiscernibility relation. Then the indiscernibility or similarity classes can be tuned to relevant ones (in order to get relevant indiscernibility classes, also called elementary granules), e.g., by selecting or extracting relevant attributes. On the basis of information systems (as data sets) data models are induced using different methods, in particular based on rough sets.

In this chapter, we propose some generalizations of information systems.

First we will consider a bit more general definition of the value sets for attributes. In particular, together with the value set V_a for any attribute a, we will also consider a relational structure $\mathscr{R}_a$ over V_a. These $\mathscr{R}_a$'s are not restricted to the case of the relational structure consisting of only the equality relation on V_a's, as it was originally considered by Pawlak. More general cases can include a linear order over V_a as well as more complex relations with arity greater than 2 [34]. Together with the relational structure $\mathscr{R}_a$ we consider a language $\mathscr{L}_a$ of formulas defining (under a given interpretation over $\mathscr{R}_a$) subsets of V_a. It is to be noted, that such formulas can be obtained from formulas with many free variables by substituting a constant for each of them except one. Some relevant formulas from this set of formulas become useful as they can play a role in inducing data models. For example, one can consider an attribute with real values and formalize a discretization problem. In this case for any real-valued attribute a we can consider a set of formulas $\{x \in [c, d]\}$, where x is a free variable corresponding to the attribute a taking real values and c, d are constants defining an interval. Then we search for a minimal set of such formulas discerning in the optimal by decisions labeling the attribute values and defining the partition of the real numbers [15]. Another example may be related to the dominance rough set approach (see, e.g., [3, 35]), where linear orders are considered on attribute value sets.

In this chapter, we also introduce a network of information systems over such generalized information systems. This is done analogous to the notion of information flow approach proposed by Barwise and Seligman [1, 32, 33]. However, first we consider different kinds of aggregation of relational structures corresponding to attributes from a given set of attributes A. Then we define a set of formulas $\mathscr{L}$ which can be interpreted over such relational structures. In this context, one may introduce relations with many arguments. Discovery of such kinds of relevant relations, based on purpose, is the task of relational learning [2].

Our final stage of generalization of information systems concerns of interactions of information systems with the environment. This issue is strongly related to the discussed interactive granular computations (see e.g., [4–7, 28–31]), where information systems are treated as open objects, which are continuously evolving based on the interactions with the environment. This extension can be used as a basis for developing Perception Based Computing (PBC) [17, 36] and for developing the foundations of Interactive Granular Computing (IGrC) [4–7, 28–31].

The chapter is structured as follows. In Sect. 2, we first discuss the roles of relational structures over the value sets corresponding to attributes of an information system. We present different examples to elucidate the fact that aggregation of such relational structures plays an important role in representation and granulation of data of an information system, which often contains huge and scattered data. Section 3 introduces the notion of interactive information system as a generalization of the notion of *IS* (cf. Eq. (1)) presented at the beginning of this chapter. In the last section, as concluding remarks, we add some discussion regarding incorporation of some other finer aspects of interactive information systems.

2 Role of Relational Structures in Aggregation of Information Systems

Depending on purpose we need to gather information of different nature, such as images of some object as well as quantitative values for some features of the same object, together in order to make an overall understanding about the object. So, values corresponding to different features as well as the intra-relational structures among the values become important. The aim of this section is to present different kinds of aggregation of relational structures, which we need to perform in order to aggregate information collected from, and for, different perspectives.

The chapter is organized so that, in one aspect we would talk about relations over the value sets of the attributes of an information system, and in another aspect we also would like to address the issue of the relational objects lying in the real world, about which we only able to gather some information through some attributes and their values. This aspect of real world will be discussed in the next section where we propose to introduce interaction with physical reality in the process of obtaining an information system. A physical object o, being in a complex relation with other objects in the real physical world, sometimes cannot be directly accessed. We sometimes identify the object with some of its images or with some of its parts or components, and try to gauge information about the object with respect to some parameters. One possible way of measuring the real state of an object through some other state is proposed through the notion of complex granule in [4–7, 28–31]. Here we will address this introducing a notion of infomorphism in the line of [ ], and call that *interaction with physical reality*. In this section, we only stick to the relations among the values of attributes using which we learn about objects in the physical world. Let us start with some examples in order to make the issue more lucid.

Let $\mathscr{A}_{rect} = \{a, b, c\}$ be a set of attributes representing respectively *length*, *breadth*, and *angle between two sides* of a rectangle. Clearly, a and b are of the same nature and can assume values from the same set, say $V_a = [0, 300]$ in some unit of length, and be endowed with the same relation $\leq$. Let us call the relational structure over the values for the attribute a as $\mathscr{R}_a = (V_a, \leq)$, which is same as $\mathscr{R}_b$ too, in this context. Let $V_c = [0°, 180°]$ and $\mathscr{R}_c = (V_c, =)$. Now we can construct a language $\mathscr{L}_{a,b}$ (cf. Table 1).

Table 1 Language $\mathscr{L}_{a,b}$

Variable: x_1
Constants: any value from V_a
Function symbol: a, b
Relational symbol: $\leq$
Terms: (i) Variable and constants are terms (ii) $a(x_1)$ and $b(x_1)$ are terms
Examples of wffs: $b(x_1) \leq a(x_1)$ is an atomic wff

Table 2 Language $\mathscr{L}_c$

Variable: x_1
Constants: any value from V_c
Function symbol: c
Relational symbol: $=$
Terms: (i) Variable and constants are terms (ii) $c(x_1)$ is a term
Examples of wffs: $c(x_1) = 90°$ is an atomic wff

In particular, we may call $b(x_1) \leq a(x_1)$, which represents *breadth of* x_1 *is less or equal to the length of* x_1, as $\phi_{11}(x_1)$. Considering that the variable x_1 is ranging over a set of objects, say $\mathscr{O}$, values from $\mathscr{R}_a = (V_a, \leq)$ can be assigned to $a(x_1), b(x_1)$, and thus the semantics of $\mathscr{L}_{a,b}$ can be given over the relational structure $\mathscr{R}_a$. In the similar way we can have the language $\mathscr{L}_c$, semantics of which can be given over the relational structure $\mathscr{R}_c$ (cf. Table 2).

Before passing on to the next table for $\mathscr{L}_c$, it is to be noted that, the values of terms $a(x_1), b(x_1)$, belonging to V_a and V_b respectively, are obtained by some agent *ag* observing a complex granule (c-granule, for short) [4–7, 28–31] grounded on a configuration of physical objects. Relations among the parts of the configuration can be perceived partially by the c-granule through $a(x_1), b(x_1)$. Some objects in the configuration have states which may be directly measurable, and those can be encoded by elements of V_a and V_b. They can be be treated as values, e.g., $a(x_1), b(x_1)$ of the example, under the assumption that they represent states of one distinguished object o in the configuration. They considered as a current value of x_1, identified by some mean with o. However, the states of o may not be directly measurable. Information about not directly measurable states may be obtained using relevant interactions with physical objects pointed by the c-granule, and making it possible to transmit information about such states and encode it using measurable states. In this chapter we represent interactions of agents with the physical reality using infomorphisms [1].

Like previous case, here also we can assume $\phi_{12}(x_1)$ as the formula $c(x_1) = 90°$, which represents *angles between two sides of* x_1 is 90°. So, we have two relational structures, namely $\mathscr{R}_a = (V_a, \leq)$ and $\mathscr{R}_c = (V_c, =)$, on which respectively the formulas $\phi_{11}(x_1)$ and $\phi_{12}(x_1)$ are interpreted with respect to the domain of interpretation of x_1, which can be considered as a set of objects. The value of the term $c(x_1)$ is obtained in an analogous way as mentioned before for $a(x_1), b(x_1)$. Now, the question arises how can we combine these two relational structures to gather information about whether an object is rectangle or not. Here, as the attributes a, b, c are relevant for the same sort of objects, we may simply extend the language combining all the components of $\mathscr{L}_{a,b}$ and $\mathscr{L}_c$ together (cf. Table 3).

In $\mathscr{L}_{rect}$ instead of relational symbols $\leq$ and $=$, one can also consider a new three-place relational symbol r_1^3 such that $r_1^3(a(x_1), b(x_1), c(x_1))$ holds for some object from the domain of x_1 if $b(x_1) \leq a(x_1)$ (*i.e.*, $\phi_{11}(x_1)$) is true over $\mathscr{R}_a$ and $c(x_1) = 90°$ (*i.e.*, $\phi_{12}(x_1)$) is true over $\mathscr{R}_c$. So, assuming a set of objects as the domain of interpretation

Table 3 Language $\mathscr{L}_{rect}$: combination of $\mathscr{L}_{a,b}$ and $\mathscr{L}_c$

Variable: x_1
Constants: any value from $V_a \cup V_c$
Function symbol: a, b, c
Relational symbol: $\leq$, $=$
Terms: (i) Variable and constants are terms (ii) $a(x_1)$, $b(x_1)$, $c(x_1)$ are terms
Examples of wffs: $b(x_1) \leq a(x_1)$, $c(x_1) = 90°$ are atomic wffs

Table 4 Language $\mathscr{L}_{tri}$: combination of $\mathscr{L}_d$ and $\mathscr{L}_e$

Variable: x_2
Constants: any value from $V_d \cup V_e$
Function symbol: d, e
Relational symbol: $\leq$, $\preceq$
Terms: (i) Variable and constants are terms (ii) $d(x_2)$, $e(x_2)$ are terms
Examples of wffs: $d(x_2) = 1$, $e(x_2) = 180°$ are atomic wffs.

for x_1, this new language $\mathscr{L}_{rect}$ can be interpreted over the combined relational structure $\mathscr{R}_{rect} = (V_{rect}, \{\leq, =\})$, where $V_{rect} = V_a \cup V_c$. We may call $r_1^3(a(x_1), b(x_1), c(x_1))$ as $\phi_{13}(x_1)$.

Let us consider another context where the attributes are relevant for a triangle-shaped object. So, we consider $\mathscr{A}_{tri} = \{d, e\}$, where d stands for *three-sided* and e stands for *sum of the angles*. Again the relational structures suitable for the values of the attributes are respectively $\mathscr{R}_d = (\{0, 1\}, \leq)$ and $\mathscr{R}_e = ([0°, 180°], \preceq)$. It is to be noted that $\preceq$ is the same relation as that of the real numbers (*i.e.*, $\leq$). We use different symbol in order to emphasize that the values relevant for d and e are of different types. Now as shown in the previous case, we can construct different languages over the different relations from $\mathscr{R}_d$ and $\mathscr{R}_e$, and combining them together we can have the language $\mathscr{L}_{tri}$ (cf. Table 4).

In this context too, in $\mathscr{L}_{tri}$, instead of two relation symbols $\leq$, $\preceq$, one can take a two-place relation symbol r_1^2 such that for some object from the domain of x_2, $r_1^2(d(x_2), e(x_2))$ holds if with respect to that object $d(x_2) = 1$ and $e(x_2) = 180°$ are true over $\mathscr{R}_{tri} = (V_{tri}, \{\leq, \preceq\})$ where $V_{tri} = V_d \cup V_e$. As above, $r_1^2(d(x_2), e(x_2))$ may be called $\phi_{23}(x_2)$, where the values of $d(x_2)$ and $e(x_2)$ are obtained in an analogous way as before.

In the above two cases we have obtained the extended relational structures $\mathscr{R}_{rect}$ and $\mathscr{R}_{tri}$ by combining the respective relational structures for each attribute from $\mathscr{A}_{rect}$ and $\mathscr{A}_{tri}$. In some context, we need to gather information about objects whose domain consists of tuples of elements of different natures. As an example we can consider a situation where we need to collect information about objects which are prisms with rectangular bases and triangular faces. So, we need to have a language over $\mathscr{A} = \mathscr{A}_{rect} \cup \mathscr{A}_{tri}$, and contrary to the earlier cases of combining languages here

Table 5 Language $\mathscr{L}_{prism}$: aggregation of $\mathscr{L}_{rect}$ and $\mathscr{L}_{tri}$

Variable: $x = (y, z)$
Constants: any value from $V_{rect} \cup V_{tri}$, x_1, x_2, and $\phi_{13}(x_1)$, $\phi_{23}(x_2)$
Relational symbol: r_{11}, r_{21}
Terms: (i) Variable and constants are terms
Wffs: $r_{11}((x_1, x_2), \phi_{13}(x_1))$, $r_{21}((x_1, x_2), \phi_{23}(x_2))$ are atomic wffs

we need to aggregate information of two different languages $\mathscr{L}_{rect}$ and $\mathscr{L}_{tri}$ with different domains of concern focusing on different parts of an object. In this context, we would construct the language $\mathscr{L}_{prism}$ one level above the languages $\mathscr{L}_{rect}$ and $\mathscr{L}_{tri}$, and the variables, constants, and wffs of those languages will be referred to as constant symbols of the language of $\mathscr{L}_{prism}$ (cf. Table 5).

Here we have introduced a pair of variables (y, z) to represent a single object with respectively first component for the base and the second component for the face, and $(y,\ z)$ is assumed to range over a set of objects of the form $o = (o_b, o_f)$ where o_b's are taken from the domain of interpretation of $\mathscr{L}_{rect}$ (i.e., objects on which x_1 ranges), and o_f's from that of $\mathscr{L}_{tri}$ (i.e., objects on which x_2 ranges). So, $r_{11}((x_1, x_2), \phi_{13}(x_1))$ is introduced to represent that an object, characterized by the pair of components base and face (x_1, x_2), has the property of a prism with rectangular base. On the other hand, $r_{21}((x_1, x_2), \phi_{23}(x_2))$ is introduced to represent that (x_1, x_2) is a prism with triangular face. Let us call $r_{11}((x_1, x_2), \phi_{13}(x_1)) = \alpha$ and $r_{21}((x_1, x_2), \phi_{23}(x_2)) = \beta$. Then with respect to the set of objects of the form (o_b, o_f)'s, α and β are true over $\mathscr{R}_{\mathscr{A}}$ for the subsets of objects given by $\{(o_b, o_f) = o : o_b \in ||\phi_{13}(x_1)||_{\mathscr{R}_{\mathscr{A}_{rect}}}\}$ and $\{(o_b, o_f) = o : o_f \in ||\phi_{23}(x_2)||_{\mathscr{R}_{\mathscr{A}_{tri}}}\}$ respectively.

Let us now come to the discussion of how these relational structures over the values for attributes and the respective languages help in the representation of different purposes of information systems.

Let us assume that we have a technical set-up to abstract out images of some parts of the objects appearing in front of a system. Two cameras are set up in a way that any object appearing to the system through a specified way can have their images recorded in the database of the system through some way of measurements. Let the first camera be able to capture the image of the base of the object and the second camera be able to capture the face of the object. So, there are two information systems, namely $IS_{rect} = (B, \mathscr{A}_{rect}, \{\mathscr{R}_a\}_{a \in \mathscr{A}_{rect}}, \{f_a : B \mapsto V_a\}_{a \in \mathscr{A}_{rect}})$ where $\mathscr{R}_a = (V_a, \leq) = \mathscr{R}_b$ and $\mathscr{R}_c = (V_c, =)$, and $IS_{tri} = (F, \mathscr{A}_{tri}, \{\mathscr{R}_d\}_{d \in \mathscr{A}_{tri}}, \{f_d : F \mapsto V_d\}_{d \in \mathscr{A}_{tri}})$ with $\mathscr{R}_d = (V_d, \leq)$ and $\mathscr{R}_e = (V_e, \preceq)$. At the first level we may need to gather information from both IS_{rect} and IS_{tri} in a single information system, say IS_{prism}, in a way that a copy of each of IS_{rect} and IS_{tri} is available. So, we construct a sum of information systems as $IS_{prism} = (B \times F,\ \mathscr{A}_{prism},\ \mathscr{R}_{prism},\ \{f_a : B \times F \mapsto V_{prism}\}_{a \in \mathscr{A}_{prism}})$ where $\mathscr{A}_{prism} = (\{1\} \times \mathscr{A}_{rect}) \cup (\{2\} \times \mathscr{A}_{tri})$, $V_{prism} = V_{rect} \cup V_{tri}$, and the relational structure $\mathscr{R}_{prism} = (V_{prism}, \{r\}_{r \in \mathscr{R}_{rect} \cup \mathscr{R}_{tri}})$.

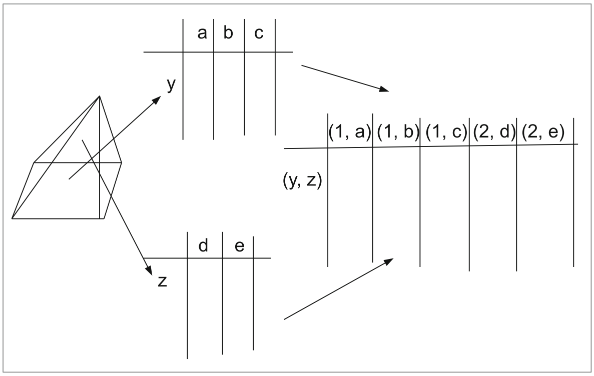

Fig. 5 IS_{prism}: sum of the information systems IS_{rect} and IS_{tri}

Table 6 Sum of information systems

	$(1,a)$	$(1,b)$	$(1,c)$	$(2,d)$	$(2,e)$
(o_b,o_f)	$f_a(o_b)$	$f_b(o_b)$	$f_c(o_b)$	$f_d(o_f)$	$f_d(o_f)$
⋮	⋮	⋮	⋮	⋮	⋮
(o'_b,o'_f)	$f_a(o'_b)$	$f_b(o'_b)$	$f_c(o'_b)$	$f_d(o'_f)$	$f_d(o'_f)$
⋮	⋮	⋮	⋮	⋮	⋮

Language over the relational structure comes into play when we want to have an information system with an added constraint. Let us assume that from all possible three-dimensional objects from $B \times F$, in the above information system IS_{prism}, we are interested in the chunk which has objects with rectangular bases and triangular faces. We can construct an information system imposing a constraint from the language $\mathscr{L}_{prism}$, and thus have the *constraint-based information system*, viz., CIS_{prism} $= ((B \times F) \cap (||\alpha||_{\mathscr{R}_{\mathscr{A}}} \cap ||\beta||_{\mathscr{R}_{\mathscr{A}}}),\ \mathscr{A}_{prism},\ \mathscr{R}_{prism},\ \{f_a : B \times F \mapsto V_{prism}\}_{a \in \mathscr{A}_{prism}})$.

Simultaneous consideration of each relational structure included in $\mathscr{R}_{prism} = (V_{prism}, \{r\}_{r \in \mathscr{R}_{rect} \cup \mathscr{R}_{tri}})$ becomes useful when looking at values for one object, say (o_b, o_f) one needs to predict about another object, say (o'_b, o'_f). Let us consider the following situation where we have the information aggregated from IS_{rect} and IS_{tri} in the information system IS_{prism} (cf. Fig. 5 and Table 6).

Let us assume that the values corresponding to each attribute for $o = (o_b, o_f)$ are known. A new object $o' = (o'_b, o'_f)$ appears to the system for which some of the values are missing or because of some technical error measurements of values are not precise. So, one may need to check how value corresponding to each attribute of o is related to the respective value of the other object o'. Let $(v^1_o, v^2_o, v^3_o, v^4_o, v^5_o)$ be the tuple of values corresponding to the attributes (a, b, c, d, e) for object o, and $(v^1_{o'}, v^2_{o'}, v^3_{o'}, v^4_{o'}, v^5_{o'})$ be that of o'. Now on $\Pi_{a \in \mathscr{A}} V_a$, the cartesian product of the value sets corresponding to each attributes of $\mathscr{A}$, we can define a relation r such that $r((v^1_o, v^2_o, v^3_o, v^4_o, v^5_o), (v^1_{o'}, v^2_{o'}, v^3_{o'}, v^4_{o'}, v^5_{o'}))$ if and only if the following relations hold among their components.

(i) $r_{1,2}((v_o^1, v_o^2), (v_{o'}^1, v_{o'}^2))$ iff $v_o^1 \leq v_o^2$ and $v_{o'}^1 \leq v_{o'}^2$,
(ii) $r_3(v_o^3, v_{o'}^3)$ iff $|v_o^3 - v_{o'}^3| \leq \epsilon$ for some $\epsilon > 0$,
(iii) $r_4(v_o^4, v_{o'}^4)$ iff $v_o^4 = v_{o'}^4$, and
(iv) $r_5(v_o^5, v_{o'}^5)$ iff $|v_o^5 - v_{o'}^5| \leq \delta$ for some $\delta > 0$.

So, based on the relational structure $\mathscr{R}_{\mathscr{A}} = (\Pi_{a\in\mathscr{A}} V_a, r)$ we can have the information system $(B \times F,\ \Pi_{a\in\mathscr{A}} a,\ \mathscr{R}_{\mathscr{A}}, f : B \times F \mapsto \Pi_{a\in\mathscr{A}} V_a)$, which will allow us to cluster objects together, satisfying the relation r.

So, in this section we observe different types of aggregation of relational structures, starting from simple combination of all value sets and their relations in a single aggregated relational structure to cartesian product of value sets and some new relations defined over the relations of the component value sets. We also notice that how based on the requirement of presentation of data in a form of a table, aggregation of those relational structures, and languages interpreted over them become useful. This gives a hint that we need to depart from the classical way of presenting an information system as IS $= (U, \mathscr{A}, \{f_a : U \mapsto V_a\}_{a\in\mathscr{A}})$ to $(U, \mathscr{A}, \{\mathscr{R}_a\}_{a\in\mathscr{A}}, \{f_a : U \mapsto V_a\}_{a\in\mathscr{A}})$, where $\mathscr{R}_a = (V_a, \{r_{ai}\}_{i=1}^k)$.

In the next section, we would present another aspect of generalizing the classical notion of information systems bringing in a component of interaction of an information system, via the respective agent, with the physical reality, and letting the information system to evolve with time.

3 Interactive Information Systems

Information system (cf. Eq. (1)) allow us to present the information about the real world phenomenon in the form a table with object satisfying certain properties to certain degrees. Moreover, as discussed in the previous section, through information systems we can also present how two objects are related based on the interrelation among the values/degrees they obtain for different parameters/attributes/properties. But presentation of the reality through an information system is subjective to the agent's perspective towards viewing, perceiving the reality, which can change with time. At some time t an agent can manage to access some parts of a real object or phenomenon through some process of interactions with the real physical world and abstract out some relevant information, which then can be presented in the form of an information system. That this interaction with reality, based on the factors of time and accessibility, plays a great role in the presented form of an information system cannot be ignored. Two information systems approximating the same real phenomenon may yield quite different views. Thus, incorporating the process of interactions, through which one obtains a particular information system, may help in understanding the background of the presented data. In this regard, below we propose a notion of interactive information system.

As presented the definition of *IS* in Sect. 1, we start with an information system $IS_t = (\mathscr{O}_t, \mathscr{A}_t, \{f_a : U_t \mapsto V_a\}_{a\in\mathscr{A}_t})$ at time t, and trace back to the interactions with the physical reality causing the formation of such an IS_t. The idea is to incorporate those interactions with reality in the mathematical model of IS and have an interactive information system, which we may call *ISAPR*, an *information system approximating the physical reality*. Let at time t, there be an existing physical reality which is nothing but a complex network of objects and their interrelations.

1. We assume *physical reality at time t*, denoted as PR_t, as $PR_t = (U'_t, \mathscr{R})$, where U'_t is the set of real objects at time t, and $\mathscr{R}$ is a family of relations of different arities on U'_t. The time point t can be considered as the time related to the local clock of the agent involved in the process of interaction with the physical reality.
2. A part of PR_t is a subrelational structure $(U_\#, \mathscr{R}_\#)$ where $U_\# \subseteq U'_t$ and $\mathscr{R}_\#$ $(\subseteq \mathscr{R})$ is a subfamily of relations over $U_\#$.
3. The set of all subrelational structures at time t is denoted by S_{PR_t}. It is to be noted that for any subset of U'_t endowed with the set of relations from $\mathscr{R}$, restricted to that subset, is a member of S_{PR_t}. So, $S_{PR_t} = \{(U_\#, \mathscr{R}_\#) : U_\# \in P(U'_t), \mathscr{R}_\# = \mathscr{R}_{|U_\#}\}$.
4. At time t, by some means, we can access some information about some parts of the reality. As an instance we can consider a real tree as an object of the physical world, and some houses, park surrounding it as a description of a relational structure among objects in the reality. This we may call $(U_\#, \mathscr{R}_\#)$, a fragment of the real world. Now when an agent captures some images of the tree using a camera, which is another physical object, some states of the real tree are recorded. The real object tree may then be identified with those states or images in the agent's information system. So, we introduce a function $AR_t : S_{PR_t} \mapsto P(S^*)$, where $\mathscr{O}_t \subseteq S^*$, and call it a *function accessing reality at time t*. S^* may be interpreted as a set of states, which can be accessible by some means, and $\mathscr{O}_t$ is the set of states which is possible to access at time t. The role of the camera here is like a tool, which mathematically can be thought of as the function AR_t, through which we access the reality. If instead of a standard camera, one uses a high-resolution camera, then the same fragment of a real physical world may be accessed better than before. So, change of AR_t may give different perspectives of the same physical object.
5. The pair (S_{PR_t}, AR_t) can also be viewed as an information system. For any pair $(U_\#, \mathscr{R}_\#) \in S_{PR_t}$, the first component of the pair can be considered as object and the second can be considered as the set of relations characterizing the object in the physical reality. Given such a pair $(U_\#, \mathscr{R}_\#)$, the function AR_t, which may be a tool (like camera) to interact with the physical reality, basically selects out a set of states, say $\{s_1, s_2, \ldots, s_m\}$ $(\subseteq S^*)$ representing the object $(U_\#, \mathscr{R}_\#)$. That is, we can visualize the information system as follows (Table 7).

Table 7 Information system corresponding to (S_{PR_t}, AR_t)

	AR_t	$\ldots$
$(U_\#, \mathscr{R}_\#)$	$\{s_1, s_2, \ldots, s_m\}$	$\ldots$
$\vdots$		

6. When we fix such a function AR_t, i.e., the mean by which a part of the real world is accessed, we can identify different parts of the real world, say $(U_\#, \mathscr{R}_\#)$ with the respective set of states $\{s_1, s_2, \ldots, s_m\}$ obtained through the function.
   Let us call $\text{States}_{AR_t}(S_{PR_t}) = \cup_{(U_\#, \mathscr{R}_\#) \in S_{PR_t}} AR_t(U_\#, \mathscr{R}_\#)$. So, we may consider the physical reality with respect to a specific accessibility function AR_t as an information system $PR_{AR_t} = (S_{PR_t}, States_{AR_t}, \models_{PR_t})$, where $(U_\#, \mathscr{R}_\#) \models_{PR_t} s$ iff $s \in AR_t(U_\#, \mathscr{R}_\#) \cap \mathscr{O}_t$.
7. On agent's side there is $EIS_t = (P(S^*), \mathscr{A}_t, \models_{IS_t})$, which is grounded on the information system $IS_t = (\mathscr{O}_t, \mathscr{A}_t, \{f_a : \mathscr{O}_t \mapsto V_a\}_{a \in \mathscr{A}_t})$. In EIS_t agent also considers states belonging to $S^* - \mathscr{O}_t$, which are potentially measurable with respect to some parameters, but not measured at time t. So, the satisfaction relation is defined with respect to the information available at IS_t, and given by $\{s_1, s_2, \ldots, s_l\} \models_{IS_t} a$ iff for all $i = 1, 2, \ldots, l, f_a(s_i) \in V_a$.
8. Now an *interaction of the agent with* PR_t, the physical reality at time t, can be presented as an infomorphism from $EIS_t = (P(S^*), \mathscr{A}_t, \models_{PR_t})$, an extension of IS_t to $PR_{AR_t} = (S_{PR_t}, States_{AR_t}(S_{PR_t}), \models_{PR_t})$ following the sense of Barwise and Seligman [1].
   The infomorphism from $(P(S^*), \mathscr{A}_t, \models_{PR_t})$ to $(S_{PR_t}, States_{AR_t}(S_{PR_t}), \models_{PR_t})$ is defined as follows. An infomorphism

$$I_t : (P(S^*), \mathscr{A}_t, \models_{IS_t}) \rightleftarrows (S_{PR_t}, States_{AR_t}(S_{PR_t}), \models_{PR_t})$$

   consists of a pair of functions $(\hat{I}_t, \check{I}_t)$ where for any $a \in \mathscr{A}_t, \hat{I}_t(a) \in States_{AR_t}(S_{PR_t})$ and for any $(U_\#, \mathscr{R}_\#) \in S_{PR_t}, \check{I}_t(U_\#, \mathscr{R}_\#) \in P(S^*)$, and $(U_\#, \mathscr{R}_\#) \models_{PR_t} \hat{I}_t(a)$ iff $\check{I}_t(U_\#, \mathscr{R}_\#) \models_{IS_t} a$.
9. An *ISAPR* at time point t, denoted as $ISAPR_t$, is represented by the tuple (PR_{AR_t}, I_t, IS_t). In the classical sense of information system, $ISAPR_t$ is also an information system consisting of a single object PR_{AR_t} representing the information at the physical world with respect to the accessibility function AR_t, a single parameter I_t representing a specific interaction, and an outcome of the interaction with the physical reality viz., $IS_t = (\mathscr{O}_t, \mathscr{A}_t, \{f_a : \mathscr{O}_t \mapsto V_a\}_{a \in \mathscr{A}_t})$ (cf. Table 8).
   Here we use the same symbol I_t considering an interaction as a parameter. Thus there is a function f_{I_t} corresponding to the parameter I_t such that $f_{I_t}(PR_{AR_t}) = IS_t$ if $I_t : (P(S^*), \mathscr{A}_t, \models_{IS_t}) \rightleftarrows (S_{PR_t}, States_{AR_t}(S_{PR_t}), \models_{PR_t})$.
10. An *interactive information system approximating the physical reality*, denoted as *IISAPR*, represents an information system of the following kind. (Fig. 6)

Table 8 An information system as an outcome of an interaction with the physical reality

	I_t
PR_{AR_t}	$(\mathscr{O}_t, \mathscr{A}_t, \{f_a : \mathscr{O}_t \mapsto V_a\}_{a \in \mathscr{A}_t})$

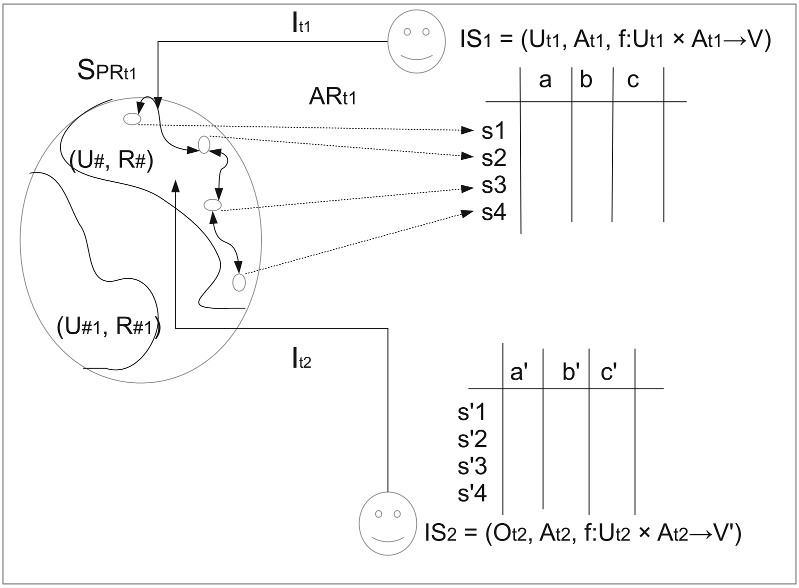

Fig. 6 Interaction between agent and physical reality through an infomorphism from the agent's information system to the physical reality

Table 9 An interactive information system approximating the physical reality

	I_{j_1}	I_{j_2}	...
$PR_{AR_{t_1}}$	$(\mathscr{O}_{t_1}, \mathscr{A}_{t_1}, \{f_a : \mathscr{O}_{t_1} \mapsto V_a\}_{a \in \mathscr{A}_{t_1}})$	$(\mathscr{O}^1_{t_1}, \mathscr{A}^1_{t_1}, \{f_a : U^1_{t_1} \mapsto V_a\}_{a \in \mathscr{A}^1_{t_1}})$	...
$PR_{AR^1_{t_2}}$	$(\mathscr{O}'_{t_2}, \mathscr{A}_{t_2}, \{f_a : \mathscr{O}'_{t_2} \mapsto V_a\}_{a \in \mathscr{A}_{t_2}})$	...	...
⋮	⋮	⋮	⋮

$$IISAPR = (\{PR_{AR_t}\}_{AR_t \in A_f, t \in T}, \{I_j\}_{j \in J}, \{f_{I_j} : \{PR_{AR_t}\}_{AR_t \in A_f, t \in T} \mapsto \{IS_l\}_{l \in L}\}_{j \in J}),$$

where $\{PR_{AR_t}\}_{AR_t \in A_f, t \in T}$ is a family of fragments of reality indexed by both time $t \in T$ and possible accessibility functions $AR_t \in A_f$, $\{I_j\}_{j \in J}$ is a family of possible interactions of agents with the physical world, and corresponding to each interaction I_j, f_{I_j} is a function assigning a unique information system from $\{IS_l\}_{l \in L}$ to each of $\{PR_{AR_t}\}_{AR_t \in A_f, t \in T}$. That is, we have Table 9 for *IISAPR*.

So, the whole process of arriving at relevant information systems with the passage of time and different interactions may be visualized through the picture presented in Fig. 7. There can be different cases when the time factor is considered. At some point

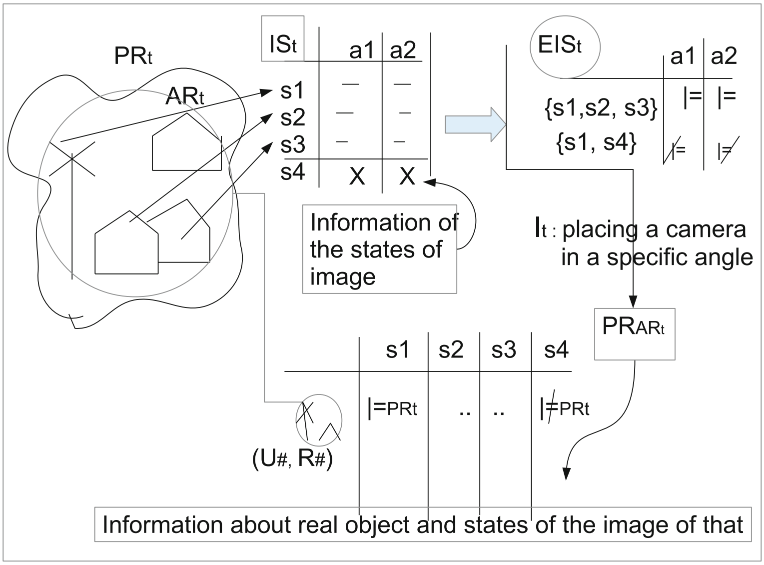

Fig. 7 Example: the process of interaction between agent and physical reality and obtaining an information system

of time t' ($> t$), $PR_{t'}$ may remain the same as PR_t, but with time the agent can manage to access more or something different than before. That is, $AR_{t'}$ may change; that is $PR_{AR_{t'}}$ may become different from PR_{AR_t}, and one can arrive at a different set of states $\{s'_1, s'_2, \ldots, s'_n\}$ from the same fragment of the physical world $(U_\#, \mathscr{R}_\#)$. On the other hand, with time the physical reality itself can change, and we may have $PR_{t'}$ different from PR_t (Fig. 6).

It is to be noted that apart from the time point t there are two more factors, namely the *function accessing the reality at time t* (AR_t) and the *interaction of agent with reality at time t* (I_t). Let us take an attempt to explain the role of the different components we have introduced in the model for an interactive information system.

Let there be a tree surrounded by a number of houses and a park, which may be considered as a fragment of reality $(U_\#, \mathscr{R}_\#)$. An agent can gather some information about $(U_\#, \mathscr{R}_\#)$ from an image of the fragment of the physical world taken by some particular camera. So, the camera works as a tool for accessing the information about the real physical world. That is, mathematically the functionality of such a tool or mode of accessing reality is availed by a function AR_t. Let through the camera the agent become able to get a good overview of the tree and two houses close to the tree, which are respectively captured by some states s_1, s_2, s_3 of the image in terms of brightness of colours (a_1), pixel-points (a_2) etc. Let us assume that some other state s_4, representing another object in the vicinity of the tree, appeared blurred in the image. So, though s_4 seems to be a possible measurable state, is not measured properly at time t. So, following the terminologies used above $s_4 \in S^* - \mathscr{O}_t$. The agent manages to get an image of $(U_{\#}, \mathscr{R}_{\#})$ as there was a specific interaction I_t between the agent and the reality at time t, which might be considered as *placing a camera in a specific angle*. At some further point of time t' the agent may initiate a different interaction $I_{t'}$ by changing the angle of the same camera, or replacing the earlier camera by a high-resolution camera. In the former case, interaction $I_{t'}$ changes but the tool for accessing the reality $AR_{t'}$ remains the same as AR_t. On the other hand, in the latter case, both $I_{t'}$ and $AR_{t'}$ change.

4 Concluding Remarks

As concluding remarks, let us present some important aspects, which can come up naturally from the proposed idea of interactive information systems, as future issues of investigation.

- One is, how can we assure that the set of states, about which we learn through AR_t, depicts the relational structure present in $(U_{\#}, \mathscr{R}_{\#})$ properly. In this context, we need to concentrate on the relations over the sets of values *i.e.*, the range sets of the functions AR_t as well as f_a's for each $a \in \mathscr{A}_t$. To illustrate the point we can think about the example of image of a real object captured by a camera. The expectation to AR_t is that when applied on a pair $(U_{\#}, \mathscr{R}_{\#})$ as an outcome it should produce a set of states $\{s_1, s_2, \ldots, s_m\}$ which together represents a prototypical image of the real object $(U_{\#}, \mathscr{R}_{\#})$ preserving the relations among the real parts of the object. So, we may consider that if two objects $o_i, o_j \in U_{\#}$ are such that for some $r_{\#} \in \mathscr{R}_{\#}$, $r_{\#}(o_i, o_j)$, then the relation should also be preserved somehow under the transformation of $(U_{\#}, \mathscr{R}_{\#})$ to $AR_t(U_{\#}, \mathscr{R}_{\#}) = \{s_1, s_2, \ldots, s_m\}$. So, the relational structures as well as the languages having interpretation over them would come into play in the context of interactive information systems.
- The above point leads us towards another important aspect. The question is, what happens if for two object $o_i, o_j \in U_{\#}$, $r_{\#}(o_i, o_j)$ holds for some $r_{\#} \in \mathscr{R}_{\#}$ but the relation is not preserved among the states which represent $(U_{\#}, \mathscr{R}_{\#})$ under the accessibility function AR_t. In the context of an human agent, it is quite natural that such a situation would generate some action to initiate new interaction with the physical reality. So, satisfaction or dissatisfaction of some interrelations between the relational structures lying in the physical object, and that of in its representation with respect to the the agent's information system may generate some typical actions. These actions in turn, with the progress of time, generates new interaction and keeps on modifying the agent's information system characterizing some fragment of the physical world. So, for each interactive information system approximating the physical reality at some time point t, viz., $IISAPR_t$, we also need to count a set of decision attributes consisting of actions, say A_c, so that depending on the level of accuracy of an agent's information system in characterizing a fragment of the physical world which action to be taken can be determined.

Acknowledgements This work was partially supported by the Polish National Science Centre (NCN) grants DEC-2011/01/D /ST6/06981, DEC-2013/09/B/ST6/01568 as well as by the Polish National Centre for Research and Development (NCBiR) under the grant DZP/RID-I-44 / 8 /NCBR/2016.

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