Chapter 5
An Integration of Mixed Contact Formulation with Model-Reduction Techniques

Blaž Starc, Gregor Čepon, and Miha Boltežar

Abstract A new method for efficient dynamics simulations of flexible systems with unilateral contacts is presented. The procedure is based on the Craig-Bampton model-reduction technique together with a contact formulation using single- and set-valued force laws. For the computation of system response, an event-driven integration scheme is proposed, which allows transitions among different contact states. An updating algorithm for derivation of the reduced-system matrices is developed as well as the formulation of new initial conditions. The applicability of the developed method is demonstrated on a simple structure with varying contact conditions.

Keywords Penalty method • System response • Time integration • Unilateral contacts • Dynamics

5.1 Introduction

The dynamic simulations of contacts between flexible bodies are usually conducted by applying a dense mesh and the use of the classical finite element method in combination with the penalty method [1]. This requires a large set of Degrees of Freedom (DoF) to represent the structure and the use of explicit analyses with small-time step-sizes to analyse the system response. This typically leads to long computation times and requires large computational resources. In this paper, a new method for efficient dynamics simulations of flexible systems with unilateral contacts is proposed. The procedure proposes the integration of single- and set-valued force laws together with the Craig-Bampton model reduction technique [2]. According to the contact state, an event-driven integration scheme is used, which allows the updating of reduced-system matrices as well as formulating new reduced-space initial conditions. The applicability of the developed method is demonstrated on a clamped-beam structure with a harmonic force on the free end and a varying contact condition.

5.2 Craig-Bampton Method

The classic finite-element approach requires a large number of nodes, which leads to large models and long computation times. In order to reduce the time, a coarser mesh needs to be applied, which is not always possible due to the convergence of the solution. A possible alternative is the Craig-Bampton [2] model-reduction technique. It retains the dense finite-element mesh, but replaces the physical DoF by a much smaller set of generalized DoF.

The model-reduction techniques are closely connected to the substucturing field, where a substructure dynamical model is defined as:

$$\mathbf{M}^{(s)}\,\ddot{\mathbf{u}}^{(s)}(t) + \mathbf{C}^{(s)}\,\dot{\mathbf{u}}^{(s)}(t) + \mathbf{K}^{(s)}\,\mathbf{u}^{(s)}(t) = \mathbf{f}^{(s)}(t) + \mathbf{g}^{(s)}(t), \tag{5.1}$$

The matrices $\mathbf{M}^{(s)}$, $\mathbf{C}^{(s)}$ and $\mathbf{K}^{(s)}$ represent the mass, the damping and the stiffness matrix of a substructure s, $\mathbf{u}^{(s)}(t)$ is the displacement vector, $\mathbf{f}^{(s)}$ is the external excitation vector and $\mathbf{g}^{(s)}$ is the vector of connection forces with the surrounding substructures. The Craig-Bampton method divides the physical DOF $\mathbf{u}$ into the internal $\mathbf{u}_i$ and the boundary DOF $\mathbf{u}_b$, which gives Eq. (5.1) the following shape:

B. Starc (✉) • G. Čepon • M. Boltežar
Faculty of Mechanical Engineering, University of Ljubljana, Aškerceva cesta 6, 1000, Ljubljana, Slovenia
e-mail: blaz.starc@fs.uni-lj.si

M.S. Allen et al. (eds.), *Dynamics of Coupled Structures, Volume 4*, Conference Proceedings of the Society for Experimental Mechanics Series, DOI 10.1007/978-3-319-54930-9_5

$$\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ib} \\ \mathbf{M}_{bi} & \mathbf{M}_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_b \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_i \\ \mathbf{f}_b \end{Bmatrix} + \begin{Bmatrix} \mathbf{g}_i \\ \mathbf{g}_b \end{Bmatrix}, \tag{5.2}$$

where the index i denotes the internal DOF and b the boundary DOF. Note that the internal excitation forces $\mathbf{g}_i$ are assumed to be $\mathbf{0}$, since there is no contact with the neighbouring substructures.

The internal DOF are approximated as:

$$\mathbf{u}_i \approx \boldsymbol{\Psi}_c\, \mathbf{u}_b + \boldsymbol{\Phi}_i\, \boldsymbol{\eta}_i \tag{5.3}$$

Here, $\boldsymbol{\Psi}_c$ are the static constraint modes and $\boldsymbol{\Phi}_i$ are a reduced set of fixed interface vibration modes with the corresponding modal DOF $\boldsymbol{\eta}_i$. If Eq. (5.3) is inserted into Eq. (5.2) and the orthogonality between the vibration modes with respect to the mass or stiffness matrix [3] is taken into account, the following reduced equations of motion are obtained:

$$\begin{bmatrix} \mathbf{I} & \mathbf{M}_{\phi b} \\ \mathbf{M}_{b\phi} & \tilde{\mathbf{M}}_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{\eta}}_i \\ \ddot{\mathbf{u}}_b \end{Bmatrix} + \begin{bmatrix} \boldsymbol{\Omega}_i^2 & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{K}}_{bb} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\eta}_i \\ \mathbf{u}_b \end{Bmatrix} = \begin{Bmatrix} \tilde{\mathbf{f}}_i \\ \tilde{\mathbf{f}}_b \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{g}_b \end{Bmatrix}, \tag{5.4}$$

where:

$$\begin{aligned} \tilde{\mathbf{K}}_{bb} &= \mathbf{K}_{bb} - \mathbf{K}_{bi}\, \mathbf{K}_{ii}^{-1}\, \mathbf{K}_{ib} \\ \tilde{\mathbf{M}}_{bb} &= \mathbf{M}_{bb} - \mathbf{M}_{bi}\, \mathbf{K}_{ii}^{-1}\, \mathbf{K}_{ib} - \mathbf{K}_{bi}\, \mathbf{K}_{ii}^{-1}\, \mathbf{M}_{ib} + \mathbf{K}_{bi}\, \mathbf{K}_{ii}^{-1}\, \mathbf{M}_{ii}\, \mathbf{K}_{ii}^{-1}\, \mathbf{K}_{ib} = \\ &= \mathbf{M}_{bb} - \mathbf{M}_{bi}\, \boldsymbol{\Psi}_c - \boldsymbol{\Psi}_c^{\mathrm{T}}\, \mathbf{M}_{ib} + \boldsymbol{\Psi}_c^{\mathrm{T}}\, \mathbf{M}_{ii}\, \boldsymbol{\Psi}_c \\ \mathbf{M}_{\phi b} &= \boldsymbol{\Phi}_i^{\mathrm{T}} (\mathbf{M}_{ib} - \mathbf{M}_{ii}\, \mathbf{K}_{ii}^{-1}\, \mathbf{K}_{ib}) \\ \mathbf{M}_{b\phi} &= \mathbf{M}_{\phi b}^{\mathrm{T}} \\ \tilde{\mathbf{f}}_i &= \boldsymbol{\Phi}_i^{\mathrm{T}}\, \mathbf{f}_i \\ \tilde{\mathbf{f}}_b &= \mathbf{f}_b - \mathbf{K}_{bi}\, \mathbf{K}_{ii}^{-1}\, \mathbf{f}_i = \boldsymbol{\Psi}_c^{\mathrm{T}}\, \mathbf{f}_i \end{aligned} \tag{5.5}$$

Here, $\boldsymbol{\Omega}_i^2$ represents a diagonal matrix of squared fixed-interface frequencies $\omega_{i,j}^2$.

5.3 Contact Formulation

The modelling of contacts between flexible bodies is usually formulated using the penalty method [1]. Using a large penalty stiffness may lead to an ill-conditioned stiffness matrix and consequently to a poor convergence. On the other hand, the method that imposes a non-penetration condition originates from the formulation between rigid-bodies [1, 4]. Here, a combination of single- and set-valued unilateral force laws is used as proposed in [5]. The proposed contact method models the contact with three states: no-contact, continuous and impact/penalty state. This enables faster integration times, since the penalty is present only during the impact. When the contact is lost (no-contact) the stiffness matrix is classical (without penalty) and when fixed the contact DOF is fixed. The contact states in addition with the event-driven integration scheme can be seen on case of a clamped beam with a varying contact and a harmonic force on the free end in Fig. 5.1.

Note, that the contact states influence the accompanying reduced-models and also have an influence on the definition of integration events. The three reduced models can be computed in advance and therefore do not influence the integration process. Also the events need to be defined in reduced space. The constraint force in Event 1 (see Fig. 5.1) is derived from Eq. (5.4):

$$\mathbf{g}_b = \mathbf{M}_{b\phi}\, \ddot{\boldsymbol{\eta}}_i + \mathbf{M}_{bb}\, \ddot{\mathbf{u}}_b + \mathbf{K}_{bb}\, \mathbf{u}_b - \tilde{\mathbf{f}}_b \tag{5.6}$$

and further simplified if boundary displacements and accelerations are zero:

$$\mathbf{g}_b = \mathbf{M}_{b\phi}\, \ddot{\boldsymbol{\eta}}_i - \tilde{\mathbf{f}}_b \tag{5.7}$$

Other events (2–7, see Fig. 5.1) are defined classically.

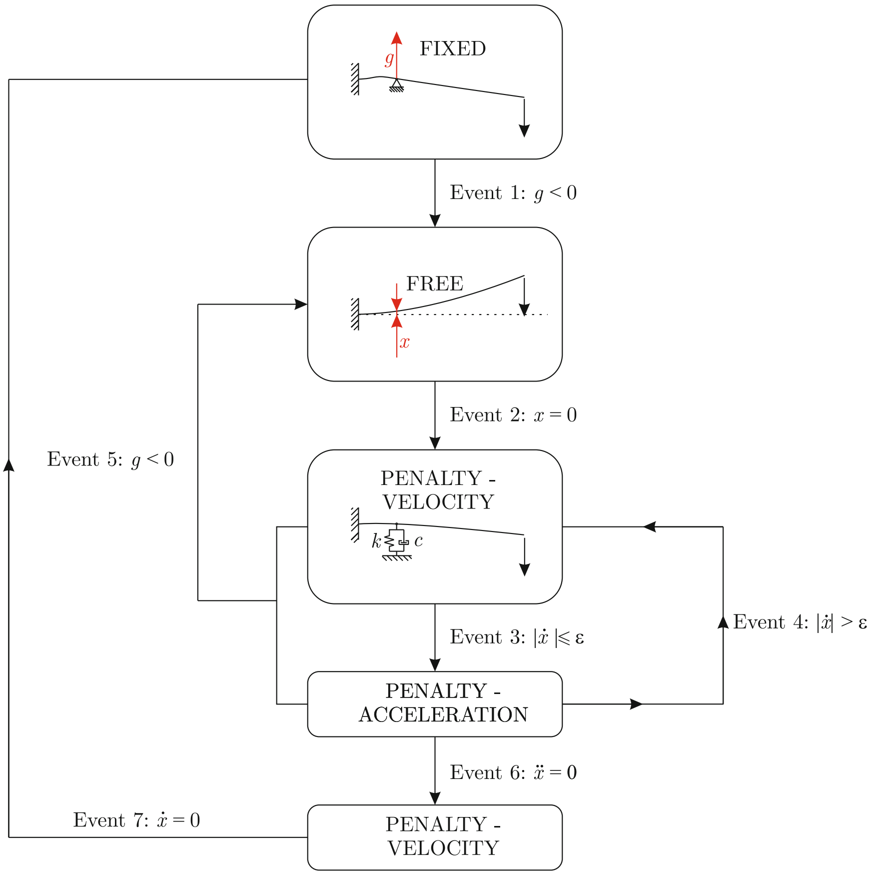

Fig. 5.1 Contact states and integration events on a clamped beam

In addition to events, also new initial conditions need to be defined in order to define transition among the three reduced states. The transition between two reduced spaces (e.g. no-contact and continuous) is done by equalizing the physical coordinates (Eq. 5.3) and since the boundary conditions are already in the physical space, the new initial conditions can be obtained:

$$\boldsymbol{\eta}_{i,free} = \boldsymbol{\Phi}^{+}_{i,free}(\boldsymbol{\Phi}_{i,fixed}\,\boldsymbol{\eta}_{i,fixed} + \boldsymbol{\Psi}_{c,fixed}\,\mathbf{u}_b - \boldsymbol{\Psi}_{c,free}\,\mathbf{u}_b). \tag{5.8}$$

Equation (5.8) can be further simplified, due to the orthogonal properties of the fixed-interface modes ($\boldsymbol{\Phi}^{+}_{i,free}\boldsymbol{\Phi}_{i,fixed} = \mathbf{I}$):

$$\boldsymbol{\eta}_{i,free} = \boldsymbol{\eta}_{i,fixed} + \boldsymbol{\Phi}^{+}_{i,free}\,(\boldsymbol{\Psi}_{c,fixed} - \boldsymbol{\Psi}_{c,free})\,\mathbf{u}_b \tag{5.9}$$

Note, that the above equation is derived for the no-contact and continuous reduced models, but the same conclusion is valid also for other combinations with the impact/penalty model.

5.4 Conclusions

A new method for modelling the dynamic response of systems with contacts is proposed. The Craig-Bampton model-reduction technique is used to reduce the number of DoF and therefore enable faster and more efficient computations of the system response. Moreover, the reduced model is upgraded with a contact formulation consisting of three main states: no-contact, continuous and impact/penalty state. This enables faster computation times than pure penalty method, since the continuous contact is modelled without the additional penalty stiffness. The use of the method is showcased on a clamped beam structure with a varying contact and a harmonic force on the free end.

References

1. Pfeiffer, F., Glocker, C.: Multibody Dynamics with Unilateral Contacts, vol. 1996, 1st edn. Springer, Wien (1996)
2. Bampton, M.C.C., Craig, R.R., Jr.: Coupling of substructures for dynamic analyses. AIAA J. **6**(7), 1313–1319 (1968)
3. Geradin, M., Rixen, D.J.: Mechanical Vibrations: Theory and Application to Structural Dynamics, 3rd edn. Wiley, Hoboken (2015)
4. Glocker, C.: Formulation of spatial contact situations in rigid multibody systems. Comput. Methods Appl. Mech. Eng. **177**(3–4), 199–214 (1999)
5. Čepon, G., Boltežar, M.: Dynamics of a belt-drive system using a linear complementarity problem for the belt-pulley contact description. J. Sound Vib. **319**(3–5), 1019–1035 (2009)