Chapter 19
Dynamic Substructuring of Geometrically Nonlinear Finite Element Models Using Residual Flexibility Modes

Morteza Karamooz Mahdiabadi, Erhard Buchmann, Duo Xu, Andreas Bartl, and Daniel Jean Rixen

Abstract A nonlinear dynamic substructuring approach using Residual Flexibility method for geometrically nonlinear structures is investigated. In order to reduce the model order of a geometrically nonlinear structure, the closed form equations of motion for the substructures are classically required. However, in industrial applications these equations are often not available, because the model of interest is constructed in a commercial finite element software package. As a result, a non-intrusive reduction method is applied. To do so, the Rubin method which contains Residual Flexibility effect is used as a linear basis and the nonlinear terms with unknown coefficients – due to geometric nonlinearity – are added to them. These nonlinear stiffness coefficients of the reduced substructure are calculated using the Implicit Condensation and Expansion Method (ICE). Then, the nonlinear reduced substructures are assembled using Component Mode Synthesis (CMS). The performance of the free-interface method of Rubin as a linear basis for nonlinear substructuring is examined by implementing the proposed methods on an academic example developed in a commercial finite element software. The accuracy of the reduced order model is assessed by comparing the Nonlinear Normal Modes (NNMs) of the reduced order model with the ones of the original model before reduction.

Keywords Nonlinear substructuring • Residual flexibility modes • Model reduction • Geometric nonlinearity • Nonlinear normal modes

19.1 Introduction

In the design procedure of dynamic structures, engineers often deal with large finite element models (FEMs) containing an enormous number of Degrees of Freedom (DOFs). On the other hand, they have to solve dynamic responses of the structure (such as eigenfrequency and eigenmodes, transient or harmonic response, etc.) many times (e.g. in optimization procedures), which leads to inadmissible computational time. In order to reduce these costs and simultaneously take the advantage of FEMs with very accurate and fine meshes, component model reduction methods are proposed. These methods are based on the idea that the motion of a system is written in terms of a superposition of the modes, and afterwards a truncated set of these modes are taken to approximate the response of the system. Model reduction techniques can be applied on a monolithic structure [4] as well as on components (substructures) of a structure, which are assembled then through substructuring techniques to build the full reduced model [1, 12, 19]. In the later case, one should also take the static deformation caused by connecting forces (or displacements) into account. These reduction methods are also called Component Mode Synthesis (CMS) [6].

While the linear reduction techniques are already well established in application, methods in nonlinear framework are actively under consideration. One of the typical kinds of nonlinearities that many lightweight structures face, is geometric nonlinearity, caused by large deflections in the structures. Nonlinear model reduction techniques for monolithic structures are basically classified into direct and indirect methods. In the former one, a closed form equations of motion is required in order to construct the nonlinear reduced order model (NLROM). Whereas, in the later, the NLROM is developed without requiring to deal with the nonlinear tensors of the FEM. In other words, giving a series of input forces (displacements) to an FE Modeling package and getting the requested output displacements (forces), the NLROM is built, without knowing the details of nonlinear terms in the primary FEM [13]. Therefore, the indirect methods (also known as non-intrusive methods) seem to be more efficient in case the structures are developed in commercial finite element (FE) packages (which is the case for most of the industrial applications), since there is no need to build the nonlinear FE tensors of the full structure [3, 5, 16].

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Furthermore, it is very cumbersome to reduce a nonlinear FEM containing a fine mesh with millions of DOFs at once. Alternatively, nonlinear substructuring methods are proposed in order to deal with such models [21–23]. Kuether et al. [10] developed a nonlinear substructuring approach, in which they first reduced each substructure by means of a non-intrusive method. They took two different linear basis in the procedure of indirect reduction of each substructure, namely, free interface modes and Craig-Bampton modes. Furthermore, they checked the accuracy of their NLROMs using Nonlinear Normal Modes (NNMs).

Convergence validation of NLROMs with NNMs facilitates the path of nonlinear model reduction, as time integration of the whole nonlinear structure is no more required [11].

In this paper efficient dynamic substructuring of geometrically nonlinear FEMs is investigated. A set of truncated free interface modes in companion with residual flexibility modes (instead of free interface, solely used in [9]) are taken as a linear basis for each nonlinear substructure to increase the accuracy of NLROMs. Employing this linear basis, the substructures are reduced using an indirect method with input forces (which is also known as Implicit Condensation and Expantion (ICE)). In order to compute the nonlinear coefficients for the NLROMs with ICE, a commercial FE package (Abaqus®) is used in this study. Afterwards, the NLROM substructures are coupled using primal assembly. NNMs are employed for the sake of validation of the reduced order model with the primary structure. In order to evaluate the performance of the studied method, a nonlinear FEM is developed in the FE package, divided into two substructures and the proposed method is implemented on it.

19.2 Indirect Nonlinear Model Reduction with Load Input

This section explains briefly the indirect NLROM with applied loads procedure (or ICE method) [3, 5]. Consider a structure (or substructure) with geometrical nonlinearity effect consists of n DOFs. The governing equations of motion (EOMs) in matrix form can be written as

$$M\ddot{u}(t) + Ku(t) + f_{nl}(u(t)) = f(t) \tag{19.1}$$

where M and K are the mass and linear stiffness matrices, respectively. The nonlinear restoring force vector is denoted by $f_{nl}(u(t))$, representing the geometric nonlinear effects caused by large deformations in the system, and $f(t)$ is the external forces applied on the structure (If the EOMs are written for a substructure, $f(t)$ also includes connecting forces imposed by neighboring substructures). In order to reduce the nonlinear FEM, first a proper linear reduced model for the FEM should be constructed. Therefore, the displacement of the system, $u(t)$, is approximated as superposition of a truncated number of modes

$$u(t) = T_{red}\,\eta(t) \tag{19.2}$$

Here, $\eta(t)$ is the reduced generalized coordinate vector, and T_{red} is the transformation matrix, resulting in the reduced number of DOFs in the system. For an elastic FEM with quadratic strain-displacement relation, the nonlinear restoring force due to geometric nonlinearity can be written as a function of quadratic and cubic terms including all combinations of the modal coordinates [2]. Consequently, the NLROM for the investigated structure with EOMs (19.1) is formed as

$$\hat{M}\ddot{\eta} + \hat{K}\eta + \sum_{i=1}^{m}\sum_{j=i}^{m}\alpha(i,j)\eta_i\eta_j + \sum_{i=1}^{m}\sum_{j=i}^{m}\sum_{k=j}^{m}\beta(i,j,k)\eta_i\eta_j\eta_k = T_{red}^{T}F_c \tag{19.3}$$

where, $\alpha_r(i,j)$ and $\beta_r(i,j,k)$ are the unknown coefficients of the quadratic and cubic terms formed from multiplication of ith and jth or ith, jth and kth modal DOFs, respectively. The number of DOFs of the reduced model is denoted by m. The linear reduced mass and stiffness matrices are denoted by $\hat{M}$ and $\hat{K}$ and are obtained as follows

$$\begin{aligned}\hat{M} &= T_{red}^{T}MT_{red}\\ \hat{K} &= T_{red}^{T}KT_{red}\end{aligned} \tag{19.4}$$

In order to compute the unknown coefficient tensors corresponding to the quadratic and cubic terms, a set of static forces are applied on the system (through the FE package) to calculate the corresponding displacements. The static forces are defined using a combination of one, two or three scaling factors as

$$F_c = K(T_{red_k}\eta_k + T_{red_l}\eta_l + T_{red_m}\eta_m) \tag{19.5}$$

here, η_k, η_l and η_m are the scaling factors corresponding to the kth, lth and mth reduced modal coordinates, respectively. The subscripts k, l and m refer to the kth, lth and mth columns of the transformation matrix T_{red}, correspondingly. Gordon et al. [3] introduced these scaling factors to be the forces needed to reach a maximum displacement in the linear system as

$$\eta_q = \frac{w_{q,\max}}{T_{red_{q,\max}}}\omega_q^2 \tag{19.6}$$

where ω_q and $T_{red_{q,\max}}$ are the qth eigenfrequency of the reduced linear system and the maximum translational component in the corresponding mode (column) of the reduction basis. The desired maximum displacement in physical domain is denoted by $w_{q,\max}$. Gordon et al. [3] discussed different criteria to choose the value of $w_{q,\max}$. In this study, this value is chosen to be in the order of the structure's thickness. After importing the defined forces, F_c, as static loads in the FE package (i.e. Abaqus® in this work), the corresponding displacements, u_c, can be computed. These displacements are then transformed into the modal coordinate, η_c, as

$$(T_{red}^T M T_{red})^{-1} T_{red}^T M u_c = \eta_c \tag{19.7}$$

Now, applying several static forces to the FE package and solving for the corresponding displacements, an overdetermined problem should be solved to estimate the unknown stiffness coefficients. Therefore, the following static equation for the rth modal coordinate becomes:

$$\sum_{i=1}^{m}\sum_{j=i}^{m}\alpha_r(i,j)\eta_i\eta_j + \sum_{i=1}^{m}\sum_{j=i}^{m}\sum_{k=j}^{m}\beta_r(i,j,k)\eta_i\eta_j\eta_k = T_{red}^T F_c - \hat{K}_{rq}\eta_q \tag{19.8}$$

Afterwards, the reduced EOMs are reformulated in the form proposed by [14]

$$\hat{M}\ddot{\eta} + \hat{K}\eta + \frac{1}{2}N_1(\eta)\,\eta + \frac{1}{3}N_2(\eta)\,\eta = T_{red}^T f(t) \tag{19.9}$$

where $N_1(\eta)$ and $N_2(\eta)$ are Jacobian matrices constructed from quadratic and cubic terms, respectively.

19.3 Component Mode Synthesis with Residual Flexibility Modes

19.3.1 Rubin Reduction Basis

Consider the equations of motion for the undamped substructure, s, with n DOFs as

$$M^{(s)}\ddot{u}(t)^{(s)} + K^{(s)}u(t)^{(s)} = f(t)^{(s)} + g(t)^{(s)} \tag{19.10}$$

where, $M^{(s)}$ and $K^{(s)}$ are mass and stiffness matrices of the substructure. The acceleration and displacement vectors are shown by $\ddot{u}(t)^{(s)}$, and $u(t)^{(s)}$, respectively. The external force vector applied on the substructure is denoted by $f(t)^{(s)}$ and $g(t)^{(s)}$ is the interface force vector imposed by neighboring substructures.

An efficient way for model reduction of structures with substructuring is approximating the motion of each substructure as a truncated number of free interface modes in addition to a static deformation caused by connecting forces from neighboring substructures [12, 17, 19].

$$u^{(s)} \approx u_{static}^{(s)} + \sum_{i=1}^{k-r}\phi_{free,i}^{(s)}\,\eta_{free,i}^{(s)} \tag{19.11}$$

Here, $\phi^{(s)}_{free,i}$ is the ith vibration mode when the interface DOFs are left free, and $\eta^{(s)}_{free,i}$ is the corresponding generalized modal coordinate. The total number of modes kept in the substructure and number of Rigid Body (RB) modes are represented with k and r, correspondingly. Equation (19.10) implies that if RB modes exist in the system, they should be included in the static deformation $u^{(s)}_{static}$. Therefore, in order to represent the static deformation of the substructure s caused by neighboring substructures, $u^{(s)}_{static}$ is introduced as follows

$$u^{(s)}_{static} = \Psi^{(s)}_{RFA}\, g^{(s)} + \Phi^{(s)}_{RB}\, \eta^{(s)}_{RB} \tag{19.12}$$

where $\Phi^{(s)}_{RB}$ and $\Psi^{(s)}_{RFA}$ are matrices including RB and Residual Flexibility Attachment (RFA) modes, respectively. The generalized coordinate vector corresponding to RFA modes is denoted by $\eta^{(s)}_{RB}$.

In order to obtain Residual Flexibility Attachment modes, first the attachment modes are defined as the displacement vector caused by applying a unit force at an interface DOF, u_b. Therefore, the columns of the flexibility matrix $G^{(s)}$ containing the attachment modes are obtained as

$$G^{(s)} = (K^{(s)})^{+} \tag{19.13}$$

The superscript $+$ is the generalized inverse (or pseudo-inverse) operation, implying the fact that if the substructure has RB modes, the stiffness matrix $K^{(s)}$ is singular. Afterwards, the RFA modes are defined as

$$\Phi^{(s)}_{RFA} = (G^{(s)} - \sum_{j=1}^{r-m} \frac{\phi^{(s)}_{free\_j} \phi^{(s)^T}_{free,j}}{\omega_j^2}) B^{(s)} \tag{19.14}$$

where $B^{(s)}$ is the signed Boolean matrix, specifying interface DOFs of the substructure s. The RFA modes are the static deflection of the deleted modes (which are truncated in the substructure) due to the unit force at each interface DOF. Now, having the RFA modes, $\Psi^{(s)}_{RFA}$, the Eq. (19.12) is substituted into (19.11) giving the approximation for displacement vector $u^{(s)}$

$$u^{(s)} \approx \Psi^{(s)}_{RFA}\, g^{(s)} + \Phi^{(s)}_{RB}\, \eta^{(s)}_{RB} + \Phi^{(s)}_{free}\, \eta^{(s)}_{free} \tag{19.15}$$

In order to provide the assembly of substructures with interface DOFs, Eq. (19.15) is pre-multiplied by $B^{(s)^T}$ and solved for the interface forces $g^{(s)}$:

$$u^{(s)}_b = B^{(s)} u^{(s)} = B^{(s)} (\Psi^{(s)}_{RFA}\, g^{(s)} + \Phi^{(s)}_{RB}\, \eta^{(s)}_{RB} + \Phi^{(s)}_{free}\, \eta^{(s)}_{free}) = \Psi^{(s)}_{RFA,b}\, g^{(s)} + \Phi^{(s)}_{RB,b}\, \eta^{(s)}_{RB} + \Phi^{(s)}_{free,b}\, \eta^{(s)}_{free} \tag{19.16}$$

$$g^{(s)} = \Psi^{+}_{RFA,b}{}^{(s)} (u^{(s)}_b - \Phi^{(s)}_{RB,b}\, \eta^{(s)}_{RB} - \Phi^{(s)}_{free,b}\, \eta^{(s)}_{free}) \tag{19.17}$$

where the subscript b for matrices denote they are pre-multiplied by $B^{(s)^T}$, and therefore, the part of the matrices corresponding the interface DOFs are included. The next step is to substitute (19.17) in (19.15) and one can obtain the final reduction basis for the internal DOFs, $u^{(s)}_i$ as [19, 20]

$$\begin{aligned} u^{(s)}_i = {} & \Psi^{(s)}_{RFA,i} \Psi^{+}_{RFA,b}{}^{(s)} u^{(s)}_b + (\Phi^{(s)}_{RB,i} - \Psi^{(s)}_{RFA,i} \Psi^{+}_{RFA,b}{}^{(s)} \Phi^{(s)}_{RB,b}) \eta^{(s)}_{RB} + \\ & (\Phi^{(s)}_{free,i} - \Psi^{(s)}_{RFA,i} \Psi^{+}_{RFA,b}{}^{(s)} \Phi^{(s)}_{free,b}) \eta^{(s)}_{free} \end{aligned} \tag{19.18}$$

The reduction basis can also be written in the following matrix form

$$u^{(s)} = \begin{bmatrix} u^{(s)}_i \\ u^{(s)}_b \end{bmatrix} = T_{red} \begin{bmatrix} \eta^{(s)}_{free} \\ \eta^{(s)}_{RB} \\ u^{(s)}_b \end{bmatrix} \tag{19.19}$$

where T_{red} is the (Rubin) reduction basis by taking the Residual Flexibility modes into account and is obtained as

$$T_{red} = \begin{bmatrix} \Phi^{(s)}_{free,i} - \Psi^{(s)}_{RFA,i}\Psi^{+}_{RFA,b}{}^{(s)}\Phi^{(s)}_{free,b} & \Phi^{(s)}_{RB,i} - \Psi^{(s)}_{RFA,i}\Psi^{+}_{RFA,b}{}^{(s)}\Phi^{(s)}_{RB,b} & \Psi^{(s)}_{RFA,i}\Psi^{+}_{RFA,b}{}^{(s)} \\ 0 & 0 & I \end{bmatrix} \tag{19.20}$$

Finally, in order to compute the reduced mass and stiffness matrices, $\hat{M}^{(s)}$ and $\hat{K}^{(s)}$, the obtained reduced basis (19.20) should be substituted in the expressions of Eq. (19.4).

19.3.2 Primal Assembly

The final step to achieve the whole NLROM, is to assemble the NLROMs of all substructures. The assembly of substructures can be performed using either interface displacements or forces. If the substructures are coupled by satisfying first the compatibility boundary conditions the assembly is called *primal*. In contrast, if the equilibrium boundary conditions are imposed to the assembly procedure first, and the compatibility is satisfied alongside the EOMs, the assembly is called *dual* [8]. This study only considers the primal assembly, which is also called Component Modes Synthesis (CMS). Considering the assembly problem of two reduced nonlinear substructures s_1 and s_2, the EOMs of the substructures in a block diagonal form is written as

$$\begin{bmatrix} \hat{M}^{(s_1)} & 0 \\ 0 & \hat{M}^{(s_2)} \end{bmatrix} \begin{Bmatrix} \eta^{(s_1)} \\ \eta^{(s_2)} \end{Bmatrix} + \begin{bmatrix} \hat{K}^{(s_1)} & 0 \\ 0 & \hat{K}^{(s_2)} \end{bmatrix} \begin{Bmatrix} \eta^{(s_1)} \\ \eta^{(s_2)} \end{Bmatrix} + \frac{1}{2} \begin{bmatrix} N_1^{(s_1)}(\eta^{(s_1)}) & 0 \\ 0 & N_1^{(s_2)}(\eta^{(s_2)}) \end{bmatrix} \begin{Bmatrix} \eta^{(s_1)} \\ \eta^{(s_2)} \end{Bmatrix} + \cdots \frac{1}{3} \begin{bmatrix} N_2^{(s_1)}(\eta^{(s_1)}) & 0 \\ 0 & N_2^{(s_2)}(\eta^{(s_2)}) \end{bmatrix} \begin{Bmatrix} \eta^{(s_1)} \\ \eta^{(s_2)} \end{Bmatrix} = \begin{Bmatrix} T^{(s_1)^T} \left\{ f(t)^{(s_1)} + g^{(s_1)} \right\} \\ T^{(s_2)^T} \left\{ f(t)^{(s_2)} + g^{(s_2)} \right\} \end{Bmatrix} \tag{19.21}$$

then the compatibility condition is expressed by

$$B\,\eta = 0 \tag{19.22}$$

where B is the signed Boolean matrix, specifying the corresponding connecting DOFs (for conforming meshes) for all substructures. The equilibrium boundary condition is then defined as

$$L^T g = 0 \tag{19.23}$$

Here, L is the Boolean localization matrix which puts the Substructures' local DOFs into the global fashion of DOFs

$$\eta = L\,\eta_c \tag{19.24}$$

The subscript c denotes the coupled global set of DOFs. It is shown in [8] that the localization matrix L is the null space of the B matrix. In other words

$$\begin{aligned} BL &= 0 \\ L^T B^T &= 0 \end{aligned} \tag{19.25}$$

Substituting Eq. (19.24) into (19.22), the compatibility condition is satisfied first

$$BL\,\eta_c = 0 \tag{19.26}$$

Finally, substituting Eq. (19.24) into (19.21) and pre-multiplying (19.21) by L^T lead to the assembled EOMs as

$$L^T \begin{bmatrix} \hat{M}^{(s_1)} & 0 \\ 0 & \hat{M}^{(s_2)} \end{bmatrix} L\,\ddot{\eta}_c + L^T \begin{bmatrix} \hat{K}^{(s_1)} & 0 \\ 0 & \hat{K}^{(s_2)} \end{bmatrix} L\,\eta_c + \tfrac{1}{2} L^T \begin{bmatrix} N_1^{(s_1)}(\eta^{(s_1)}) & 0 \\ 0 & N_1^{(s_2)}(\eta^{(s_2)}) \end{bmatrix} L\,\eta_c + \\ \ldots \tfrac{1}{3} L^T \begin{bmatrix} N_2^{(s_1)}(\eta^{(s_1)}) & 0 \\ 0 & N_2^{(s_2)}(\eta^{(s_2)}) \end{bmatrix} L\,\eta_c = L^T \begin{Bmatrix} T^{(s_1)^T} \left\{ f(t)^{(s_1)} + g^{(s_1)} \right\} \\ T^{(s_2)^T} \left\{ f(t)^{(s_2)} + g^{(s_2)} \right\} \end{Bmatrix} \tag{19.27}$$

19.4 Validation

The accuracy of the NLROM depends strongly on the modes included in the reduction basis as well as the external load applied to the model when one is performing ICE. Therefore, they have to be chosen very carefully. Since there's no concrete standard about how these factors can be properly chosen so far, a validation of the NLROM model is necessary. The common procedure to validate NLROMs, is to compare the time integration of a certain DOFs of them to a given load with the ones of the full model. This method brings two obvious shortcomings. On the one hand, the computational cost can be prohibitively expensive when implementing it on models with many DOFs. On the other hand, it only validates the model at one or few load levels or response amplitudes.

Instead, Kuether et al. used Norlinear Normal Modes (NNMs) to validate NLROM in [11]. NNMs provide an insight into how the system responds to different load scenarios and can capture a wide range of response amplitudes experienced by the structure, which makes it the ideal metric for the validation of the NLROM. NNMs were first defined by Rosenberg in [18] as a periodic motion of the system, in which all DOFs reach the equilibrium position at the same time. The definition was further extended by Kerschen et al. in [7] as *not necessarily synchronous periodic responses*. Although orthogonality and modal superposition are no longer applicable for NNMs, they show other unique properties compared to the linear normal modes, such as modal interactions, bifurcation, and frequency-energy dependency.

Kerschen proposed to compute NNMs with numerical continuation methods in [15]. In order to find the periodic motion of the system, a shooting function is defined to indicate the difference between the system motion at $t = 0$ and $t = T$. The free response of the nonlinear system to a prescribed initial condition is only then periodic, when the shooting function is smaller than a given tolerance. In this way, the boundary value problem is transferred into initial value problem that can be solved with a shooting method. The branch of an NNM is followed with the pseudo-arclength algorithm proposed by Peeters et al. [15]. Prediction and correction steps are used to find the periodic solution, where Jacobian matrices are employed in prediction and correction steps and play a huge roll in computational efficiency.

The further development of this method by Kuether et al. in [11] not only improved the efficiency, more importantly, they also made it feasible to compute the NNMs of models built in commercial FE package. The Applied Modal Force method (AMF) defines the initial displacement as the nonlinear static response of the system to a prescribed external force, which is a weighted truncated subset of linear modes shapes. The weighting factors, termed as modal force amplitude, are to be determined with pseudo-arclength continuation. The individual variables are thus reduced from n to the number of truncated weighting factors m. In this way, the computational burden caused by Jacobian matrices, is largely reduced. Only one mode is used to define the initial displacement at first. The contribution of rest modes is monitored, while the energy in the system rises and corresponding modes will be activated if necessary.

In this work, the NNMs of the full order model were computed with AMF, while those of the reduced model with the method described in [15]. Due to its frequency-energy dependence, the NNMs are represented in Frequency-Energy Plot (FEP) as in [7], where the fundamental frequency of the periodic motions is plotted on the vertical axis, and the total energy in the system, including kinetic and potential energy, is plotted on the horizontal axis. Each point on an NNM curve corresponds to a periodic solution of the equation of motion.

19.5 Numerical Example

In order to evaluate the performance of the proposed method on a geometrically nonlinear structure, the two simply supported beams (also shown in Fig. 19.1) – the same as the ones studied in [10] – are investigated. Table 19.1 shows the parameters used to build the model of the substructures in Abaqus®.

The NLROMs of the substructures one and two are developed using the ICE method explained in Sect. 19.2. In order to have a more accurate linear reduction basis (compare to the free interface modes used in [9]), the method of Rubin [19] is employed. Namely, a set of truncated free interface modes in addition to the Residual Flexibility Attachment modes are taken for each substructure to span the effect of the connecting forces imposed by neighboring structures. The number of free interface modes kept for substructures one and two are 3 (out of 119 modes) and 2 (out of 80 modes), respectively.

Furthermore, the developed NLROMs of the substructures are coupled using the primal CMS assembly approach (Sect. 19.3.2) and the NNMs of the assembled structure is computed with Shooting and Pseudo-Arclength Continuation approaches developed in [15]. For the sake of efficient validation of the achieved NLROM, the NNMs of the whole geometrical nonlinear structure are also computed with the AMF method of Kuethers et al. [11]. The first computed NNM for NLROM as well as the corresponding one for the full beam is shown in Fig. 19.2. As it can be seen in this figure, until a certain energy range, the NNM computed from the NLROM converges well to the one of the full model. However, for higher energies, the difference between the two curves becomes more obvious. This can be due to the fact that quite a few number of modes (5 out of 199) are taken into account to develop the NLROM. The NNMs comparison of the NLROM with the full model for higher modes (NNMs 2, 3 and 4) is shown in Fig. 19.3 In this study, the NNMs of higher modes are only compared for one of the branches (the subbranches of the NLROM are not plotted). As it is shown in Fig. 19.3 the NNMs of the NLROM match to the ones of full model which expresses the proper performance of the proposed method.

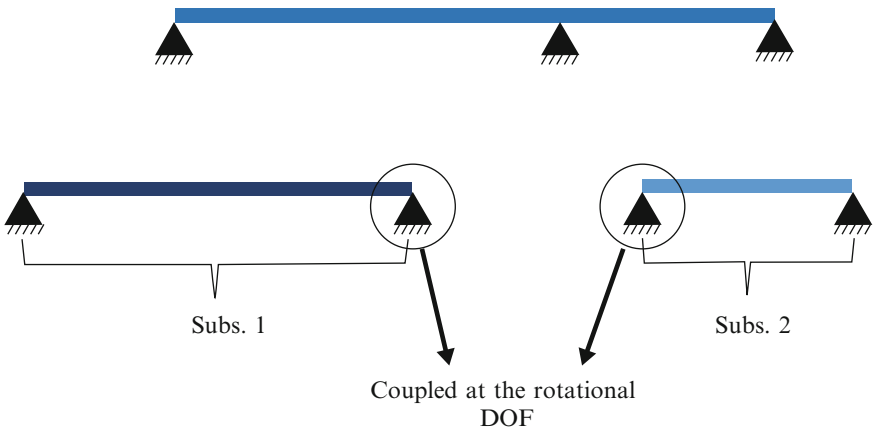

Fig. 19.1 Two simply supported beams coupled at the rotational DOF

Table 19.1 Properties of investigated model

		Substructure 1	Substructure 2
Geometry	Length	229 mm	152 mm
	Width	12.7 mm	12.7 mm
	Heigth	0.787 mm	0.787 mm
Material	Mass density	7870 kg/m^3	7870 kg/m^3
	Young's modulus	204.8 GPa	204.8 GPa
	Poisson's ratio	0.28	0.28
Discretization	Element-type	Euler-Bernoulli ABAQUS B31	Euler-Bernoulli ABAQUS B31
	Number of elements	40	27
	Number of degrees of freedom	119	80

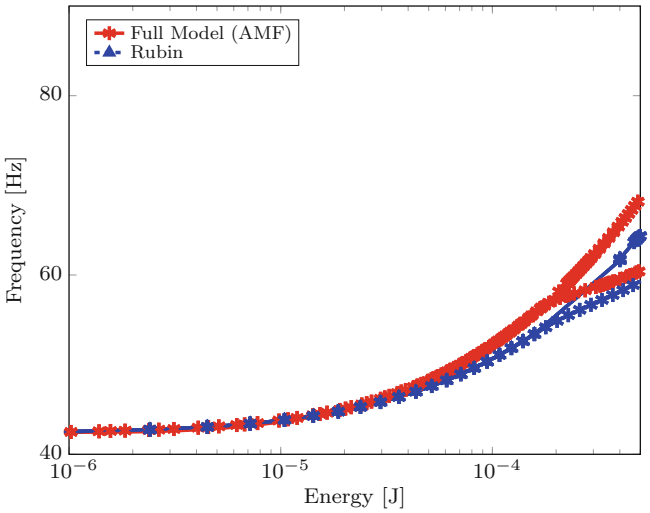

Fig. 19.2 Comparison of the first NNMs. The *red circle curve* is the full model NNM calculated with AMF, the *blue triangle* is the NNM of the NLROM with five modes (up to 500 Hz)

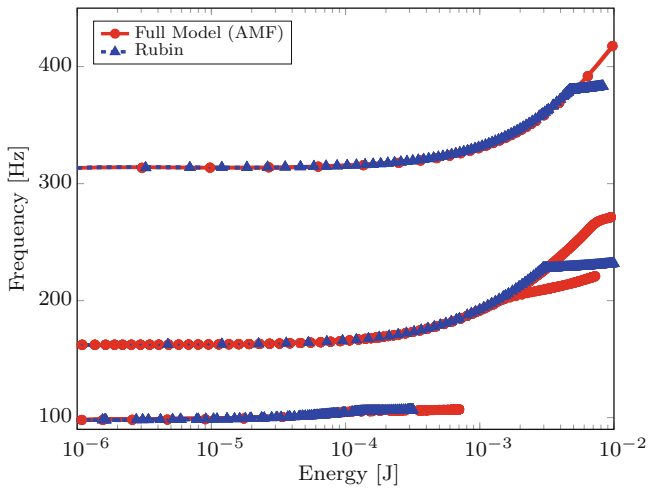

Fig. 19.3 Comparison of the second to forth NNM. The *red circles curve* are the full model NNMs calculated with AMF, the *blue triangles* are the NNMs of the NLROM with five modes (up to 500 Hz)

19.6 Conclusion

This work studies model reduction and substructuring of geometrically nonlinear finite element models proposing the Rubin reduction basis. Namely, a set of truncated free interface modes in addition to residual flexibility attachment modes are taken as a linear basis for each substructure. Nonlinear coefficients of the reduced order model due to large deflections are compute employing the non-intrusive method of Implicit Condensation and Expansion, which is efficient in case the nonlinear finite element model (FEM) is only available in a commercial FE package. Furthermore, the nonlinear reduced order substructures are assembled using Component Mode Synthesis. For the sake of validation of the nonlinear reduced order model developed with the aforementioned approach, the nonlinear normal modes (NNMs) of the primal structure and the reduced model are compared. In fact using NNMs as a convergence check let us avoid expensive time integration of the whole nonlinear structure for validation. Finally, the procedure is implemented on a geometrically nonlinear FEM divided into two substructures. Taking just 2.5% of the linear modes (5 modes out of 199 modes), the NNMs resulted from substructuring converged to the NNMs of the whole model, expressing the efficiency of the proposed method.

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