Chapter 17
Dynamic Decoupling of Nonlinear Systems

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Abstract Structural decoupling problem has been well investigated for three decades and led to several decoupling methods. In spite of the inherent nonlinearities in a structural system in various forms all decoupling studies are for linear systems. In this study, decoupling problem for nonlinear systems is addressed for the first time and a method is proposed for calculating the frequency response functions of a substructure decoupled from a coupled nonlinear structure where nonlinearity can be modelled as a single nonlinear element. The method proposed is validated through simulated case studies.

Keywords Nonlinear decoupling • Nonlinear uncoupling • Nonlinear inverse substructuring • Nonlinear subsystem identification • Nonlinear substructure decoupling

17.1 Introduction

Since engineering structures are generally designed as an assembly of several components, considerable effort has been devoted to structural decoupling of linear systems, some of those worth mentioning is listed in references [1–3]. However, the problem where system to be decoupled includes a nonlinear element such as clearance, friction and nonlinear stiffness remains untouched. In this paper, a method is developed for the decoupling problem of nonlinear systems. The method is tested on simple lumped parameter systems by using simulated experimental data.

17.2 Theory

The uncoupling problem is studied as three separate problems, depending on the location of the nonlinear element in the coupled system: The nonlinearity can be either in the unknown subsystem or in the known subsystem, or it can connect two subsystems. The method proposed for the solution of this problem is mainly based on the application of the following techniques:

- The controlled displacement amplitude testing technique Additional Comment #4 for nonlinear systems.
- The decoupling technique proposed by D'Ambrogio et al. [1] for linear systems.
- The parametric modal identification technique proposed by Arslan et al. [4] for nonlinear systems.

The method proposed is applicable to systems where the nonlinearity can be modelled as a single nonlinear element. It is also assumed that the location of this nonlinear element is known.

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17.2.1 Nonlinearity in the Unknown Subsystem

In this case, it is assumed that known subsystem is linear whereas unknown subsystem is nonlinear and the location of the nonlinear element is known. Firstly, the complete FRF matrix of the known subsystem for the coordinates we are interested in is obtained by using the known system parameters. Then, various different sets of linear FRFs of the coupled system for the coordinates we are interested in are obtained by keeping the amplitude of the relative harmonic displacement between the end coordinates of the nonlinear element constant at a different value for each set of FRFs. Note that, depending on the location of the nonlinearity, the number of coordinates at which FRFs should be measured in the coupled system can be reduced. Using available FRFs, sets of linear FRF curves for the unknown subsystem can be obtained by applying the decoupling formulation proposed by D'Ambrogio et al. [1] for linear systems, each set corresponding to a different response level. Then, by applying the modal identification technique developed by Richardson and Formenti [5], a set of modal parameters will be obtained from each FRF curve. As the identified modal parameters vary with the response amplitude, they can be expressed as a function of the amplitude of the relative harmonic displacement between the end coordinates of the nonlinear element [4]. Then, the FRFs of the unknown subsystem can be calculated at different response levels by using the modal parameter variations obtained.

17.2.2 Nonlinearity in the Known Subsystem

In this case, it is assumed that known subsystem is nonlinear whereas unknown subsystem is linear. Nonlinear element may be located at any place in the known subsystem. Firstly, the point and transfer FRFs of the coupled system as well as of the known subsystem at coordinates that belong to the known subsystem should be obtained by keeping the amplitude of the relative harmonic displacement between the end coordinates of the nonlinear element at a specific value throughout the desired frequency range. FRFs of the known system will be calculated whereas those of the coupled system should be measured by making controlled displacement amplitude testing. This will yield a set of linear FRF curves for the coupled system, as well as for the known subsystem. Note that the nonlinearity matrices, first introduced by Tanrıkulu et al. [6] and then used in many applications, added to the dynamic stiffness matrices of the coupled system and the known subsystem will have the same values at each frequency throughout the desired frequency range. Hence, the existence of nonlinearity will be the same as adding a linear stiffness matrix to the known part of the system, and thus the problem will be reduced into decoupling of linear systems. Consequently, the FRFs of the unknown subsystem at its coupling DOFs can be calculated using the FRFs of the known and coupled systems obtained above by applying the decoupling formulation proposed by D'Ambrogio et al. [1].

17.2.3 Nonlinearity in the Connection of Two Subsystems

When the nonlinear element connects two subsystems, the problem can be reduced into the one of those defined in sect. 17.2.1 or sect. 17.2.2, depending on the availability of the properties of the nonlinear element. If the parameters of the nonlinear connection element are not known, it should be taken as a part of the unknown subsystem with a massless node at the other end, which is rigidly connected to the connection node of the known subsystem. A similar approach has been followed by the authors in [7]. Thus, the system will be reduced into the system considered in sect. 17.2.1. In case where the parameters of the nonlinear element are known, the system can be reduced into the system considered in sect. 17.2.2 in the same vein.

17.3 Simulated Case Studies

In this section, applications of the proposed decoupling method to a lumped parameter system are presented in order to demonstrate the validity and the efficiency of the method developed.

17.3.1 Decoupling of a Lumped Parameter Nonlinear System: Nonlinearity at the Unknown Subsystem

In this case study, decoupling of a 2 DOF nonlinear subsystem from a 3 DOF lumped parameter system is demonstrated by applying the decoupling method proposed. The nonlinear element is assumed to be a grounded cubic stiffness connected to the coupling DOF of the unknown subsystem. Firstly, the complete FRF matrix (for the DOFs we are interested in) of the known subsystem are theoretically obtained from the known subsystem parameters. Secondly, we need to obtain point and transfer FRFs of the coupled system at coordinates that belong to the known subsystem experimentally through a controlled displacement amplitude test in the frequency range of interest for different constant harmonic displacement amplitudes of the second DOF of the coupled system. These values are theoretically obtained, but in order to include the effect of experimental errors, they are polluted by adding a complex random number. Then the decoupling formulation proposed by D'Ambrogio et al. [1] for linear systems is applied for each 20 different sets of point and transfer FRF curves of the coupled subsystem at coordinates that belong to the known subsystem in order to obtain the point FRF of the unknown subsystem at its coupling DOF. The results are given in Fig. 17.1.

Note in Fig. 17.1 that, each FRF curve shows linear behavior, as it is obtained for a constant harmonic displacement amplitude of the nonlinear element. Firstly, the variation of modal parameters with respect to the amplitude of the relative harmonic displacement between the end coordinates of the nonlinear element is obtained by first fitting FRF curves to the calculated FRF values and then identifying modal parameters for each FRF curve by applying linear modal identification. Then, harmonic response of the unknown subsystem at its coupling DOF is calculated for a harmonic excitation of magnitude 1 N applied at the same point, by employing the approach proposed by Arslan et al. [4] and using the modal parameters calculated above (as a function of response amplitude). The same calculation is also performed through the application of the Harmonic Balance Method (HBM) by using the actual data for the unknown subsystem. These results are compared in Fig. 17.2. An excellent agreement of two response curves for both forward and backward sweeps in the whole frequency range demonstrates the validity of the method proposed.

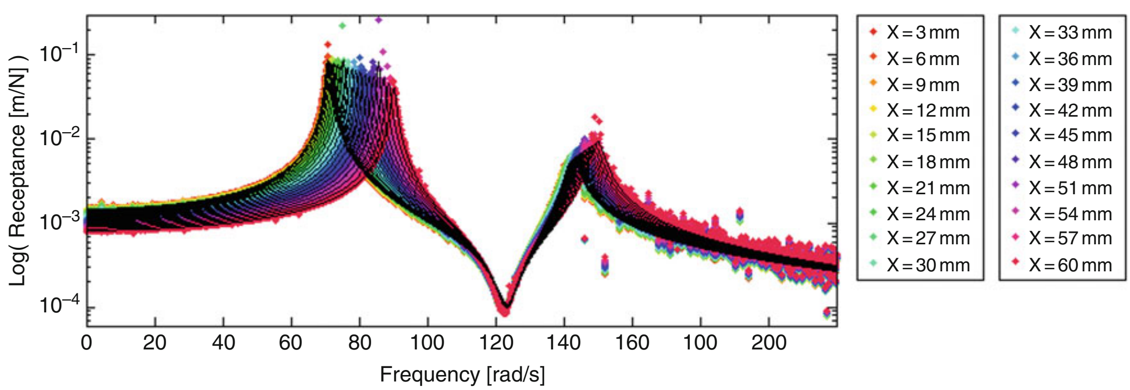

Fig. 17.1 Point FRFs of the unknown subsystem at its coupling DOF (*colored points*) and fitted FRF curves (—, *black*)

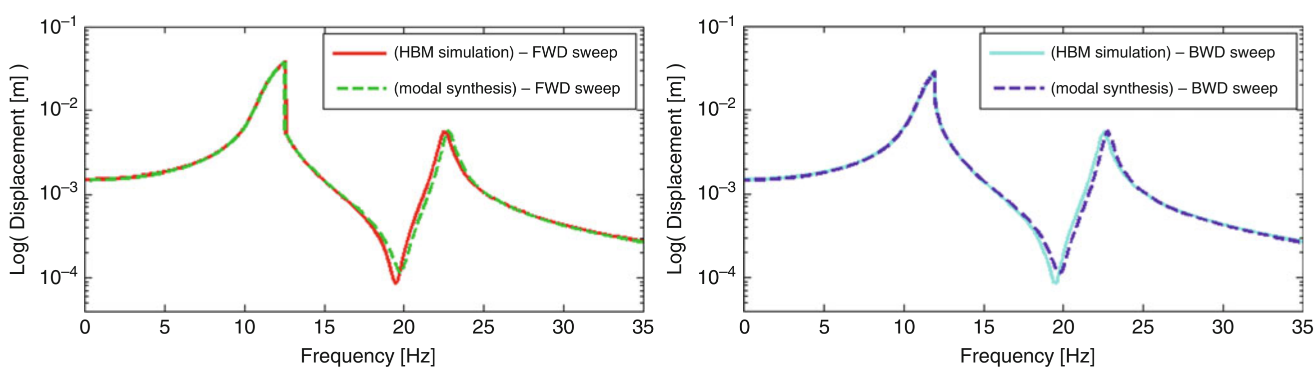

Fig. 17.2 Forward (FWD) and backward (BWD) frequency sweep responses of the unknown subsystem at its coupling DOF

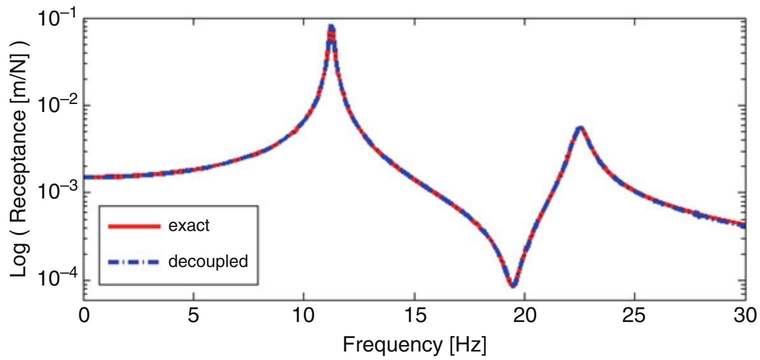

Fig. 17.3 Exact and decoupled point receptances of the unknown subsystem at its coupling DOF

17.3.2 Decoupling of a Lumped Parameter Nonlinear System: Nonlinearity at the Known Subsystem

In this case study, decoupling of a 2 DOF linear subsystem from a 3 DOF lumped parameter nonlinear system is demonstrated by applying the decoupling method proposed. The nonlinear element is again assumed to be a grounded cubic stiffness connected to the internal DOF of the known subsystem. Then, the exact point and transfer FRFs of the coupled system and the known subsystem at coordinates that belong to the known subsystem are calculated by keeping the amplitude of the relative harmonic displacement between end coordinates of the cubic nonlinearity at a specific constant value (20 mm for both systems). In order to include the effect of noise in real testing, a complex random perturbation is added to the calculated FRFs. Finally, the decoupling formulation proposed by D'Ambrogio et al. [1] for linear systems is applied to obtain the point receptance of the unknown subsystem at its coupling DOF by using the FRF curves fitted to the point and transfer receptances of the coupled system obtained through simulated experiment, and receptances of the known subsystem. The results are given in Fig. 17.3.

Fig. 17.3 shows that the FRF obtained using the decoupling method proposed almost the same as the exact FRF. Then it can be concluded that the decoupling method developed yields very good results for the case where the nonlinearity is in the known subsystem.

17.4 Discussion and Conclusions

Although there are some accuracy problems, the dynamic decoupling problem of linear structures is well addressed in literature. However, there has been almost no effort to tackle the dynamic decoupling problem of nonlinear structures. This paper presents the first attempt to solve this problem by suggesting a method that can be applied when the nonlinearity can be modelled as a single element. It is also assumed in this method that the location of nonlinearity is known. The approach proposed can be applied for all possible cases as far as the location of the nonlinear element is concerned, i.e. nonlinearity can be either in the known or unknown subsystem, or it can be at the connection. The method proposed is validated through simulated case studies.

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