Chapter 1
Techniques for Verification of Structural Acoustic Models

Hasan G. Pasha and Rajendra Gunda

Abstract When designing mechanical structures, the desired acoustic performance and efficiency are often achieved by employing upfront CAE driven design and development process. The advantages of this approach are multi-fold compared to a purely testing based approach. However, making key design decisions based solely on the results obtained from these CAE models requires that the models be verified and validated by comparing the model predictions with known solutions of simple sources and experimental data respectively. The boundary element method is typically used for modeling and predicting the radiated sound-field from a vibrating structure. This involves discretizing the surface of the vibrating structure with discrete boundary elements, applying the appropriate boundary conditions and solving the Helmholtz integral equations to predict the sound-field. The theoretical background is presented along with verification examples involving simple and complex sound sources.

Keywords Model verification • Code verification • Solution verification • Boundary element method • CHIEF • Burton-Miller formulation • Gear whine predictions

Notation

$|\epsilon_{rel}|$	Relative error, %
λ	Wavelength, m
ρ	Density, $\mathrm{kg\,m^{-3}}$
ω	Circular frequency, $\mathrm{rad\,s^{-1}}$
Ω	Solid angle
c	Sound wave propagation velocity, $\mathrm{m\,s^{-1}}$
j_m	Spherical Bessel function of the first kind of order m
k	Wavenumber, $\mathrm{m^{-1}}$
n	Outward normal vector
$p(r, t)$	Time dependent acoustic pressure, Pa
$\bar{p}(r, \omega)$	Frequency dependent acoustic pressure, Pa
r	Receiver point location
r_0	Source point location
v_n	Normal velocity, $\mathrm{m\,s^{-1}}$
$C(r)$	Integration constant of the Kirchoff-Helmholtz Integral Equation
D	Domain of wave propagation
∂D	Boundary of the domain of wave propagation
$G(r, r_0)$	Free-field Green's function

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$H_0^{(1)}$	Hankel function of the first kind of order zero
$H_0^{(2)}$	Hankel function of the second kind of order zero
J_m	Bessel function of the first kind of order m
N	Number of nodes

Abbreviations

Symbol	Description
BEM	Boundary Element Method
CAE	Computer Aided Engineering
FEM	Finite Element Method
FMM	Fast Multipole Method
SPL	Acoustic Pressure Level
V&V	Verification and Validation

1.1 Background

1.1.1 Introduction

Characterizing the sound-fields due to mechanical or acoustic (internal sources) excitation is a critical part of designing mechanical structures. These sound-fields are complicated functions of geometry, source distribution and noise control treatment. Hence, these cannot be computed in closed form. The desired acoustic performance and efficiency are often achieved by employing upfront CAE driven design and development process.

Typically CAE techniques such as, *boundary element method (BEM)* and *finite element method (FEM)*, are used in structural acoustic analysis. A detailed account on BEM and its application to acoustics can be found in [1–4]. In both FEM and BEM the problem domain is divided into finite elements. However, in FEM the *entire problem domain* is divided into elements, but in BEM only the *bounding surface* of the domain is discretized.

The advantages of a CAE driven approach are multi-fold compared to a purely testing based approach. However, making key design decisions based solely on the results obtained from these CAE models requires that the models be verified and validated by comparing the model predictions with known solutions of simple sources and experimental data respectively.

The verification and validation (V&V) methods and procedures were originally developed to improve the credibility of simulations in several high-consequence fields, such as nuclear technology. This was driven by a ban on nuclear testing for safety purposes [5]. Although there may not be safety concerns in many other engineering applications, conducting extensive testing as part of product development may be very expensive and time consuming. In addition, it may not be possible to conduct testing in certain configurations. Applying V&V to improve the credibility of CAE models for structural design can potentially address these concerns.

1.1.2 Verification and Validation

AIAA [6] and Oberkampf and Trucano [7] in the context of assessing the accuracy and reliability computer models, verification and validation are defined as:

Verification: refers to process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model. Frequently, the verification portion of the process answers the question *"Are the model equations correctly implemented?"* and deals with mathematics.

Validation: refers to the process of determining the degree to which a model is an accurate representation of the real world system from the perspective of the intended uses of the model. Frequently, the validation process answers the question *"Are the correct model equations implemented?"* and deals with physics.

Verification should always precede validation. The discussions in this paper are limited to the verification part of the V&V process. The mathematical model for structural systems can be defined by a set of partial differential or integro-differential equations, along with the required initial and boundary conditions. The mathematical model is then translated in to a computational (discrete-mathematics) model and solved by a computer implementation of relevant algorithms. The goals as part of the verification process are to identify, quantify and reduce errors caused by the mapping of the mathematical model to a computer implementation.

The verification process can be further classified as *code verification* and *solution verification*. Code verification deals with assessing the reliability of the software coding, whereas solution verification deals with assessing the numerical accuracy of the solution to a computational model.

1.1.3   Mathematical Models

1.1.3.1   The Acoustic Wave Equation

Sound is a *vibrational* transmission of mechanical energy that propagates through matter as a wave (as a compression wave through fluids and as both compression and shear waves through solids).

The equation for the transmission of a sound wave in a *homogeneous isotropic fluid* can be evolved by deriving the relationship between the pressure variations within the fluid and the deformation of the fluid. This can be achieved by using the thermodynamic properties of the fluid and law of conservation of mass. The linear acoustic wave equation in homogeneous isotropic medium at rest is,

$$\nabla^2 p(r, t) - \frac{1}{c^2} \frac{\partial^2 p(r, t)}{\partial t^2} = 0. \tag{1.1}$$

1.1.3.2   The Helmholtz Equation

In the *steady-state* case, the acoustic pressure fluctuation is *time harmonic* as expressed in Eq. (1.2), the time dependence, $e^{-i\omega t}$, is *suppressed*.[1]

$$p(r, t) = \text{Re}\left(\bar{p}(r, \omega) e^{-i\omega t}\right) \tag{1.2}$$

The Helmholtz equation (1.3) is obtained by substituting Eq. (1.2) in the wave equation (1.1).

$$(\nabla^2 + k^2)\bar{p}(r, \omega) = 0, \tag{1.3}$$

where, the constant k, is termed as the *wavenumber*,

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}. \tag{1.4}$$

For a complete solution, the wave equation is written as a series of Helmholtz problems by expressing the boundary conditions as a Fourier series with components of the form of Eq (1.2). For each wavenumber and its associated boundary condition, the Helmholtz equation is then solved. The time-dependent acoustic pressure, $p(r, t)$, can then be constituted from the separate solutions.

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[1] Alternatively, the Helmholtz equation can be obtained by applying *Fourier transform* to the acoustic wave equation (1.1).

1.1.3.3 The Free-Field Green's Function: Fundamental Solution of the Helmholtz Equation

The *fundamental solution* of the Helmholtz equation in 3D (interior), $G(r, r_0)$, is the sound-field due to a Dirac delta excitation, $\delta(r - r_0)$, due to a *monopole source* present at r_0. $G(r, r_0)$ is also termed as the *free-field Green's function*, which means that it is the (unique) solution, the sound-field generated by a point source excitation in free space.

$$(\nabla^2 + k^2)G(r, r_0) = -\delta(r - r_0) \tag{1.5}$$

$$G(r, r_0) = \frac{e^{-ik|r - r_0|}}{4\pi|r - r_0|}; \quad r, r_0 \in R^3 \tag{1.6}$$

The fundamental solutions of the Helmholtz equation in 3D for exterior boundary value problems are given by Eq. (1.7).

$$G(r, r_0) = \frac{e^{ik|r-r_0|}}{4\pi|r - r_0|}; \quad r, r_0 \in R^3. \tag{1.7}$$

1.2 BEM Formulation and the Computational Model

Acoustic problems can be formulated as *boundary integral equations* (BIE). BEM is a numerical method used for obtaining approximate solutions to BIE. This work concerns with the application of BEM to *time-harmonic* acoustic radiation problems.

The formulation of a boundary value problem into a BIE representation has several advantages as compared to FEM. The most important of these is the *reduction of the problem dimension*; three-dimensional problems are solved on two-dimensional surfaces enclosing the domain. In addition, for exterior problems, the *Sommerfeld radiation condition* is automatically incorporated in the integral formulation.

Copley [8] applied the BIE formulations to acoustics to predict sound-fields. There are two fundamental approaches to derive the boundary integral equation formulation for acoustic problems.

Direct Boundary Integral Formulation: It is based on the *Kirchoff-Helmholtz integral equation*. When at least one closed surface is present, direct formulation is possible. The acoustic problem is solved directly to yield the primary variables – *acoustic pressure* and *particle velocity*, on one side of the domain.
Indirect Boundary Integral/Simple-source Formulation: It is adapted from potential theory where the potential (acoustic pressure) is represented as being due to a *distribution of simple sources* over a surface represented by an integral of an unknown source-density function. The indirect formulation is applicable to both closed and open boundary problems. Both the sides of the of the boundary surface can be considered. The acoustic problem is solved to yield a secondary variable – *fictitious distribution of simple sources* at the boundary. The primary variables can then be obtained using a post-processing operation.

1.2.1 Direct BIE Formulation for Helmholtz Equation

The acoustic pressure or the particle velocity in a steady-state sound-field in a domain D, bounded by a closed surface ∂D, is governed by the Helmholtz equation and is subjected to the prescribed boundary conditions. Direct boundary integral formulation is based on deriving an equivalent integral equation – the *Kirchhoff-Helmholtz integral equation*, which is obtained using *Green's second identity* to the unknown field pressure and the fundamental solution.[2] Equation (1.8) can be used to evaluate the pressure at any point in the domain in interior or exterior problems.

$$C(r)p(r) = \int\int_S \left(i\rho ckG(r, r_0)v_n(r) - p(r)\frac{\partial G(r, r_0)}{\partial n}\right) dS \tag{1.8}$$

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[2]Direct BIE formulation can also be obtained using the weighted residual technique.

$$C(r) = \begin{cases} 1, & \text{r in the fluid domain D} \\ \frac{1}{2}, & \text{r on smooth boundary } \partial D \text{ of the fluid domain D} \\ \frac{\Omega}{4\pi}, & \text{r on non} - \text{smooth boundary } \partial D \text{ of the fluid domain D} \\ 0, & \text{r outside the fluid domain D} \end{cases} \tag{1.9}$$

where, C is the integration constant of the Kirchhoff-Helmholtz integral equation, ρ is the density of the wave propagation medium, c is the wave propagation velocity, k is the wavenumber, $G(r, r_0)$ is the free-field Green's function, v_n is the normal component of the acoustic particle velocity, p is the acoustic pressure, $\frac{\partial}{\partial n}$ is the normal derivative and Ω is the solid angle.

1.2.2 Non-uniqueness and Regularization

The sound-field predicted in certain acoustic radiation problems (exterior domain) using integral equation approach is in error. For certain *characteristic frequencies/wavenumbers*, the straightforward formulation of the exterior problem in terms of a single integral equation using either of the formulations mentioned above yields a *non-unique* solution. Unique solutions for such problems exist and the anomaly is *non-physical*. This anomaly is purely due to the integral equation formulation.

Uniqueness can be restored by deriving another integral equation and combining it with the original equation. Schenck [9] proposed the *Combined Helmholtz Integral Equation Formulation* (CHIEF). In CHIEF, the integral equations for the *surface* and that for the *interior* are combined to form an *over-determined* system with which the deficiencies and the undesirable computational characteristics of the direct boundary integral formulation are overcome. Randomly distributed points (a.k.a. CHIEF points) are added inside the closed surface and the pressure is set to zero at these points.

Burton and Miller [10] suggested combining the integral equation and a multiple of its *normal derivative*, which addressed the non-uniqueness for the simple-source formulation. Although the Burton and Miller approach is computationally intensive compared to CHIEF, it is a more robust formulation.

1.2.3 Solution Schemes for the Boundary Integral Equation

For numerically approximating the Kirchhoff-Helmholtz integral equation, the surface is divided into a set of *distinct* boundary elements using standard boundary element procedures such as the *collocation scheme*, where the nodes of discretization represent the collocation points.

Let,

$$p^* = \frac{e^{-ik|r - r_0|}}{4\pi |r - r_0|}; \tag{1.10}$$

$$\frac{\partial p}{\partial n} \equiv q = i\rho c k v_n; \tag{1.11}$$

$$q^* = \frac{\partial p^*}{\partial n}; \tag{1.12}$$

where p^* is the fundamental solution.

It is assumed that the boundary variables, the acoustic pressure and particle velocity on each element, can be approximately represented by a linear combination of interpolation functions,

$$p(r) = \sum_{j=1}^{N} p_j \phi_j(r); \tag{1.13}$$

where, p_j is the pressure at a discrete boundary node and $\phi(r)$ are the shape functions. Also, the acoustic velocity at the boundary point can be represented as,

$$v_n(r) = \sum_{j=1}^{N} v_j \phi_j(r). \tag{1.14}$$

When the interpolations are used in Eq. (1.8), the integration over the surface is approximated by a sum of integrals over all the boundary elements such that,

$$C(r)\,p(r) = \sum_{j=1}^{N} p_j \int\!\!\int_S \phi_j(r)q^*(r, r_0)dS - \sum_{j=1}^{N} q_j \int\!\!\int_S \phi_j(r)p^*(r, r_0)dS. \tag{1.15}$$

This can be rewritten in a compact form as,

$$C(r)\,p(r) = \sum_{j=1}^{N} p_j a_j - \sum_{j=1}^{N} q_j b_j; \tag{1.16}$$

where,

$$a_j = \int\!\!\int_S \phi_j q^*(r, r_0)dS \tag{1.17}$$

$$b_j = \int\!\!\int_S \phi_j p^*(r, r_0)dS. \tag{1.18}$$

This formulation gives a system of N equations with N unknowns. The N imposed boundary conditions can be used to solve the system of equations to yield the unknown nodal acoustic pressure or particle velocity. When the nodal pressure or velocity for all nodes is known, the field pressure at any point in the domain D can be obtained using Eq. (1.15).

The boundary element method is considered as very promising for most of the acoustic problems as only the boundary needs to be meshed. Despite this advantage, BEM leads to dense non-symmetric complex valued linear systems. For N unknown field parameters this requires $O(N^2)$ storage and $O(N^3)$ computational costs. Use of iterative methods reduce the cost to $O(nN^2)$ operations, where n is the number of iterations required. As this is still quite large, a strategy that minimizes n is also needed.

Of the various techniques that have been proposed to increase the scalability of the BEM by accelerating the matrix-vector multiplication, the fast multipole method (FMM) is most widely accepted method in fast BEM implementations. The key idea in FMM is a multipole expansion of the kernel in which the connection between the collocation point and the source point is separated. By incorporating the FMM in a quickly convergent iterative scheme, rapid solution can be achieved with reduced computational and storage costs of the order, $O(nN \log N)$ [11].

1.3   Code Verification Examples

The *code verification* of the BEM implementation was performed by solving benchmarks problems that consisted of known analytical solutions. The results of code verification are presented in the following subsections [12].

1.3.1   *Interior Pressure Due to a Circular Vibrator*

This model problem consists of a *circular vibrator* of radius, $a = 1\,m$, which is vibrating with a uniform normal velocity, $v_n = 1\,ms^{-1}$, due to mechanical excitation. The boundary element mesh is uniform and consisted 16 constant elements (Fig. 1.1a). Numerical results of the interior surface pressure and the pressure at a location, $r = 0.5\,m$, are presented in Figs. 1.2 and 1.3 respectively up to $k = 20\,m^{-1}$.

The *characteristic wavenumbers* for the circular cavity are given as the roots or the *zero-crossings* of the Bessel function of order 0.

$$\begin{aligned} J_0(ka) &= 0 \\ k &= 2.4, 5.5, 8.7, 11.8, 15.0 \text{ and } 18.1. \end{aligned} \tag{1.19}$$

From Fig. 1.2 it is observed that the characteristic wavenumbers correspond to the values predicted in Eq. (1.19).
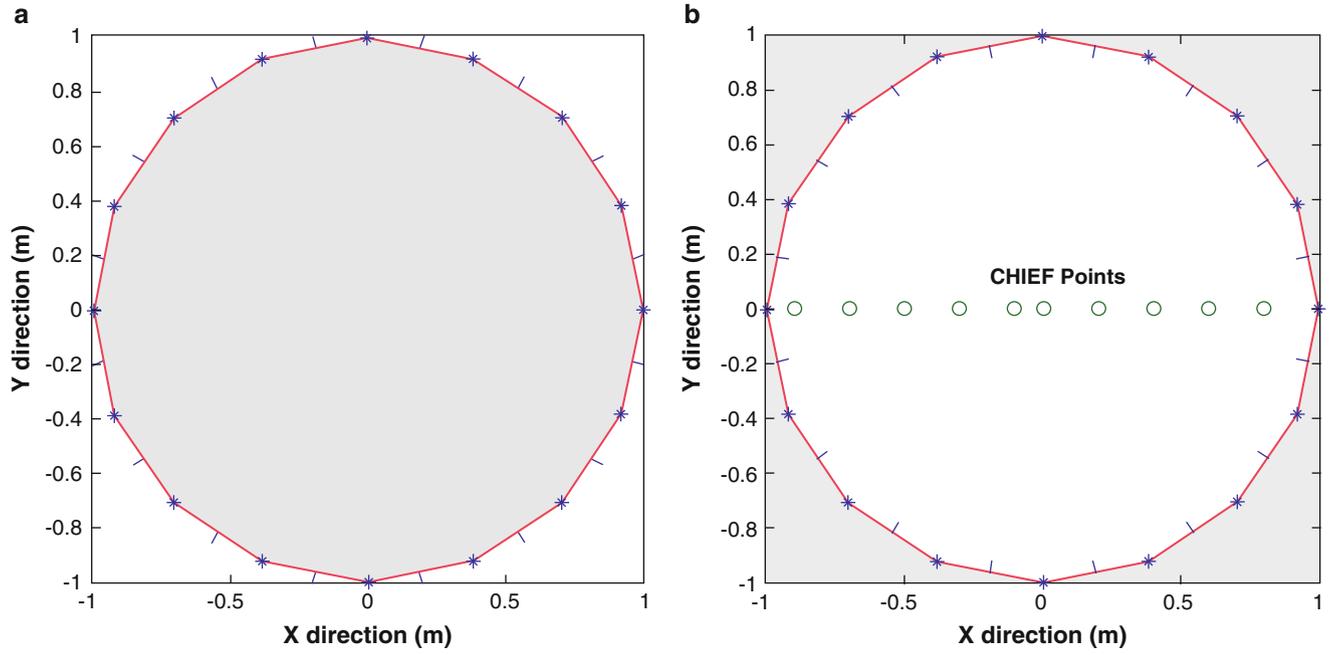
Fig. 1.1 Circular vibrator. (**a**) Interior domain. (**b**) Exterior domain – with 10 CHIEF points
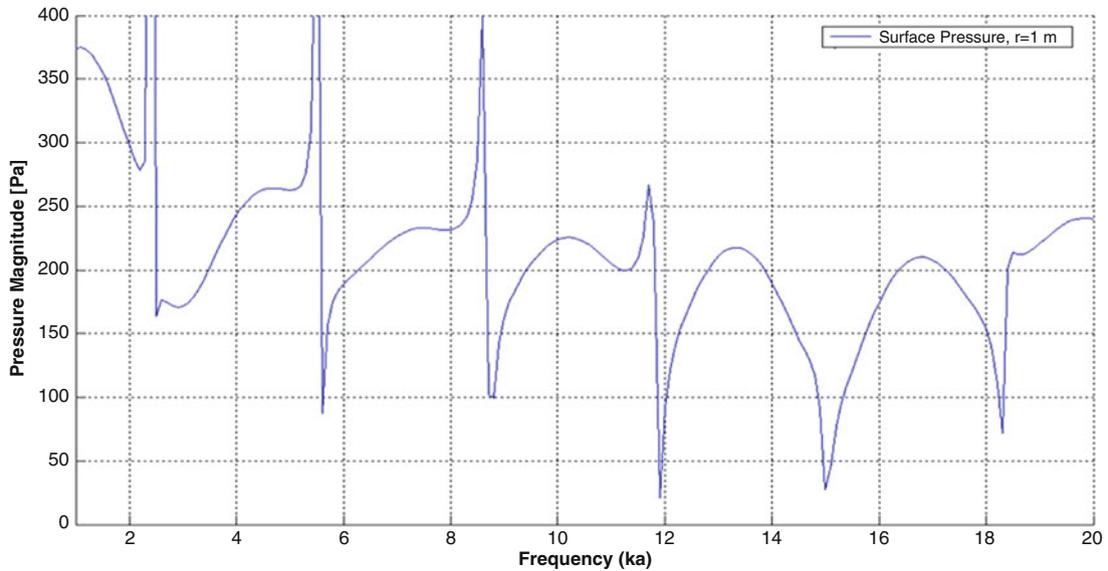

Fig. 1.2 Interior surface pressure due to a circular vibrator

1.3.2 Exterior Pressure Due to a Circular Vibrator

In exterior domain problems, for certain characteristic wavenumbers, unique solution is not possible as the boundary integral formulations are rendered incapable for numerical computation of valid solution. This is illustrated by the problem of predicting the sound-field radiated by a circular vibrator.

The sound radiated by a circular vibrator of radius, $a = 1\,m$, in an infinite acoustic domain ($\rho = 1.2\,kg\,m^{-3}$ and $c = 343\,ms^{-1}$) is predicted. The boundary element mesh consisted 16 constant elements (Fig. 1.1b). The exterior pressure is governed by the Helmholtz equation and it can be computed analytically using the following expression [2],

$$p(r) = i\rho c \frac{H_0^{(2)}(kr)}{H_1^{(2)}(ka)}. \tag{1.20}$$
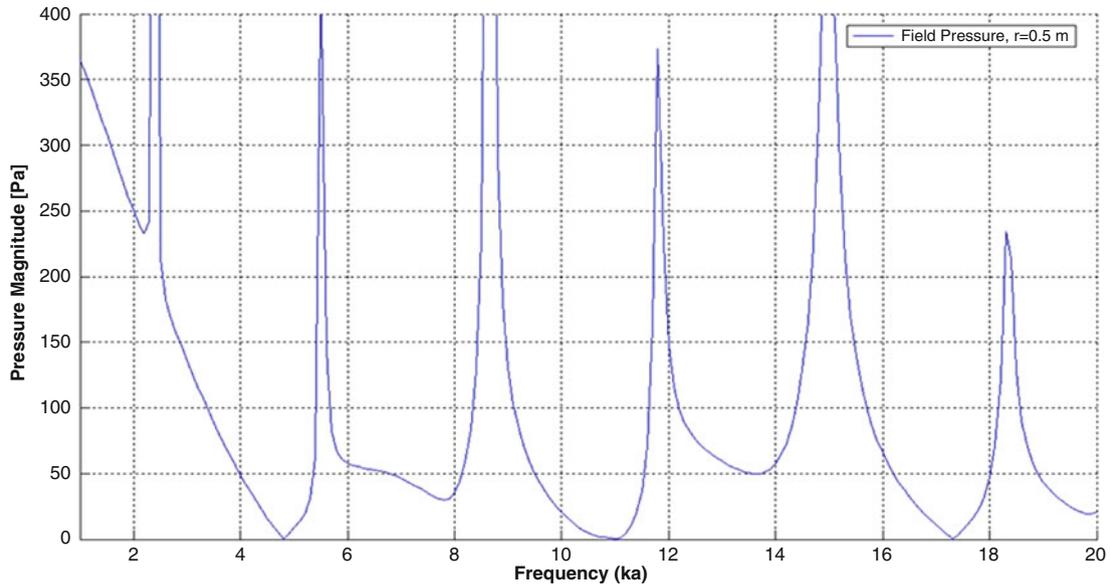
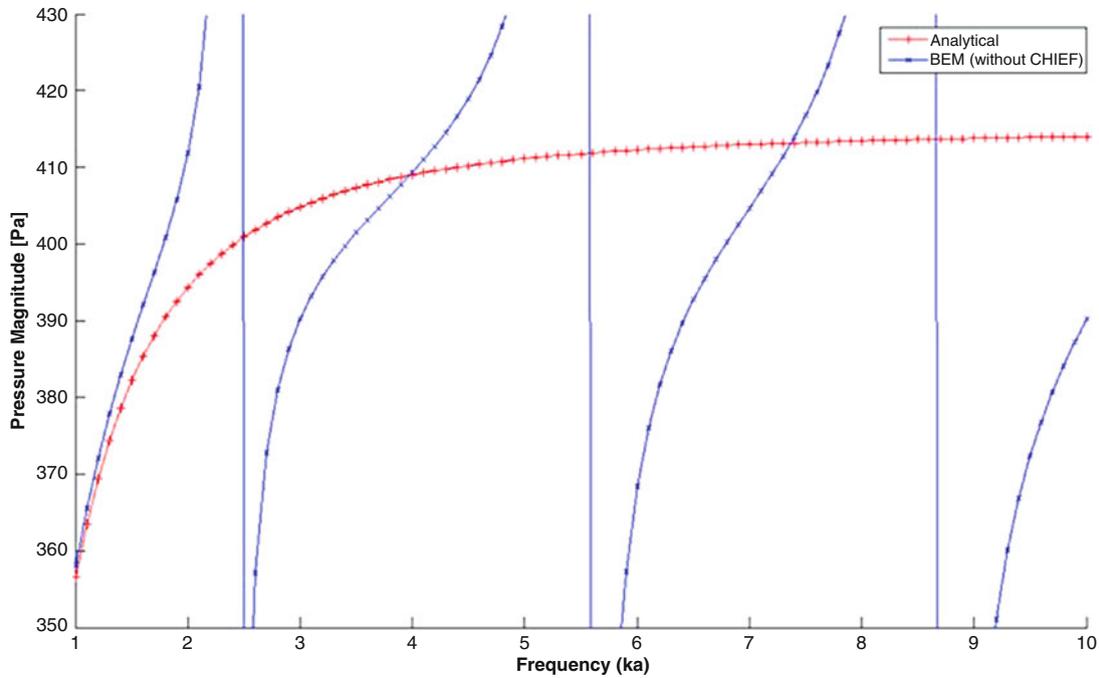**Fig. 1.3** Interior field pressure due to a circular vibrator

Fig. 1.4 Exterior pressure due to a circular vibrator

It is evident from Fig. 1.4 that the method of direct application of Eq. (1.8) fails to yield unique solution at critical wavenumbers that are given in Eq. (1.19). These wavenumbers match the characteristic wavenumbers of the corresponding interior problem. As mentioned in Sect. 1.2.2, this anomaly is purely due to the application of the BEM and does not have any physical significance.

The *Combined Helmholtz Integral Equation Formulation* (CHIEF) was applied for regularization. In this algorithm, the integral formulations for the *surface* and the *interior* are combined to form an *over-determined* system (substituting $C(r) = 1/2$ for the boundary nodes and $C(r) = 0$ for the interior CHIEF points, in the exterior domain BIE, Eq. (1.8)). Solving this over-determined system yields unique values for the field parameters – acoustic pressure and particle velocity.

While it is suggested that choosing just one CHIEF point would yield a unique solution, the effect of choosing more than one CHIEF point is also demonstrated. Sound-field predicted using an over-determined system with 1, 5 and 10 CHIEF points is shown in Figs. 1.5, 1.6 and 1.7 respectively.
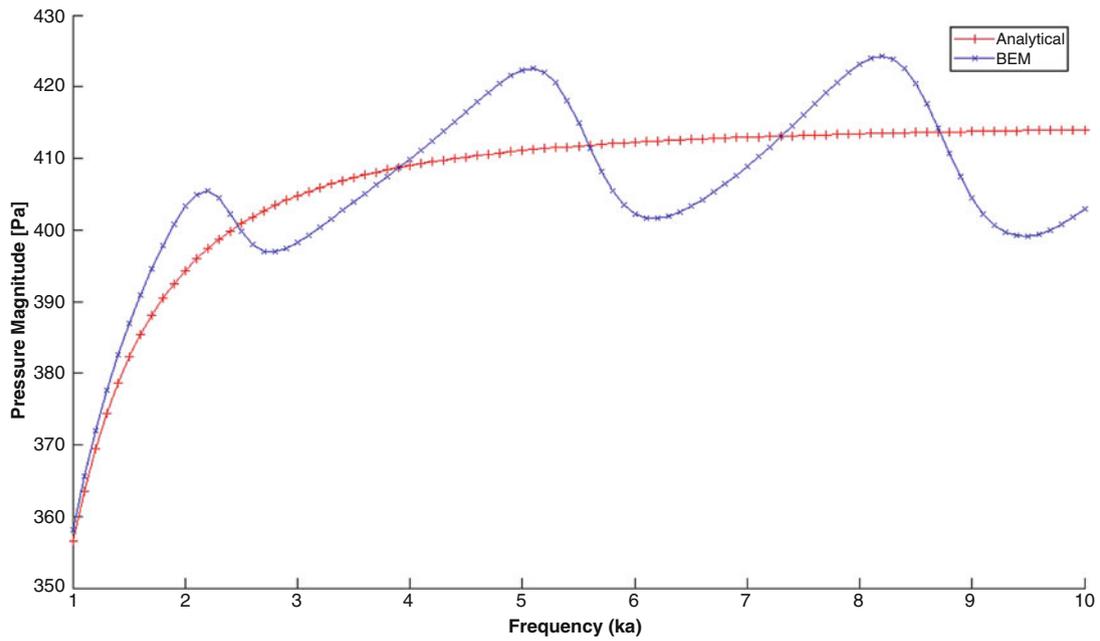

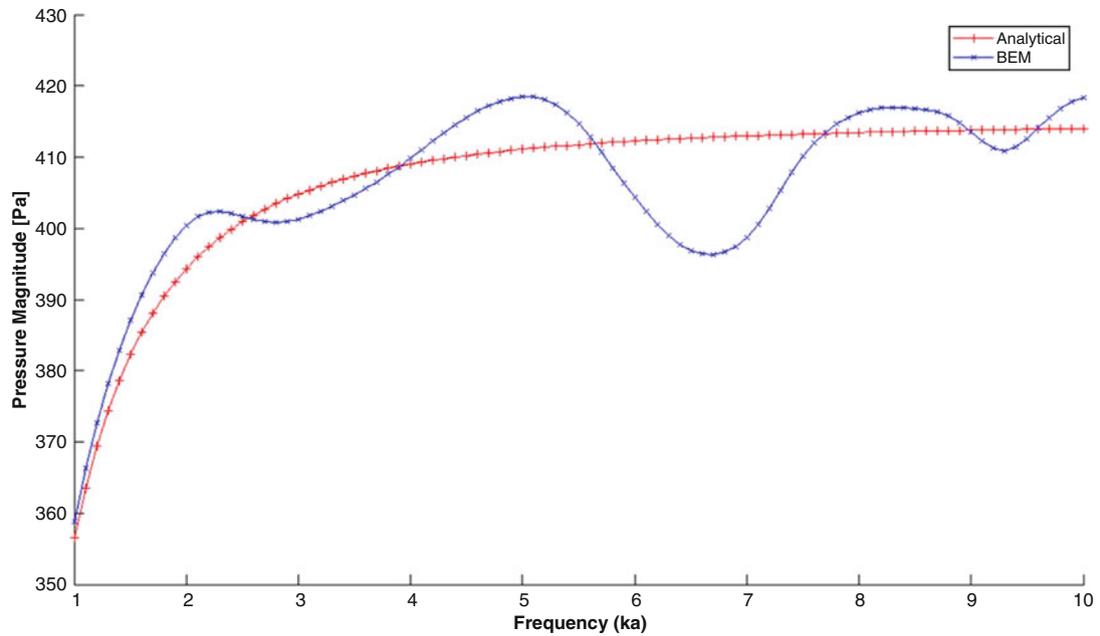**Fig. 1.5** Exterior pressure due to a circular vibrator, regularized with 1 CHIEF point

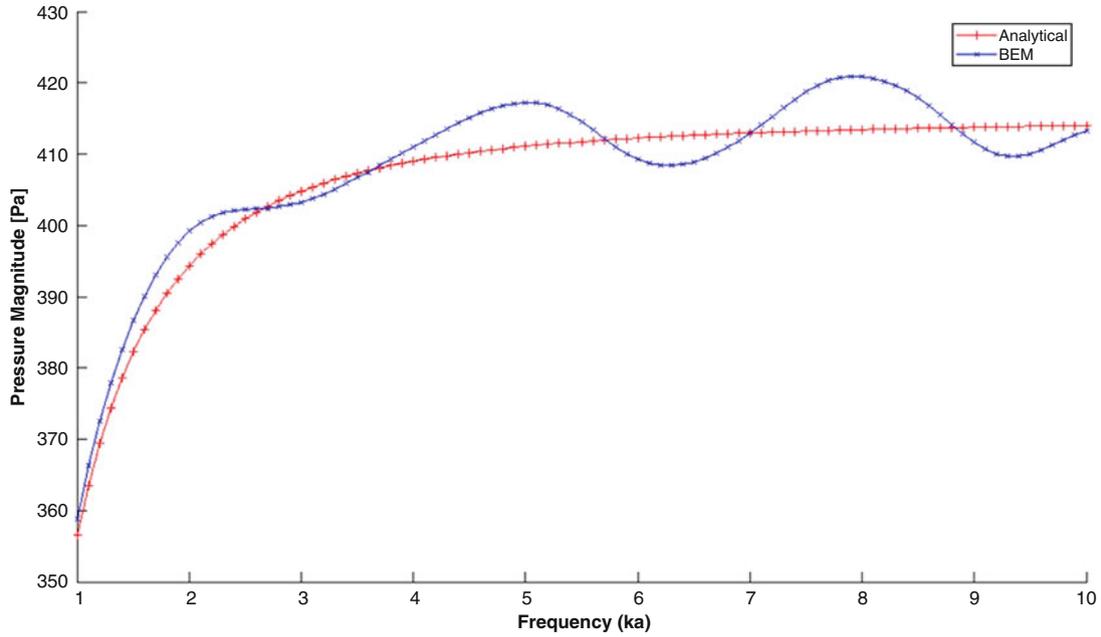**Fig. 1.6** Exterior pressure due to a circular vibrator, regularized with 5 CHIEF points

Fig. 1.7 Exterior pressure due to a circular vibrator, regularized with 10 CHIEF points
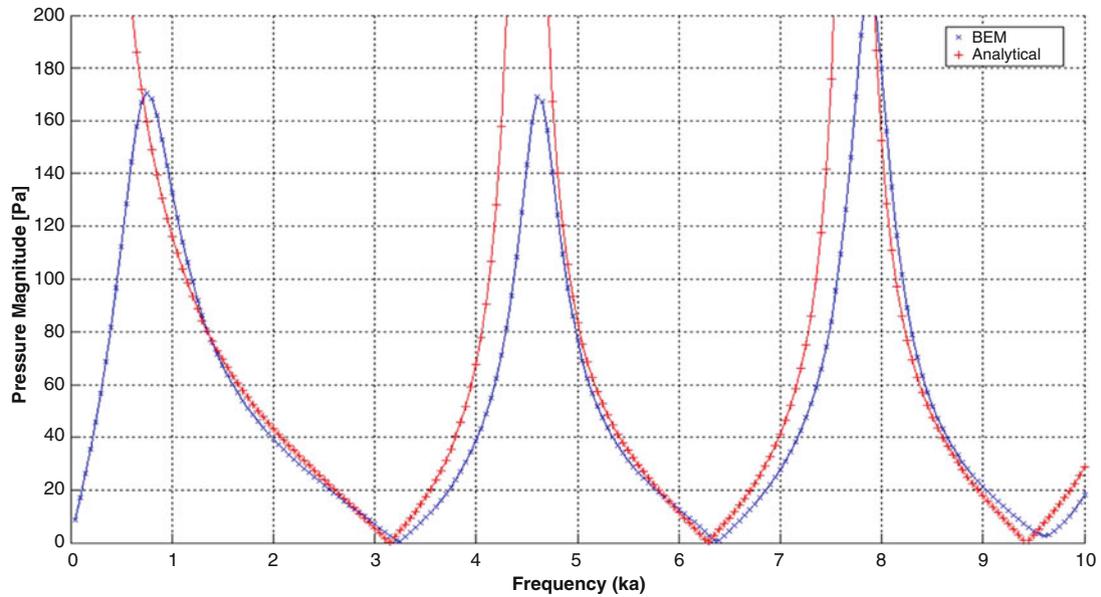

Fig. 1.8 Interior pressure due to a pulsating sphere

1.3.3 Interior Pressure Due to a Pulsating Sphere

A sphere of radius, $a = 1\,\mathrm{m}$, pulsating with a uniform normal velocity, $v_n = 1\,\mathrm{m\,s^{-1}}$, was analyzed in the frequency domain using the DBEM. The density of the acoustic medium and the wave propagation velocity were set as, $\rho = 1.2\,\mathrm{kg\,m^{-3}}$ and $c = 343\,\mathrm{m\,s^{-1}}$ respectively. Computations were performed with a wavenumber step of $0.01\,\mathrm{m^{-1}}$.

The surface mesh of the sphere is shown in Fig. 1.9. As the sphere is symmetric about the three co-ordinate axes, it is sufficient to solve for the acoustic pressure using one octant of the sphere. The mesh consisted of 19 quadrilateral elements with linear interpolation functions. The predicted and analytical values of interior surface pressure are plotted in Fig. 1.8.
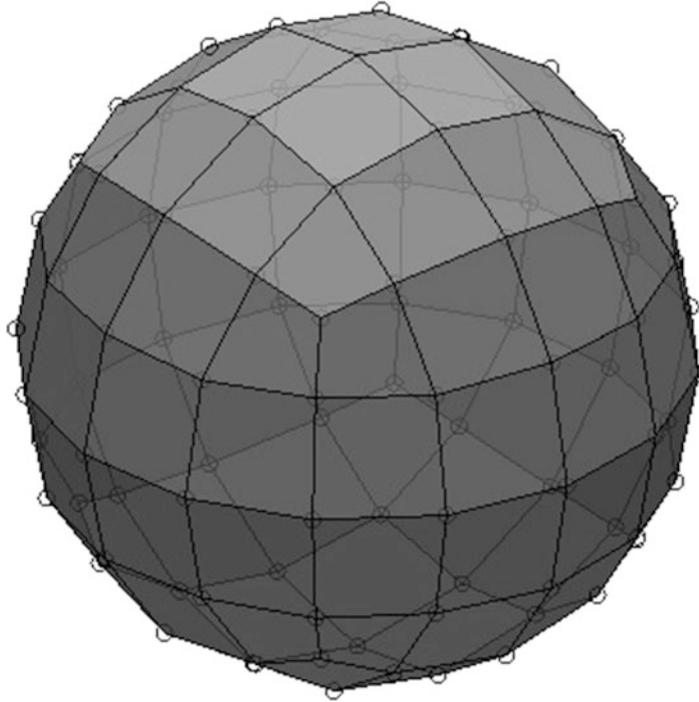
Fig. 1.9 Surface of a sphere meshed with quadrilateral elements

The pressure, p, at any location, $r \leq a$, inside the a pulsating sphere can be computed analytically using [13],

$$p(r) = -i\rho c \frac{j_0(kr)}{j_1(ka)} v_n, \tag{1.21}$$

where j_0 and j_1 are spherical Bessel functions of the first kind and order 0 and 1 respectively.

1.3.4 Exterior Pressure Due to a Pulsating Sphere

The sound-field radiated by a *pulsating sphere* of radius, $a = 1\,m$, and *uniform normal velocity*, $v_n = 1\,ms^{-1}$, prescribed on its surface, is predicted. The surrounding acoustic medium is air (density, $\rho = 1.2\,kg\,m^{-3}$ and wave propagation velocity, $c = 343\,ms^{-1}$). The computations are performed for a wavenumber step of $0.01\,m^{-1}$. The sphere is discretized as shown in Fig. 1.9. Due to axial symmetry it is sufficient to analyze just *one octant* of the sphere.

The analytical solution for the acoustic pressure, p, at a distance, $r \geq a$, from the center of the sphere of radius, a, pulsating with uniform radial velocity, v_n, is given by [13],

$$p(r) = \rho c v_n k a^2 \frac{ksa - i}{((ka)^2 + 1)\,r} e^{\left(\frac{ik(r-a)}{r}\right)} \tag{1.22}$$

The results of the analysis and the analytical solution are shown in Fig. 1.10. Several peaks occur, which coincide with the *eigenfrequencies* (also known as characteristic or irregular frequencies) of the inner volume of the sphere. As these peaks do not have any physical significance, they are eliminated by adding randomly distributed points inside the sphere at which the pressure is set to zero. The results obtained after regularization using CHIEF are shown in Fig. 1.11.
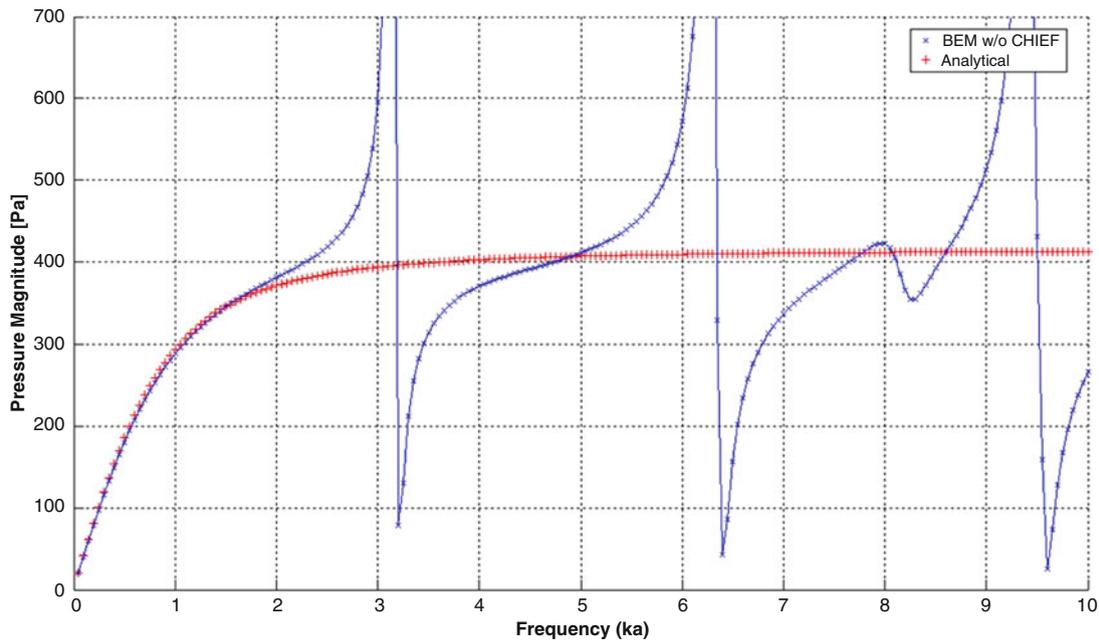
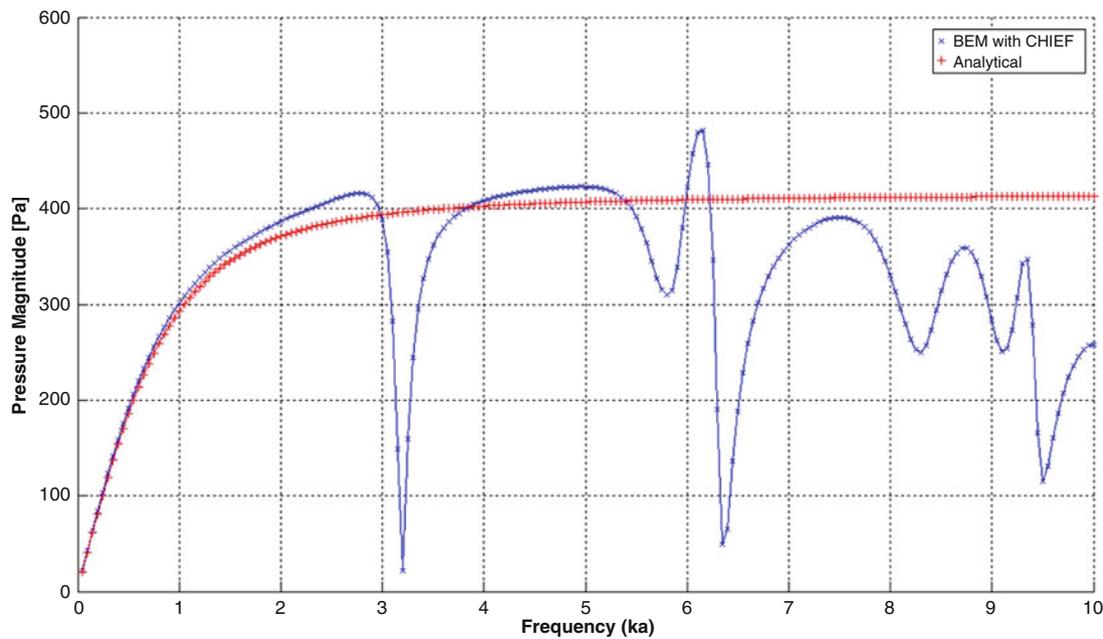**Fig. 1.10** Exterior pressure due to a pulsating sphere (without regularization)

Fig. 1.11 Exterior pressure due to a pulsating sphere regularized with 1 CHIEF point

1.4 Solution Verification Examples

1.4.1 Gear Whine Prediction

Gear whine is one of the major noise sources in electric vehicle powertrains. It is theorized that gear whine is caused by static and dynamic transmission error, gear mesh stiffness variations, sliding friction, shuttling forces, etc. Gear whine is characterized by steady-state vibrations of the gear pairs. It manifests itself as modulated tones at gear mesh frequency, its harmonics, and the side bands of the gear mesh frequencies [14]. The gearbox housing structure vibration and its acoustic
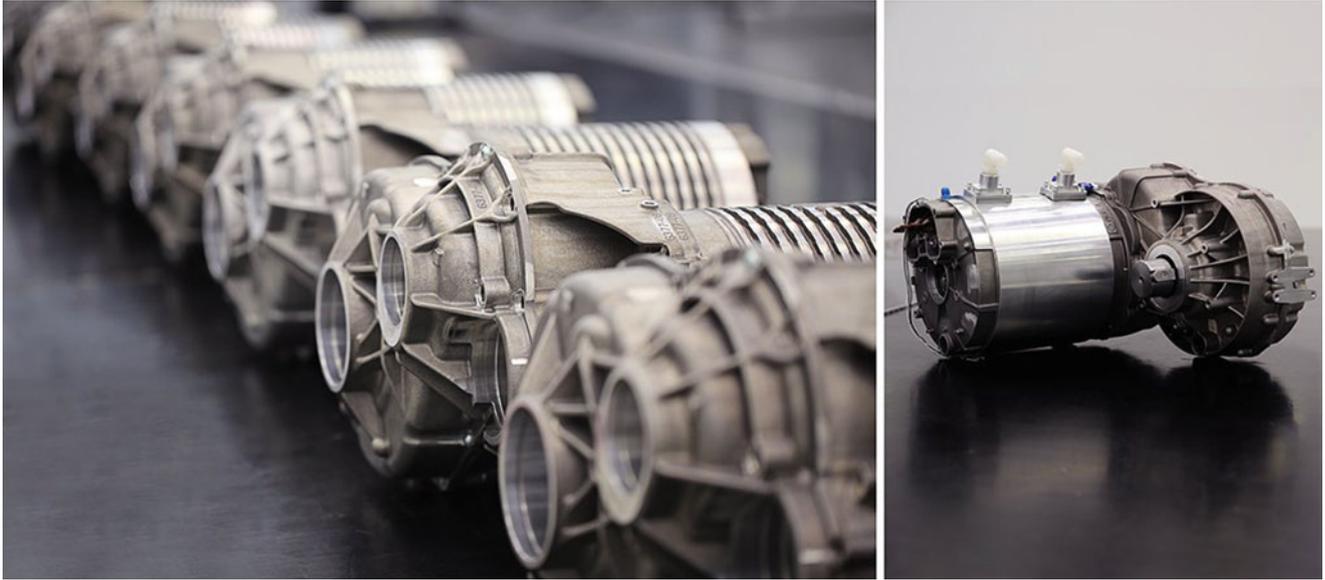
Fig. 1.12   Gearbox housing [www.lucidmotors.com]

sensitivity are critical in response to the gear whine excitations. The design objective is to optimize the gearbox housing for gear whine while achieving the desired structural efficiency by employing upfront CAE driven design and development.

The gearbox housing structure has a complicated geometry, as shown in Fig. 1.12. Closed form solution for such complicated geometries is not available. A *method of manufactured solutions* [7] using simple sound sources can be used to assist in the code and solution verification process. For example, the sound-field due to a monopole source in the far-field is given by Eq. (1.23). When the monopole source is bounded by the housing surface, the induced velocity on the housing surface due to the monopole source can be extracted. In a subsequent step, the induced velocity could be used as a boundary condition on the housing surface and the sound in far-field can be predicted. The sound predicted at a far-field point using the induced velocity boundary condition on the housing surface should be the same as that calculated using Eq. (1.23). This would serve as a code verification case. This process could be repeated for other simple sources, e.g. a dipole and quadrapole sound sources with known solutions, to generate additional code verification test cases.

$$p(r, \omega) = \frac{e^{ik|r-r_0|}}{4\pi |r - r_0|} \tag{1.23}$$

The number of elements in the computational model controls the solution time. It was desired to optimize the mesh size by studying the associated spatial discretization error. As previously discussed, the solution verification processes deals with obtaining an estimate of the error in the model predictions. By applying the method explained above, i.e. comparing the field pressure due to an induced velocity boundary condition with that calculated directly from the analytical expression. A family of surface meshes having different levels of mesh refinement were generated for the gearbox housing. An average element length of 7, 8, 10, 11 and 12 mm was chosen for the different surface meshes. The induced velocity on the housing surface due an enclosed monopole source was used to estimate the spatial discretization error. The results for the different meshes is shown in Fig. 1.13.

1.5   Results and Discussion

The goals as part of the verification process are to identify, quantify and reduce errors caused by the mapping of the mathematical model to a computer implementation. The code verification helps to identify errors due to spatial discretization and the choice of the BEM formulation. The solution verification process aided the quantification of the error associated with spatial discretization.
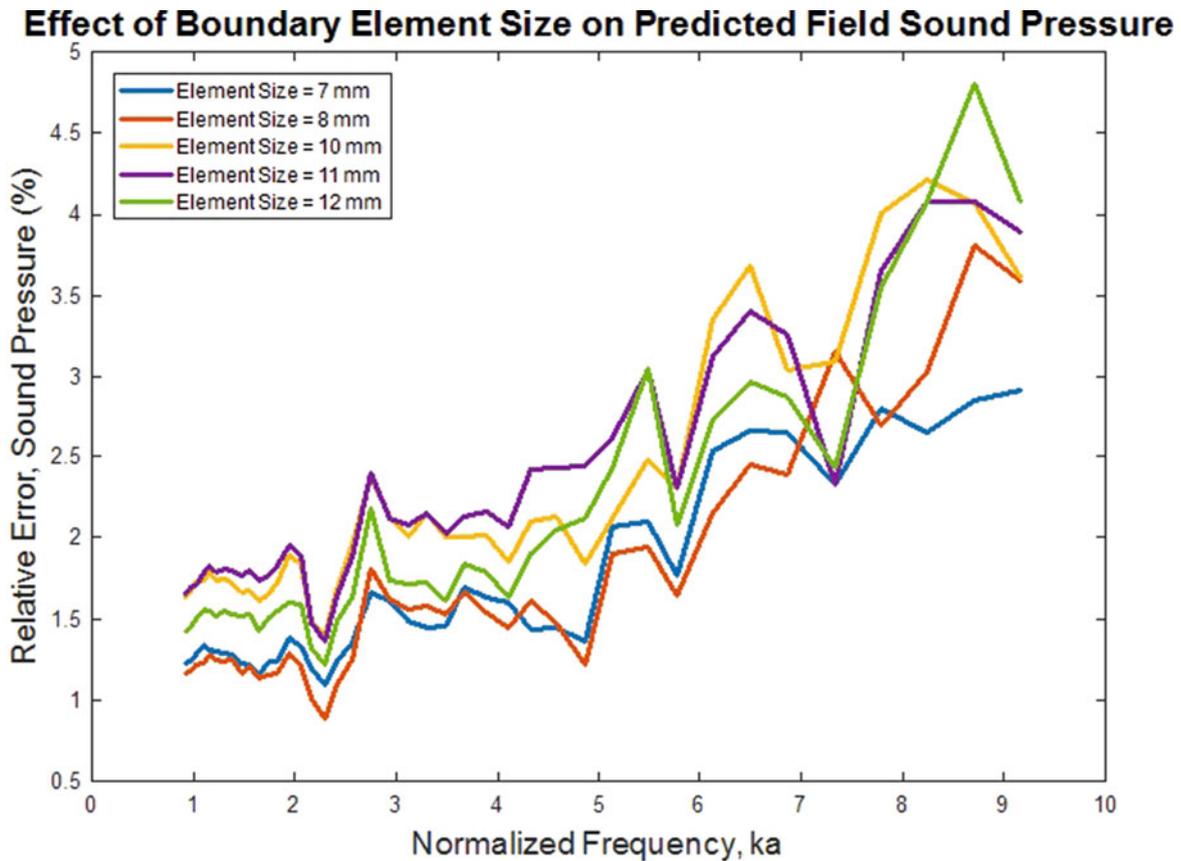
Effect of Boundary Element Size on Predicted Field Sound Pressure

Fig. 1.13 Mesh sensitivity

The discretization requirements of BEM are dictated by low- and high-frequency thresholds. The low-frequency limit is dependent on the numerical integration technique used. The high-frequency limits are established by numerical integration technique, interpolation functions and most importantly the wavelength of sound. The predicted values of the interior acoustic pressure diverge from the analytical values for $ka > 9$ for the pulsating sphere case (Fig. 1.8). It is recommended to use six elements per wavelength are required to predict an acceptable solution.

Code verification for exterior domain problems helped in identifying the non-uniqueness issue associated with the direct implementation of the Helmholtz Integral Formulation. Particularly, the relative error was significantly high at certain frequencies for the gearbox housing structure. It could be attributed to the non-uniqueness in the solution at the characteristic frequencies of the complementary interior region bounded by the closed surface. This was ascertained by determining the eigenfrequencies of the complimentary interior problem.

The model set up for solving the interior domain problem was performed using the following steps:

1. Introduce a monopole source with volume velocity one at an interior point, e.g. at the origin (0,0,0)
2. Set the boundary condition on the surface to zero
3. Predict the acoustic pressure at interior field points

The peaks in the interior pressure (Fig. 1.14) correspond to the cavity resonances. As the Direct HIE Formulation resulted in non-unique solutions. The problem was solved by applying the Burton-Miller Formulation, which addressed this issue. The results are presented in Table 1.1.
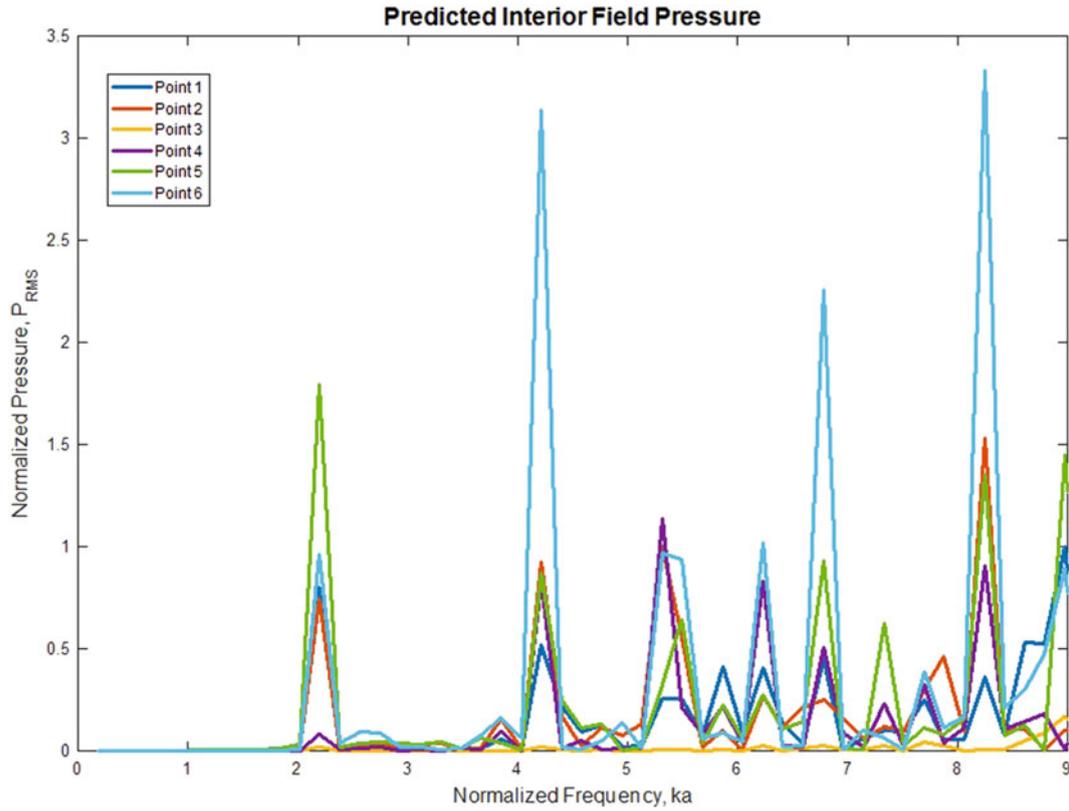
Fig. 1.14 Interior acoustic pressure

Table 1.1 Predicted exterior vs. analytical acoustic pressure

Field point	Location	Relative error $|\epsilon_{rel}|$ (%)	
		HIE	Burton-Miller
1	5a	35.30	0.94
2	10a	32.16	1.26
3	20a	30.04	1.28
4	50a	28.77	1.26
5	100a	28.04	0.84

1.6 Conclusions

Model verification and validation play a crucial role in building credibility in simulations. It is highly desirable to have a CAE lead product design and development process due to the associated time and cost advantages. Verification and validation of the models can be performed by comparing the model predictions with known solutions of simple sources and experimental data respectively. The boundary element method is applied in structural acoustics for sound-field predictions. In this paper, a background on the BEM formulations was provided. Model verification process was demonstrated with examples. The BEM implementation produced the desired results for interior domain problems. However, for exterior domain problems that have closed surfaces, the direct application of the BEM implementation failed to give unique solution at certain wavenumbers. These wavenumbers correspond to the eigenfrequencies of the complementary interior problem. This is non-physical and uniqueness can be restored using regularization techniques. Examples demonstrated the use of the Combined Helmholtz Integral Equation Formulation (CHIEF) and the Burton-Miller Formulation to restore uniqueness. In the case of complex geometries, the method of manufactured solutions to aid in the model verification was illustrated with examples.

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